Recall: $\frac{d}{dx}(b^x) = \ln(b)\cdot b^x$. We can use our knowledge of derivatives to compute integrals!

Inverse function: $f(x)$ with input X and output Y has inverse $g(y)$ with input Y and output X when:

$g(f(x)) = x$ for all x in X; $f(g(y)) = y$ for all y in Y.

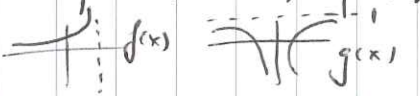
Example: $f(x) = x^3$ input $\mathbb{R}$ output $\mathbb{R}$.
$f(x) = x^2$ input $\mathbb{R}^+$ output $\mathbb{R}^+$.
$f(x) = x^2$ input $\mathbb{R}^-$ output $\mathbb{R}^+$.
$f(x) = \frac{1}{x}$ input $\mathbb{R}^+$ output $\mathbb{R}^+$.
$f(x) = \frac{1}{x}$ input $\mathbb{R}\setminus\{0\}$ output $\mathbb{R}\setminus\{0\}$.

$\frac{d}{dx}$ input function, output function. Fundamental Theorem of Calculus.

Compute inverse: $f(x) = \frac{1}{\sqrt{1-x}}$

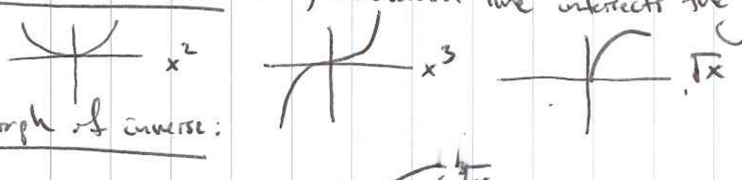
$f(x) = y$, solve for x. $\frac{1}{\sqrt{1-x}} = y$; $\frac{1}{1-x} = y^2$; $\frac{1}{y^2} = 1-x$; $x = 1 - \frac{1}{y^2}$.

$f(x)$ inputs $x<1$, outputs $\mathbb{R}^+$.
$g(y)$ inputs $\mathbb{R}^+\setminus\{0\}$, outputs $x<1$.
$g(y) = 1 - \frac{1}{y^2}$.



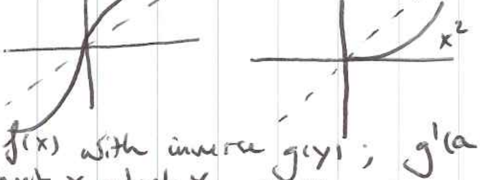
Pictorial interpretation: invertibility is given by the horizontal line test. The inverse is reflecting through $y=x$.

Horizontal line test: Every horizontal line intersects the graph once.



If $f(x)$ passes the HLT, we can trim its input to make it invertible.

Graph of inverse:



Derivative of inverse: $f(x)$ with inverse $g(y)$; input x output y, a in Y. $g'(a) = \frac{1}{f'(g(a))}$.

$f(x) = x\cdot\sin(x)$
$f'(x) = \sin(x) + x\cdot\cos(x)$

$g'(\frac{\pi}{2}) = \frac{1}{\sin(\frac{\pi}{2}) + \frac{\pi}{2}\cdot\cos(\frac{\pi}{2})} = 1$. $f(\frac{\pi}{2}) = \frac{\pi}{2}$ $g(\frac{\pi}{2}) = \frac{\pi}{2}$