Answer Key

Math 141 Final Exam Review

1. A construction company has allocated $1.92 million to buy new bulldozers, backhoes, and dumptrucks. Bulldozers cost $16,000 each, backhoes cost $24,000 each, and dumptruckcs cost $32,000 each. The company needs twice as many bulldozers as backhoes, and the total number of construction vehicles to be purchased is 100.

(a) Formulate the system of equations needed to find how many of each type of construction vehicles will be purchased. (Assume the entire budget will be used.)

x the number of bulldozers
y backhoes
z dumptrucks

$$x + y + z = 100$$
$$16000x + 24000y + 32000z = 1920000$$
$$x = 2y$$

(b) Write the system of equations as an augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 16000 & 24000 & 32000 & 1920000 \\ 1 & -2 & 0 & 0 \end{array}\right]$$

(c) Pivot the matrix about the element in the first row and the first column.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 16000 & 24000 & 32000 & 1920000 \\ 1 & -2 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{-16000 R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 8000 & 16000 & 320000 \\ 1 & -2 & 0 & 0 \end{array}\right] \xrightarrow{-R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 8000 & 16000 & 320000 \\ 0 & -3 & -1 & -100 \end{array}\right]$$

(d) Solve the system of equations from here.

The row reduced echelon form is:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 64 \\ 0 & 1 & 0 & 32 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

so

$$x = 64$$
$$y = 32$$
$$z = 4$$

2. The following augmented matrices come from systems of equations uisng x, y, and z. Solve the system of equations. If there is no solution, write no solution. If there are infinitely many solutions, find the parametric solution.

(a) $\begin{bmatrix} 1 & 2 & 4 \\ 1 & -2 & 0 \\ 4 & 3 & 12 \end{bmatrix}$

The row reduced echelon form is:

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ⟵ no solution.

(b) $\begin{bmatrix} 1 & 2 & -3 & -2 \\ 3 & -1 & -2 & 1 \\ 2 & 3 & -5 & -3 \end{bmatrix}$

The RREF is:

$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

So we parametrize $z = t$ and then $x = t$
$y = t-1$.

We have infinitely many solutions: $(t, t-1, t)$ for $t \in \mathbb{R}$.

3. The quantity demanded each month of a high end action figure is 250 when the unit price is $141. The quantity demanded each month is 1000 when the unit price is $111. The suppliers will market 700 of the action figures when the unit price is $75. If the price is $61 or lower, the supplier will make no action figures. If both supply and demand are known to be linear, what is the equilibrium quantity and price?

Demand:

$D = \frac{-1}{25}x + 151$

Supply:

$S = \frac{1}{50}x + 61$

They are equal at:

$(1500, 91)$

So 1500 toys at $91 each.

4. A manufacturer has a monthly fixed cost of $57,500 and a production cost of $9 for each unit produced. The product is then sold for $14 per unit. Find the break even quantity for the manufacturer.

$C = 9x + 57500$

$R = 14x$

They are equal at: $x = 11500$ units sold.

5. Let $A = \begin{bmatrix} 1 & x \\ 7 & 7 \\ y & -1 \end{bmatrix}$, $B = \begin{bmatrix} -6 & 1 & y \\ 2 & x & 4 \end{bmatrix}$, and $C = \begin{bmatrix} -2 & x \\ 4 & y \\ x & 0 \end{bmatrix}$. Perform the indicated matrix operations, if they exist.

(a) $A + B^T$

$$\begin{bmatrix} -5 & 2+x \\ 8 & 7+x \\ 2y & 3 \end{bmatrix}$$

(b) AC

$(3\times2) \cdot (3\times2)$ $\boxed{\text{Does not exist}}$

(c) BA

$$\begin{bmatrix} -6 & 1 & y \\ 2 & x & 4 \end{bmatrix}\begin{bmatrix} 1 & x \\ 7 & 7 \\ y & -1 \end{bmatrix} = \begin{bmatrix} 1+y^2 & -6x+7-y \\ 2+7x+4y & 9x-4 \end{bmatrix}$$

6. Bookstore A has 1663 fiction hardcover books, 2661 fiction paperback books, 2196 nonfiction hardcover books, 1524 nonfiction paperback books, 1522 reference hardcover books, and 1686 reference paperback books.

(a) Represent the inventory of Bookstore A as a matrix, A, with rows for hardcover and paperback, and columns for fiction, nonfiction, and reference.

$$A = \begin{array}{c} \text{Hardcover} \\ \text{Paperback} \end{array}\begin{bmatrix} \overset{\text{Fiction}}{1663} & \overset{\text{Non-fiction}}{2196} & \overset{\text{Reference}}{1522} \\ 2661 & 1524 & 1686 \end{bmatrix}$$

(b) Bookstore B has inventory given in matrix B:

	Fiction	Nonfiction	Reference
Hardcover	2395	1696	1691
Paperback	3033	1690	2150

$$B = \begin{array}{c} \text{Hardcover} \\ \text{Paperback} \end{array}$$

If the two bookstores merge, find matrix C that represents the total inventory of the new bookstore.

$$C = A+B = \begin{bmatrix} 4058 & 3892 & 3213 \\ 5694 & 3214 & 3836 \end{bmatrix}$$

7. A nutritionist advises a patient to take at least 2400 mg of iron, 2100 mg of vitamin B_1, and 1500 mg of vitamin B_2 over a period of time. The patient finds two suitable pills: Brand A and Brand B. Brand A pills cost 6 cents per pill and contain 40 mg of iron, 10 mg of B_1 and 5 mg of B_2. Brand B pills cost 8 cents per pill and contain 10 mg of iron and 15 mg each of B_1 and B_2. What combination of pills should the patient purchase to meet the minimum requirements at lowest cost?
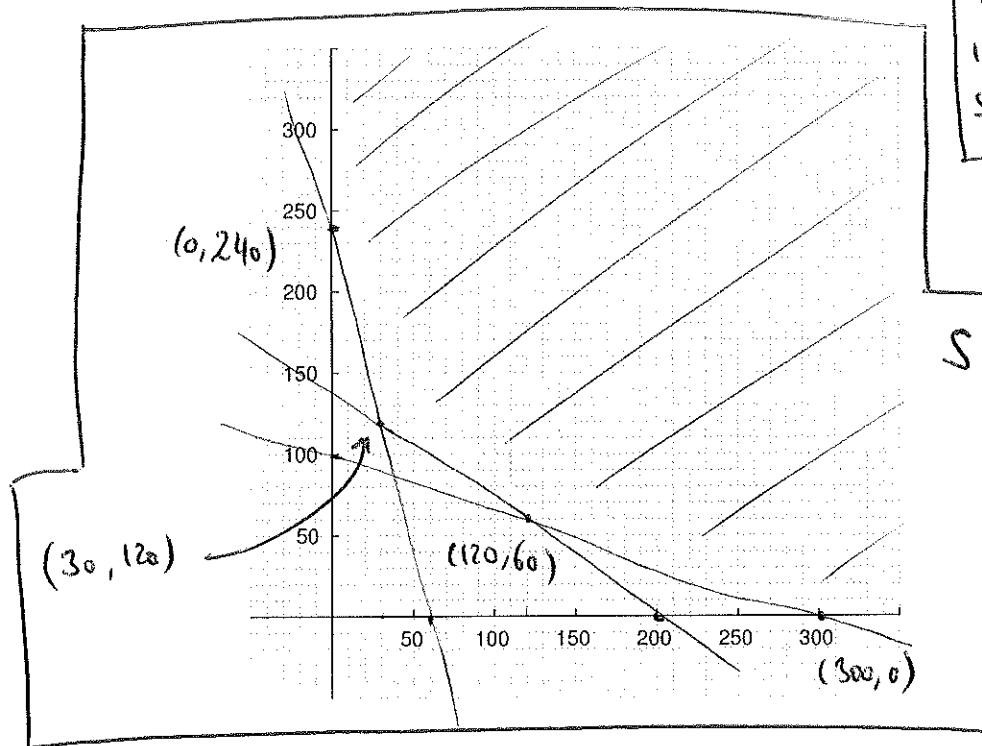
(a) Set up the linear programming problem.

$x = \#$ brand A pills.

$y = \#$ brand B pills.

Minimize : $C = 0.06x + 0.08y$

Subject to : $x \geq 0, y \geq 0$

(b) Graph the solution set for the linear programming problem.

$40x + 10y \geq 240$

$10x + 15y \geq 2100$

$5x + 15y \geq 1500$

$(0, 240)$

$(30, 120)$

$(120, 60)$

$(300, 0)$

S

(c) Solve the problem using the method of corners.

Corners	$(0, 240)$	$(30, 120)$	$(120, 60)$	$(300, 0)$
C	19.20	11.40	12.00	18.00

$x = 30$

$y = 120$

cost 11.40 dollars.

(d) How much more than the minimum nutrition requirement for iron, vitamin B_1, or vitamin B_2? (Round your answer to the nearest milligram.)
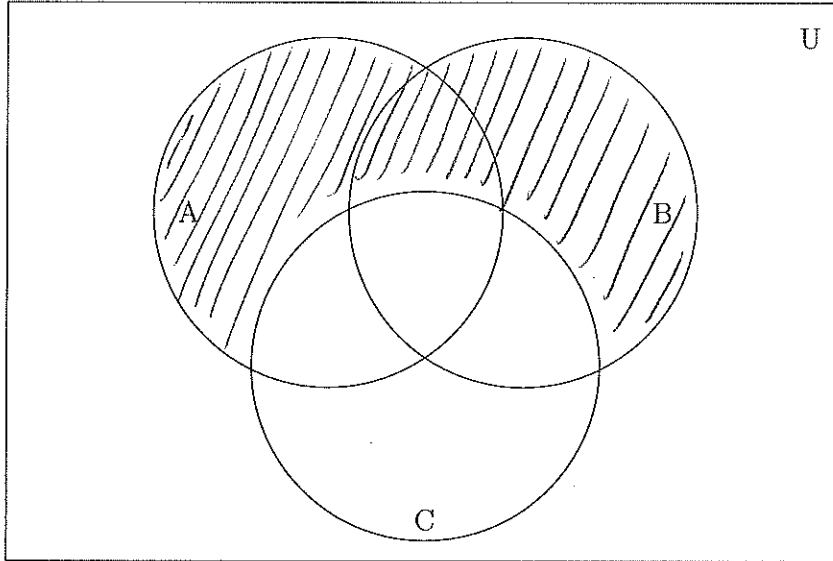
Iron : $40 \cdot 30 + 10 \cdot 120 = 2400$

B_1 : $10 \cdot 30 + 15 \cdot 120 = 2100$

B_2 : $5 \cdot 30 + 15 \cdot 120 = 1950$

no extra iron.

no extra B_1.

90 mg extra B_2.
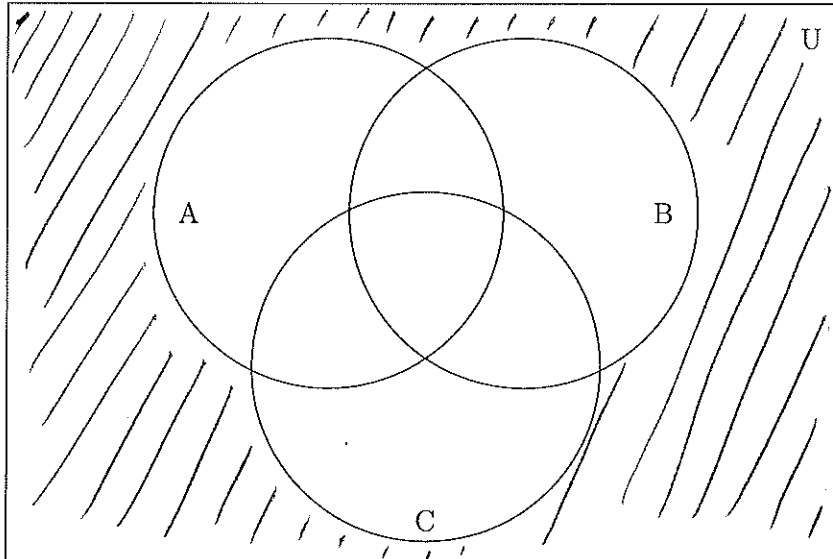
8. Shade the region of the Venn diagrams that represent the given set:

(a) $(A \cup B) \cap C^c$

(b) $A^c \cap B^c \cap C^c$

(c) Are the sets given in parts (a) and (b) above disjoint? Why or why not?

Yes, they have nothing in common.

9. Let U denote the set of all people living in Middle Earth and let

$$H = \{x \in U \mid x \text{ is a hobbit}\}$$
$$E = \{x \in U \mid x \text{ is an elf}\}$$
$$F = \{x \in U \mid x \text{ is a female}\}$$
$$M = \{x \in U \mid x \text{ knows magic spells}\}$$

(a) Write the set that represents all elves who are female or know magic spells.

$$E \cap (F \cup M)$$

(b) Use words to represent the set $H^C \cup M$.

People living in Middle Earth who are not hobbits or who know magic spells.

10. Mr. Orchard has a collection of 151 pokémon. A pokémon can be classified as fire, psychic, or ghost type. There are no pokémon that have all 3 types. He has 4 that are fire and psychic type, 20 that are only psychic type, 10 that are fire and ghost type, 7 that are only ghost type, 29 psychic types, and 44 fire types.

(a) How many of his pokémon are only fire type?
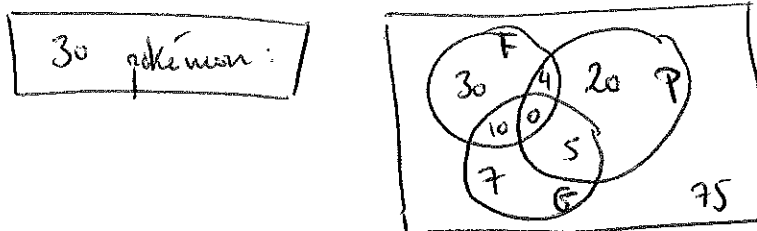
30 pokémon:

(b) What is the probability of a pokémon picked at random being psychic or fire type? (Round to two decimal places.)

$$\frac{20 + 4 + 5 + 30 + 10}{151} = 0.46$$

11. A state issues license plates using a combination of two letters from the English alphabet, followed by two digits, and then three more letters of the alphabet.

(a) How many total different license plates can be issued?

$$\underline{26}\ \underline{26} \qquad \underline{10}\ \underline{10} \qquad \underline{26}\ \underline{26}\ \underline{26}\ \underline{26} \qquad \boxed{1188137600}$$

letters   numbers   letters

(b) How many license plates can be issued if all the letters must be vowels?

$$\underline{5}\ \underline{5} \qquad \underline{10}\ \underline{10} \qquad \underline{5}\ \underline{5}\ \underline{5}\ \underline{5} \qquad \boxed{312500}$$

letters   numbers   letters

(c) What is the probability that a license plate selected at random only has vowels for its letters? (Round to four decimal places.)

$$\frac{312\ 500}{1188137600} \approx \boxed{0.0003}$$

12. In how many ways can gold, silver, and bronze medals be given if there are 18 people in a competition?

$$P(18,3) = \frac{18!}{(18-3)!} = \boxed{4896}$$

13. How many distinguishable arrangements are there of the letters in REVEILLE?

$$\frac{8!}{1!\ 3!\ 1!\ 1!\ 2!} = \boxed{3360}$$

14. A bag of Bertie Bott's Every Flavor Beans contains 12 sweaty armpit flavored beans, 8 Taco Bell Cool Ranch Doritos Locos Tacos flavored beans, and 7 crab apple flavored beans. A sample of 10 beans is picked from the bag.

(a) What is the probability that a sample contains exactly 4 sweaty armpit flavored beans and exactly 3 crab apple flavored beans? (Round to four decimal places.)

$$P(E) = \frac{n(E)}{n(S)} = \frac{970\ 200}{8436\ 285} \approx 0.1150$$

$$n(S) = C(27, 10) \qquad n(E) = \underset{armpit}{C(12,4)}\ \underset{apple}{C(7,3)}\ \underset{taco}{C(8,3)}$$

(b) How many samples contain exactly 5 Taco Bell Cool Ranch Doritos Locos Tacos flavored beans or exactly 3 crab apple flavored beans?

$$\underset{taco}{C(8,5)}\ \underset{other}{C(19,5)} + \underset{crab\ apple}{C(7,3)}\ \underset{other}{C(20,7)} - \underset{taco}{C(8,5)} \cdot \underset{crab\ apple}{C(7,3)} \cdot \underset{armpit}{C(12,2)} =$$

$$= 3235008$$

(c) How many samples contain at least 1 sweaty armpit flavored bean?

$$\underset{total}{C(27,10)} - \underset{other}{C(15,10)} \cdot \underset{armpit}{C(12,0)} = 8433282$$

15. A two stage experiment consists of flipping a coin and recording the result. If a heads is flipped a four sided die is cast and the result recorded. If a tails is flipped, the coin is flipped again and the result recorded.

(a) What is an appropriate sample space for this experiment?

$$\{ H1, H2, H3, H4, TH, TT \}.$$

(b) Write the event that a heads is flipped.

$$\{ H1, H2, H3, H4, TH \}$$

(c) How many total events are in this sample space?

$$2^6 = 64.$$

16. Let $S = \{s_1,\ s_2,\ s_3,\ s_4,\ s_5,\ s_6\}$ be the sample space associated with the experiment having the following probability distribution.

Outcome	s_1	s_2	s_3	s_4	s_5	s_6
Probability	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{6}$

(a) What is the probability of S?

$$P(S) = 1$$

(b) What is the probability of $\emptyset$?

$$P(\emptyset) = 0$$

(c) What is the probability of the event $\{s_2,\ s_3,\ s_6\}$?

$$P(\{s_2, s_3, s_6\}) = P(\{s_2\}) + P(\{s_3\}) + P(\{s_6\}) = \frac{1}{2}$$

17. A survey was conducted of 1000 people to determine the number of books they checked out from the library on that day. The results are given below.

Number of Books	0	2	3	4	5	6	7
Frequency	306	208	220	94	72	30	70

(a) Find the probability that a person checked out (round to three decimal places)

   i. exactly 2 books.

   $$\frac{208}{1000} = 0.208$$

   ii. more than 5 books.
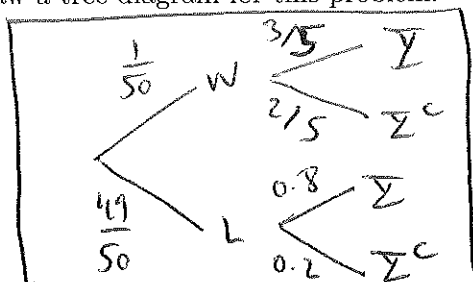
   $$\frac{30 + 70}{1000} = 0.1$$

(b) Compute the mean, median, mode, and variance of the data. (Round to two decimal places.)

$$\text{mean}: 2.48$$
$$\text{median}: 2$$
$$\text{mode}: 0$$
$$\sigma^2 = 4.47.$$

18. Megatron yells at Starscream $\frac{3}{5}$ of the time after the Decepticons win a battle (which is one in every 50 battles). If the Decepticons lose a battle, Megatron yells at Starscream 80% of the time.

    (a) Draw a tree diagram for this problem.

    (b) What is the probability that the Decepticons lose a battle and Megatron yells at Starscream? (Round to three decimal places.)

    $$P(L \cap \Sigma) = \frac{49}{50} \cdot 0.8 \approx 0.784$$

    (c) What is the probability that the Decepticons won a battle if we know Megatron is yelling at Starscream? (Round to three decimal places.)

    $$P(W|\Sigma) = \frac{\frac{1}{50} \cdot \frac{3}{5}}{\frac{1}{50} \cdot \frac{3}{5} + \frac{49}{50} \cdot 0.8} \approx 0.015$$

19. Give the range of values that the given random variable may assume and classify the random variable as finite discrete, infinite discrete, or continuous.

    (a) Z is a normally distributed random variable.

    $$\{ z \mid -\infty < z < \infty \} , \text{ continuous.}$$

    (b) X is a binomially distributed random variable with 40 independent trials. The probability of success for each trial is 0.44.

    $$\{ 0, 1, 2, \dots, 40 \} = \{ u \in \mathbb{N} \mid 0 \leq u \leq 40 \} , \text{ finite discrete.}$$

    (c) Y measures the number of times a person may take the driver's license exam and get their license.

    $$\{ 1, 2, 3, \dots \} = \{ u \in \mathbb{N} \mid u \neq 0 \} , \text{ infinite discrete.}$$

20. The probability that a TI-83 calculator is defective is approximately 0.05. A sample of 18 calculators are selected at random. (Round your answers to four decimal places.)

(a) What is the probability that the sample contains no defective calculators?

$n = 18$

$p = 0.05$

success if calculator is defective.

binompdf $(18, 0.05, 0) \approx 0.6302$.

exactly 0

(b) What is the probability that the sample contains at least two defective calculators?

"at least two" is equivalent to "everything minus one and two"

$1 -$ binomcdf $(18, 0.05, 1) \approx 0.2265$

(c) What is the expected number of calculators to be defective in this sample?

$E(X) = n \cdot p = 18 \cdot 0.05 = 0.9$

(d) What is the standard deviation of this sample?

$\sigma = \sqrt{n p (1-p)} \approx 0.9247$

21. Z is a standard normal random variable. (Round your answers to four decimal places.)

(a) Find $P(0.31 < Z < 1.4)$

normalcdf $(0.31, 1.4, 0, 1) \approx 0.2975$.

(b) Find $P(Z \geq -1.95)$

normalcdf $(-1.95, 10E99, 0, 1) \approx 0.9744$.

22. The distribution of heights of adult males is normally distributed with mean 67 inches and standard deviation 2.3 inches. What maximum height is shorter than 72% of all adult males? (Round your answer to two decimal places.)

$$P(\overline{X} > a) = 0.72 \qquad \text{so} \qquad P(\overline{X} \leq a) = 1 - P(\overline{X} > a) = 0.28$$

$$\text{InvNorm}(0.28, 67, 2.3) \approx \boxed{65.66 = a}$$

23. Arthur borrowed $5,000 from a loan shark 9 months ago with simple interest. Now he owes $8,000 to the loan shark. What was the interest rate that the loan shark charged Arthur?

$$A = 8000 \qquad A = P \cdot (1 + rt) \qquad \text{so} \qquad \boxed{r = 80\%.}$$

$$P = 5000$$
$$r = ?$$
$$t = \frac{3}{4}$$

24. Find the present value of $50,000 due in 6 years if the account is at a rate of 8% per year compounded (round your answers to the nearest cent)

(a) continuously.

$$A = Pe^{rt} \qquad \text{with} \qquad A = 50000 \qquad \text{so} \qquad \boxed{P \approx 30\,939.17}$$
$$P = ?$$
$$r = 0.08$$
$$t = 6$$

(b) quarterly.

$$m = 4 \quad , \quad \text{present value formula:}$$

$$A = P \cdot \left(1 + \frac{r}{m}\right)^{m \cdot t} \quad (\text{no payments}) \qquad \boxed{P \approx 31086.07}$$

(c) semiannually.

$$m = 2 \quad , \quad \text{and as before:} \qquad \boxed{P \approx 31229.85}$$

25. A family is looking for mortages for their home. Option A is a fixed rate of 6.5% per year compounded monthly, and option B is a fixed rate of 6.6% per year compounded quarterly. Find which loan the family should take by comparing effective rates of interest.

$r_{eff} A = 6.70\%$ ⟵ | A will accrue less interest.

$r_{eff} B = 6.77\%$

26. Lauren plans to deposit $6000 into a bank account at the beginning of next month and $200 per month into the same account at the end of that month and each subsequent month for the next 7 years. If her bank pays interest at a rate of 5% per year compounded monthly, how much will Lauren have in her account at the end of the 7 years? Round your answer to the nearest cent.

Future value: $t = 7$ with $S = R \cdot \left(\dfrac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{r/m} \right)$

$m = 12$

$r = 0.05$

$R = 200$ yields $S \approx 20065.73$

Accumulated value $P = 6000$ with $A = P \cdot \left(1 + \frac{r}{m}\right)^{mt}$

$t = 7$

$m = 12$

$r = 0.05$ yields $A \approx 8508.22$.

So the total is: $S + A \approx 28573.95$

27. After a down payment of $50,000, a family gets a loan of $150,000 for a house for 30 years at 8% per year, compounded monthly.

    (a) Compute this portion of the amortization table.

$t = 30$

$r = 0.08$

$m = 12$

$P = 150\,000$

yields (present value formula)

$\boxed{R \approx 1100.65}$

Period	Interest owed	Payment	Amount to Principal	Outstanding principal
0	--	--	--	150 000
1	1000	1100.65	100.65	149 899.35
2	999.33	1100.65	101.32	149 798.03

    (b) What is the equity on the home after 8 years? Round your answer to the nearest cent.

It is the original amount minus how much they still owe:

$200\,000 - 136526.13 \approx \boxed{63\,473.87}$

$t = 30 - 8 = 22$ Which has

$r = 0.08$ present

$m = 12$ value

$R = 1100.65$

$P \approx 136526.13$

28. A corporation creates a sinking fund in order to have $490,000 to replace machinery in 9 years.

    (a) How much should be placed in this account at the end of each week if the annual interest rate is 5.8% compounded weekly? Round your answer to the nearest cent.

The future value is:

$$S = R \cdot \left(\frac{\left(1 + \frac{r}{m}\right)^{mt} - 1}{\frac{r}{m}} \right)$$

with : $r = 0.058$

$t = 9$

$m = 52$

$S = 490000$

So $\boxed{R \approx 797.98.}$

    (b) How much interest would they earn over the life of the account? Round your answer to the nearest cent.

They pay :

$797.98 \cdot 9 \cdot 52 - 490\,000 = \boxed{116\,545.36}$

in interest.