H	25	25	25
D	15	15	15
M1	15	25	
M2	15	35	
F	30		60

(1)

Jan 31
Feb 23

7.1. Derivative of the exponential function.

$f(x) = b^x \quad b > 0,\ b \neq 1.$ base.

b<1

Intuitively: multiply b by itself x times.

Why care?
1. Population growth are exponential.
2. Nuclear decay.
3. Mortgages and interest rates.
4. Benchmark for growth: polynomial, exp.

Properties:
1. Always positive.
2. Domain: $\mathbb{R}$ Range: $\mathbb{R}^+$, positive reals but not zero.
3. Increasing: $b > 1$. Decreasing: $b < 1$.

How to work with them: (Laws of exponents).
1. $b^0 = 1$.
2. $b^x \cdot b^y = b^{x+y}$. so $b^{x-y} = \frac{b^x}{b^y}$ so $b^{-y} = \frac{1}{b^y}$.
3. $(b^x)^y = b^{x \cdot y}$ so $b^{\frac{1}{n}} = \sqrt[n]{b}$.

Example: $\sqrt[5]{\left(\frac{4^2}{2^9}\right)} = \left(\frac{(2^2)^2}{2^9}\right)^{1/5} = \left(\frac{2^4}{2^9}\right)^{\frac{1}{5}} = (2^{-5})^{\frac{1}{5}} = 2^{-1} = \frac{1}{2}$.

Derivative: $\frac{d}{dx}\left(b^x\right) = \ln(b) \cdot b^x$.

The rate of growth of b^x is proportional to b^x.
When is it equal? Exactly for base e.
The number e is the unique positive real number such that $\frac{d}{dx}(e^x) = e^x$.

Recall: Chain rule: $(f(g(x)))' = f'(g(x)) \cdot g'(x)$.

Example: $\frac{d}{dx}\left(e^{\cos(x)}\right) = e^{\cos(x)} \cdot (-\sin(x))$.

For integrals, we use derivatives! $\int e^{\cos(x)} \cdot (-\sin(x))\,dx = e^{\cos(x)} + C$.
$\int$ and $\frac{d}{dx}$ are inverses!