Numerical Simulation of Transport and Extrusion Processes of PVC Paste Flows

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Herschel–Bulkley Laminar Flow Equations in Pipes

An analytical solution exists:

$$\begin{cases} u_1 = \left(\frac{G}{2k}\right)^{\frac{1}{n}} \left(\frac{n}{n+1}\right) \left[\left(R - \frac{2\tau_y}{G}\right)^{\frac{n+1}{n}}\right] (0 \le r \le r_p) \\ u_2(r) = \left(\frac{G}{2k}\right)^{\frac{1}{n}} \left(\frac{n}{n+1}\right) \left[\left(R - \frac{2\tau_y}{G}\right)^{\frac{n+1}{n}} - \left(r - \frac{2\tau_y}{G}\right)^{\frac{n+1}{n}}\right] (r_p \le r \le R) \end{cases}$$

Where G is the pressure gradient, k is the consistency index, n is the flow behavior index for the Herschel–Bulkley model, τ_y is the yield stress, and $r_p = 2\tau_y/G$ is the radius within which the fluid behaves as a solid.

Herschel-Bulkley Constitutive Equation

There is a discontinuity in viscosity:

$$\nu = \begin{cases} \nu_0, & |\dot{\gamma}| \le \dot{\gamma}_0 \\ k\dot{\gamma}^{n-1} + \tau_0\dot{\gamma}^{-1}, & |\dot{\gamma}| \ge \dot{\gamma}_0 \end{cases}$$

It can be "regularized" and expressed as a continuous function:

$$\nu = k\dot{\gamma}^{n-1} + \tau_0\dot{\gamma}^{-1}\left(1 - e^{-m\cdot\frac{\dot{\gamma}}{\gamma_0}}\right)$$

This model is known as the **Herschel-Bulkley Papanastasiou** (HBP).

Bingham Flow Benchmark

The relative error for the HBP model implemented in OpenFOAM is 1.4%, while for the HB model in OpenFOAM it is 3.2%. The imposed conditions are shown in the following table:

G [kPa/m]	k [Pa·s]	τ_y [Pa]	ρ [kg/m^3]	Δy [m]
-22.4	0.8	350	1120	$1.57 \cdot 10^{-4}$

Table 1. Properties

In the following graph, the velocity profiles for HB, HBP, and the Analytical model are compared.

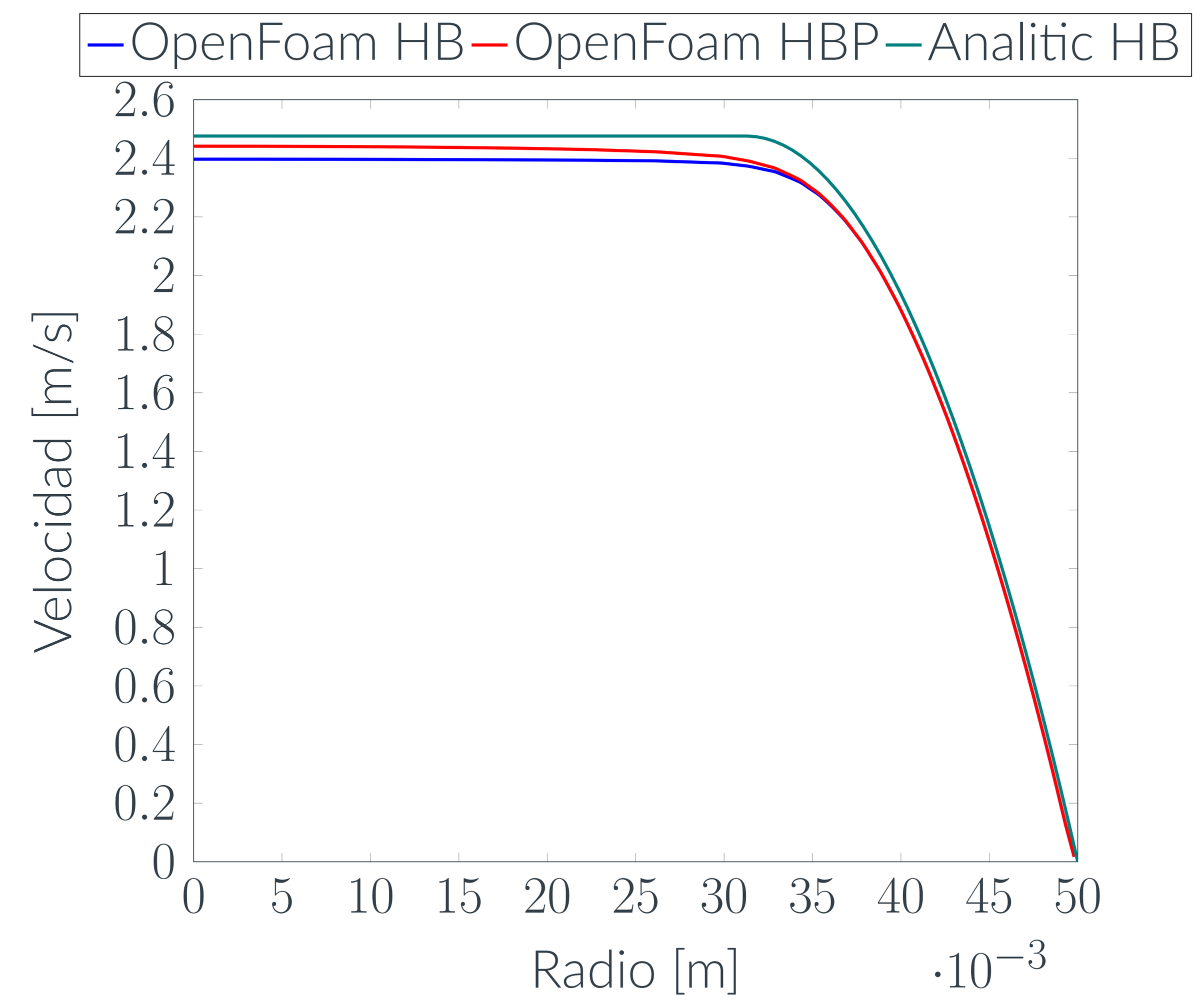

Figure 1. Velocity profiles for G=-22.4 [kPa/m]

Mesh Independence (HB)

The relative deviation of the numerical flow rate Q for different cell sizes remains below 1%. Additionally, the relative deviation of the numerical Q compared to the analytical Q is 2.2%. It can be observed that the convergence time increases exponentially as the mesh size decreases.

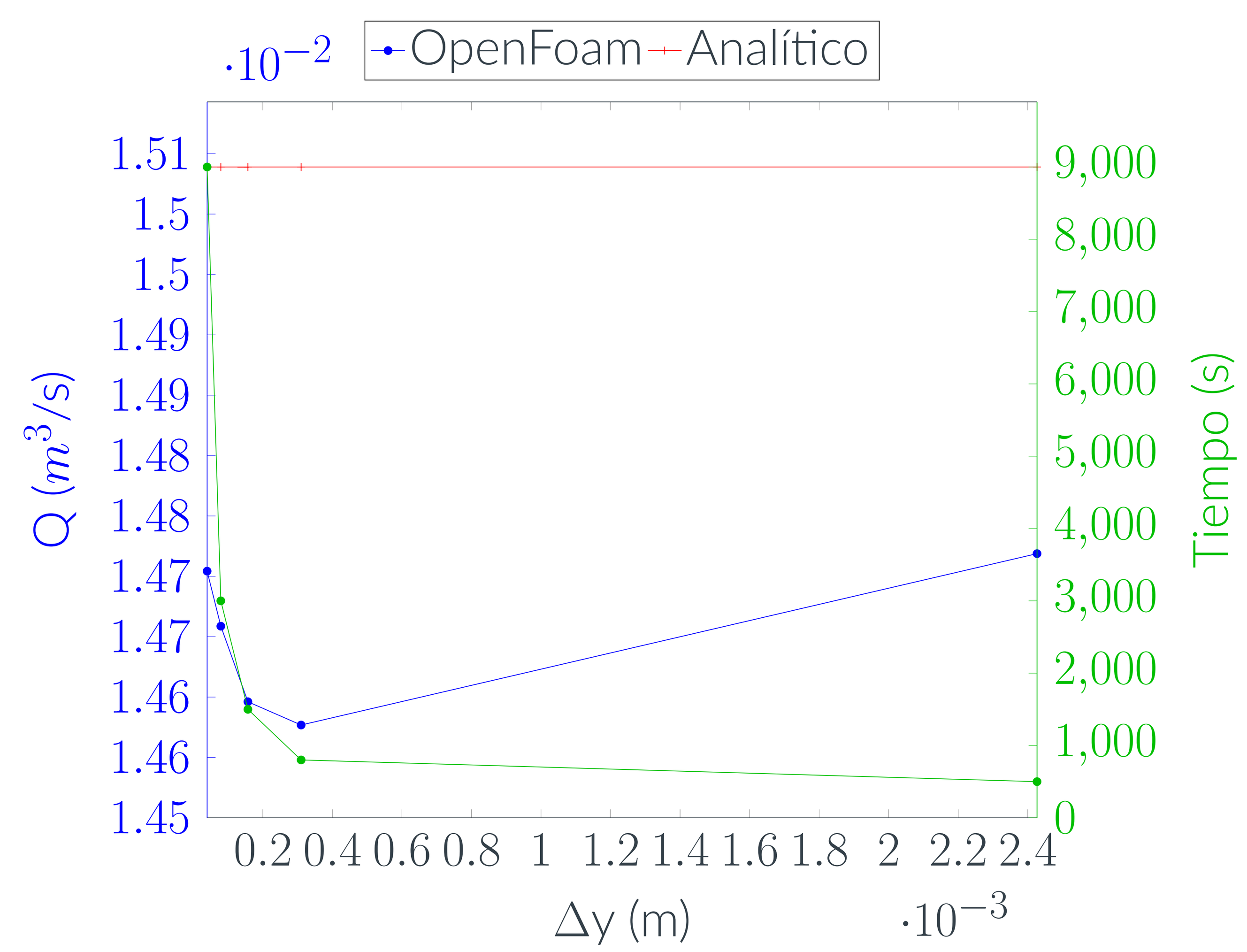

Figure 2. Mesh Sensitivity.

Parameterization (HBP)

By parameterizing G, k, and τ_y, it is deduced that there is a direct proportional relationship between G and τ_w (wall shear stress), and an approximately linear relationship between G and Q (flow rate).

G [kPa/m]	Q [m^3/s]	k [Pa·s]	τ_y [Pa]	τ_w [Pa]	t CPU [s]
-22.4	0.0146	0.8	350	0.494	2146
-22.4	0.00978	1.2	350	0.494	1632
-22.4	0.000361	0.8	550	0.503	143
-22.4	0.000299	1.2	550	0.503	123
-33.6	0.0458	0.8	350	0.742	7414
-33.6	0.0305	1.2	350	0.742	4936
-33.6	0.0188	0.8	550	0.742	2802
-33.6	0.0126	1.2	550	0.742	1979
-44.8	0.0791	0.8	350	0.989	13024
-44.8	0.0527	1.2	350	0.989	9020
-44.8	0.0490	0.8	550	0.989	7650
-44.8	0.0327	1.2	550	0.989	5180
-56	0.113	0.8	350	1.237	19222
-56	0.0754	1.2	350	1.237	

Table 2. Parameterization Results