

Webs of Virtue

A computational study
of how networks influence
moral development dynamics

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Introduction

Topic

Moral development in populations

1. **Moral development:** “the gradual [...] process by which humans become aware of what is right and wrong”
Stefanie Dorough (2011). [Moral Development](#)

Introduction

Topic

Moral development in populations

1. **Moral development:** “the gradual [...] process by which humans become aware of what is right and wrong”
Stefanie Dorrough (2011). [Moral Development](#)

2. **Morality:** “obligatory concerns with others’ welfare, rights, fairness, and justice, as well as the reasoning, judgment, emotions, and actions that spring from those concerns”
Audun Dahl (2023). [What we do when we define morality \(and why we need to do it\)](#).

Introduction

Context [1-3]

Perennial interest

Religion, philosophy

Seminal work

J. Piaget, L. Kohlberg

Scientific study

XX Century

Interdisciplinarian field

XXI Century



[4]

[1] F. C. Power (2012). [Moral Development](#)

[2] Carolyn Pope Edwards and Gustavo Carlo (2005). [Introduction: moral development study in the 21st century](#)

[3] Lene Arnett Jensen (2020). [Moral development: From paradigms to plurality](#). [4] Image by [Gordon Jonson](#)

Introduction

Demarcation

This thesis is about:

1. **Moral dynamics:** “The processes and phenomena (collective or individual) by which moral behavior and moral attitudes emerge, evolve, spread, erode or disappear”
Rainer Hegselmann (2009) [Moral Dynamics](#)

Introduction

Demarcation

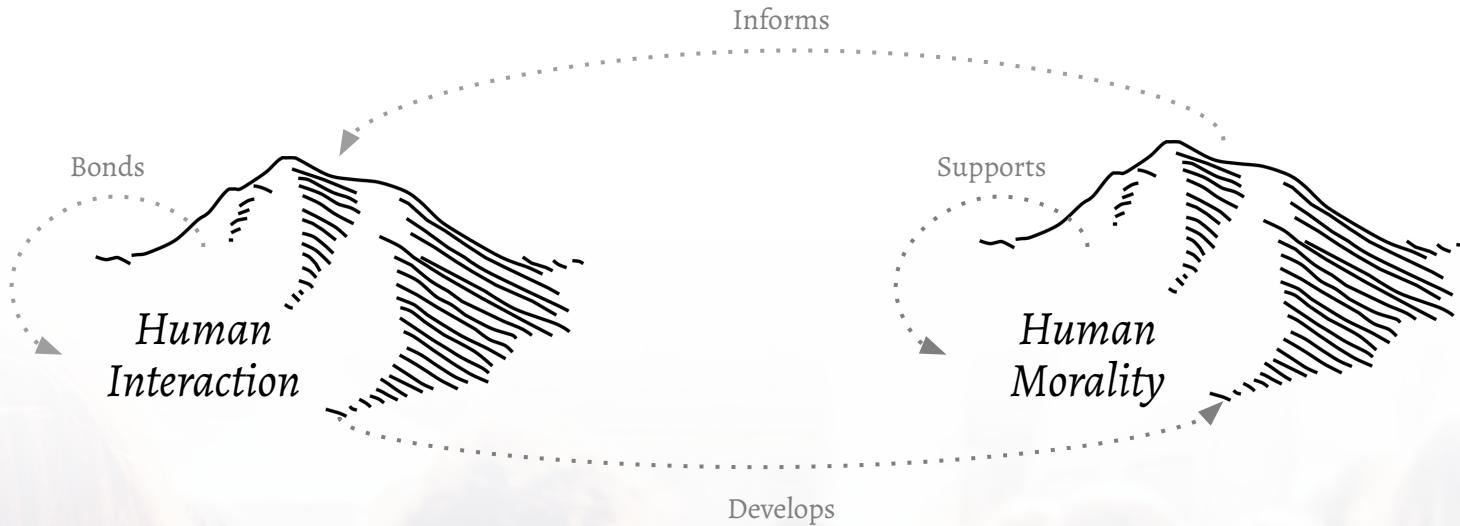
This thesis is about:

1. **Moral dynamics:** “The processes and phenomena (collective or individual) by which moral behavior and moral attitudes emerge, evolve, spread, erode or disappear”
Rainer Hegselmann (2009) [Moral Dynamics](#)

2. **Moral dynamics** that are expected to unfold in human populations as a result of their moral development, despite potential differences in their particularities
Such as the intricacies and nuances of moral standards and their evaluation, moral thought and emotional processes, bonding and relational processes, specific cultural and social rules applicable to different communities, specifics of moral actions, the consciousness of moral agents, biological and cognitive components of morality and environmental stressors.

Introduction

Scope [1-3]



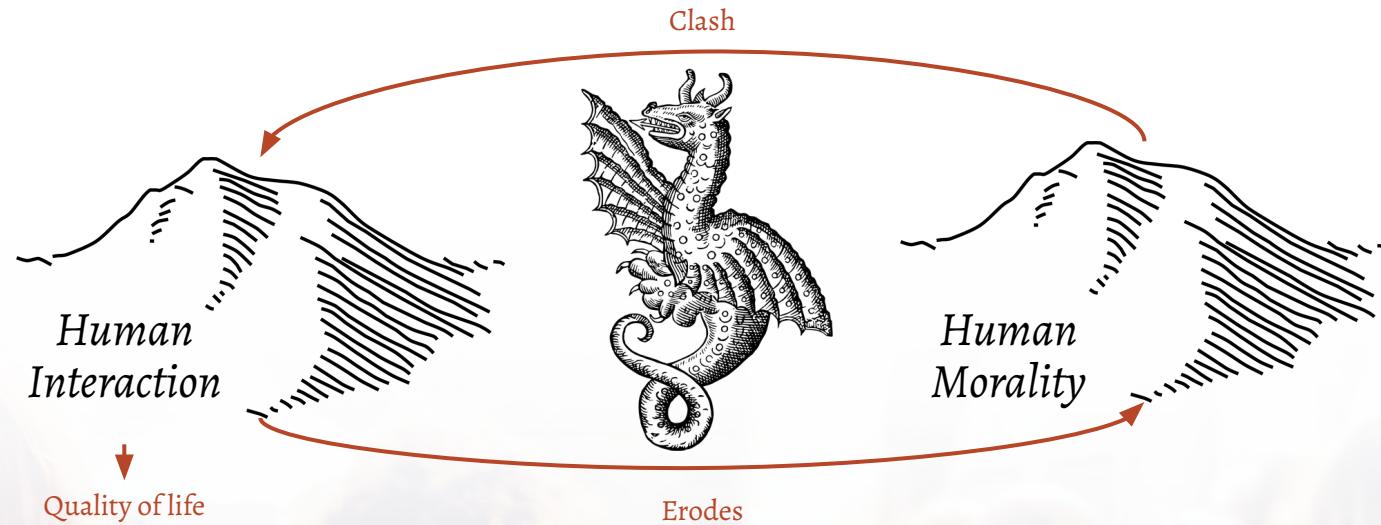
[1] Yuri A. Kuznetsov (2023). [Introduction to Dynamical Systems](#)

[2] Ted G Lewis (2011). [Network science: Theory and applications](#)

[3] Elizabeth C Vozzola (2014). [Moral development: Theory and applications](#)

Introduction

Motivation [1]

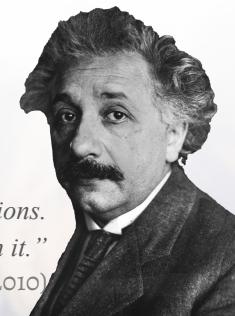


*"The most important human endeavor is the striving for **morality** in our actions.*

Our inner balance and even our very existence depend on it."

Albert Einstein. The Ultimate Quotable Einstein. (2010)

[1] Naomi Ellemers et. al. (2019) [The Psychology of Morality: A Review and Analysis of Empirical Studies Published From 1940 Through 2017](#)



Introduction

Impact



[1] Melanie Killen and Audun Dahl (2021). [Moral Reasoning Enables Developmental and Societal Change](#)

[2] UN (2016). [The sustainable Development Goals](#) [3] J. Giles (2011). [Social science lines up its biggest challenges](#) [4] K. Wilber (2024) [Finding Radical Wholeness](#)

Introduction

Impact



[1] Naomi Ellemers et. al. (2019) [The Psychology of Morality: A Review and Analysis of Empirical Studies Published From 1940 Through 2017](#)

[2] Colin Wayne Leach and Aarti Iyer (2024). [Moral improvement of self, social relations, and society](#)

Introduction

Aim



To put down a **computational framework**
for understanding how
individual-level mechanisms
shape moral development
trajectories, dynamics,
and outcomes in populations

Introduction

Research Questions

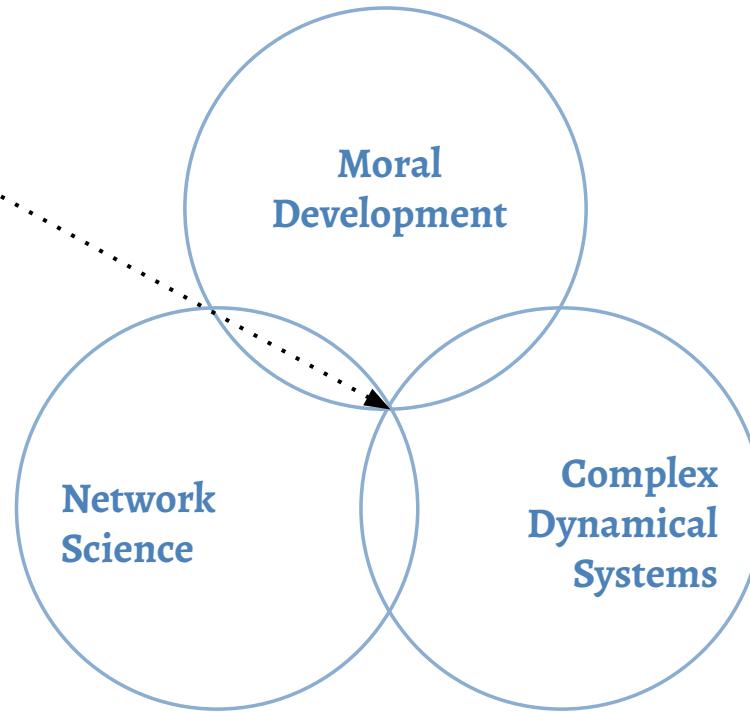


1. How do **interactions** between individual, group and societal levels **influence** the development of human morality over time?

2. What role do **individual-level factors** have in shaping **group-level** dynamics and societal outcomes?

Literature Review

Focus [1-3]



[1] Michael Lampert Commons (2007). [Introduction to the model of hierarchical complexity](#).

[2] Commons, M. L., & Kjorlien, O. A. (2016). [The Meta-Cross-Paradigmatic Order and Stage 16](#)

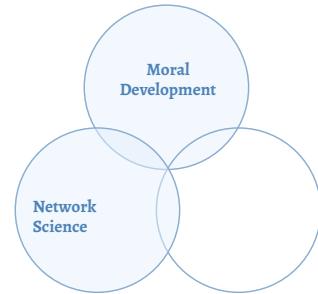
[3] Torraco, R. J. (2016). [Writing integrative literature reviews: Using the past and present to explore the future](#)

Literature Review

Main Findings

Moral development

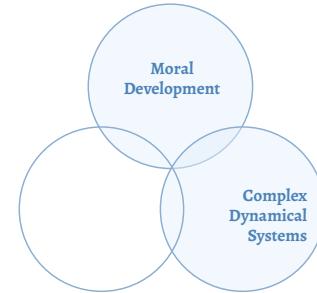
1. Follows a **singular pathway** at the individual level
 - a. Moral core: *monotonic*
 - b. Peer susceptibility: *convex*
2. Is embedded in **social relations**
 - a. Moral networks: *distinct features*
 - b. Position: *likely plays a key role*
3. Expresses itself through **decision** and **action**
 - a. Differences: *drive interaction conflict*
 - b. Collectively distributed: *drive social outcomes*



Literature Review

Main Findings (2)

A complex dynamical system perspective



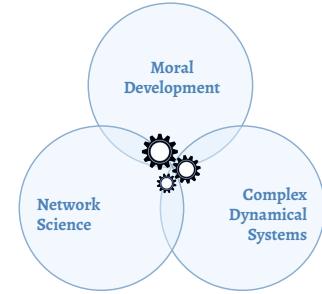
1. Makes a series of key **assumptions**
2. Provides a **solid foundation** for moral development
3. Provides a **unified account** of moral development *and* dynamics
 - a. Development: *a non-linear, self-organizing phenomenon with qualitative differences at micro, meso and macro levels of analysis*
 - b. Dynamics: *probabilistic transformation of the attractors driving development and behavior*
4. Presents many **research opportunities**

Literature Review

Main Findings (3)

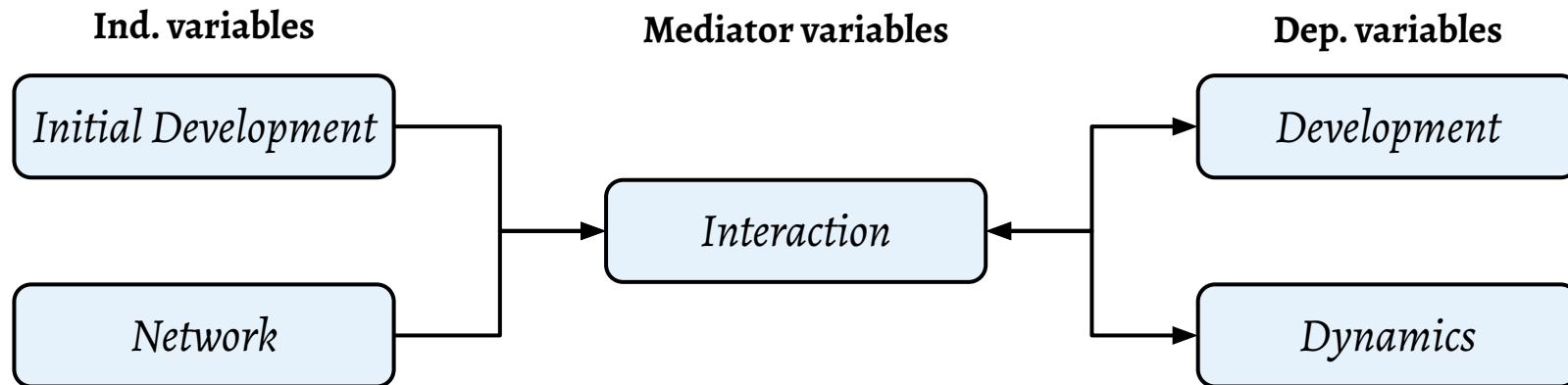
Computational approaches to the CDS of moral development

1. Are novel and underexplored [1]
2. Agent-Based-Models are a good fit
3. Present many **research opportunities**



[1] A notable precursor: Haiyan Liu, Xia Chen, and Bo Zhang (2013). [An Approach for the Accurate Measurement of Social Morality Levels](#)

Framework



Literature Review

Control Variables

Fixed to prevent interference in research outcomes

1. **Pathway:** universal and constant [1]

[1] Lockwood et. al (2024). [Moral Learning and Decision-Making Across the Lifespan](#).

2. **Social bias:** Non-existent [2]

[2] Kundu, P., & Cummins, D. D. (2013). [Morality and conformity: The Asch paradigm applied to moral decisions](#).

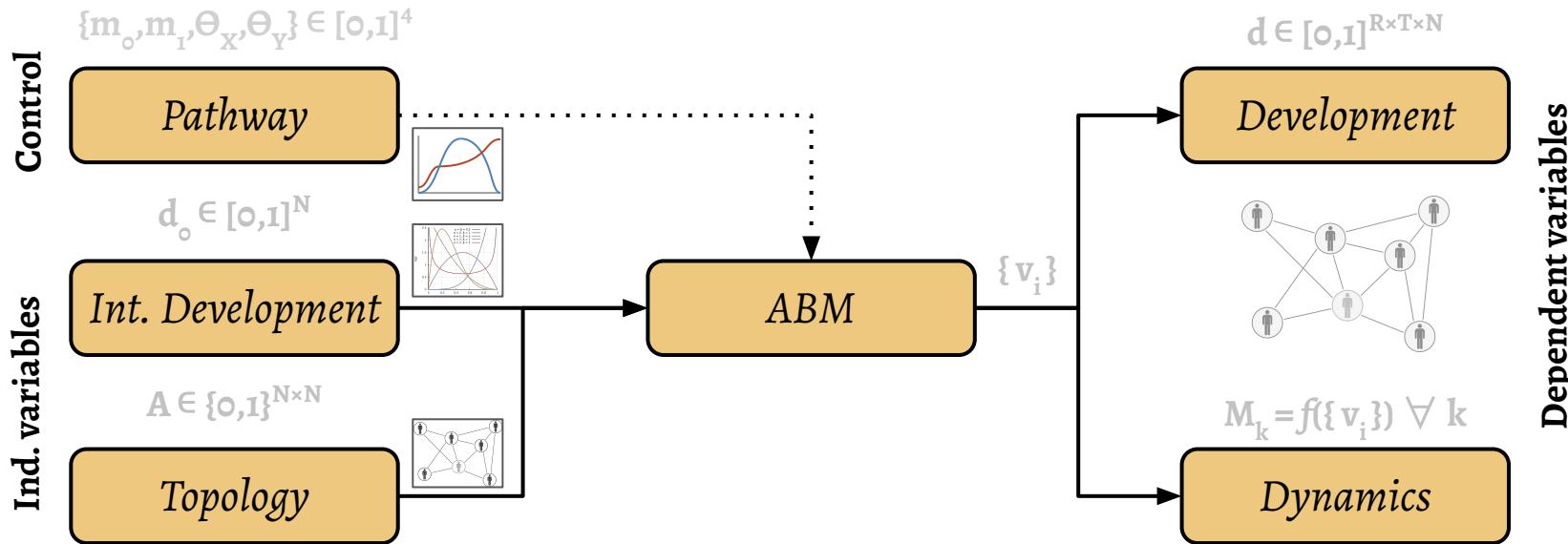
3. **Network change:** Fixed topology [3,4]

[3] Gueorgi Kossinets and Duncan J. Watts (2006). [Empirical analysis of an evolving social network](#)

[4] Pomeroy et. al (2020). [Dynamics of social network emergence explain network evolution](#)

Methodology

Overview



Methodology | Independent Variable

Moral Development

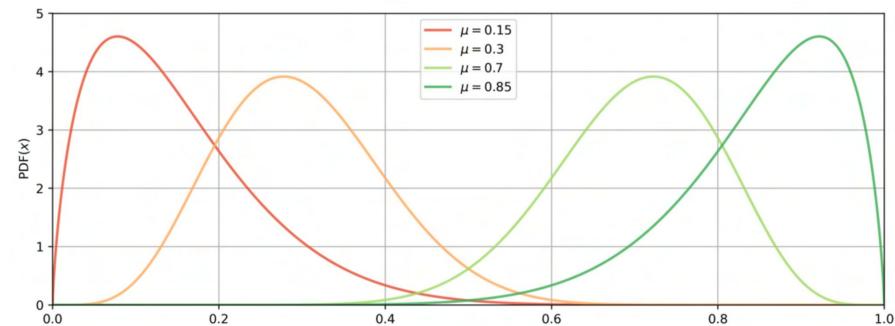
[1,2]

*Real, continuous, bounded
Beta distribution*

$$d_i \in [0,1] \\ D_o \sim \text{Beta}(\alpha, \beta)$$

$$\text{PDF}(D_o) = f(x; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha) \cdot \Gamma(\beta)} \cdot x^{\alpha-1} \cdot (1-x)^{\beta-1}$$

$$\alpha^* = \mu^2 \cdot \left(\frac{1-\mu}{\sigma^2} - \frac{1}{\mu} \right) \quad \beta^* = \alpha \cdot \left(\frac{1}{\mu} - 1 \right)$$



[1] Norman L Johnson, Samuel Kotz, and N Balakrishnan (1994). [Beta distributions. Continuous univariate distributions](#)

[2] Assumednormal (2016). [Calculating the parameters of a Beta distribution using the mean and variance](#)

Topology [1-3]

Adjacency matrix

Random networks

Scale-free networks

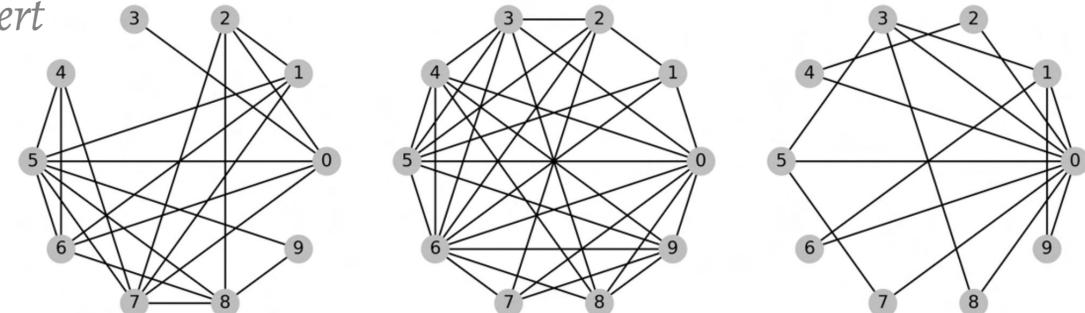
Small-world networks

A

Erdős–Rényi–Gilbert

Watts and Strogatz

Barabási-Albert



[1] Antoine Channarond (2015). [Random graph models: an overview of modeling approaches](#).

[2] Duncan J Watts and Steven H Strogatz (1998). [Collective dynamics of 'small-world' networks](#)

[2] Albert-László Barabási and Réka Albert (1999). [Emergence of scaling in random networks](#)

FIGURE 3.2: Circular spring representation of different random (left, $p = 0.3$), small-world (center, $p = 0.2, k = 6$) and scale-free (right, $m = 2$) network topologies ($N = 10$).

Pathway

Moral core

Susceptibility

Constraints

$$p_m(0) \geq 0 \text{ and } p_m(1) \leq 1$$

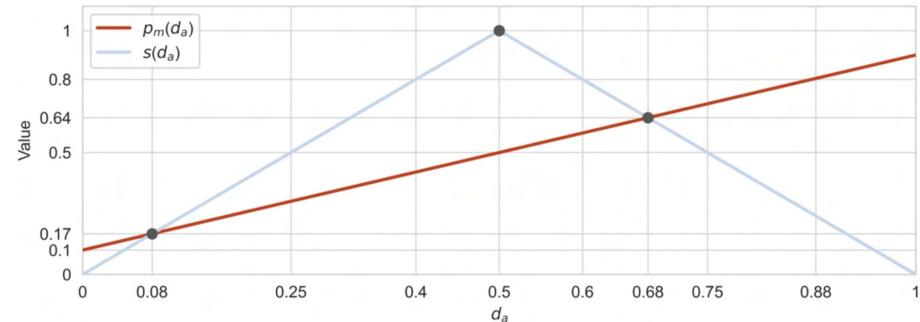
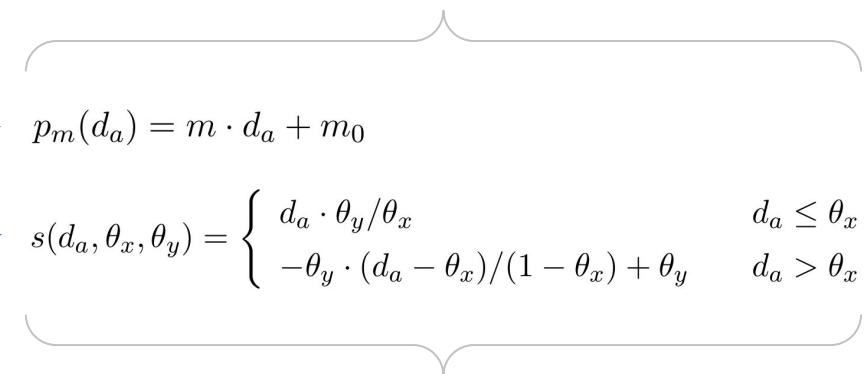
$$\theta_x \cdot m + m_0 < \theta_y$$

$$m_0 > 0 \text{ and } p_m(1) < 1$$

Tuning

$$\langle m_0 = 0.1, m = 0.8, \theta_x = 0.5, \theta_y = 1 \rangle$$

$$d_a \in [0, 1]$$



Agent-Based Model

Moral selection random (neighbor/population)

Moral deliberation

$$p_m(a) = p_m(d_a) \cdot [1 - s(d_a)] + p_m(n_a) \cdot s(d_a) \quad \text{with} \quad p_m(n_a) = \sum_{a' \in n_a} p_m(d_{a'})$$

Moral action

$$O(a) = \text{Bernoulli}(p_m(a)) \in \{0, 1\}$$

Moral development

$$d_a[t+1] = d_a[t] + \delta_m \cdot (2 \cdot O_b - 1) \quad \text{with} \quad \delta_m = 0.01$$

Results & Discussion

Experimental Set-Up

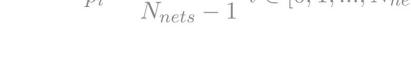
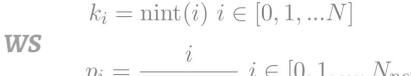
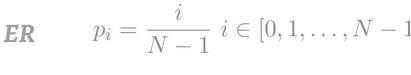
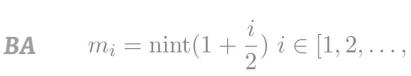
Star network

N	$N_{d,H}$	$N_{d,P}$	σ_0	$\mu_{0,min}$	$\mu_{0,max}$
10	20	20	0.01	0.1	0.9

Network density

N	N_{nets}	N_{dist}	σ_0	$\mu_{0,min}$	$\mu_{0,max}$
15	20	20	0.01	0.3	0.9

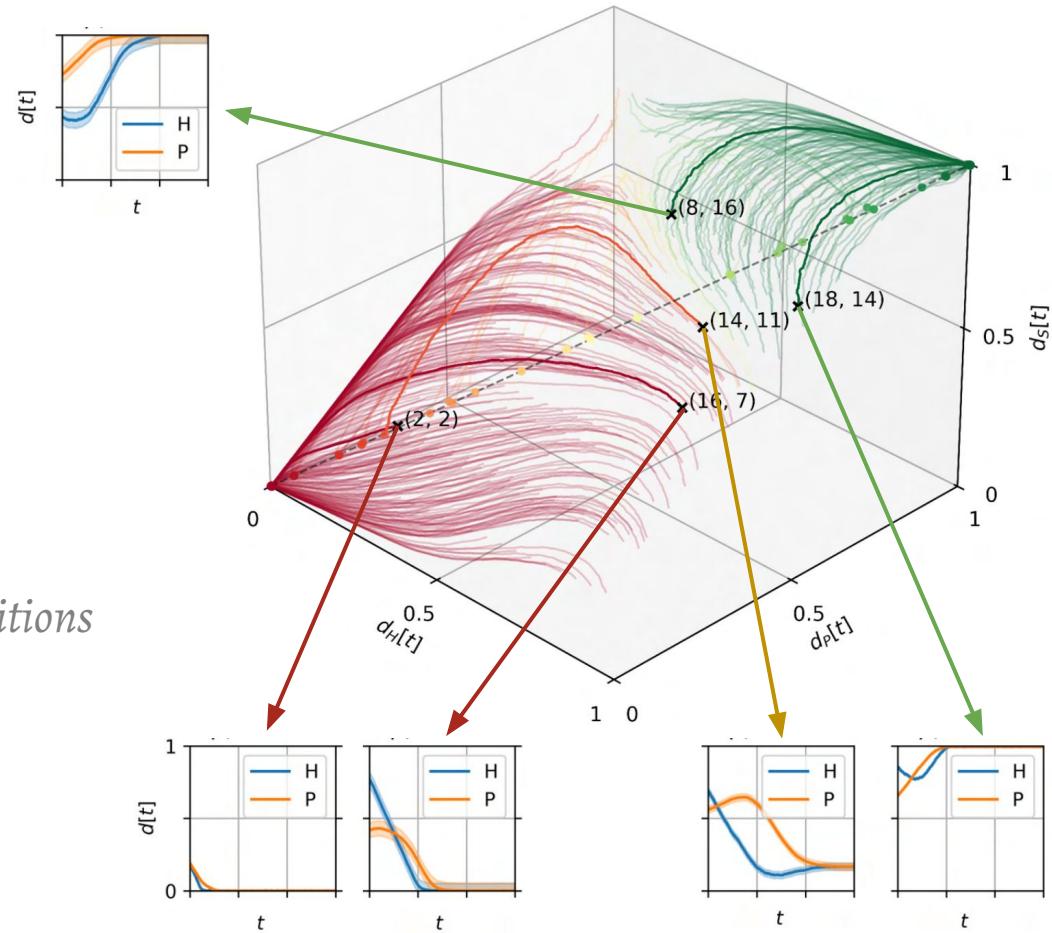
$$f(A) = \frac{1}{N^2 - N} \cdot \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} A_{i,j}$$



Regimes

Under fixed networks the population converges to a stable MD distribution

1. Moral interaction tends to push social outcomes towards the regime **prefigured by the initial system conditions**
2. Individual-level factors can contribute to the emergence of temporary and unstable developmental attractors



Thresholds

*Small changes on the input space
can yield big changes in collective outcomes*

1. In specific networks
*opposite social outcomes are
possible given the same initial MD*
2. Increases in **initial MD**
*contribute more consistently to
developmental outcomes than
increases in network density*

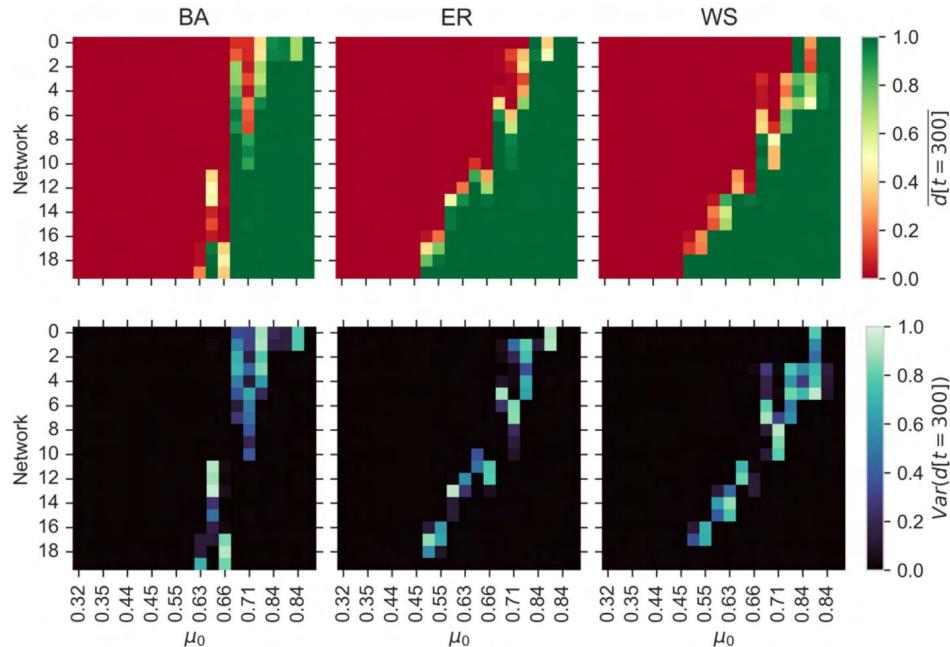


FIGURE 4.8: Final average moral development $\bar{d}[t]$, and variance $\text{Var}(d[t])$, of the population across runs at $t = 300$ for all initial network density and MD configurations.

Mixed Equilibrium

*A regime characterized by
non-trivial stable MD distributions*

1. Regime conditions:
*low network density
and high initial MD*
2. In this regime
*a plethora of stable
MD configurations emerges*

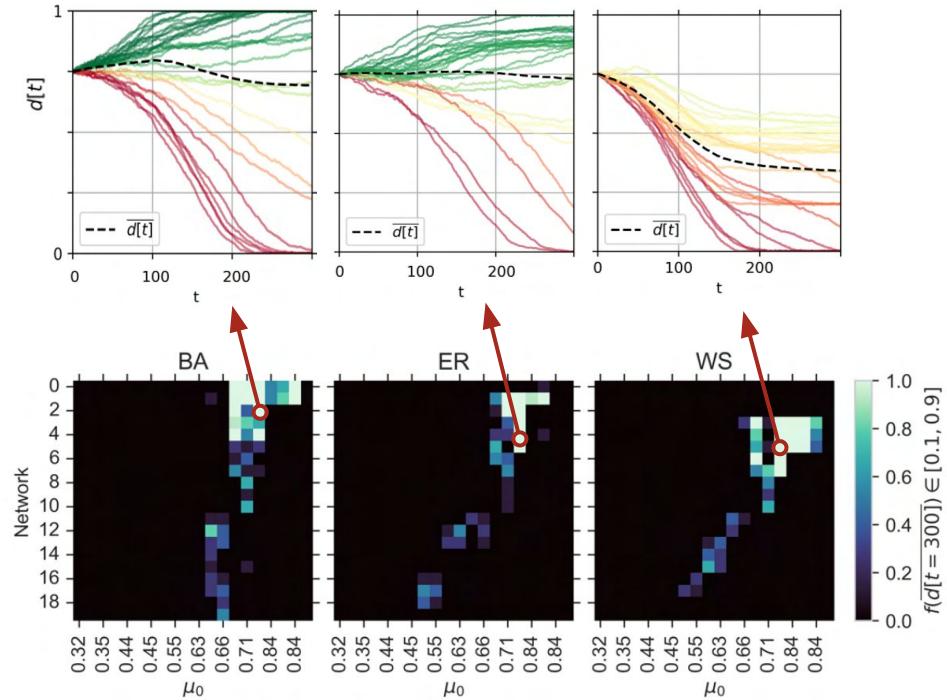


FIGURE 4.9: Fraction of runs with intermediate average moral development values at $t = 300$, $f(d[t = 300])$, for all initial network density and MD configurations.

Conclusion

Answer to RQs



- 1. How do interactions [...] influence MD over time?**
Under fixed social structures, repeated agent interaction shifts the population towards stable developmental regimes

- 2. What role do individual-level factors have in shaping [...] outcomes [...]?**
They shape them when the aggregate MD of the population and its social structure are close to these regime thresholds.

Conclusion

Contributions

1. **Integrative literature review of the complex dynamical system of MD**
*Synthesis of MD pathway, 16 dynamics,
Integrated account of MD and dynamics,
Explicitly-recognized research gaps...*
2. **Computational methodology**
to tackle this terra incognita in MD research
3. **Showcased many anticipated phenomena**
4. **Addressed many explicit research gaps**



Conclusion

Limitations



1. Approach

Vague, oversimplification, validation, generalization

2. Bias

Oversimplification, insufficient basis for modeling choices

3. Experiments

Insufficient, single variance, network size and variety

Conclusion

Future Work



1. Theoretical

Analytical work (ex. proofs on conditions for existence of regimes)

2. Methodological

In-depth LR of individual-level mechanisms, sensitivity analysis, pathway calibration, allowing the topology to evolve ...

3. Experimental

testing proposed hypothesis, mean-field approximation, exhaustive study of topology, research on gaps identified...

Conclusion

Recap

1. **Studied** Moral development in populations
2. **Approach** Modeled MD as a result of agent network interactions in a complex dynamical system
3. **Result** Identified and addressed important gaps while showcasing many anticipated phenomena
4. **Conclusion** Made a case for introducing **computational moral development** as a distinct field of study

Thank you!
Questions?

