Distortion models for the probability of rare events: an application on human reliability

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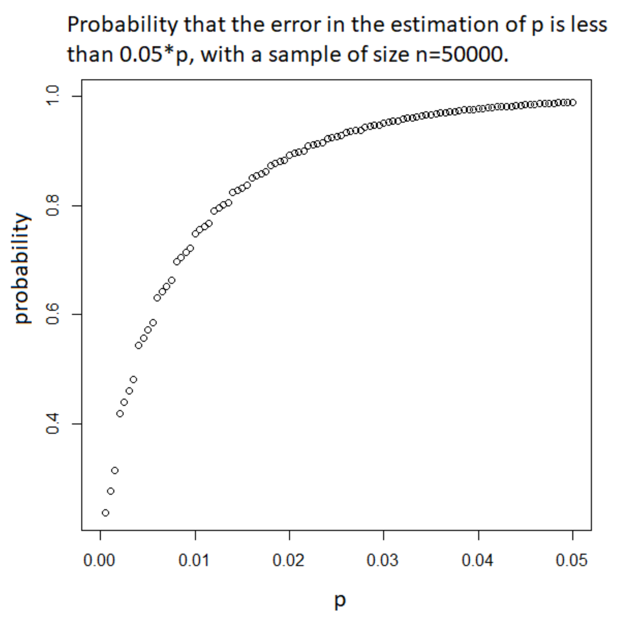
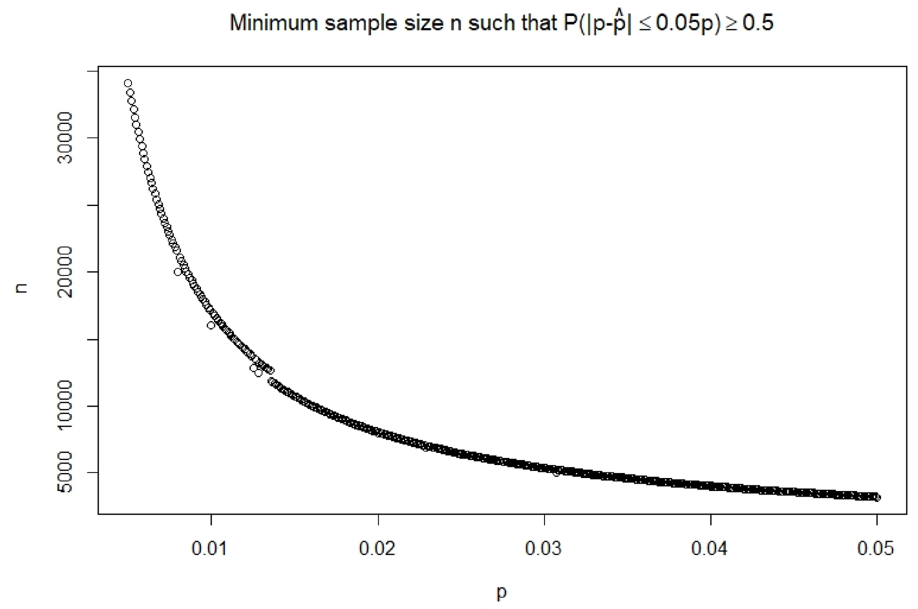
Introduction

The direct estimation of the probability of rare events is a complex problem.

Subjective assessment may be subject to bias.

Frequentist assessment may over or underestimate them.





One possible approach is the use of **distortion models** [1]:

d *Distorting function*

Estimated probability $P_0 \longrightarrow \boxed{B_d^\delta(P_0) = \{P \mid d(P,P_0) \leq \delta\}} \longleftarrow \delta$ *Distorting factor*

Example

Let us use the Kolmogorov Model to estimate tail probabilities of a normal distribution $\mathcal{N}(0,1)$.

Kolmogorov Distance:

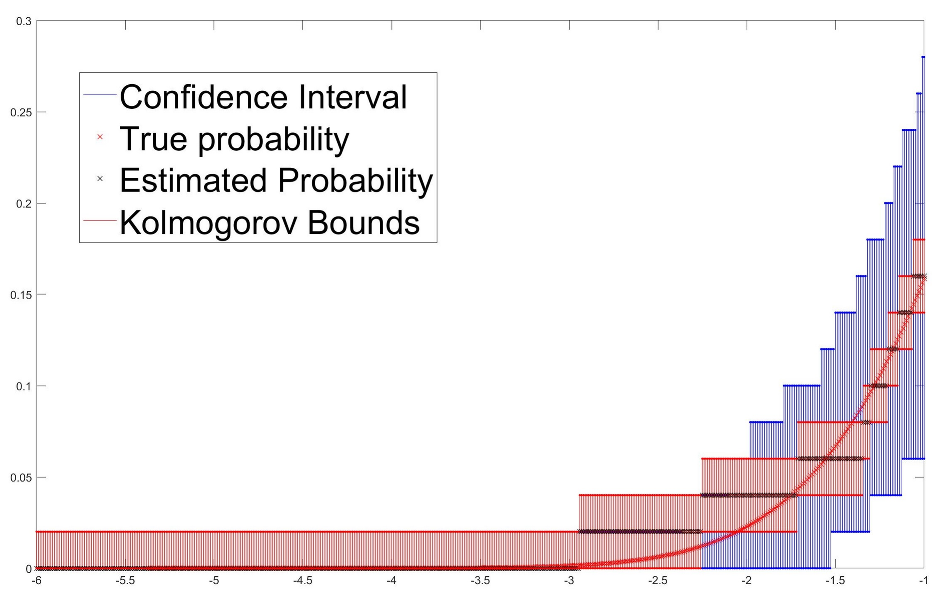
$$d_K(P,P_0) = \max_{x\in\mathcal{X}} |F_P(x) - F_{P_0}(x)|$$

Kolmogorov Model:

$B^\delta_{d_K}(P_0) = (\underline{F}_K, \overline{F}_K)$, where:

$$\underline{F}_K(x) = F_{P_0}(x) - \delta$$

$$\overline{F}_K(x) = F_{P_0}(x) + \delta$$

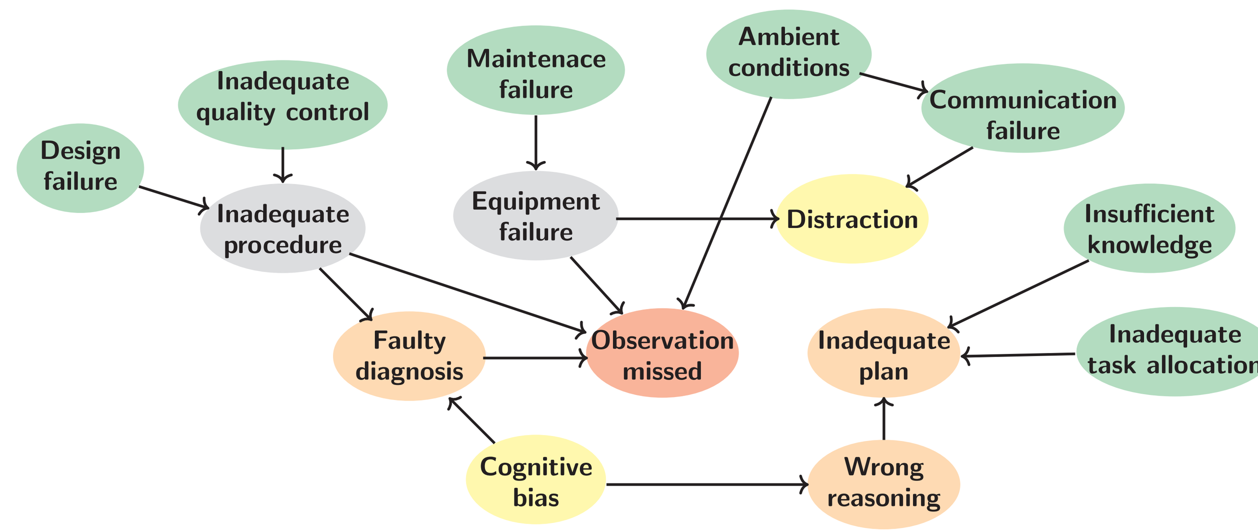


Human Reliability

Goal: to estimate the probability of human error in a major accident.

MATA-D: data from 238 major accidents [4].

Bayesian Network: the connection between the different features was represented graphically in [2]:



Challenge: in some cases, the estimated probability of a conditioning event is zero, and the conditional probabilities cannot be computed.

Credal Network: To avoid this, in [3] the Bayesian Network was turned into a Credal Network. In the cases with zero probability, the conditional model was defined as a *vacuous model*.

Problem: considering a vacuous model may make the estimations too imprecise.

Proposal

To distort the estimation P_0 from [2] by means of a **linear-vacuous model**:

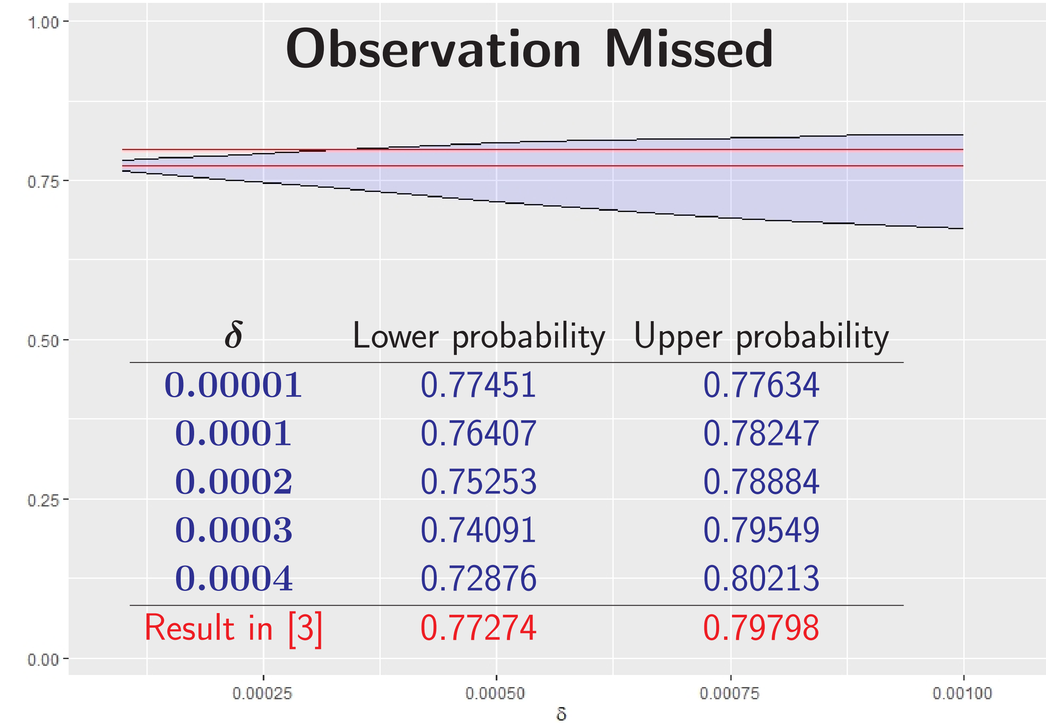
$$\overline{P}_{LV}(A) = (1-\delta)P_0(A) + \delta \quad \forall A \neq \emptyset.$$

It has some advantages w.r.t. other distortion models:

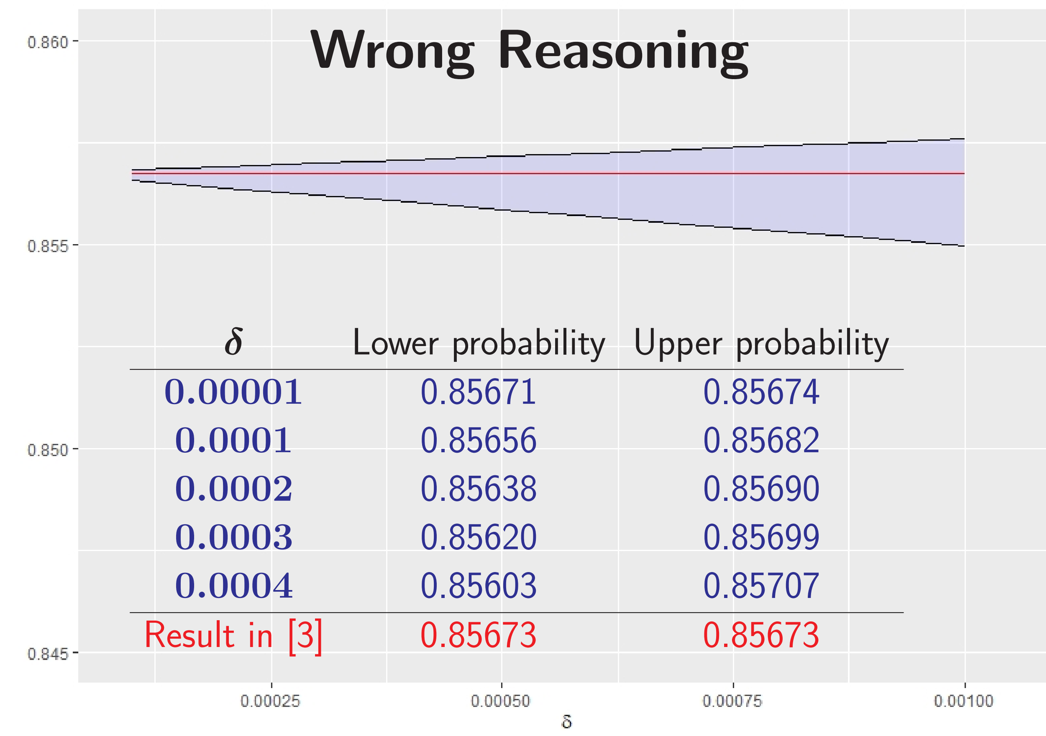
1. $\overline{P}_{LV}(A) > 0 \ \forall A \neq \emptyset$.
2. $\overline{P}_{LV}$ is 2-alternating.
3. Small complexity of the credal set (low number of extreme points).
4. Linear-vacuous models are closed under conditioning.
5. The *regular extension* is given by:

$$\overline{P}_{LV}(A|B) = \left(1 - \frac{\delta}{\overline{P}_{LV}(B)}\right) P_0(A|B) + \frac{\delta}{\overline{P}_{LV}(B)}$$

Some Results



δ	Lower probability	Upper probability
0.00001	0.77451	0.77634
0.0001	0.76407	0.78247
0.0002	0.75253	0.78884
0.0003	0.74091	0.79549
0.0004	0.72876	0.80213
Result in [3]	0.77274	0.79798



δ	Lower probability	Upper probability
0.00001	0.85671	0.85674
0.0001	0.85656	0.85682
0.0002	0.85638	0.85690
0.0003	0.85620	0.85699
0.0004	0.85603	0.85707
Result in [3]	0.85673	0.85673

Conclusions

- ↪ The use of distortion models allows us to overcome the problems of conditioning with zero probabilities.
- ↪ The results are comparable to the ones obtained with a precise model + a vacuous conditional.

Open problems

1. Comparison with other distortion models.
2. To reduce the complexity of the algorithms.
3. Study of the imprecision in terms of the parameter δ.

References

[1] I. Montes, E. Miranda, and S. Destercke. "Unifying neighbourhood and distortion models: Parts I and II". *International Journal of General Systems* 49.6 (2020).

[2] C. Morais, R. Moura, M. Beer, and E. Patelli. "Attempt to predict human error probability in different industry sectors using data from major accidents and Bayesian networks". *Proceedings of PSAM* (2018).

[3] C. Morais, S. Tolo, R. Moura, M. Beer, and E. Patelli. "Tackling the lack of data for human error probability with credal network". *Proceedings of ESREL* (2019).

[4] R. Moura, M. Beer, E. Patelli, J. Lewis, and F. Knoll. "Learning from major accidents to improve system design". *Safery Science* 84 (2016).

At a glance

- The estimation of the probability of rare events may be difficult due to the scarcity of data, and too sensitive to small changes.
- A robust approach can be obtained by introducing distortion models over the estimations.
- We apply this idea within human reliability, building on the pioneering work of Morais et al [2, 3].
- We use linear-vacuous mixtures and regular extension in order to avoid considering vacuous models.
- The (im)precision depends on the distortion parameter, but is similar to the one by Morais et al [3].
- **Future work:** election of δ, comparison with other distortion models.

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