



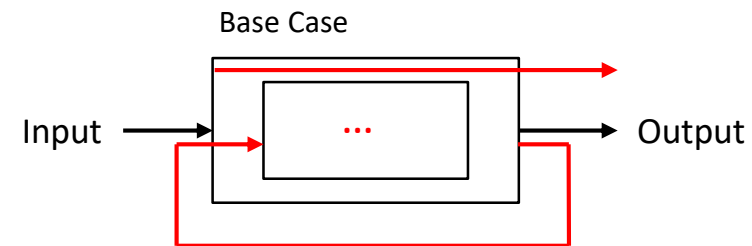
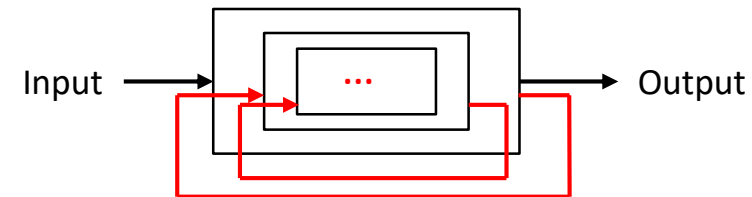
# Dynamic Programming

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# Recursive Algorithm

- One of the central ideas of computer science
- Depends on solutions to smaller instances of the same problem (= sub-problem)
- Function to call itself (it is impossible in the real world)
- Factorial example
  - $n! = n \cdot (n - 1) \cdots 2 \cdot 1$



# Dynamic Programming

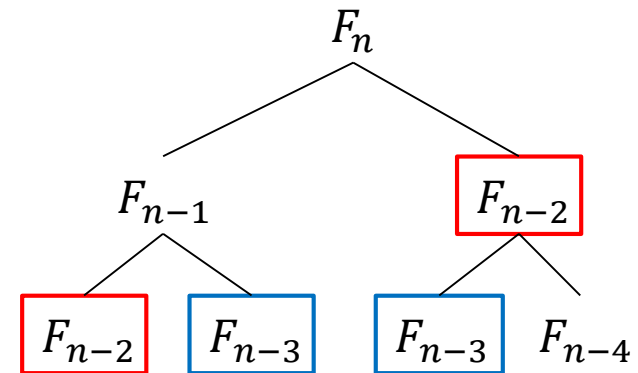
- Dynamic Programming: general, powerful algorithm design technique
- Fibonacci numbers:

$$F_1 = F_2 = 1$$
$$F_n = F_{n-1} + F_{n-2}$$

# Naïve Recursive Algorithm

```
fib( $n$ ) :  
    if  $n \leq 2$  :  $f = 1$   
    else :  $f = \text{fib}(n - 1) + \text{fib}(n - 2)$   
    return  $f$ 
```

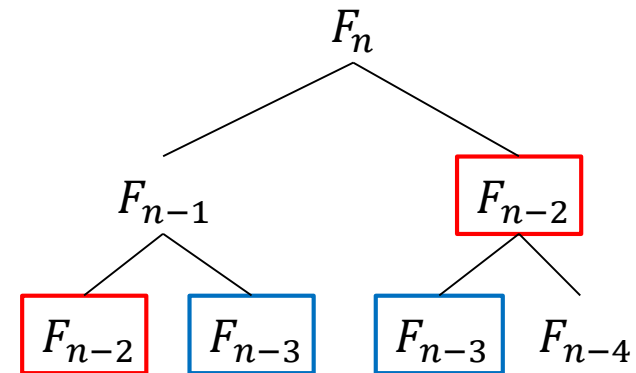
- It works. Is it good?



# Memorized Recursive Algorithm

```
memo = [ ]  
fib(n) :  
    if  $n$  in memo : return memo[ $n$ ]  
  
    if  $n \leq 2$  :  $f = 1$   
    else :  $f = \text{fib}(n - 1) + \text{fib}(n - 2)$   
  
    memo[ $n$ ] =  $f$   
    return  $f$ 
```

- Benefit?
  - fib( $n$ ) only recurses the first time it's called



# Dynamic Programming Algorithm

- Memorize (remember) & re-use solutions to subproblems that helps solve the problem
- DP  $\approx$  recursion + memorization

## Example 1: Climbing a Stair

- You are climbing a stair case. Each time you can either make 1 step, 2 steps, or 3 steps. How many distinct ways can you climb if the stairs has  $n = 30$  steps?

## Example 2: Knapsack Problem

- Burglar (or thief) can carry at most 20 kg ( = maximum capacity = 20)
- Quickly decide which item to carry

items	1	2	3	4	5	6
weight	10	9	4	2	1	20
value	175	90	20	50	10	200

- Approaches
  - Guess
  - Exhaustive search if possible
  - “smarter way” → recursive or dynamic programming

$$\text{key ideas} = \text{original problem} \rightarrow \left\{ \begin{array}{l} \text{subproblem} \rightarrow \left\{ \begin{array}{l} \text{subproblem} \rightarrow \\ \text{subproblem} \rightarrow \end{array} \right. \\ \text{subproblem} \rightarrow \left\{ \begin{array}{l} \text{subproblem} \rightarrow \\ \text{subproblem} \rightarrow \end{array} \right. \end{array} \right.$$



## Example 2: Knapsack Problem

- “smarter way” → recursive or dynamic programming

Suppose we have the following function:

```
[value, taken] = chooseBest(items(1:6),maxWeight)
```

1) item 1 is not taken

```
[v_1,t_1] = chooseBest(items(2:6),maxWeight)
```

2) item 1 is taken

```
[v_2,t_2] = chooseBest(items(2:6),maxWeight - weights(1))
```

```
v_2 = v_2 + values(1)
```

```
t_2 = [items(1),t_2]
```