Autoencoder

Industrial AI Lab.

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Unsupervised Learning

- Definition
  - Unsupervised learning refers to most attempts to extract information from a distribution that do not require human labor to annotate example
  - Main task is to find the ‘best’ representation of the data

- Dimension Reduction
  - Attempt to compress as much information as possible in a smaller representation
  - Preserve as much information as possible while obeying some constraint aimed at keeping the representation simpler
  - This modeling consists of finding “meaningful degrees of freedom” that describe the signal, and are of lesser dimension.

Autoencoders

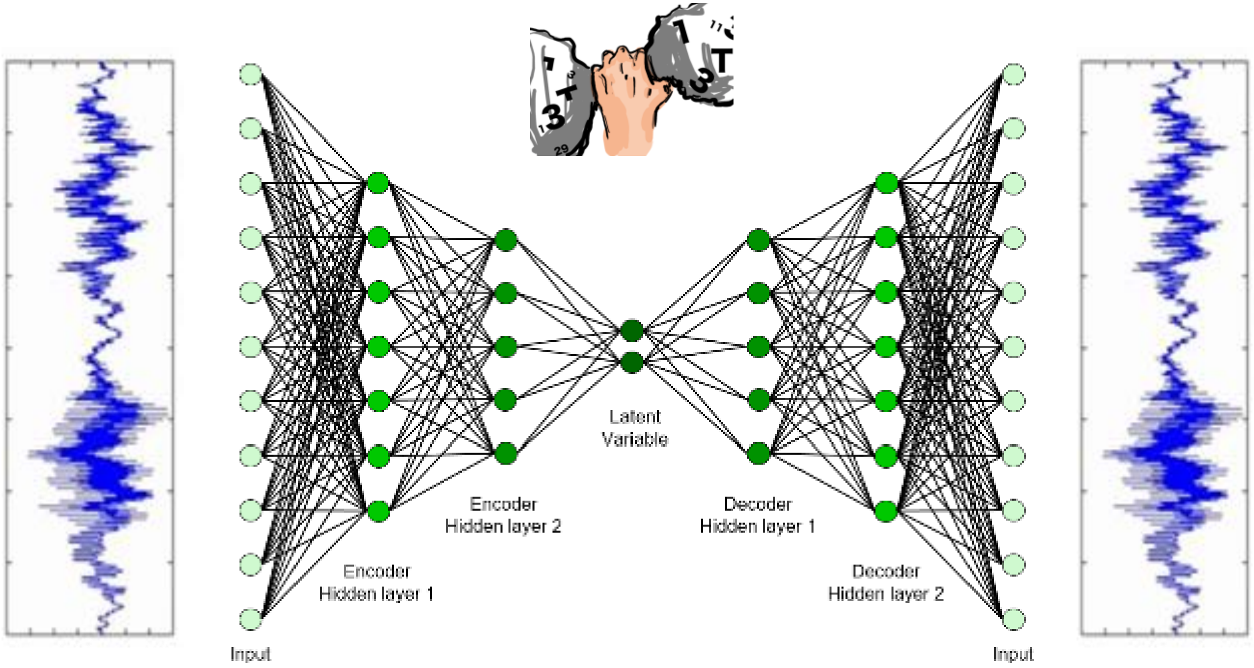
- It is like ‘deep learning version’ of unsupervised learning
- Definition
  - An autoencoder is a neural network that is trained to attempt to copy its input to its output
  - The network consists of two parts: an encoder and a decoder that produce a reconstruction
- Encoder and Decoder
  - Encoder function : $z = f(x)$
  - Decoder function : $x = g(z)$
  - We learn to set $g\left(f(x)\right) = x$

Autoencoder

- Dimension reduction
- Recover the input data

Autoencoder

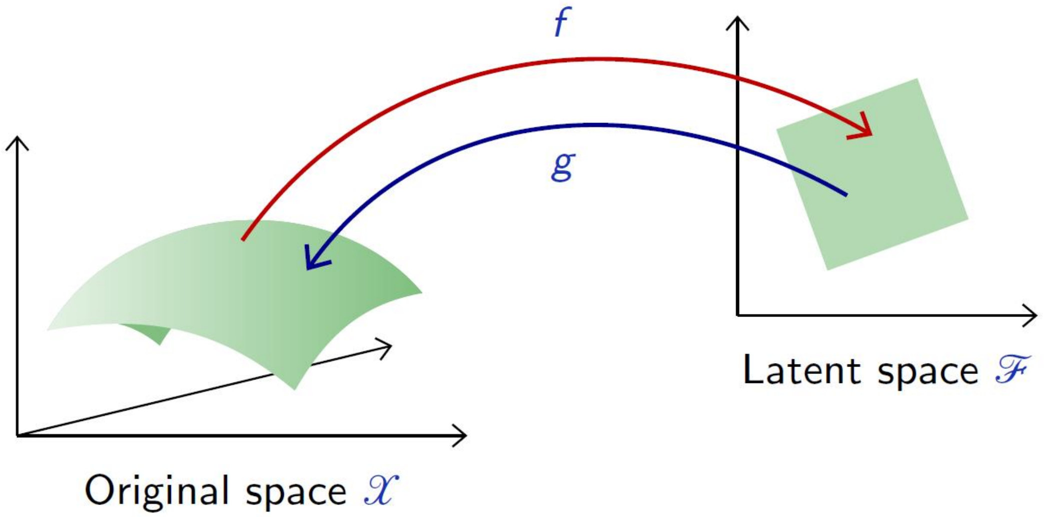
- Dimension reduction
- Recover the input data
  - Learns an encoding of the inputs so as to recover the original input from the encodings as well as possible



Original space Latent space

Autoencoder

- Autoencoder combines an encoder f from the original space $\mathcal{X}$ to a latent space $\mathcal{F}$, and a decoder g to map back to $\mathcal{X}$, such that $g \circ f$ is [close to] the identity on the data



- A proper autoencoder has to capture a "good" parametrization of the signal, and in particular the statistical dependencies between the signal components.

Source: Dr. Francois Fleuret at EPEL

Autoencoder

Let q be the data distribution over $\mathcal{X}$. A good autoencoder could be characterized with the quadratic loss

$$\mathbb{E}_{X \sim q}\left[\|X - g \circ f(X)\|^2\right] \simeq 0.$$

Given two parametrized mappings $f(\cdot\,; w)$ and $g(\cdot\,; w)$, training consists of minimizing an empirical estimate of that loss

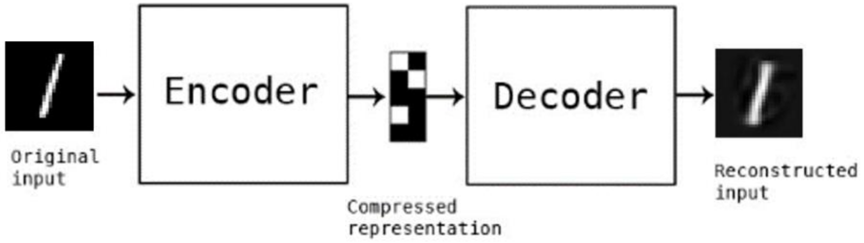
$$\hat{w}_f, \hat{w}_g = \underset{w_f, w_g}{\operatorname{argmin}} \frac{1}{N} \sum_{n=1}^{N} \|x_n - g(f(x_n; w_f); w_g)\|^2 .$$

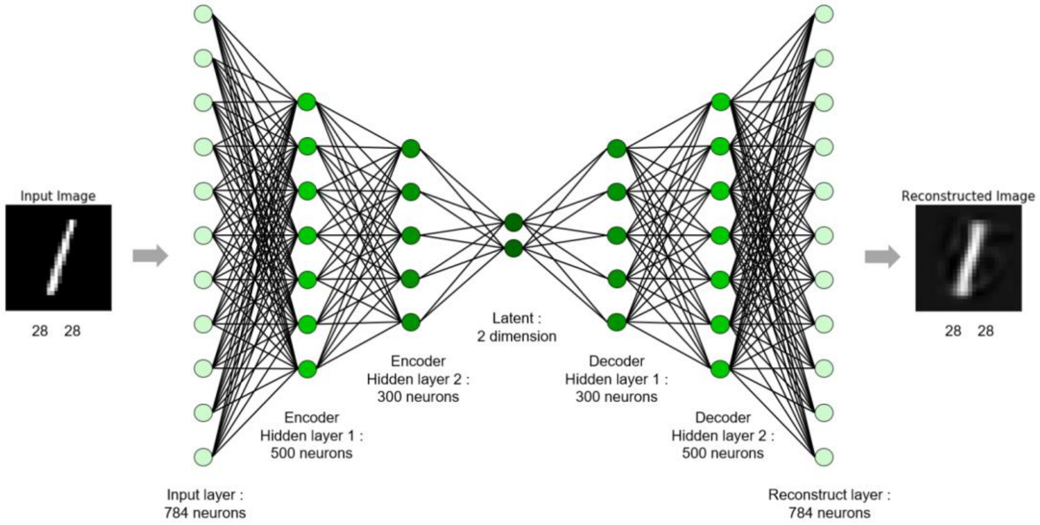
Source: Dr. Francois Fleuret at EPEL

Autoencoder with MNIST

Autoencoder with TensorFlow

- MNIST example
- Use only (1, 5, 6) digits to visualize in 2-D

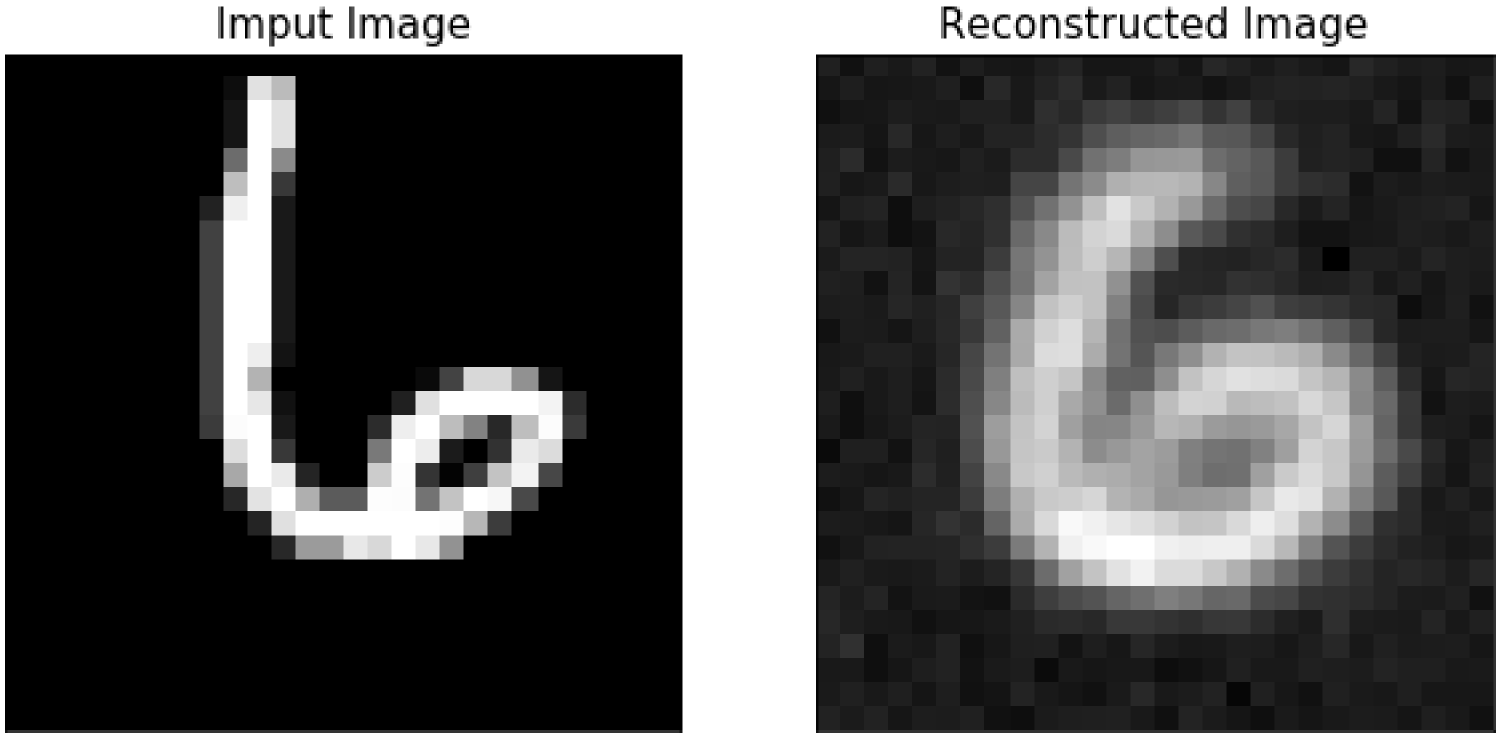




$$\frac{1}{m}\sum_{i=1}^{m}(t_i - y_i)^2$$

Test or Evaluation

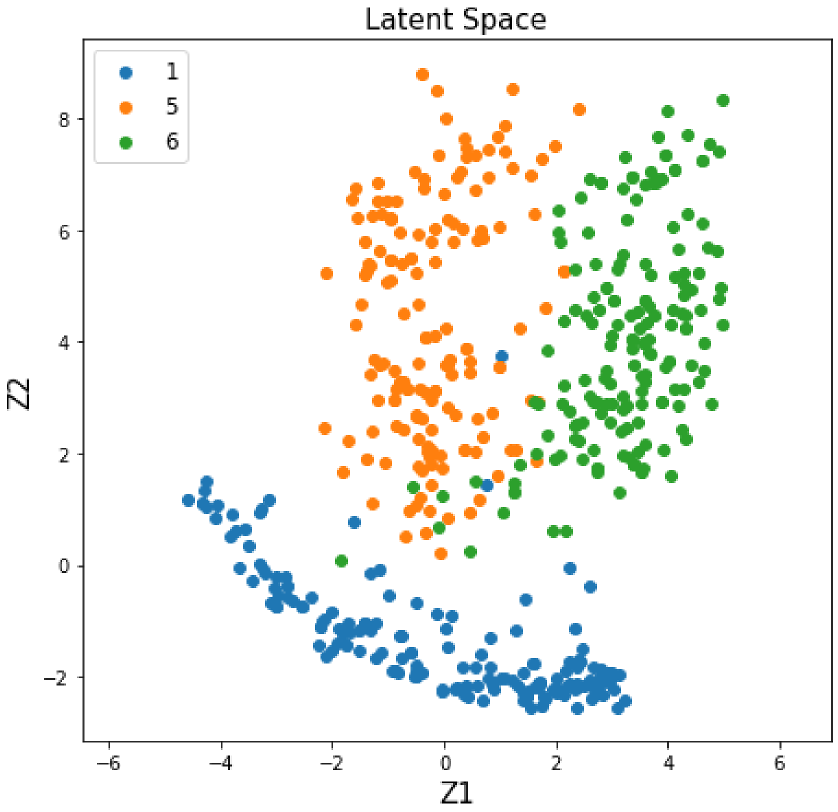
```
test_x, _ = test_batch_maker(1)
x_reconst = sess.run(reconst, feed_dict = {x: test_x})
```



Distribution in Latent Space

- Make a projection of 784-dim image onto 2-dim latent space

```
test_x, test_y = test_batch_maker(500)
test_y = np.argmax(test_y, axis = 1)
test_latent = sess.run(latent, feed_dict = {x: test_x})
```



Autoencoder as Generative Model

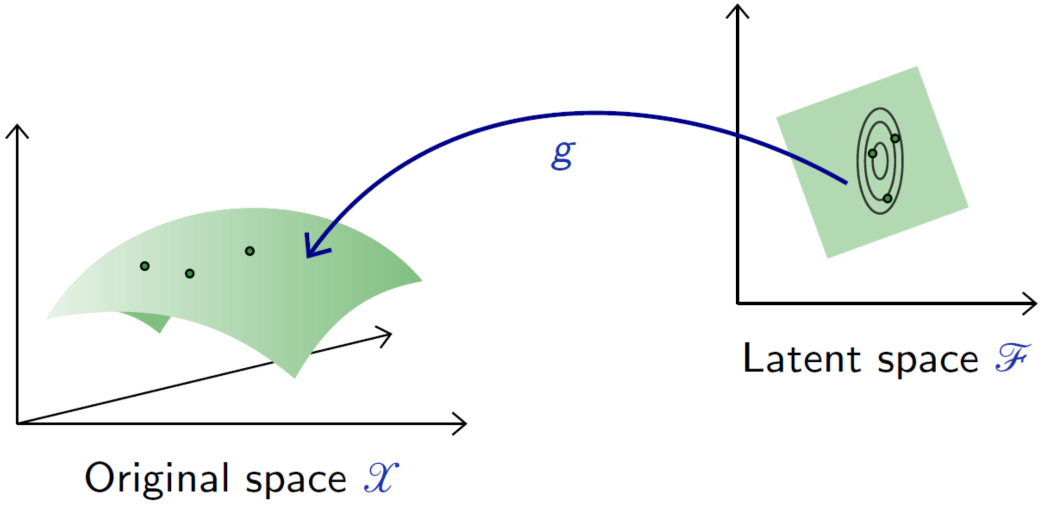
Generative Capabilities

- We can assess the generative capabilities of the decoder g by introducing a [simple] density model q^Z over the latent space $\mathcal{F}$, sample there, and map the samples into the image space $\mathcal{X}$ with g.

We can for instance use a Gaussian model with diagonal covariance matrix.

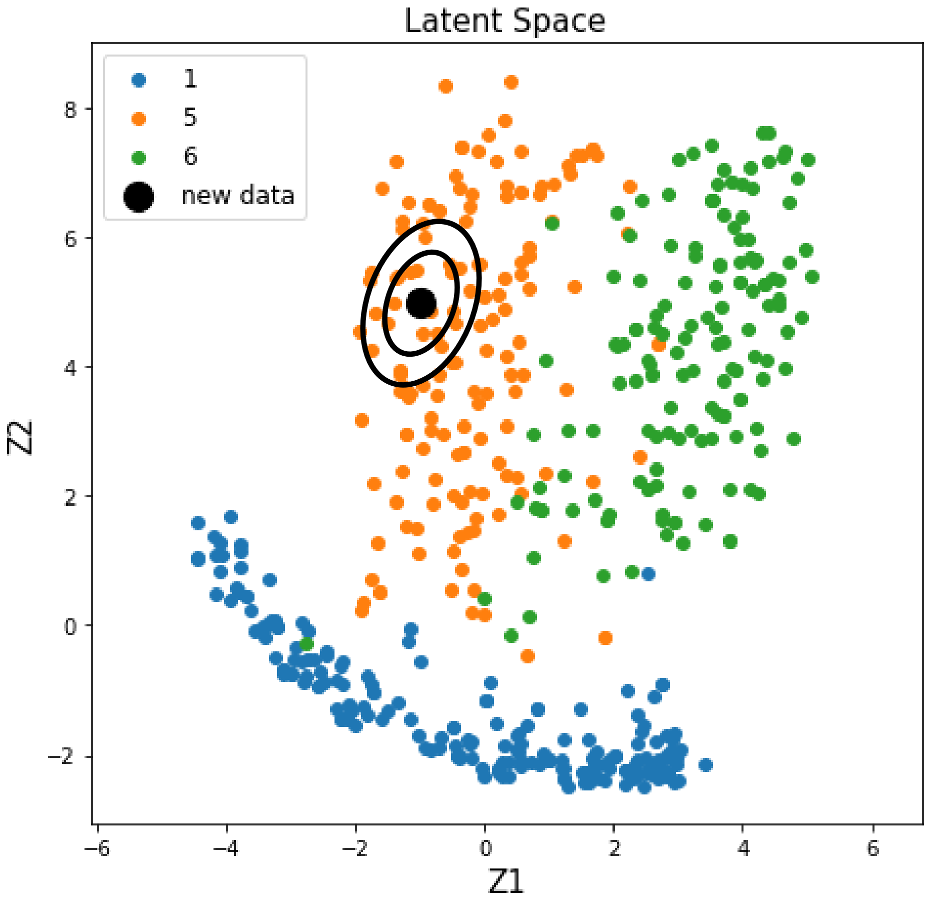
$$f(X) \sim \mathcal{N}(\hat{m}, \hat{\Delta})$$

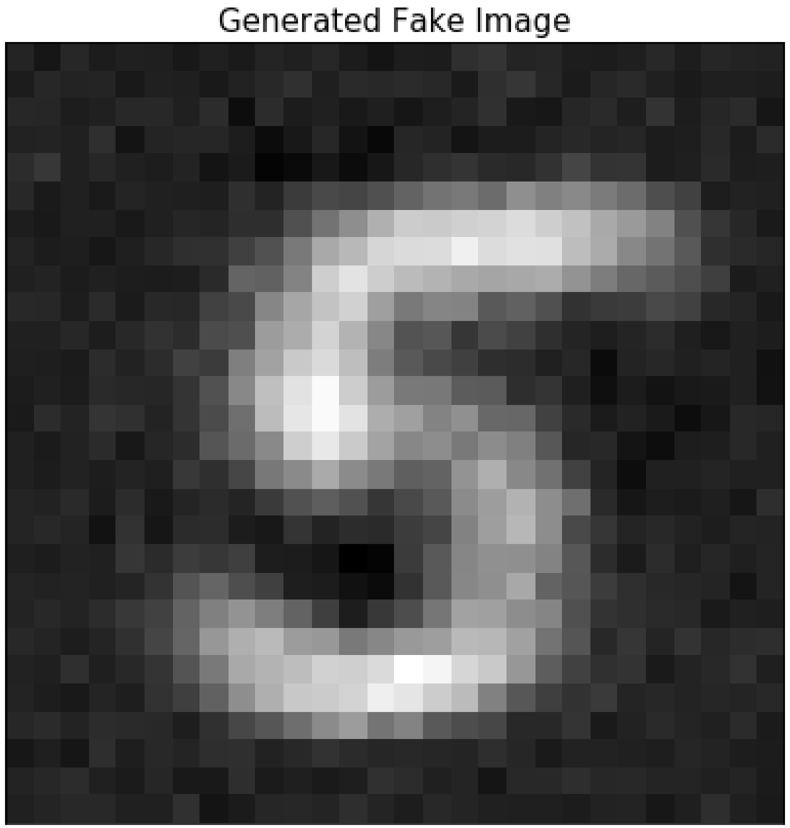
where $\hat{m}$ is a vector and $\hat{\Delta}$ a diagonal matrix, both estimated on training data.



Source: Dr. Francois Fleuret at EPEL

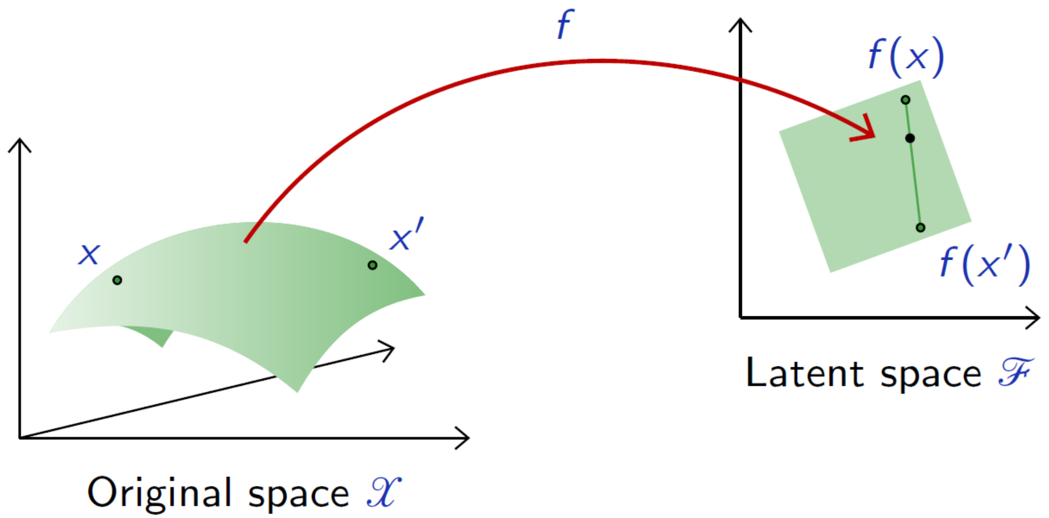
MNIST Example





Latent Representation

- To get an intuition of the latent representation, we can pick two samples x and x' at random and interpolate samples along the line in the latent space



Source: Dr. Francois Fleuret at EPEL

Latent Representation

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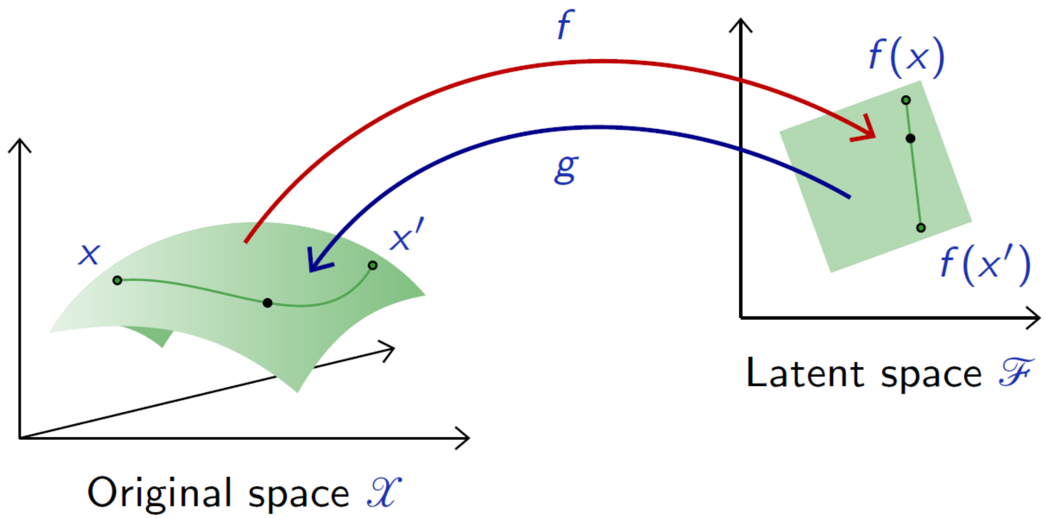
$$\forall x, x' \in \mathscr{X}^2,\ \alpha \in [0,1],\quad \xi(x, x', \alpha) = g((1-\alpha)f(x) + \alpha f(x')).$$

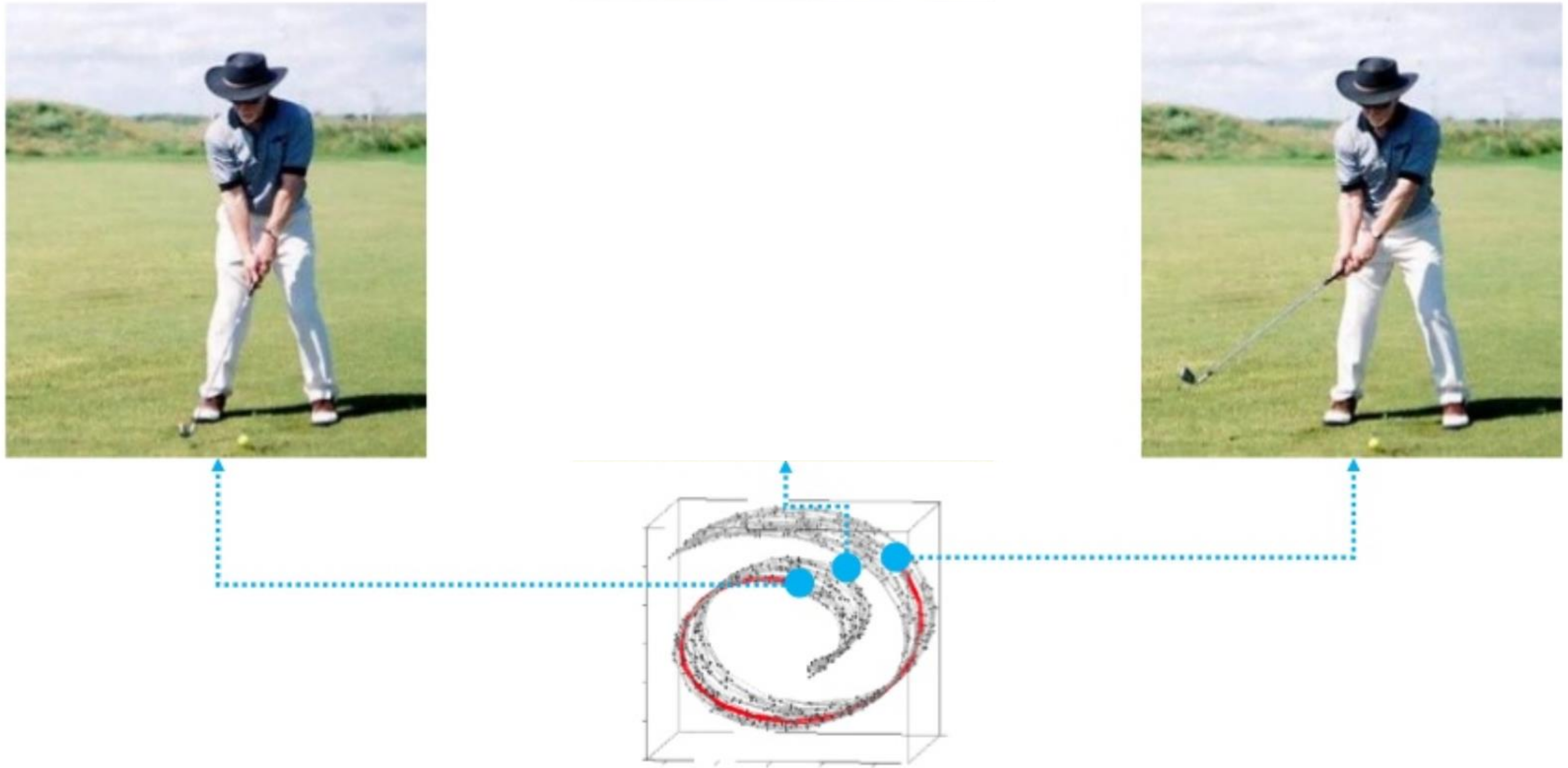


Source: Dr. Francois Fleuret at EPEL

Interpolation in High Dimension

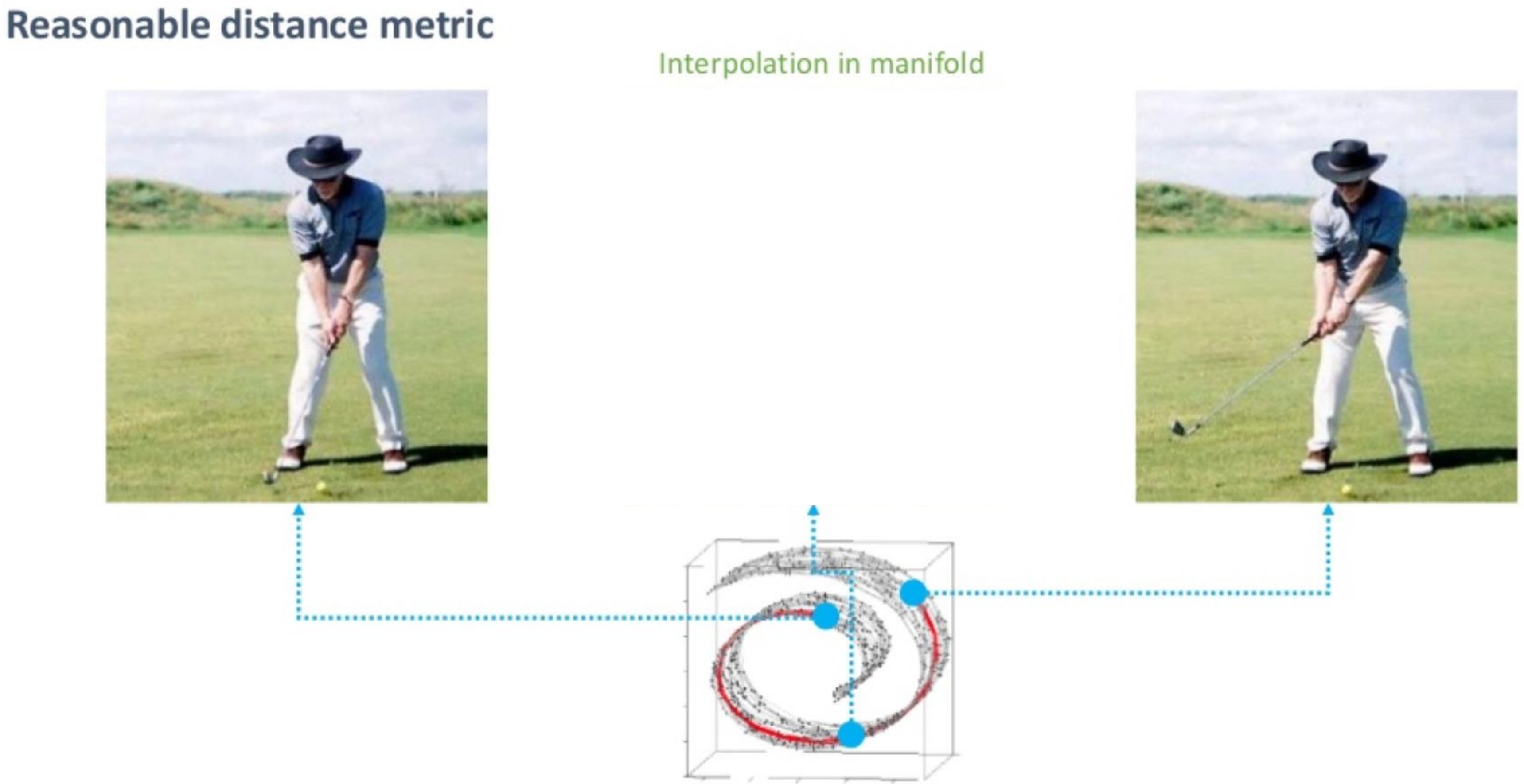
Reasonable distance metric

Interpolation in high dimension



https://www.cs.cmu.edu/~efros/courses/AP06/presentations/ThompsonDimensionalityReduction.pdf

Interpolation in Manifold



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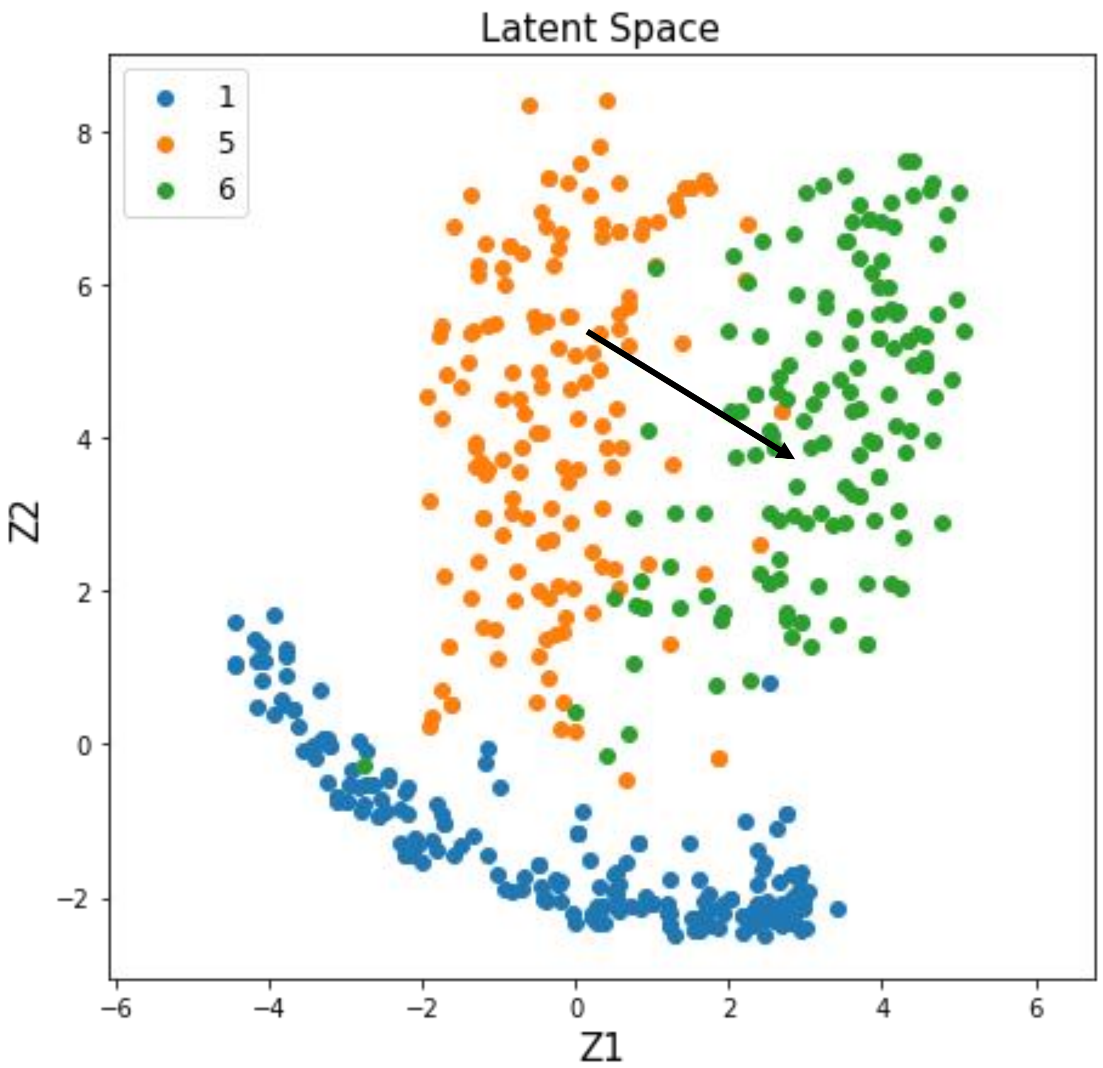
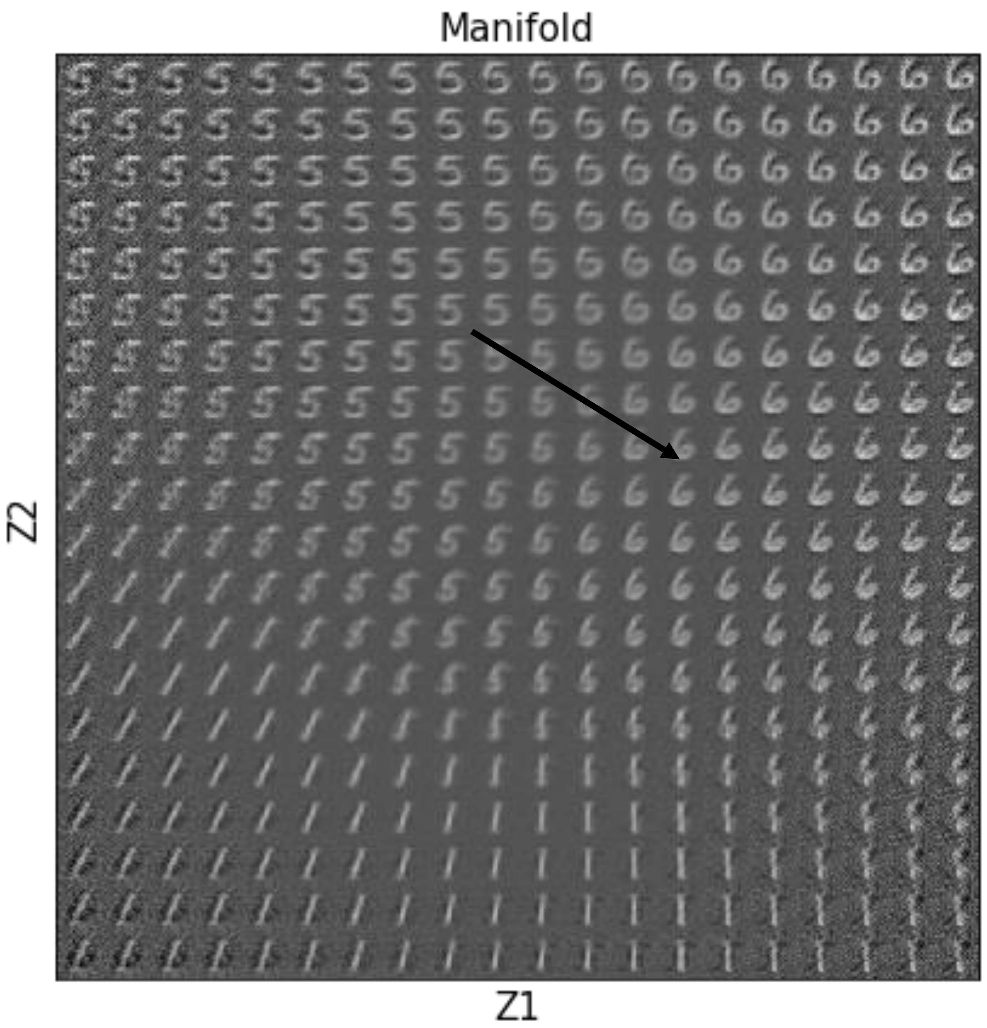
MNIST Example: Walk in the Latent Space





Generative Models

- It generates something that makes sense.

- These results are unsatisfying, because the density model used on the latent space $\mathcal{F}$ is too simple and inadequate.

- Building a "good" model amounts to our original problem of modeling an empirical distribution, although it may now be in a lower dimension space.

- This is a motivation to VAE or GAN.