



Neural Network Architectures for Time-Series: Recurrent Neural Network (RNN)

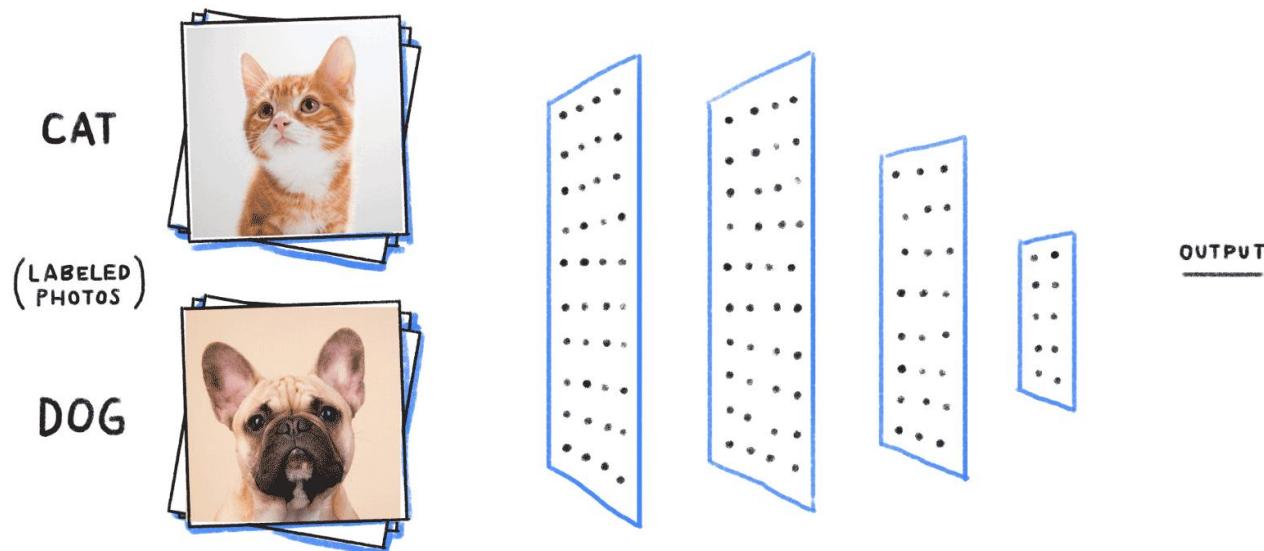
Industrial AI Lab.

Sooyoung Lee, Juhyeong Jeon, and Iljeok Kim

Time-Series Anlysis

So Far

- Regression, Classification, Dimension Reduction,
- Based on snapshot-type data



Robocup 2011

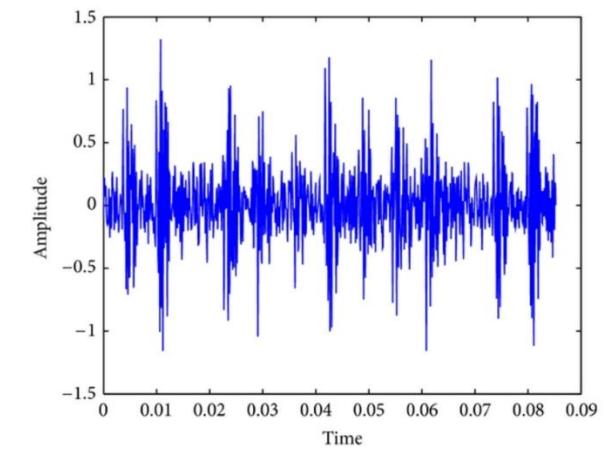
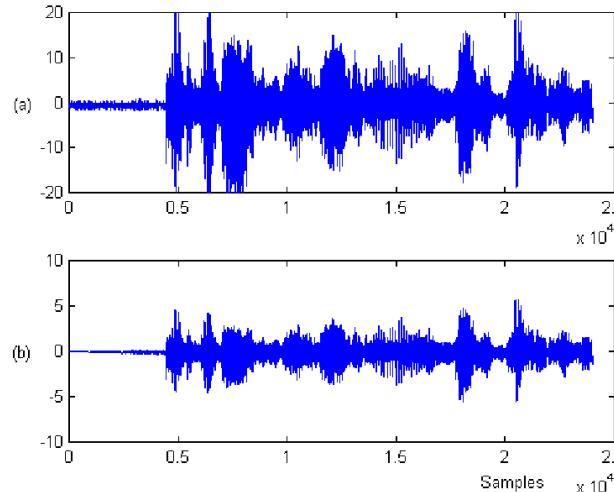
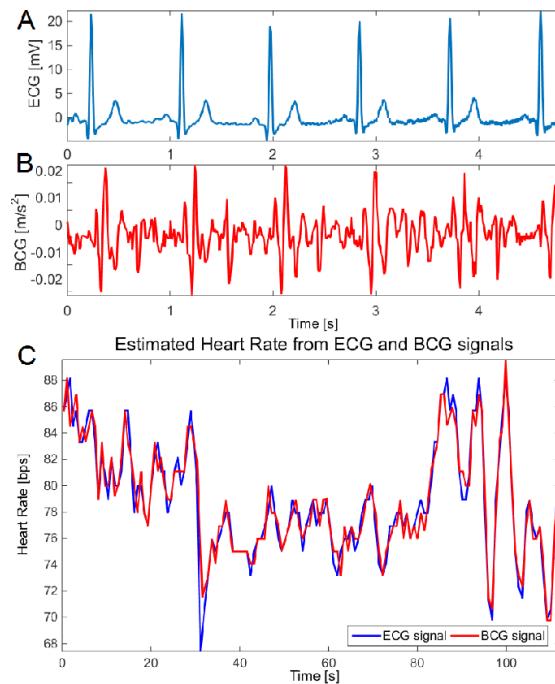


Sequence Matters



What is a Sequence ?

- Sentence
 - “This morning I took the dog for a walk.”
- Medical signals
- Speech waveform
- Vibration measurement



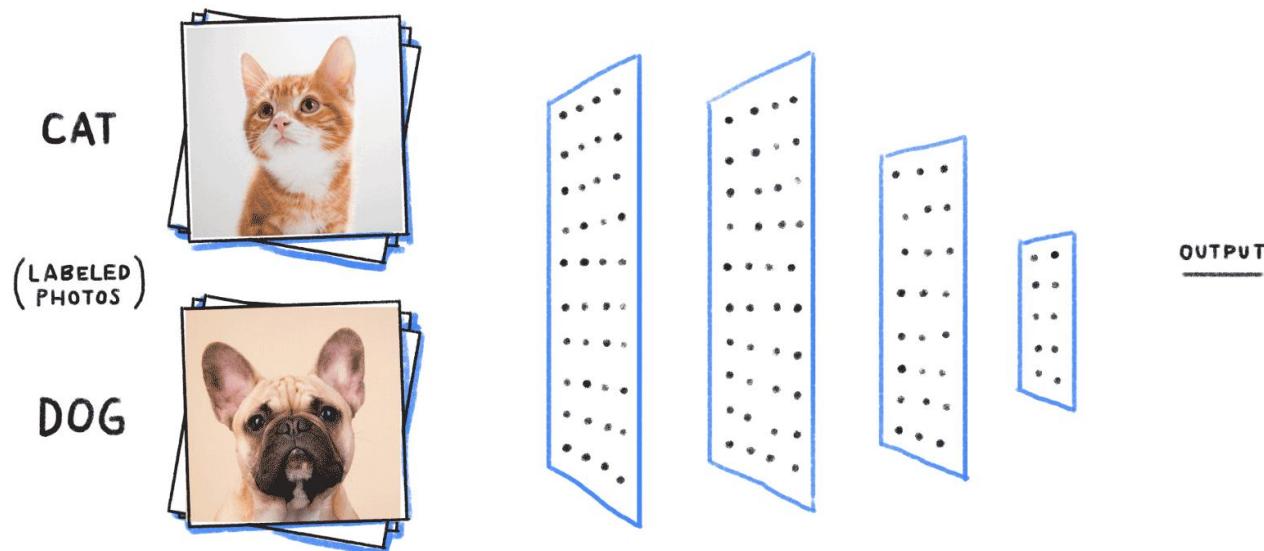
Sequence Modeling

- Most of the real-world data is time-series
- There are important bits to be considered
 - Past events
 - Relationship between events
 - Causality
 - Credit assignment
 - Learning the structure and hierarchy
- Use the past and present observations to predict the future



So Far

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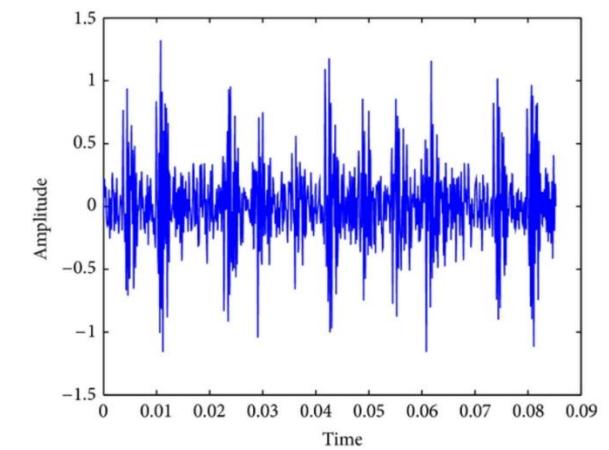
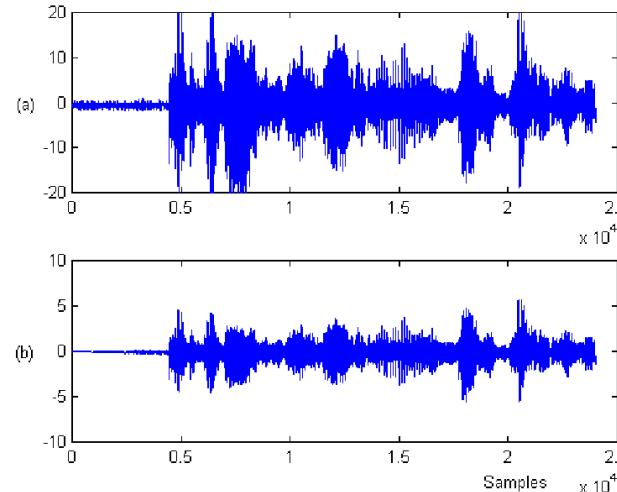
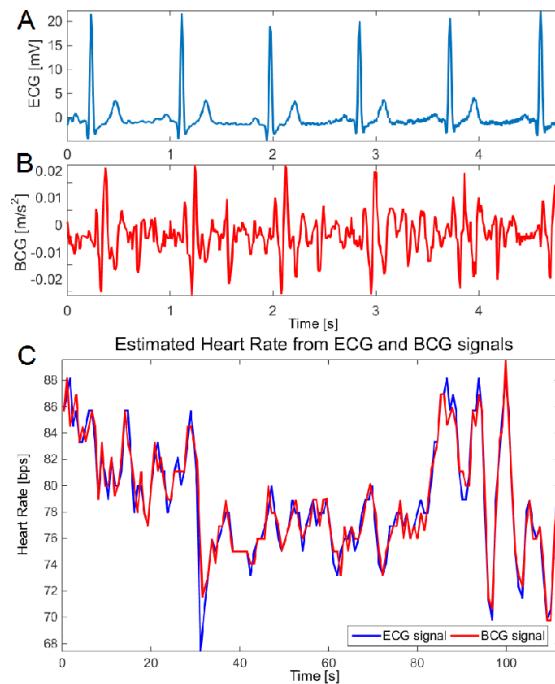


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(Deterministic) Time Series Data

- For example

$$y[0] = 1, \quad y[1] = \frac{1}{2}, \quad y[2] = \frac{1}{4}, \quad \dots$$

- Closed-form

$$y[n] = \left(\frac{1}{2}\right)^n, \quad n \geq 0$$

- Linear difference equation (LDE) and initial condition

$$y[n] = \frac{1}{2}y[n - 1], \quad y[0] = 1$$

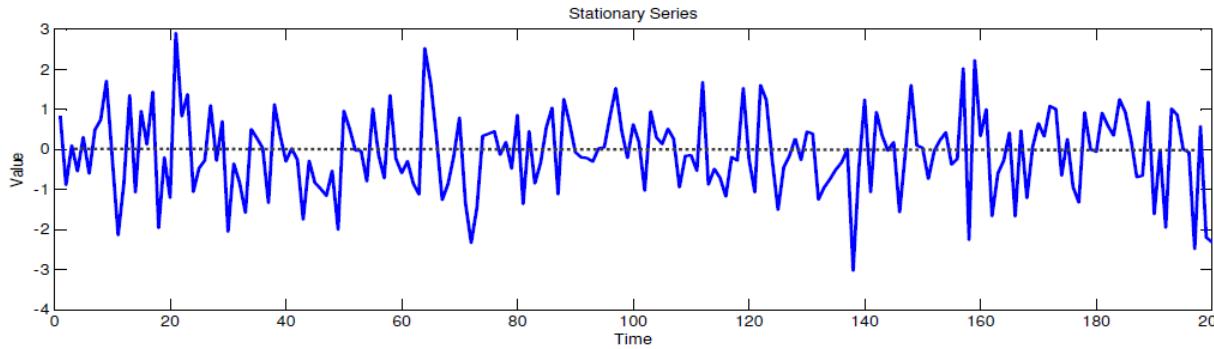
- High order LDEs

$$y[n] = \alpha_1 y[n - 1] + \alpha_2 y[n - 2]$$

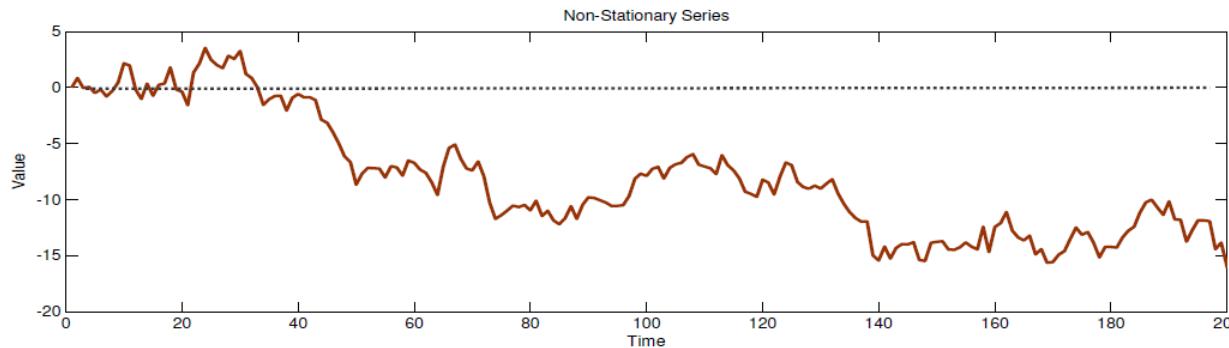
$$y[n] = \alpha_1 y[n - 1] + \alpha_2 y[n - 2] + \dots + \alpha_k y[n - k]$$

(Stochastic) Time Series Data

- Stationary

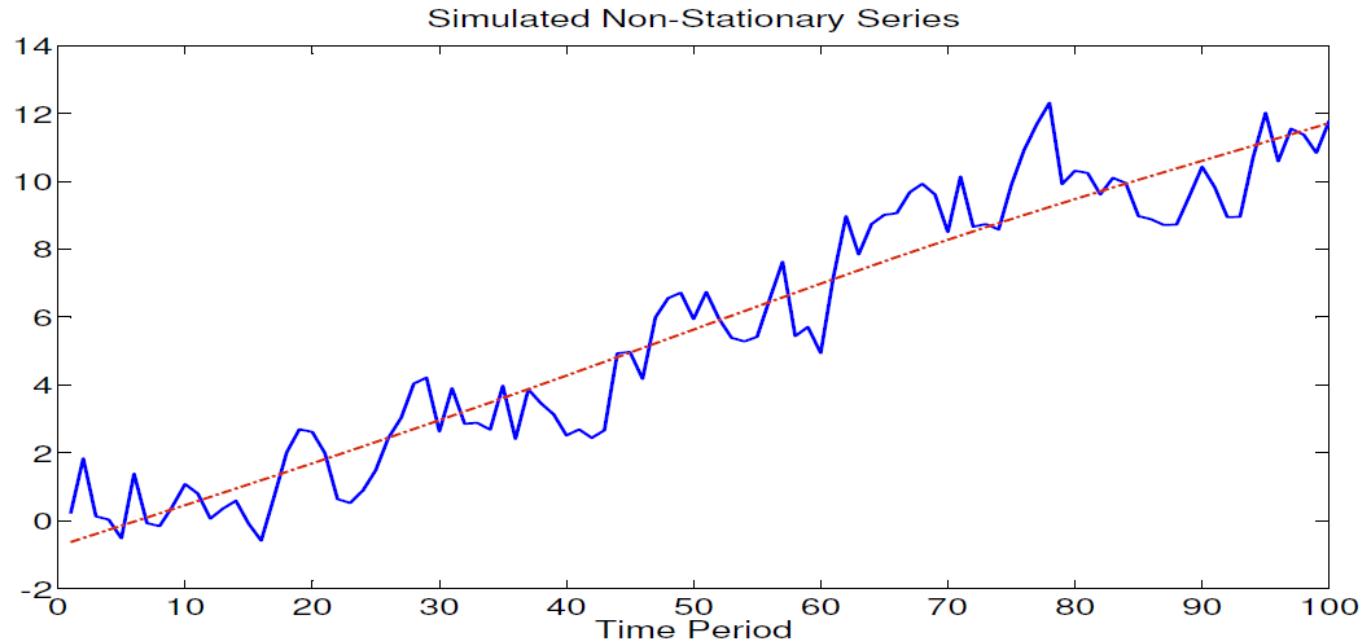


- Non-stationary
 - Mean and variance change over time



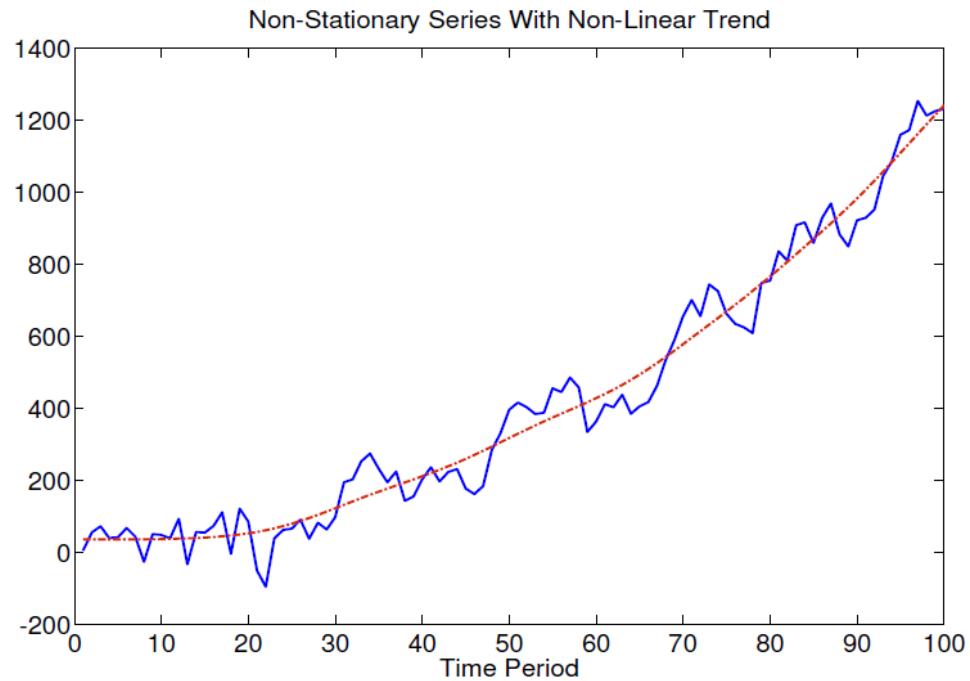
Dealing with Non-Stationarity

- Linear trends



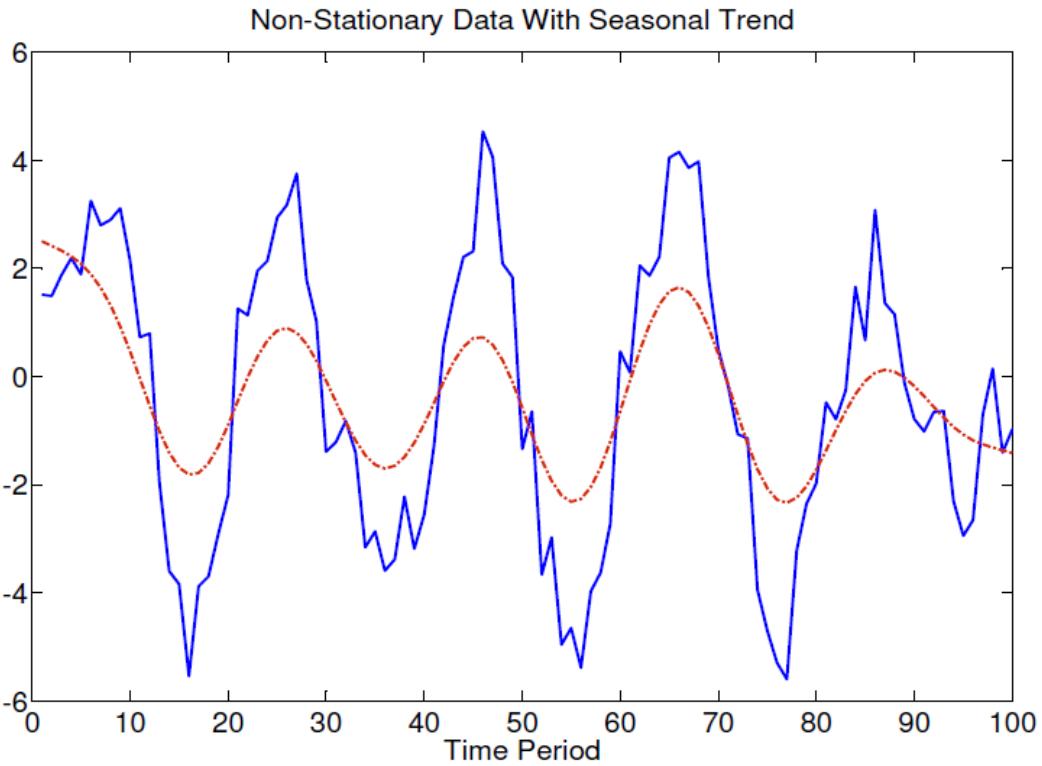
Dealing with Non-Stationarity

- Non-linear trends



Dealing with Non-Stationarity

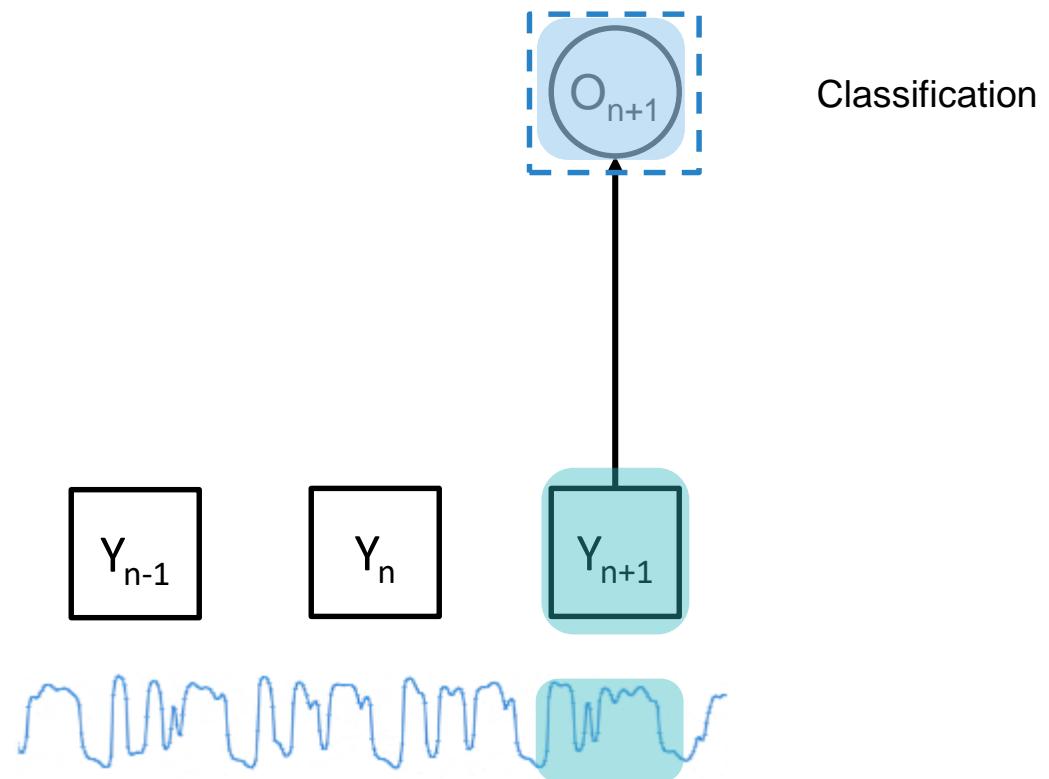
- Seasonal trends



Recurrence Representation using Neural Networks

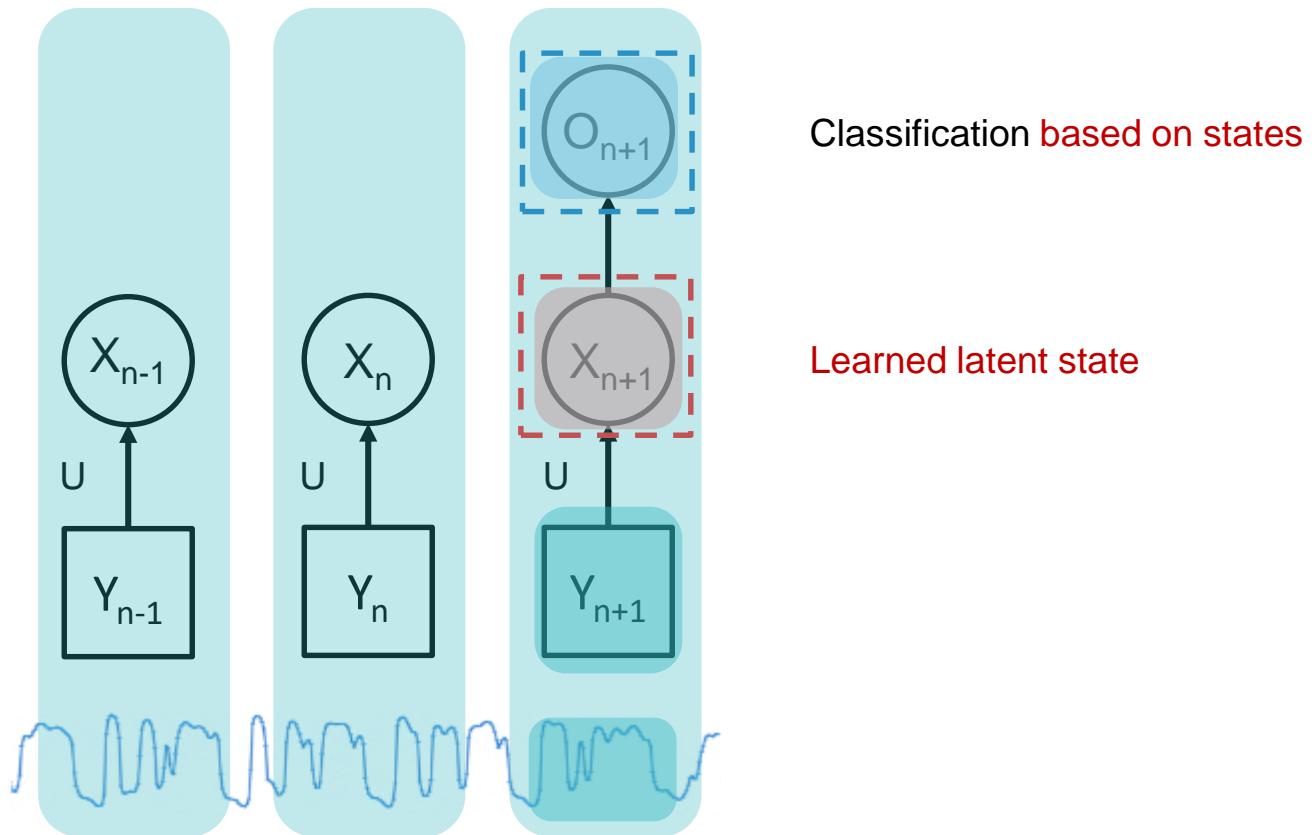
Recurrent NN (RNN)

- Hidden state extraction and transformation



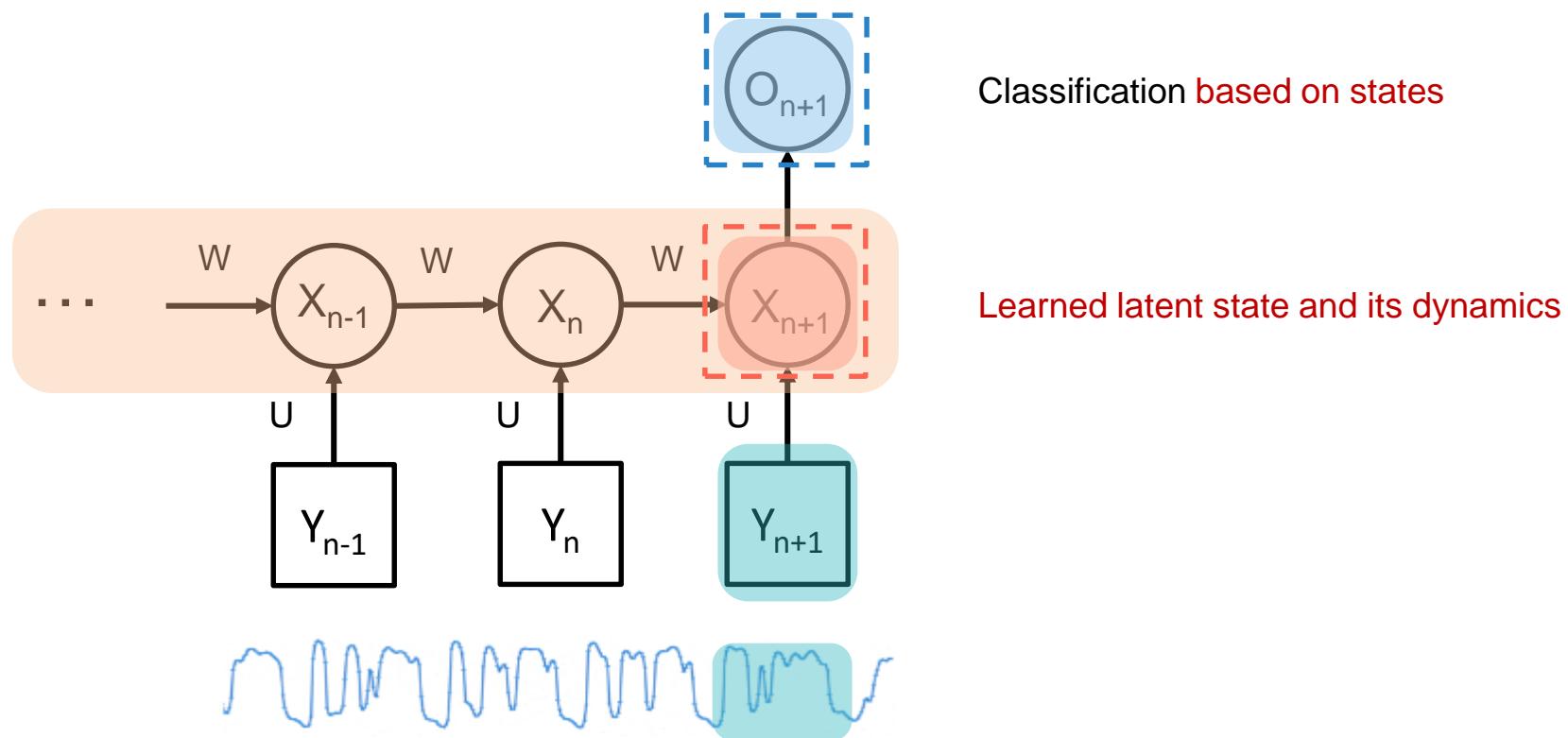
Recurrent NN (RNN)

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Recurrent NN (RNN)

- Hidden state extraction and transformation
- Good for sequential data (dynamic behavior)



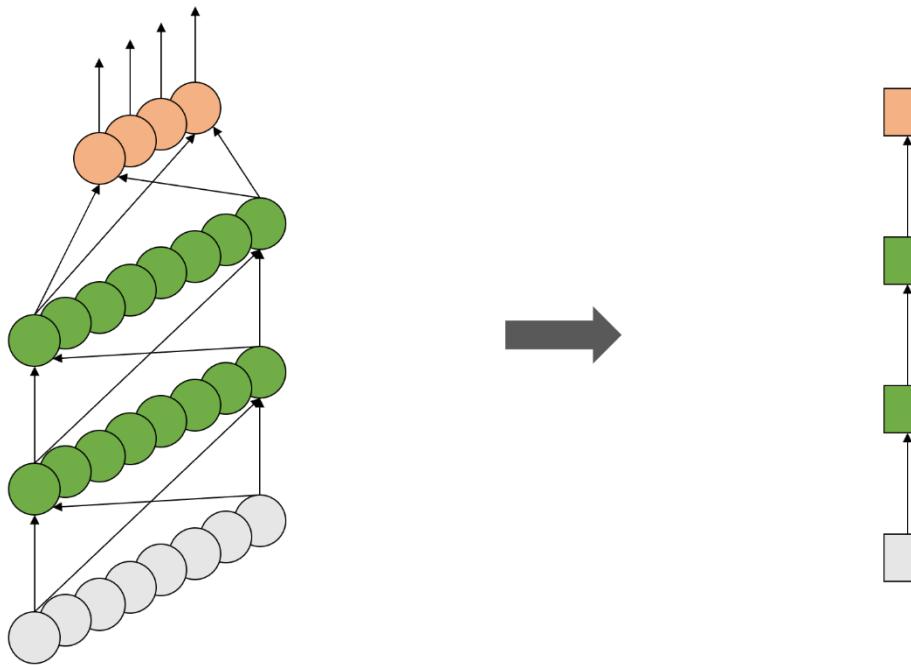
Recurrent NN

- Recurrence
 - Consider the classical form of a dynamical system:

$$s^{(t)} = f(s^{(t-1)}; \theta)$$

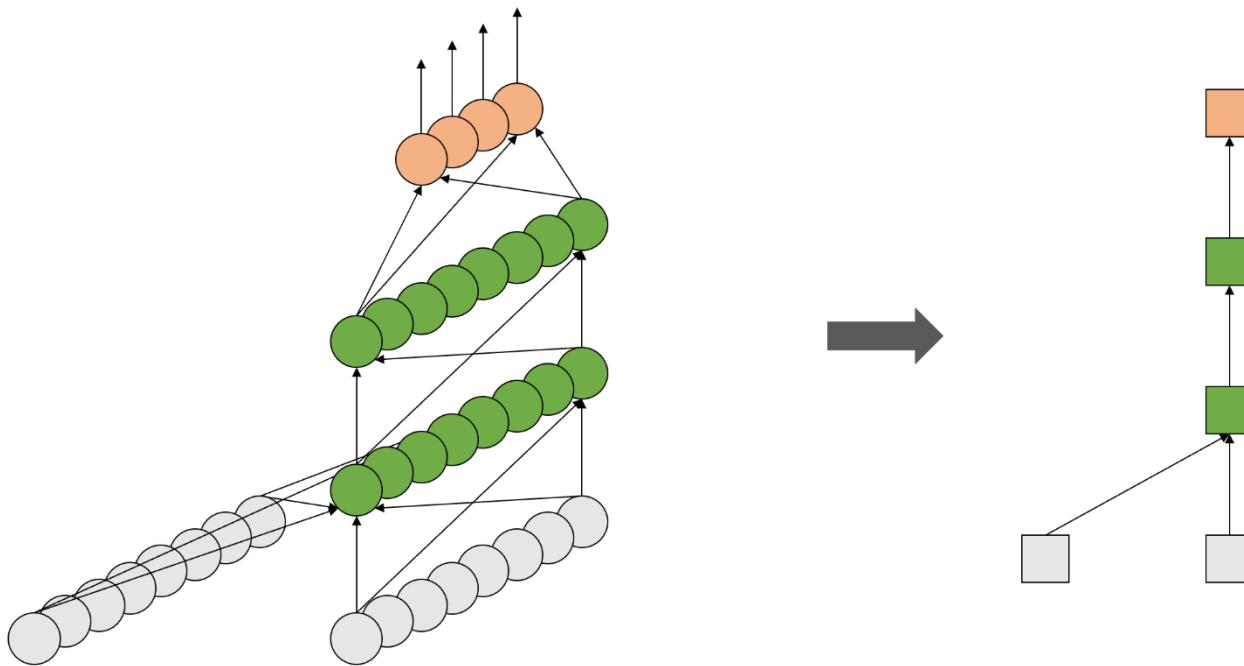
- This is recurrent because the definition of s at time t refers back to the same definition at time $t - 1$
- Hidden state representation
- Learn both from sequential data

Representation Shortcut



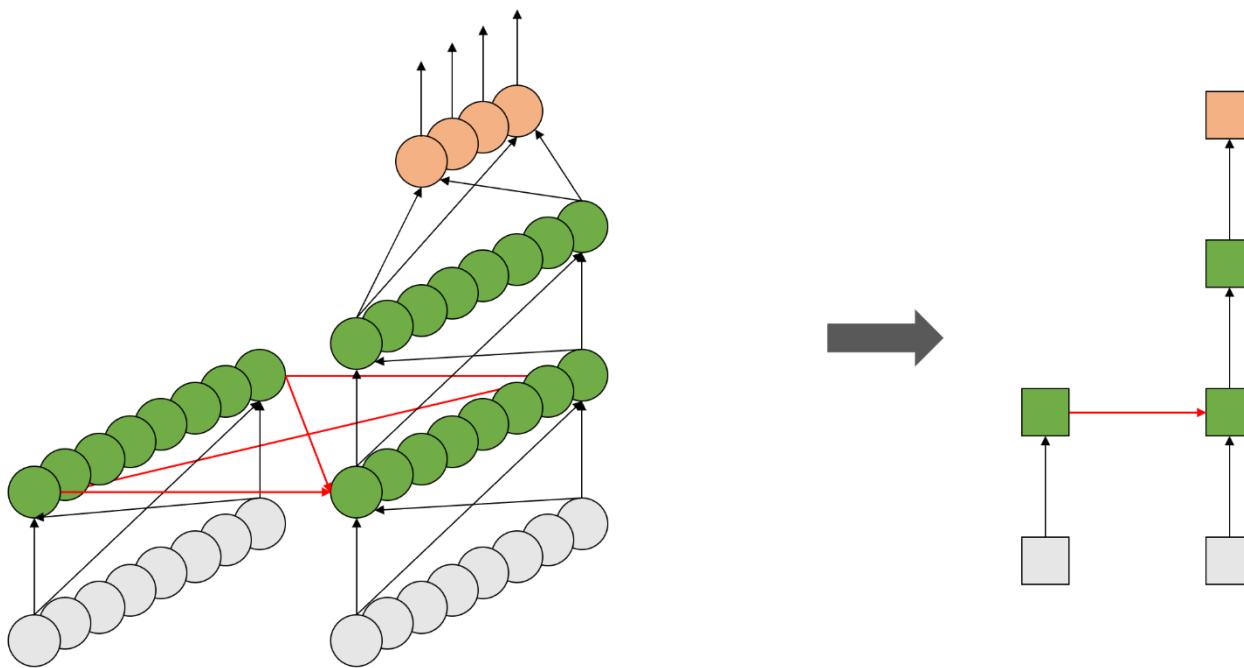
- Input at each time is a vector
- Each layer has many neurons
 - Output layer too may have many neurons
- But will represent everything simple boxes
 - Each box actually represents an entire layer with many units

Representation Shortcut



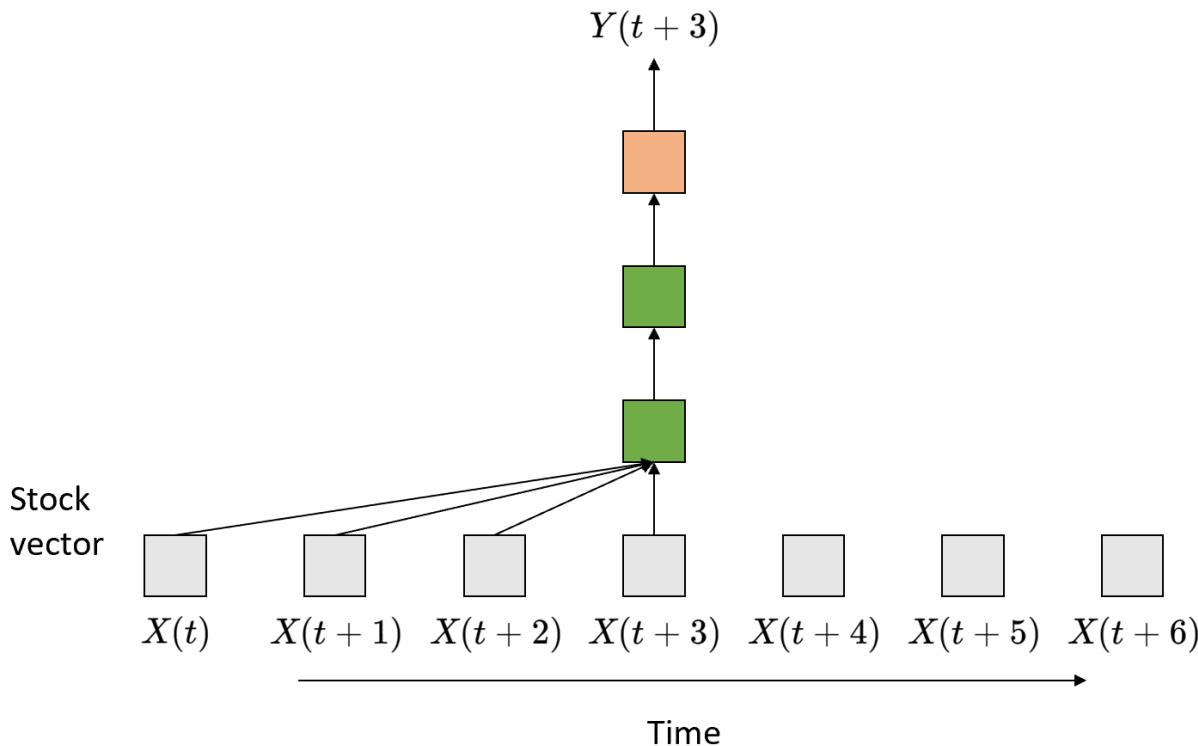
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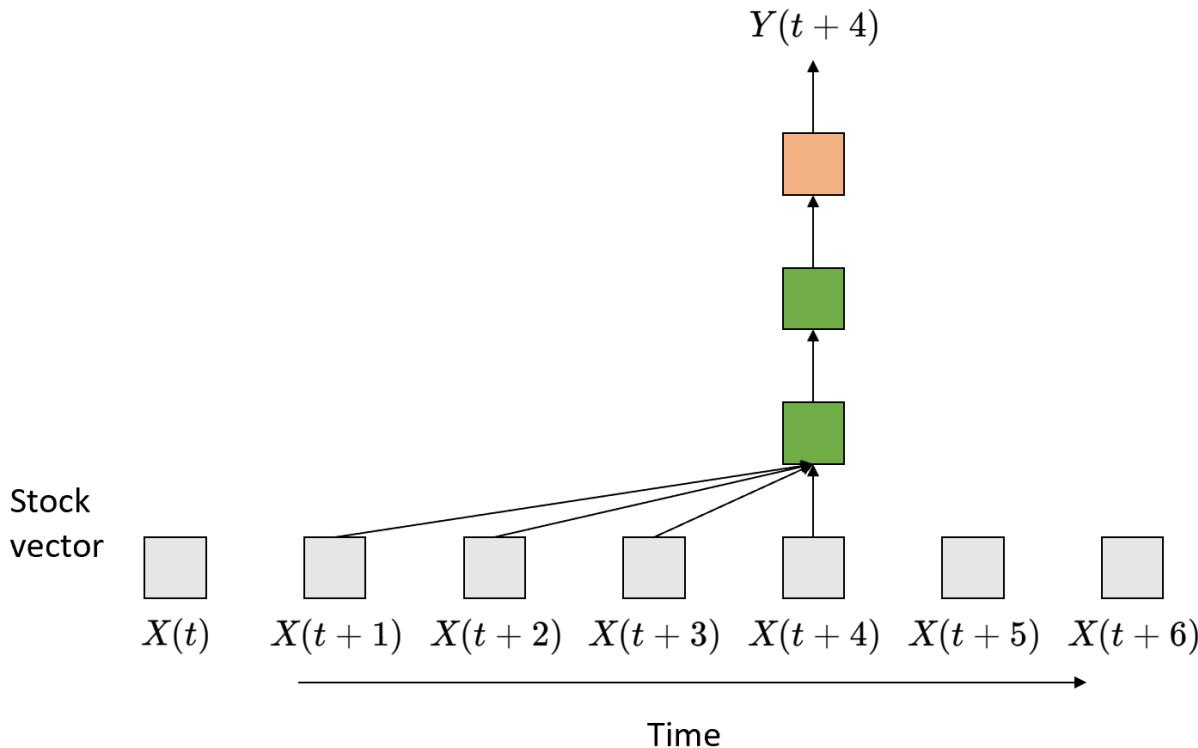
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Sliding Predictor



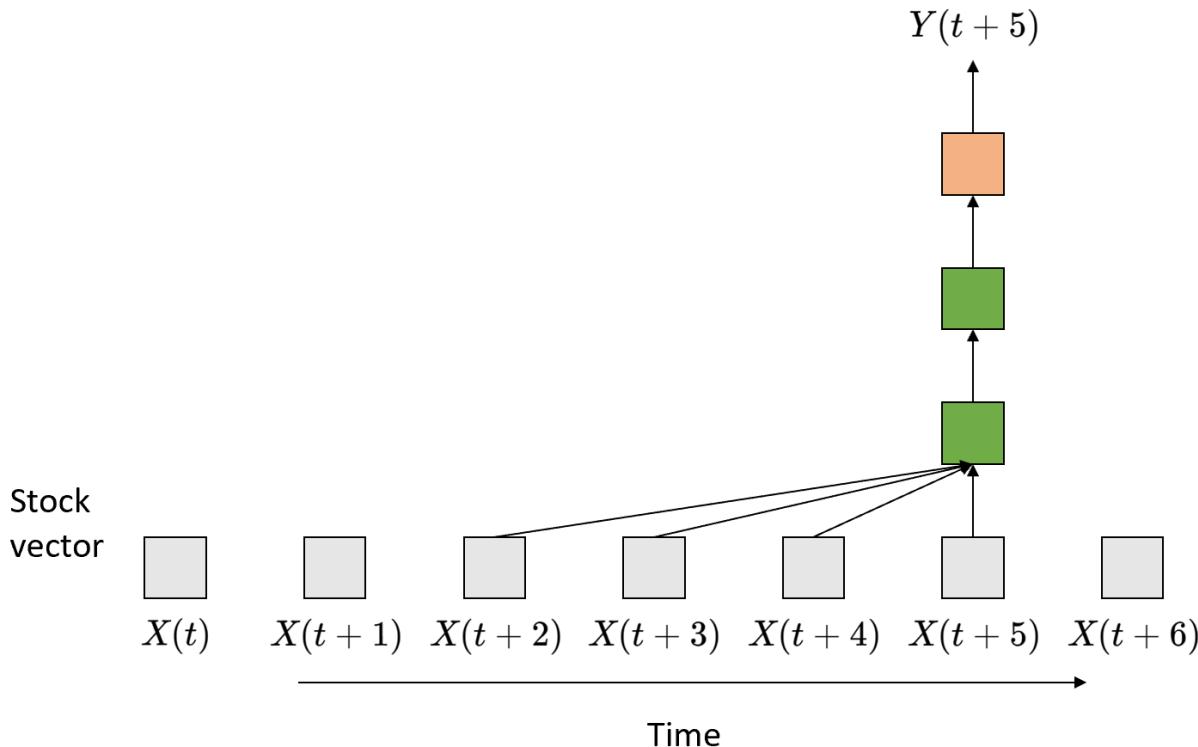
- The sliding predictor
 - Look at the last few days
 - This is just a convolutional neural net applied to sequential data

Sliding Predictor



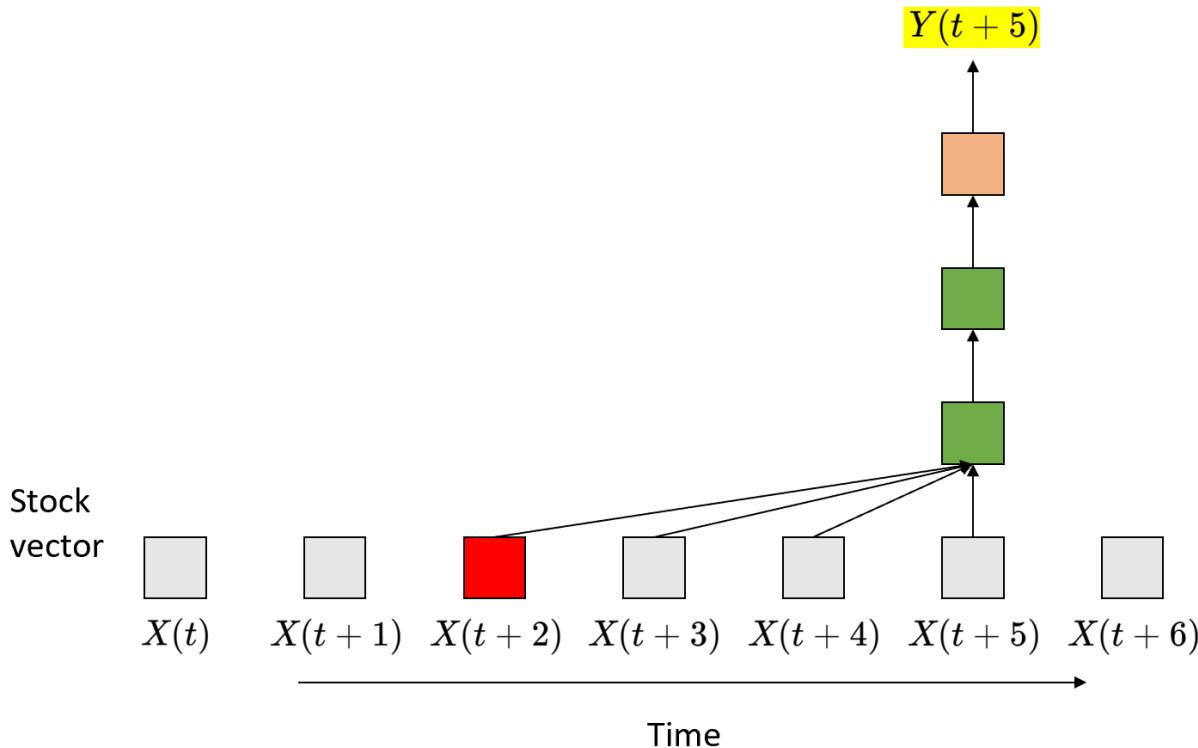
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Sliding Predictor



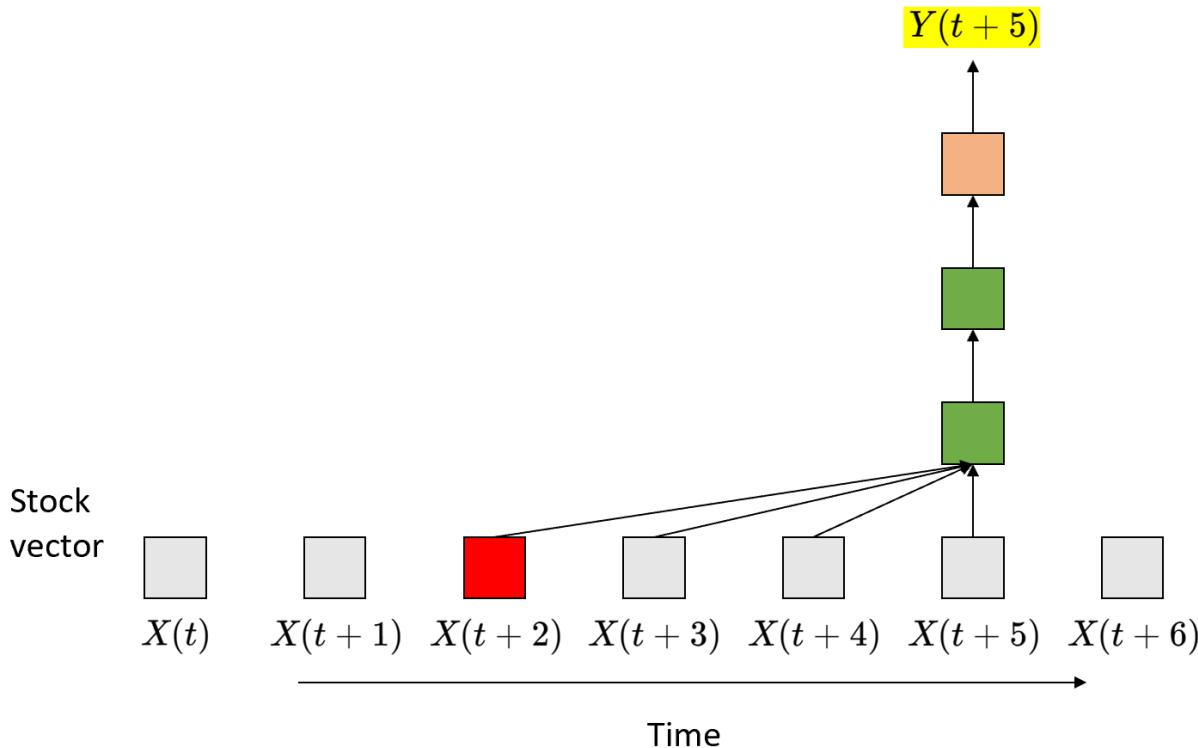
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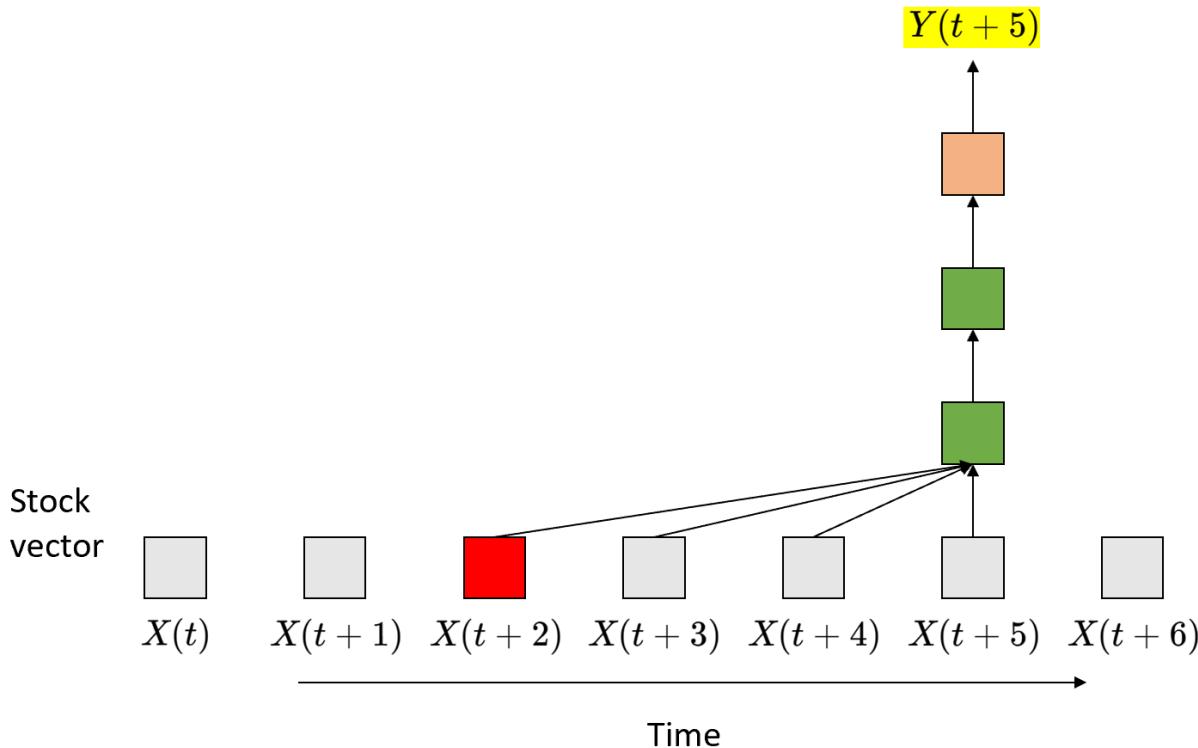
- The sliding predictor
 - Look at the last few days
 - This is just a convolutional neural net applied to sequential data

Finite-Response Model



- This is a finite response system
 - Something that happens today only affects the output of the system for N days into the future
 - N is the width of the system

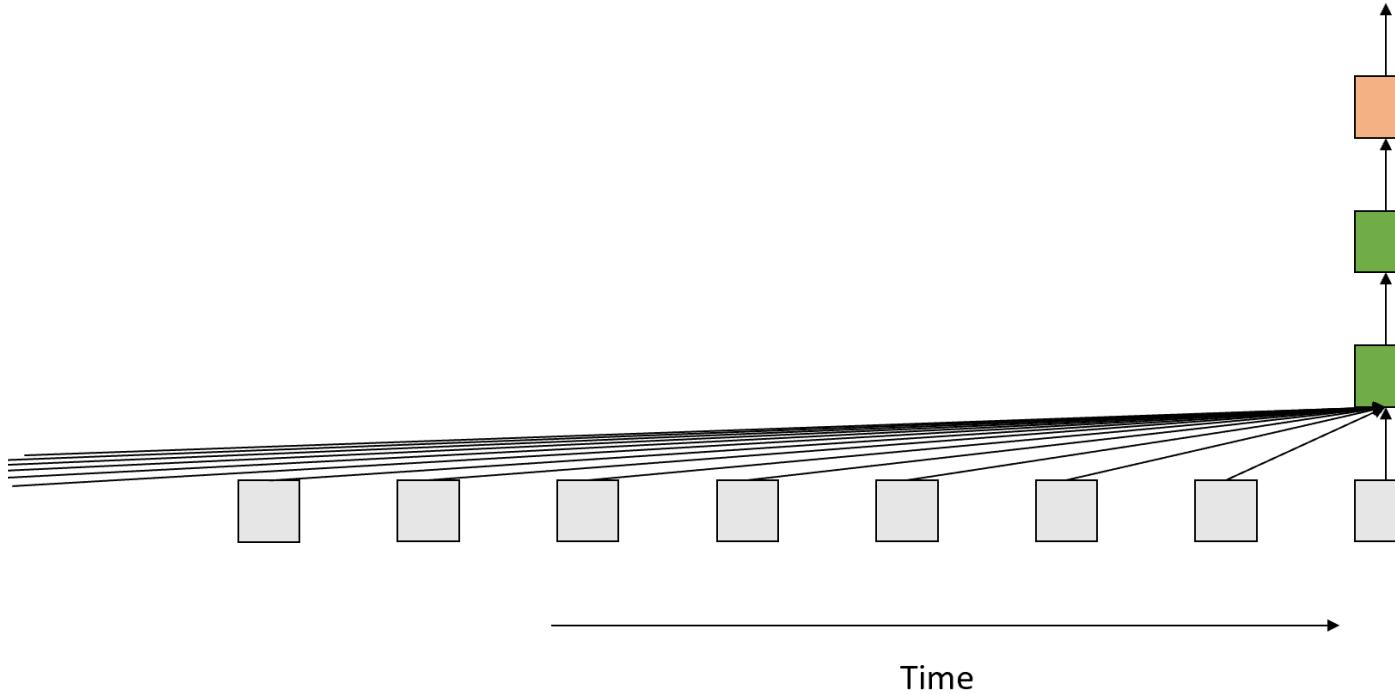
Finite-Response Model



- Problem: Increasing the “history” makes the network more complex

$$Y_t = f(X_t, X_{t-1}, \dots, X_{t-N})$$

In Theory, We Want Infinite Memory



- Required: Infinite response systems
 - What happens today can continue to affect the output forever
 - Possibly with weaker and weaker influence

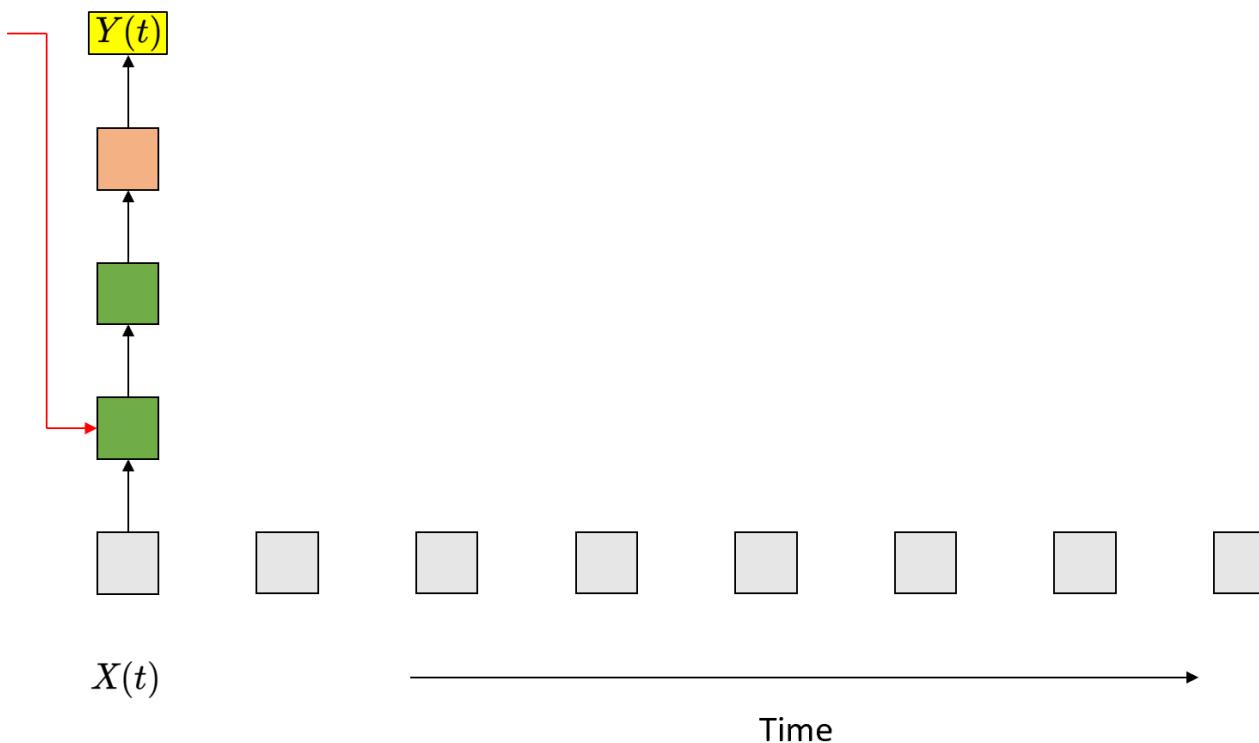
$$Y_t = f(X_t, X_{t-1}, \dots, X_{t-\infty})$$

Infinite Response Systems

$$Y_t = f(X_t, X_{t-1}, \dots, X_{t-\infty}) \implies Y_t = f(X_t, Y_{t-1})$$

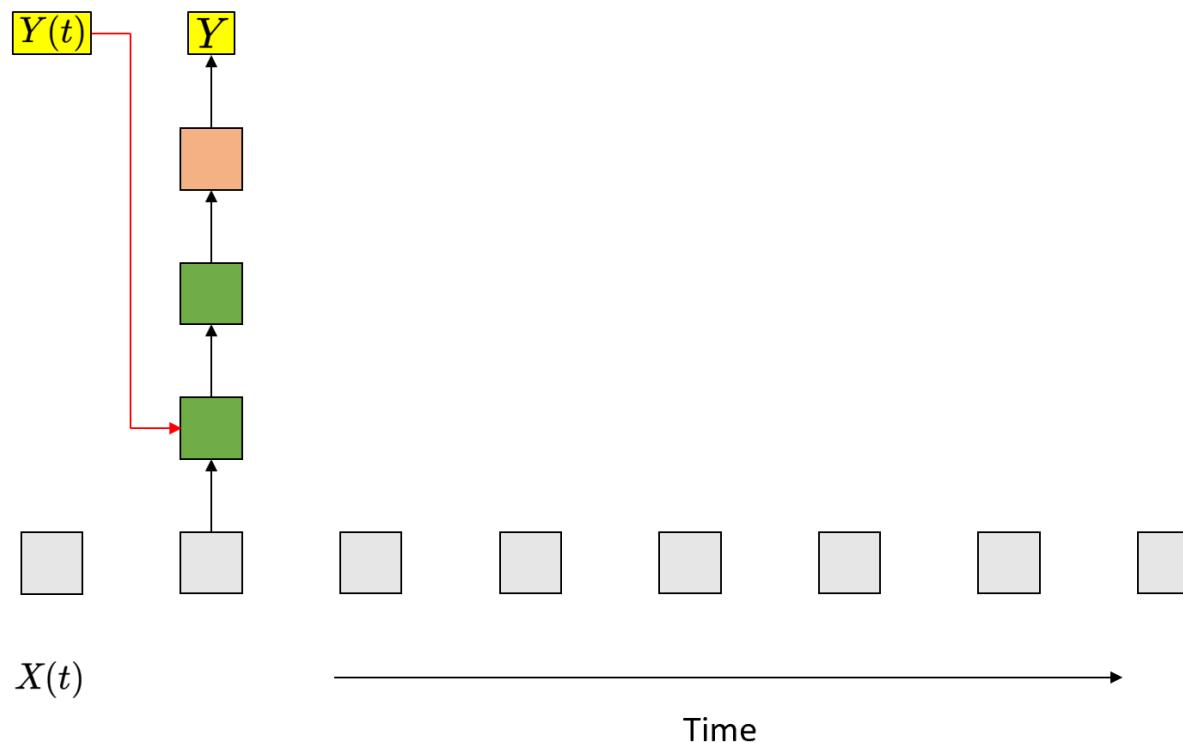
- Recursive
 - Required: Define initial output: Y_{t-1} for $t = 0$
 - An input at X_0 at $t = 0$ produces Y_0
 - Y_0 produces Y_1 which produces Y_2 and so on until Y_∞ even if X_1, \dots, X_∞ are 0
 - Nonlinear autoregressive
- Output contains information about the entire past

Autoregression



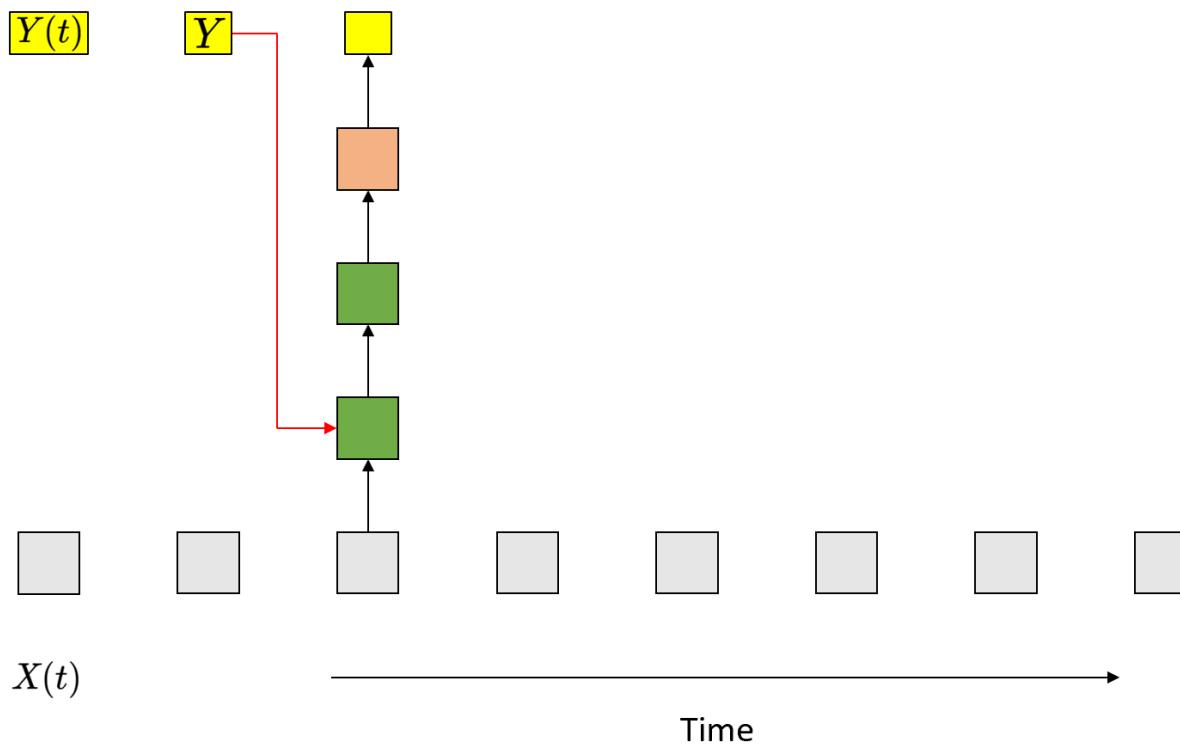
- An autoregressive net with recursion from the output

Autoregression



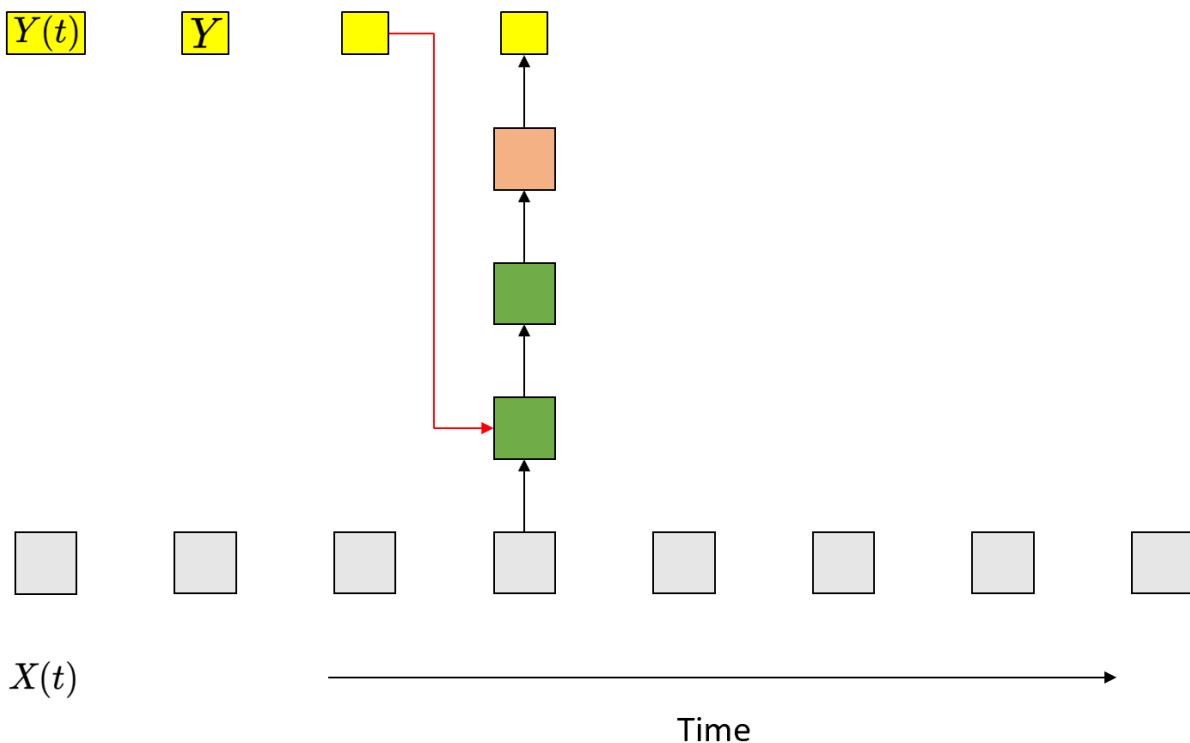
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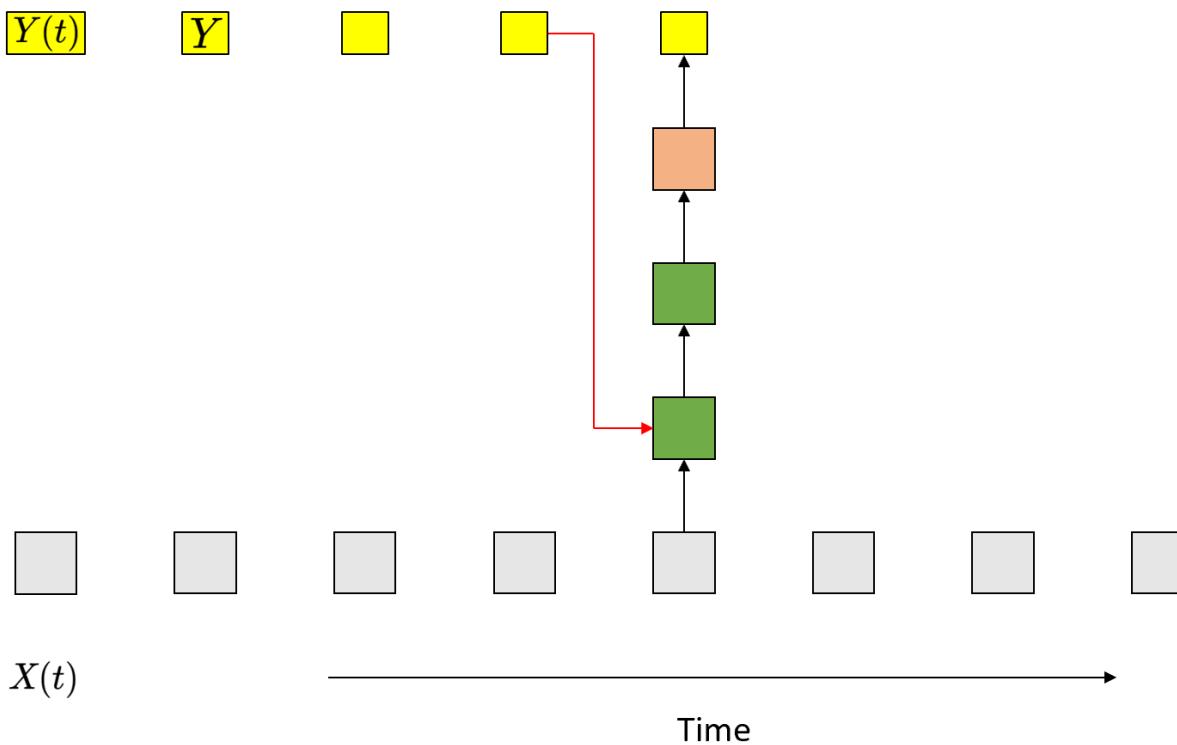
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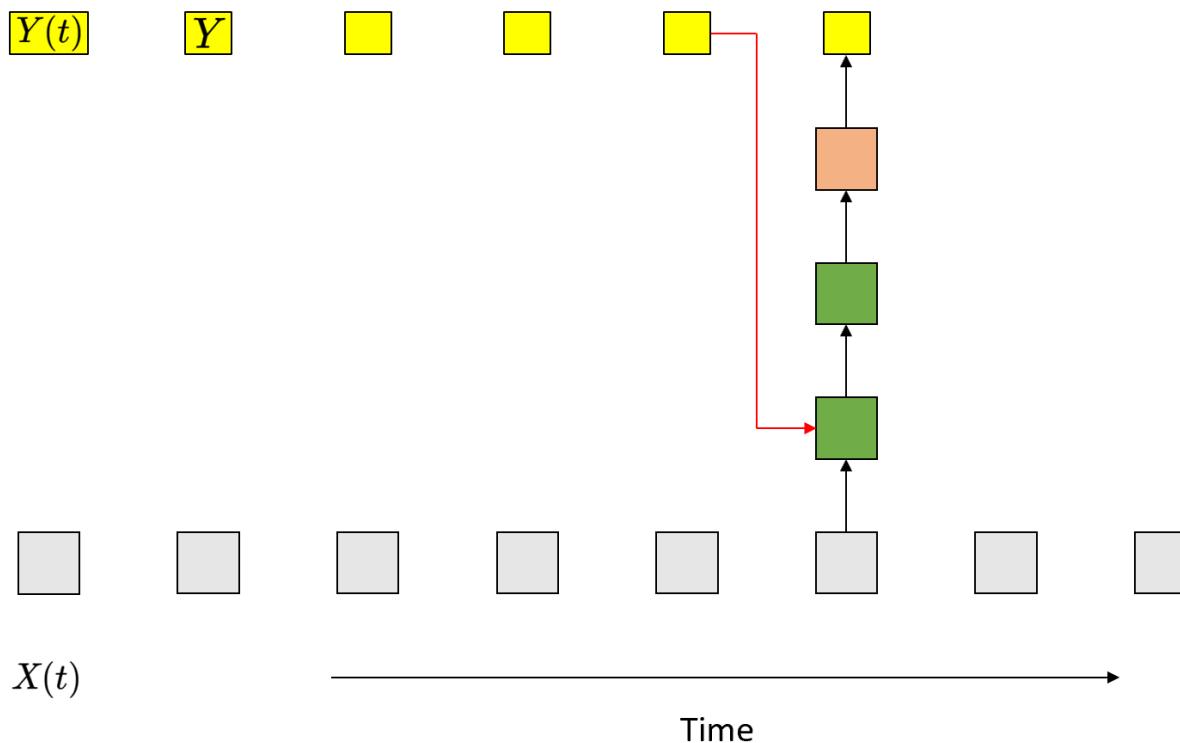
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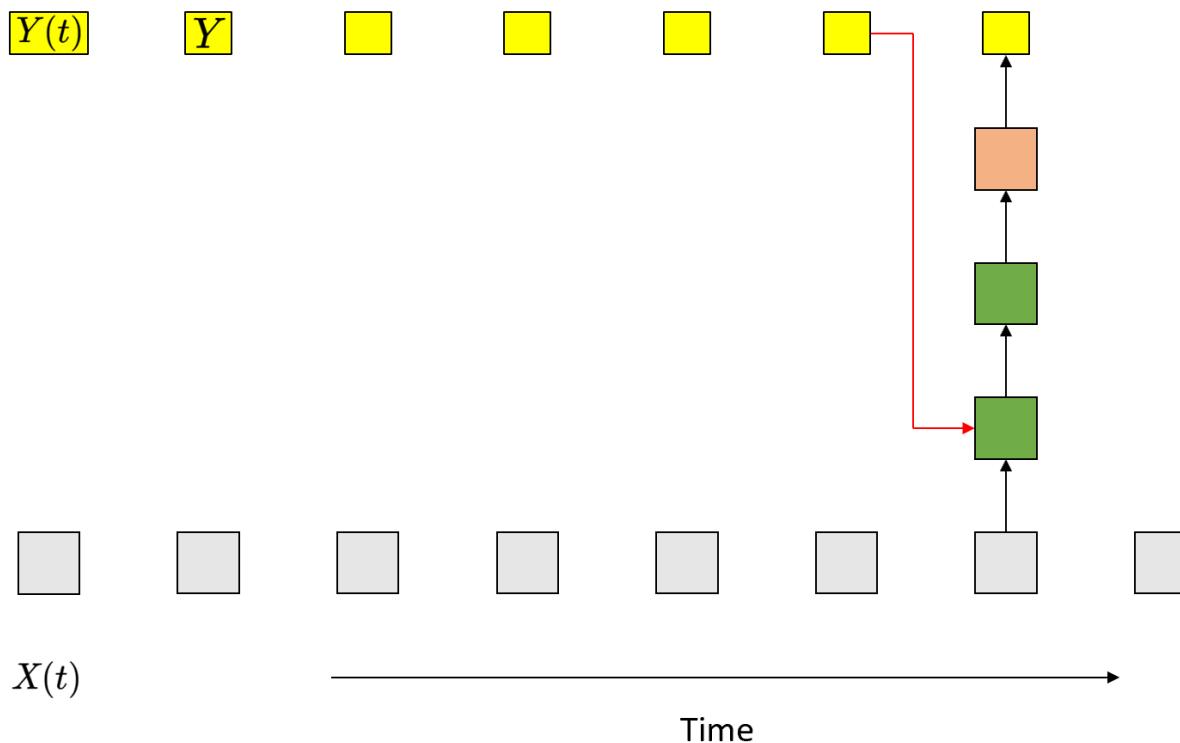
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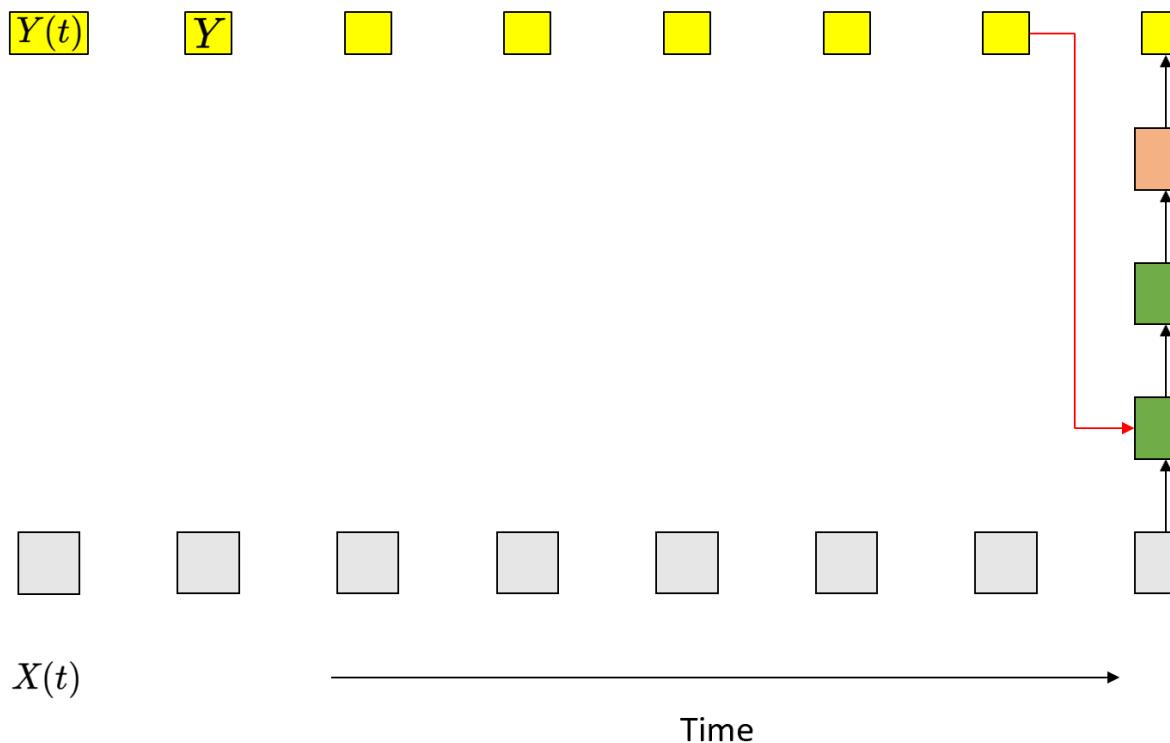
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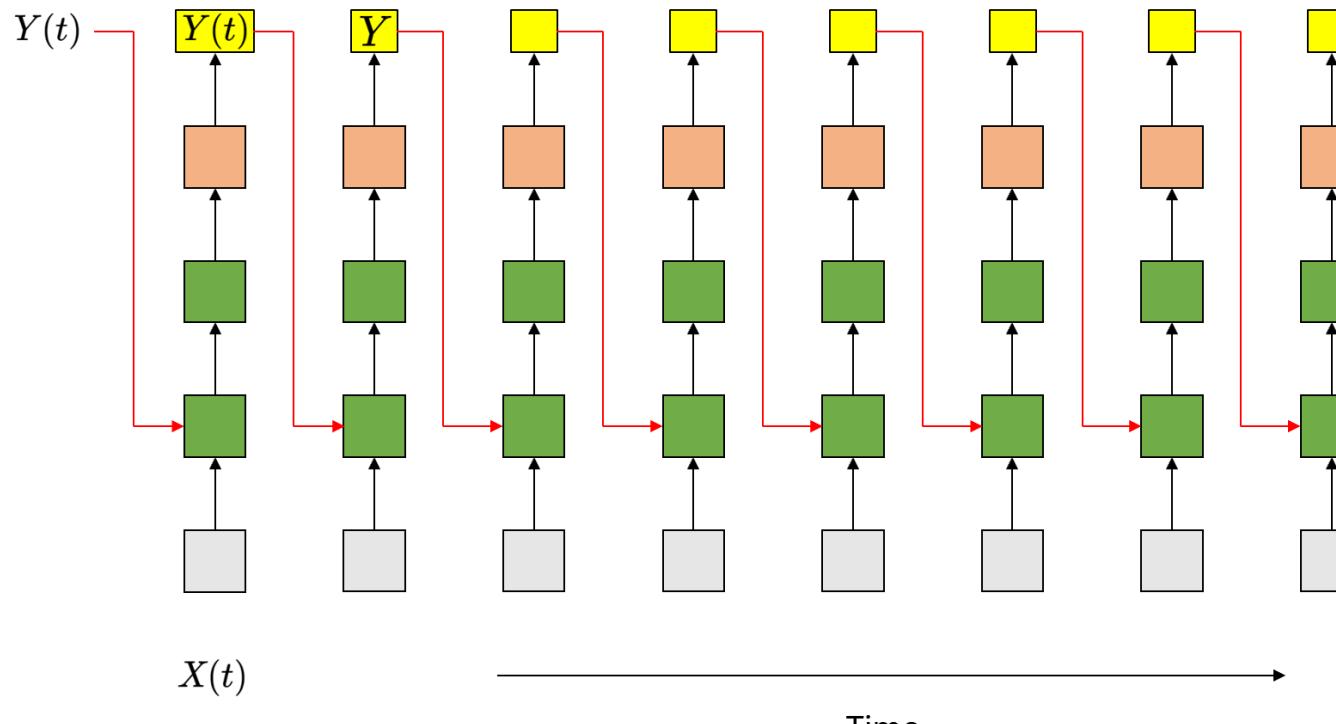
- An autoregressive net with recursion from the output

Autoregression



- An autoregressive net with recursion from the output

More Complete Representation



- An autoregressive net with recursion from the output
- Showing all computations
- All columns are identical
- An input at $t = 0$ affects outputs forever

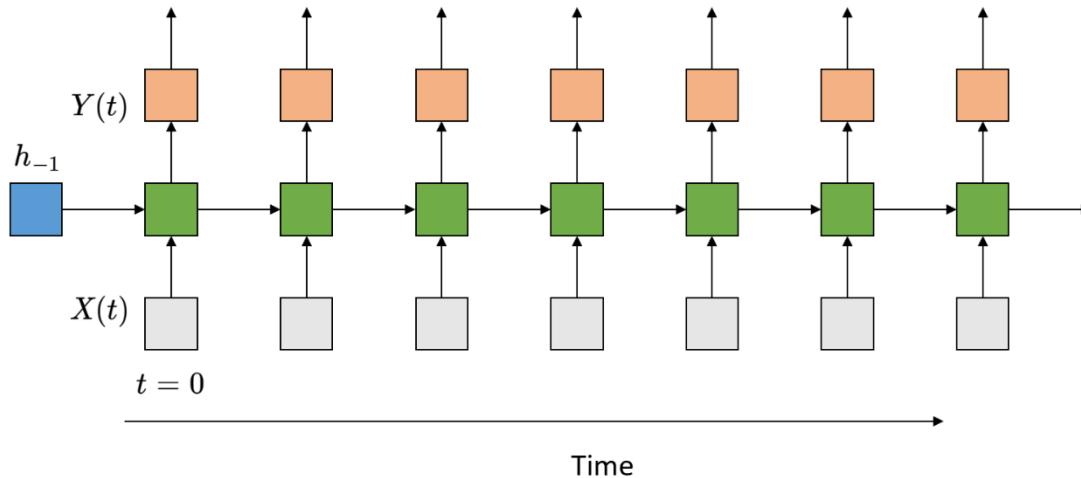
An Alternate Model for Infinite Response Systems

- the state-space model

$$h_t = f(x_t, h_{t-1})$$
$$y_t = g(h_t)$$

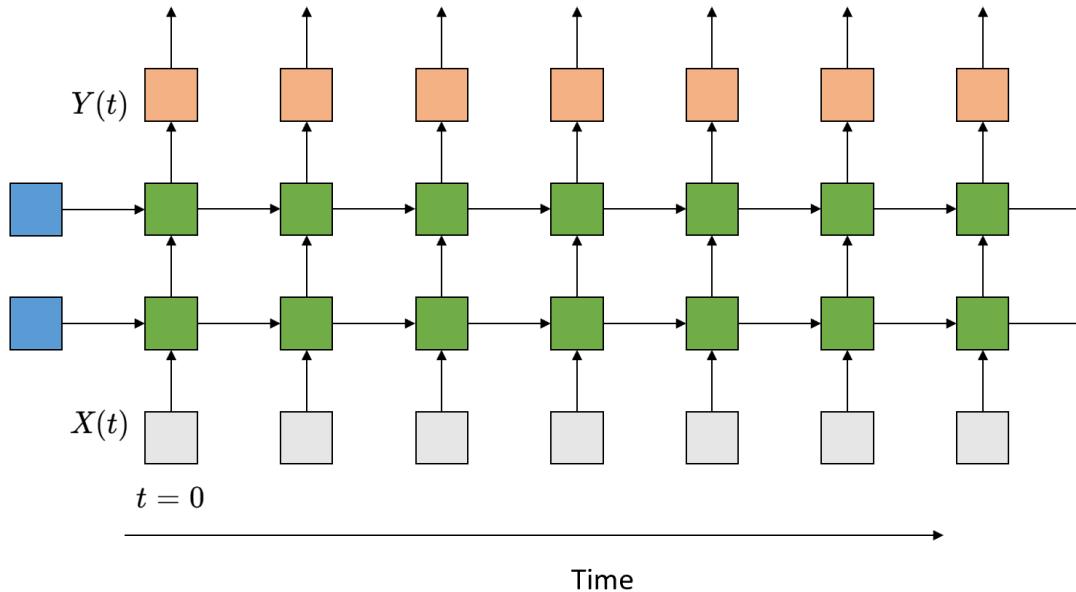
- h_t is the state of the network
- Need to define initial state h_{-1}
- This is a recurrent neural network
- State summarizes information about the entire past

Single Hidden Layer RNN (Simplest State-Space Model)



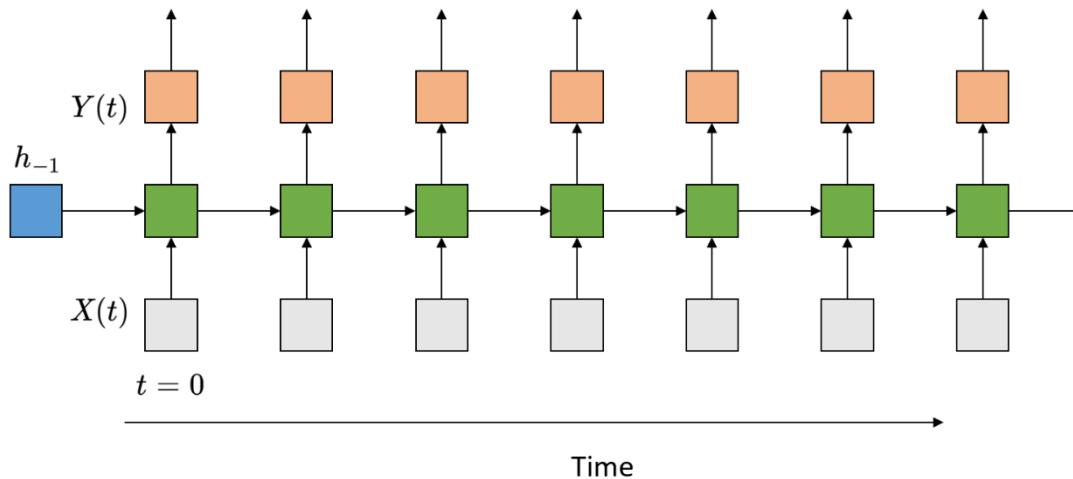
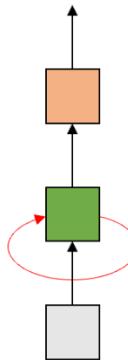
- The state (green) at any time is determined by the input at that time, and the state at the previous time
- All columns are identical
- An input at $t = 0$ affects outputs forever
- Also known as a recurrent neural net

Multiple Recurrent Layer RNN

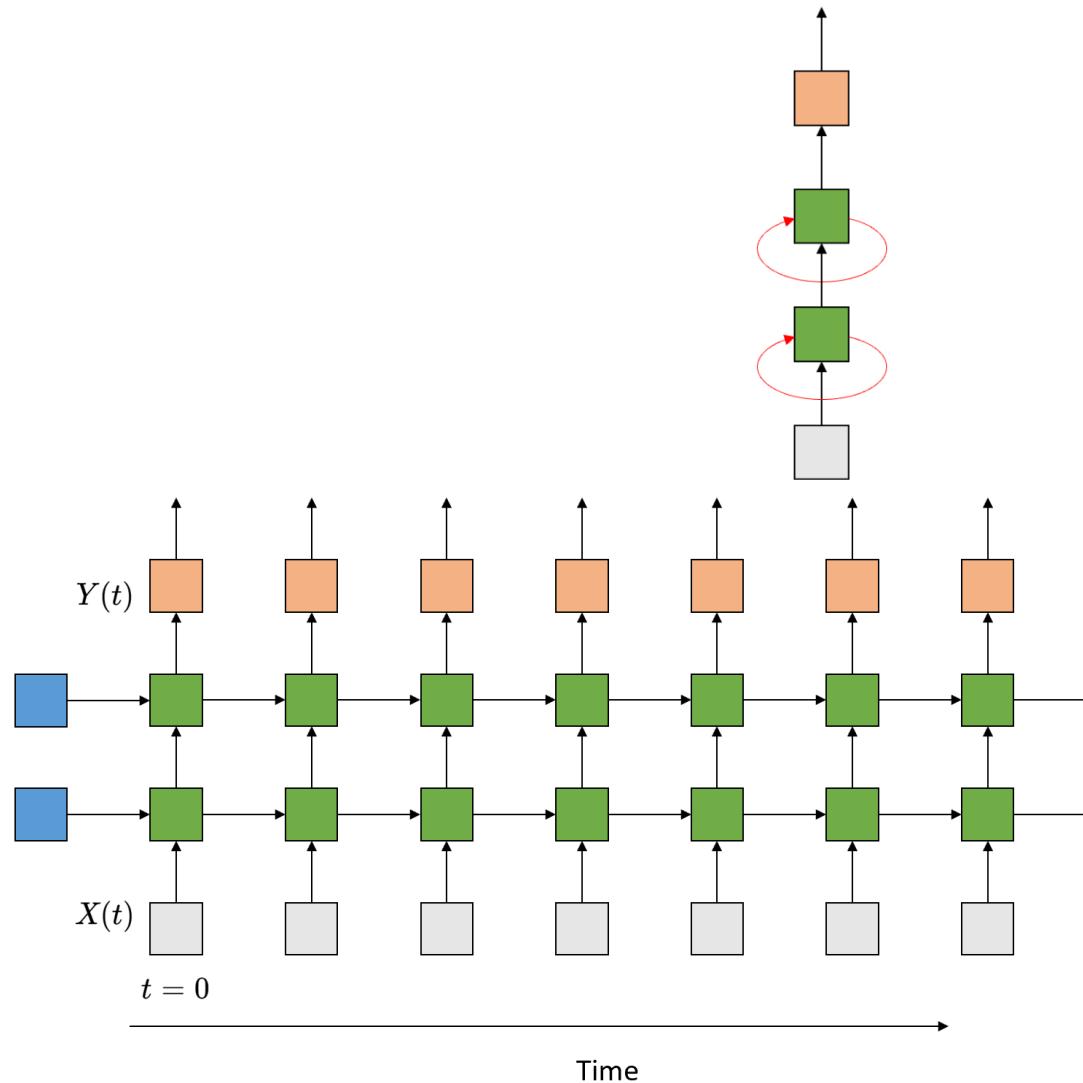


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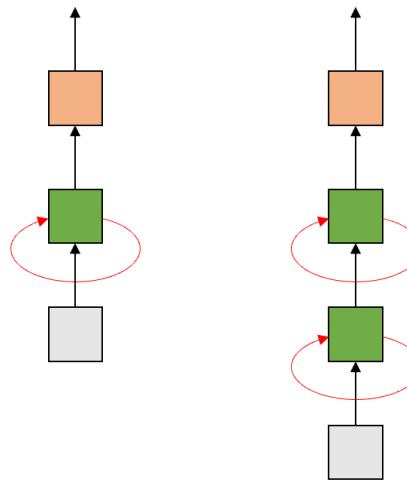
The Folded Version of RNN



The Folded Version of RNN



Recurrent Neural Network



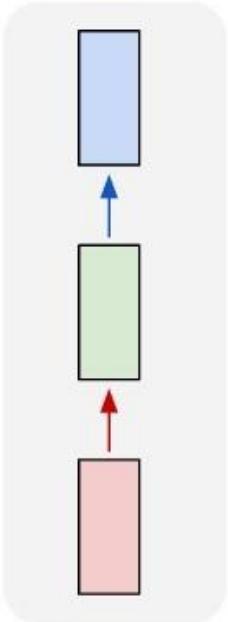
- Simplified models often drawn
- The loops imply recurrence

RNN Applications

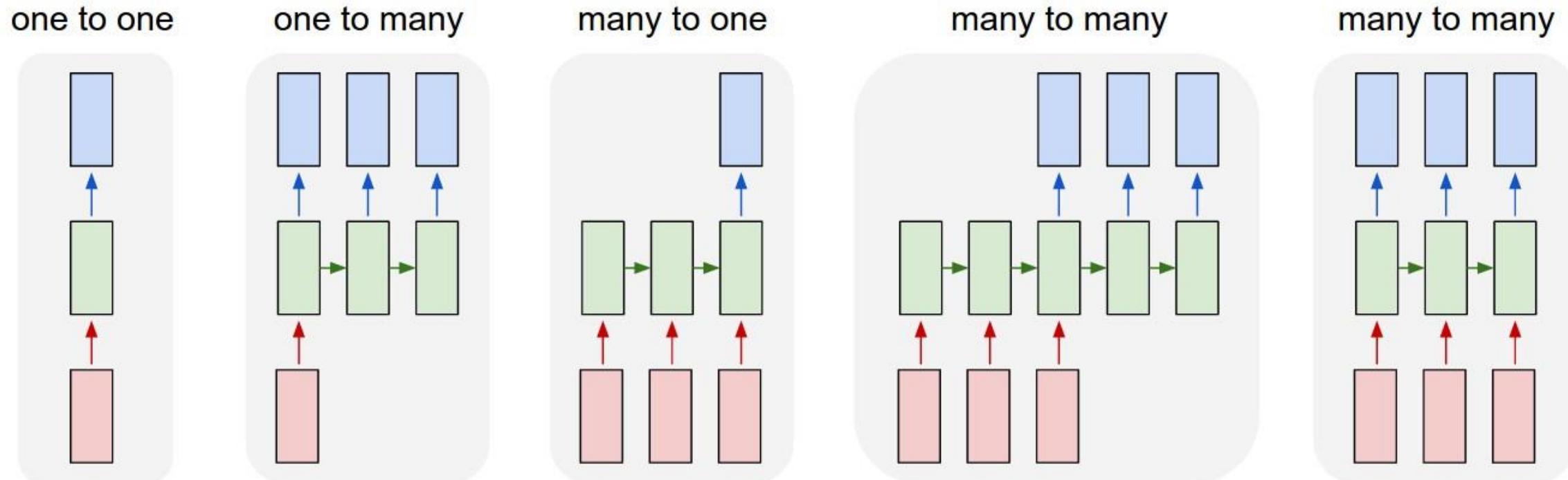
- Machine translation
- Speech recognition
- Text-to-speech
- Image captioning
- Video analysis/understanding

“Vanilla” Neural Network

one to one

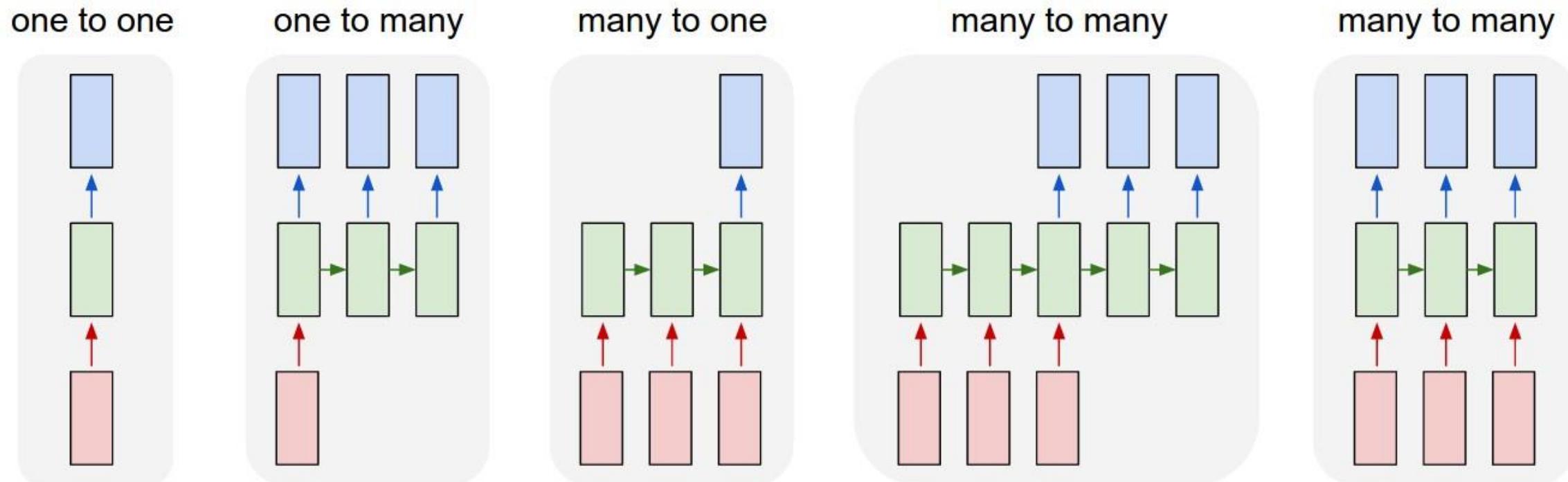


Recurrent Neural Network: Process Sequences



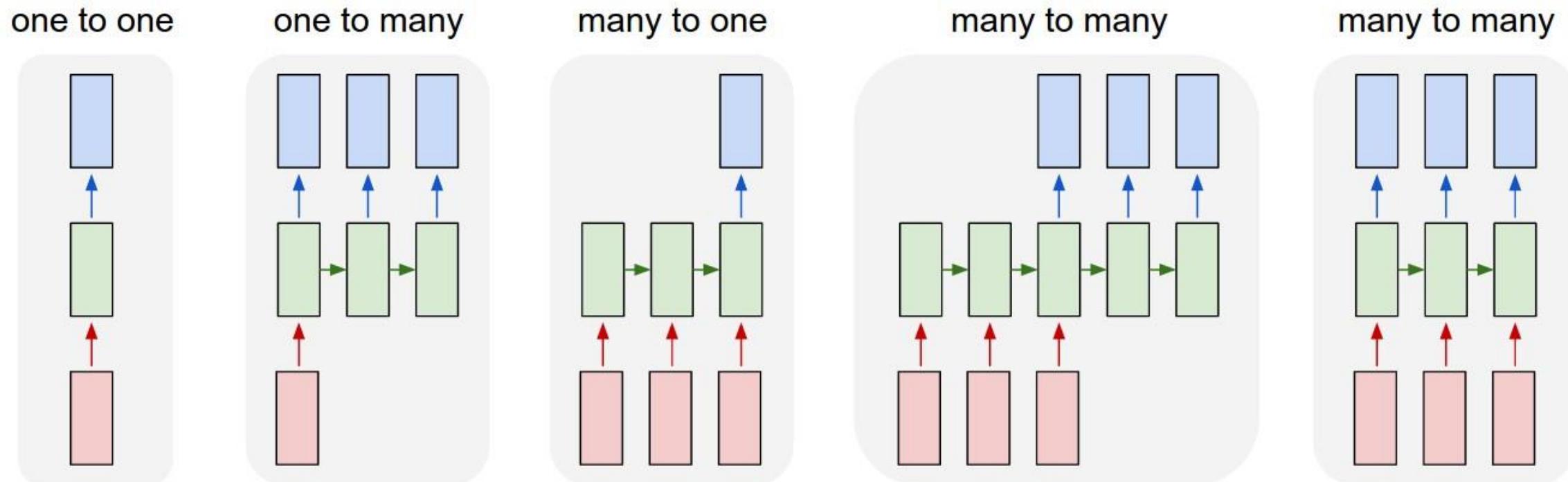
e.g. Image Captioning
image → sequence of words

Recurrent Neural Network: Process Sequences



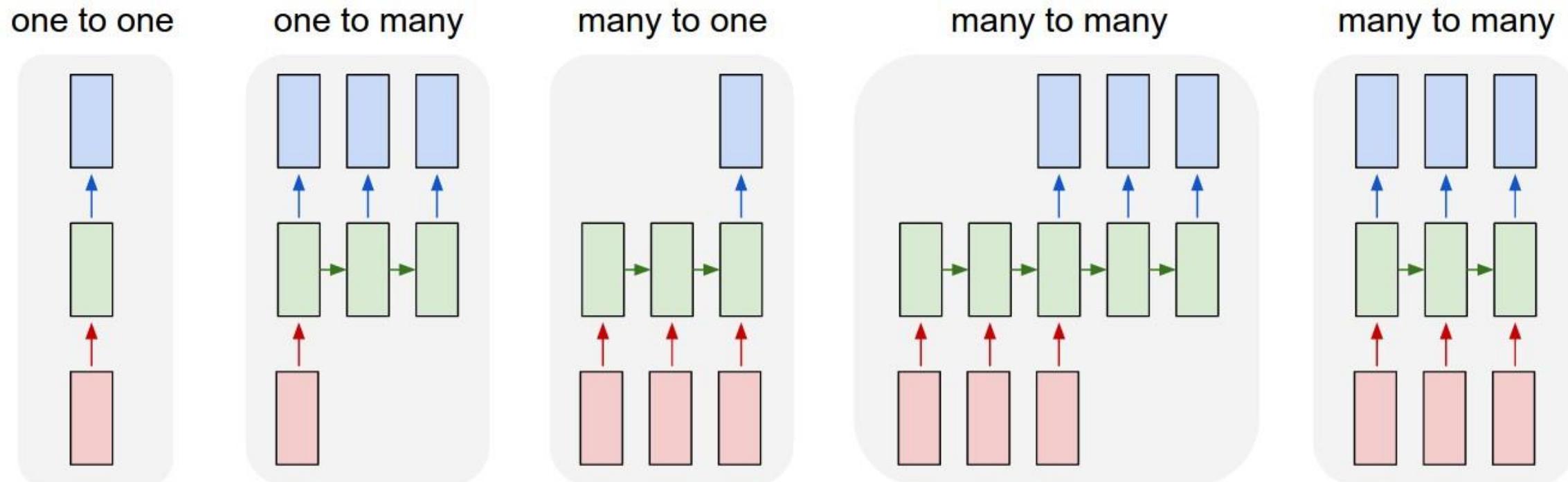
e.g. Sentiment Classification
sequence of words → sentiment

Recurrent Neural Network: Process Sequences



e.g. Machine Translation
Seq. of words → seq. of words

Recurrent Neural Network: Process Sequences



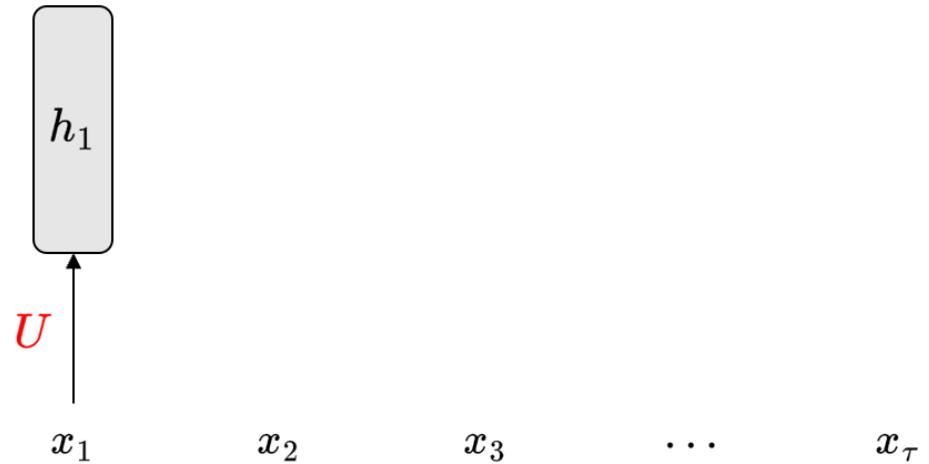
e.g. Video classification on frame level

Recurrent Neural Networks

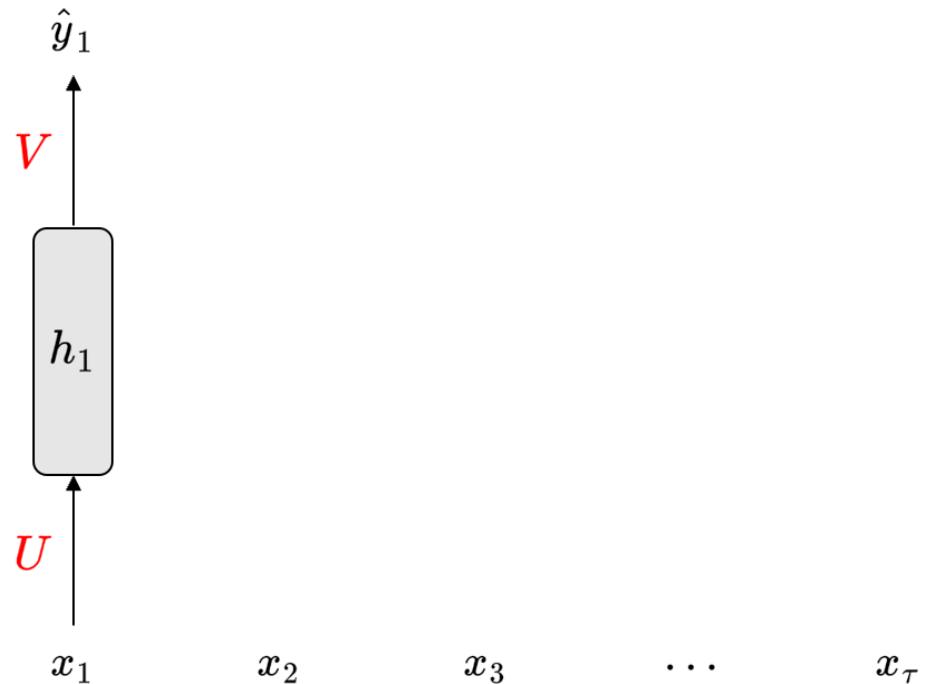
Unrolling the Recurrence

$x_1 \quad x_2 \quad x_3 \quad \dots \quad x_\tau$

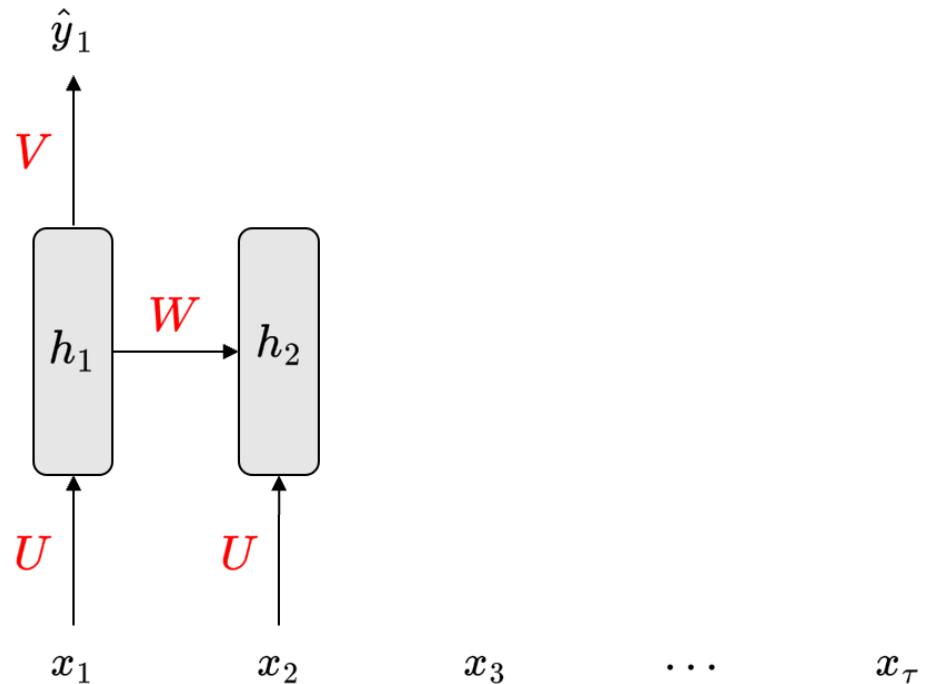
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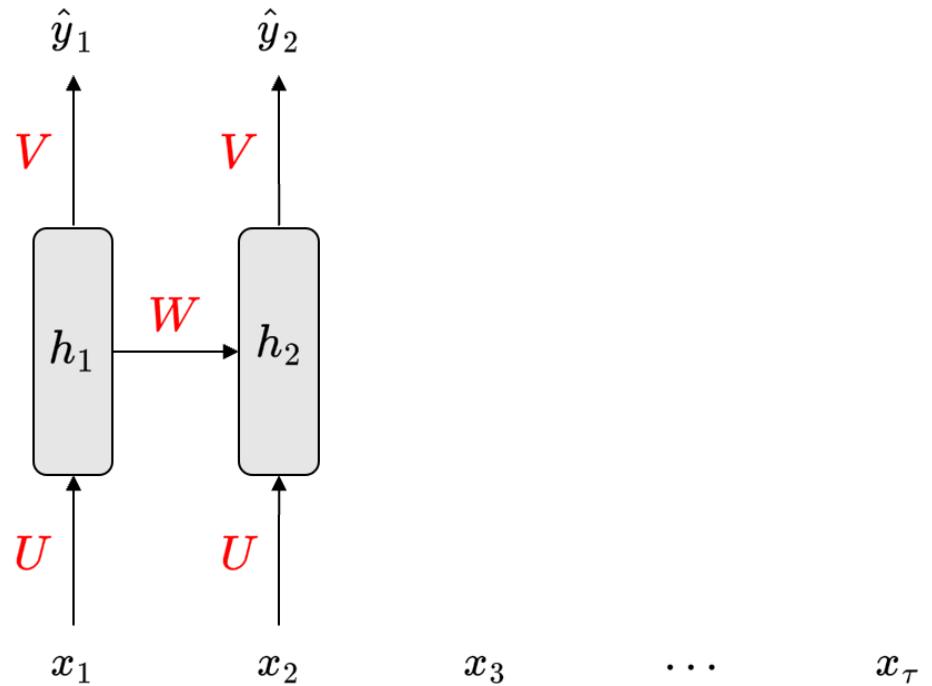
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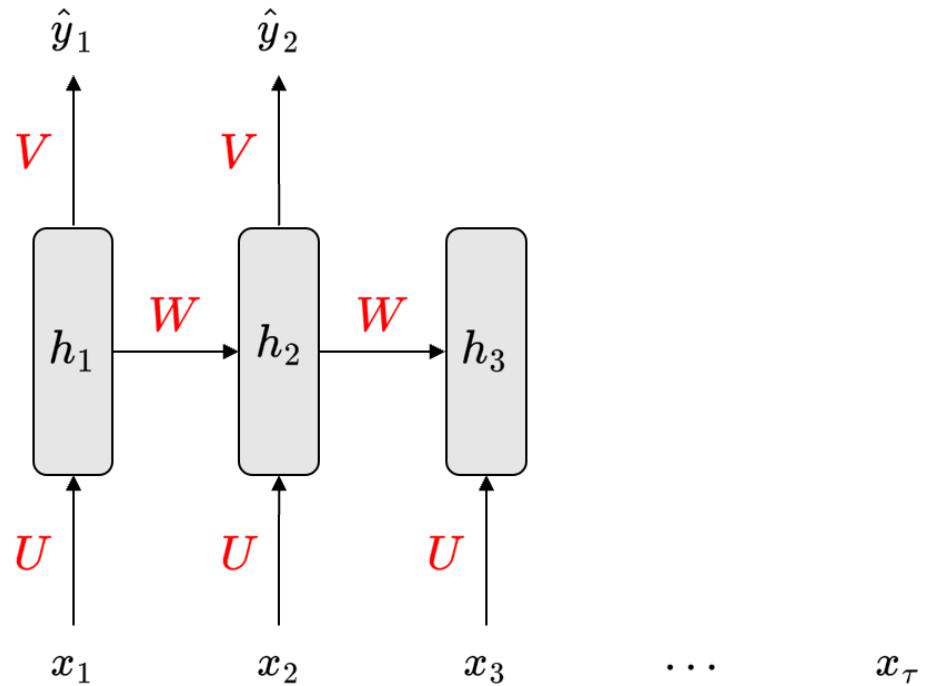
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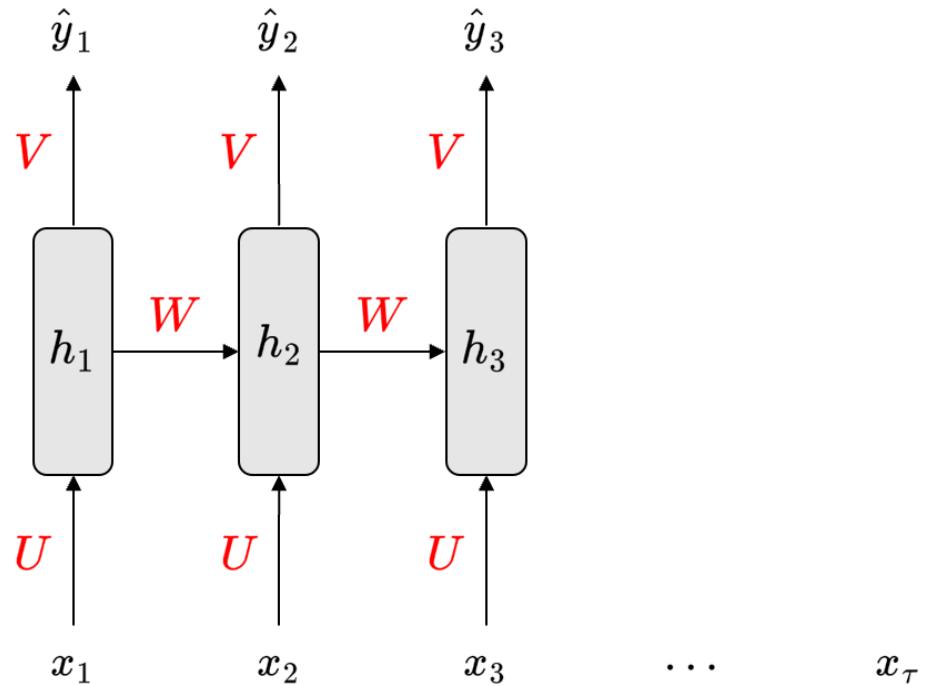
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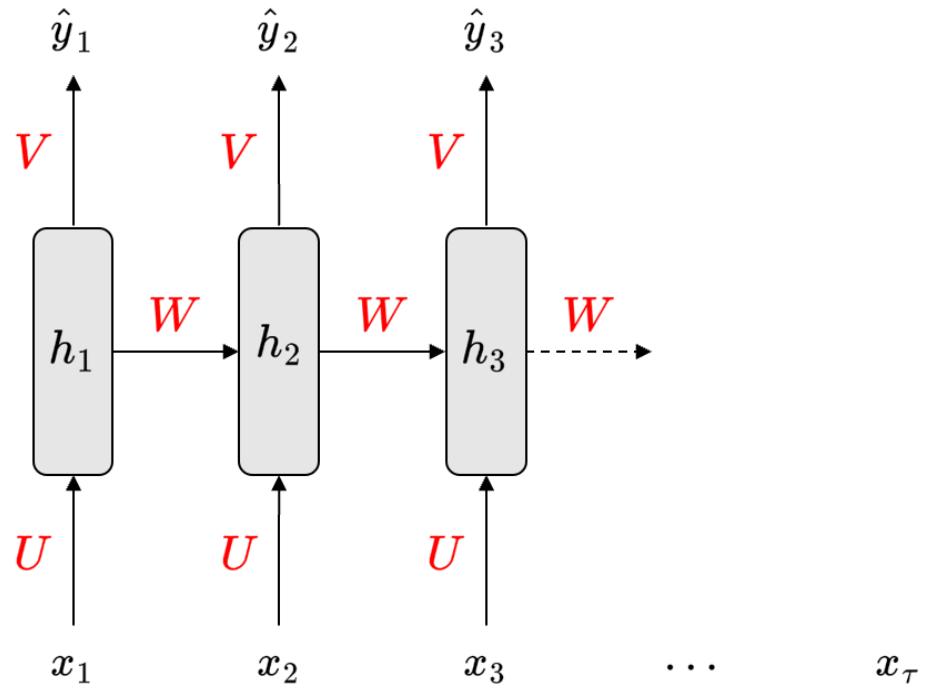
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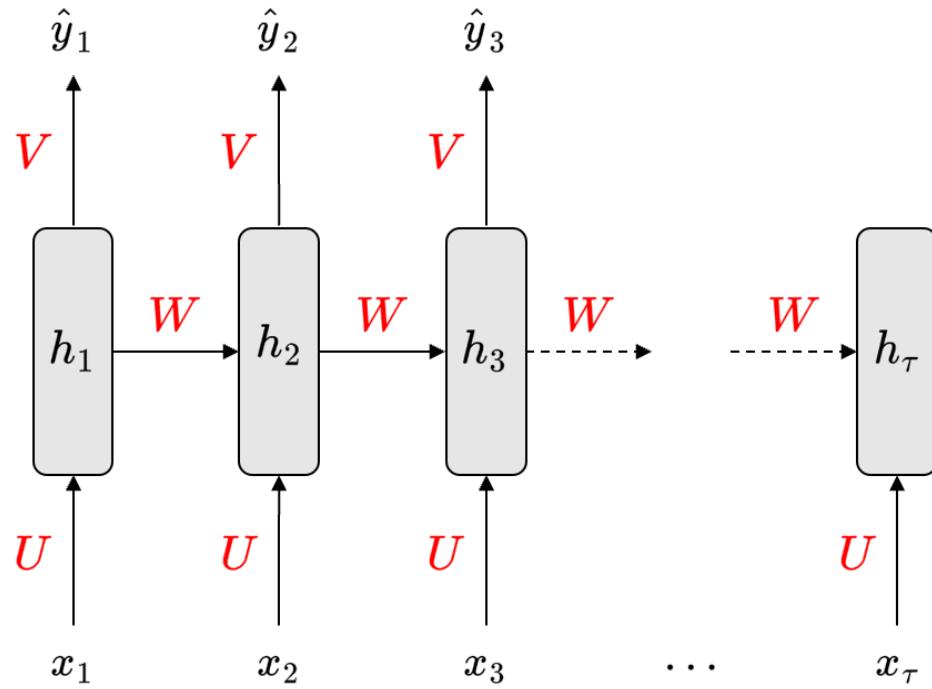
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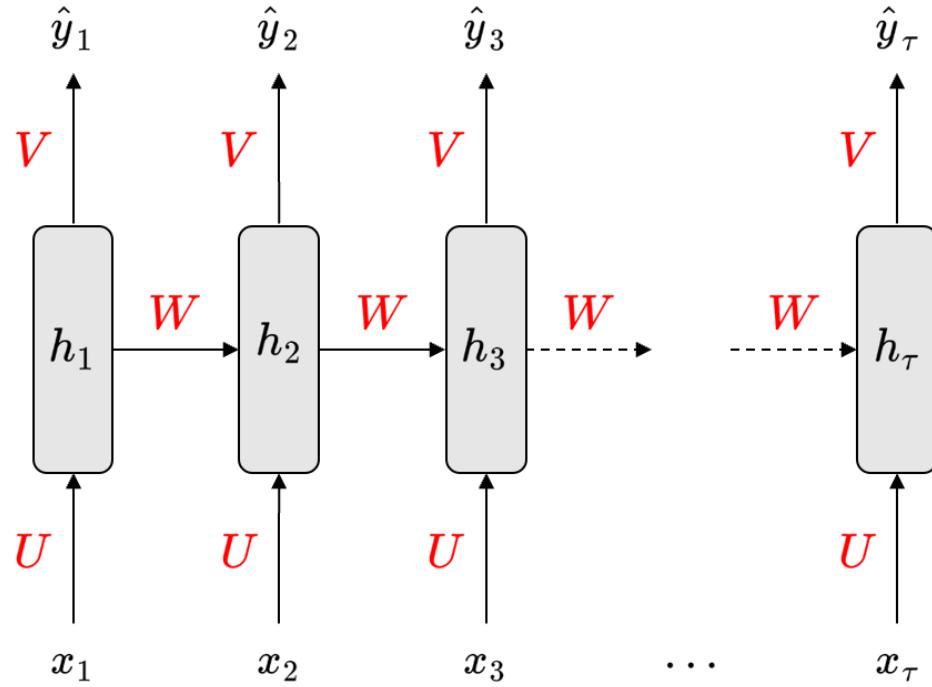
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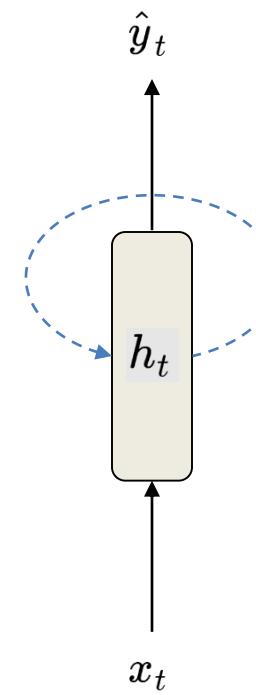
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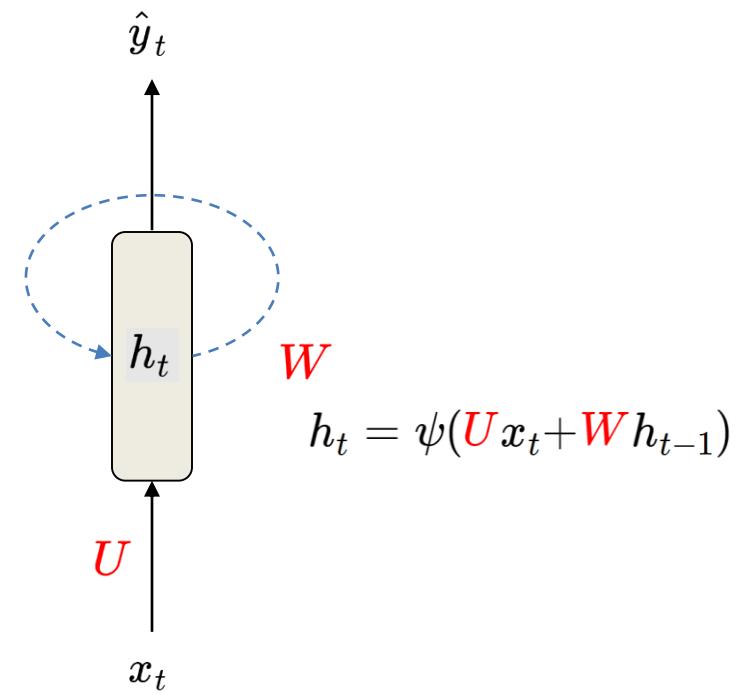
Unrolling the Recurrence



Recurrent Connections



Recurrent Connections



Feedforward Propagation

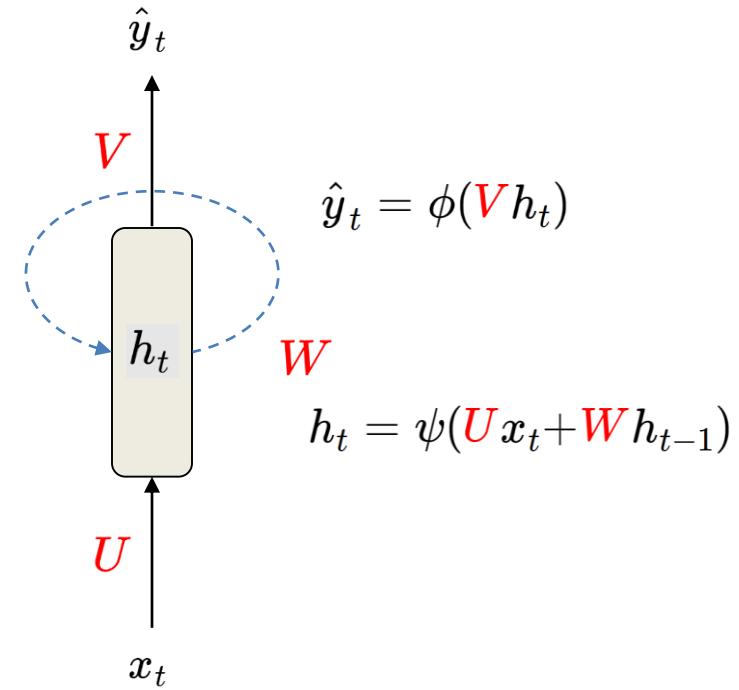
- This is a RNN where the input and output sequences are of the same length
- Feedforward operation proceeds from left to right
- Update Equations:

$$\mathbf{a}_t = b + W\mathbf{h}_{t-1} + U\mathbf{x}_t$$

$$\mathbf{h}_t = \tanh \mathbf{a}_t$$

$$\mathbf{o}_t = c + V\mathbf{h}_t$$

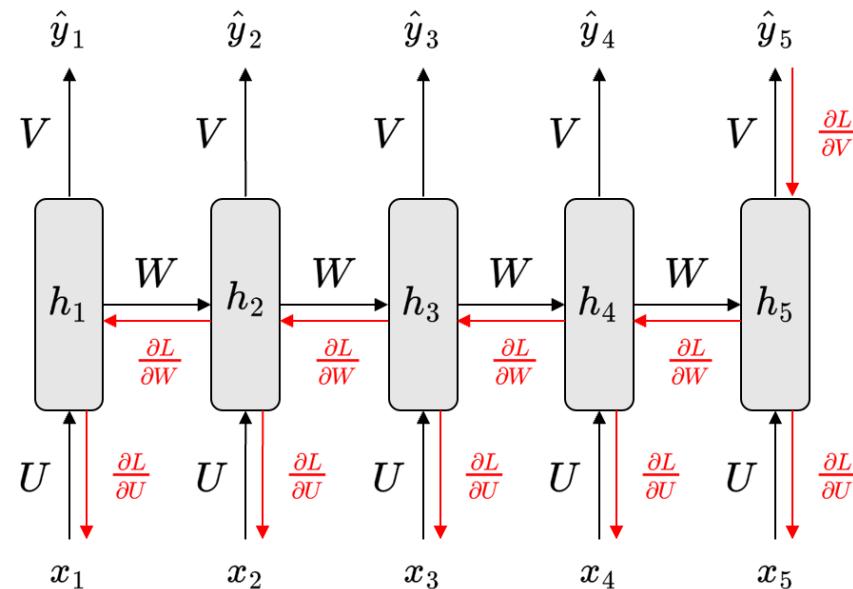
$$\hat{\mathbf{y}}_t = \text{softmax}(\mathbf{o}_t)$$



How to Train RNN

Backward Propagation

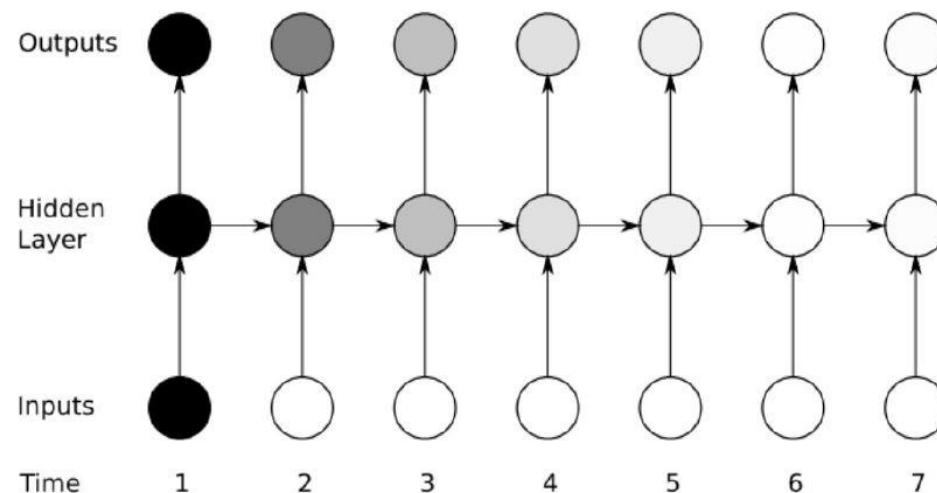
- Loss would just be the sum of losses over time steps
- Treat the recurrent network as a usual multilayer network and apply backpropagation on the unrolled network
- This is called **Backpropagation through time (BPTT)**



Long Short-Term Memory (LSTM)

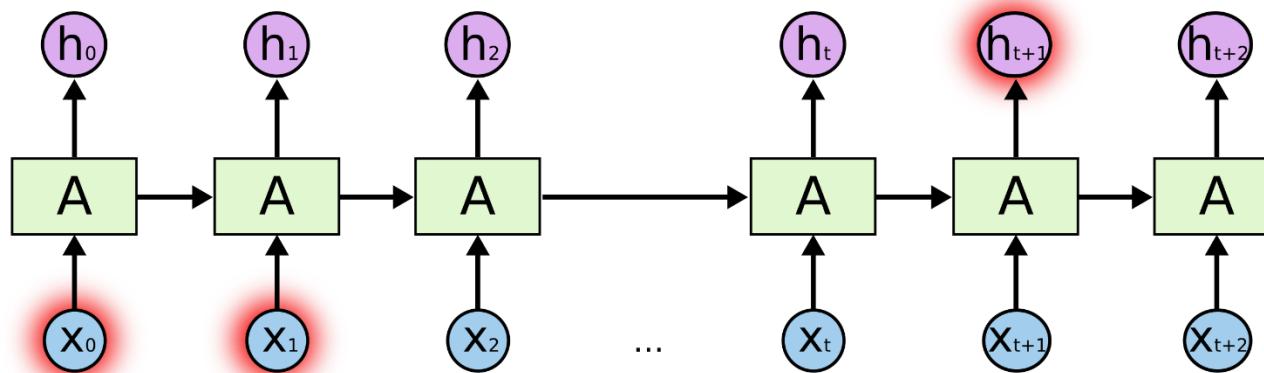
Long Short-Term Memory (LSTM)

- Long-Term Dependencies
 - Gradients propagated over many stages tend to either **vanish** or **explode**
 - Difficulty with long-term dependencies arises from the exponentially smaller weights given to long-term interactions
 - Introduce a memory state that runs through only linear operators
 - Use gating units to control the updates of the state



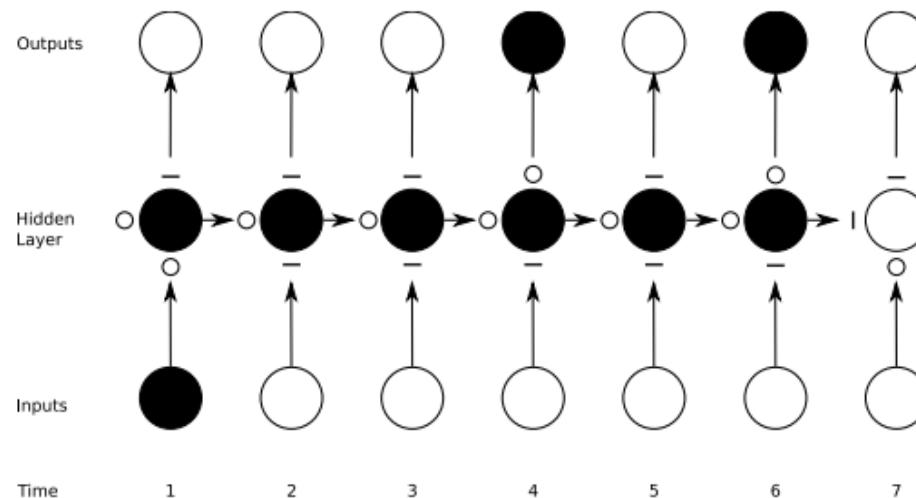
Example

- “I grew up in France... I speak fluent *French*.”



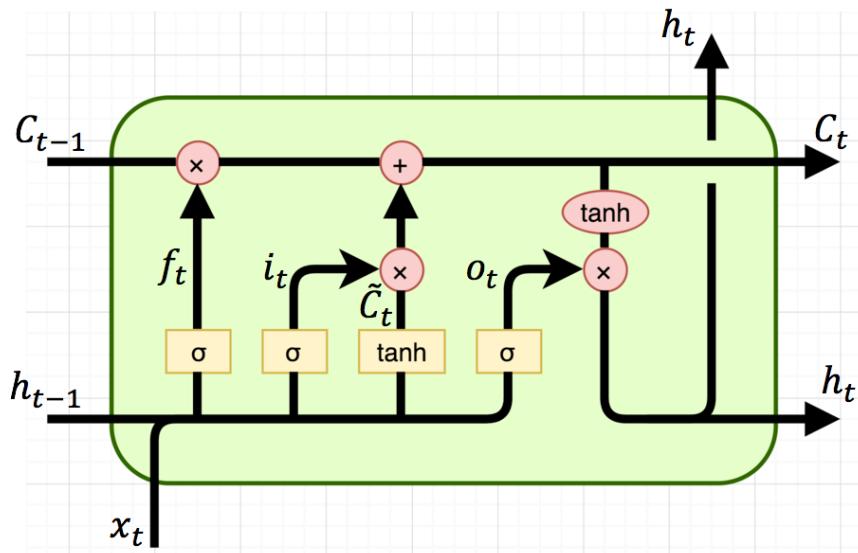
Long Short-Term Memory (LSTM)

- Consists of a memory cell and a set of gating units
 - Memory cell is the context that carries over
 - Forget gate controls erase operation
 - Input gate controls write operation
 - Output gate controls the read operation



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$$i_t = \sigma(x_t U^i + h_{t-1} W^i)$$

$$f_t = \sigma(x_t U^f + h_{t-1} W^f)$$

$$o_t = \sigma(x_t U^o + h_{t-1} W^o)$$

$$\tilde{C}_t = \tanh(x_t U^g + h_{t-1} W^g)$$

$$C_t = \sigma(f_t * C_{t-1} + i_t * \tilde{C}_t)$$

$$h_t = \tanh(C_t) * o_t$$

Element-by-Element

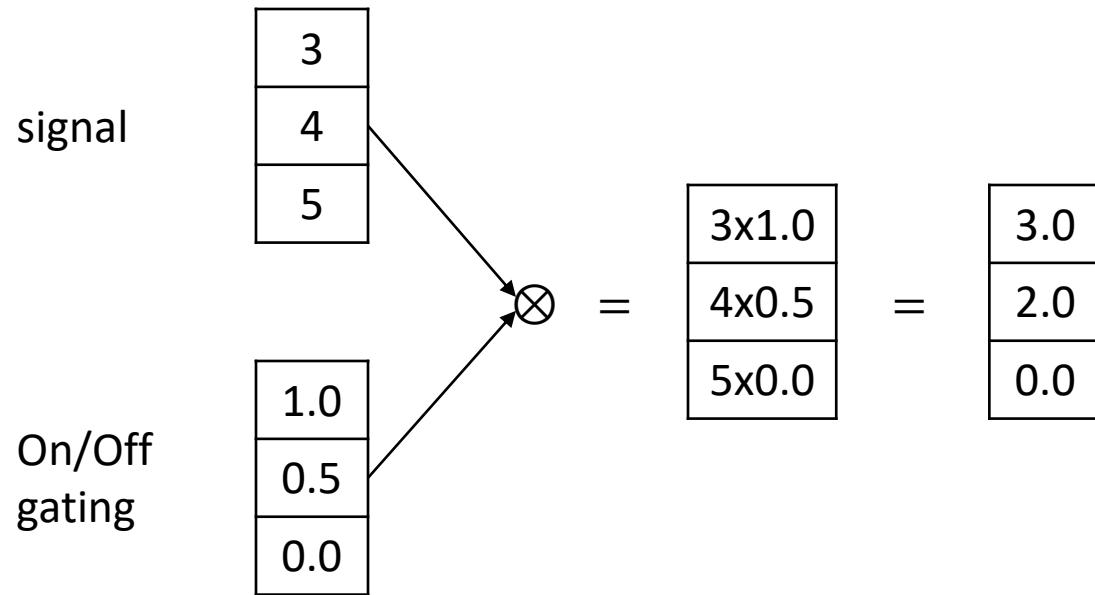
Element-by-Element Addition

$$\begin{array}{c} \begin{array}{|c|} \hline 3 \\ \hline 4 \\ \hline 5 \\ \hline \end{array} \\ \oplus \\ \begin{array}{|c|} \hline 6 \\ \hline 7 \\ \hline 8 \\ \hline \end{array} \end{array} = \begin{array}{|c|} \hline 3+6 \\ \hline 4+7 \\ \hline 5+8 \\ \hline \end{array} = \begin{array}{|c|} \hline 9 \\ \hline 11 \\ \hline 13 \\ \hline \end{array}$$

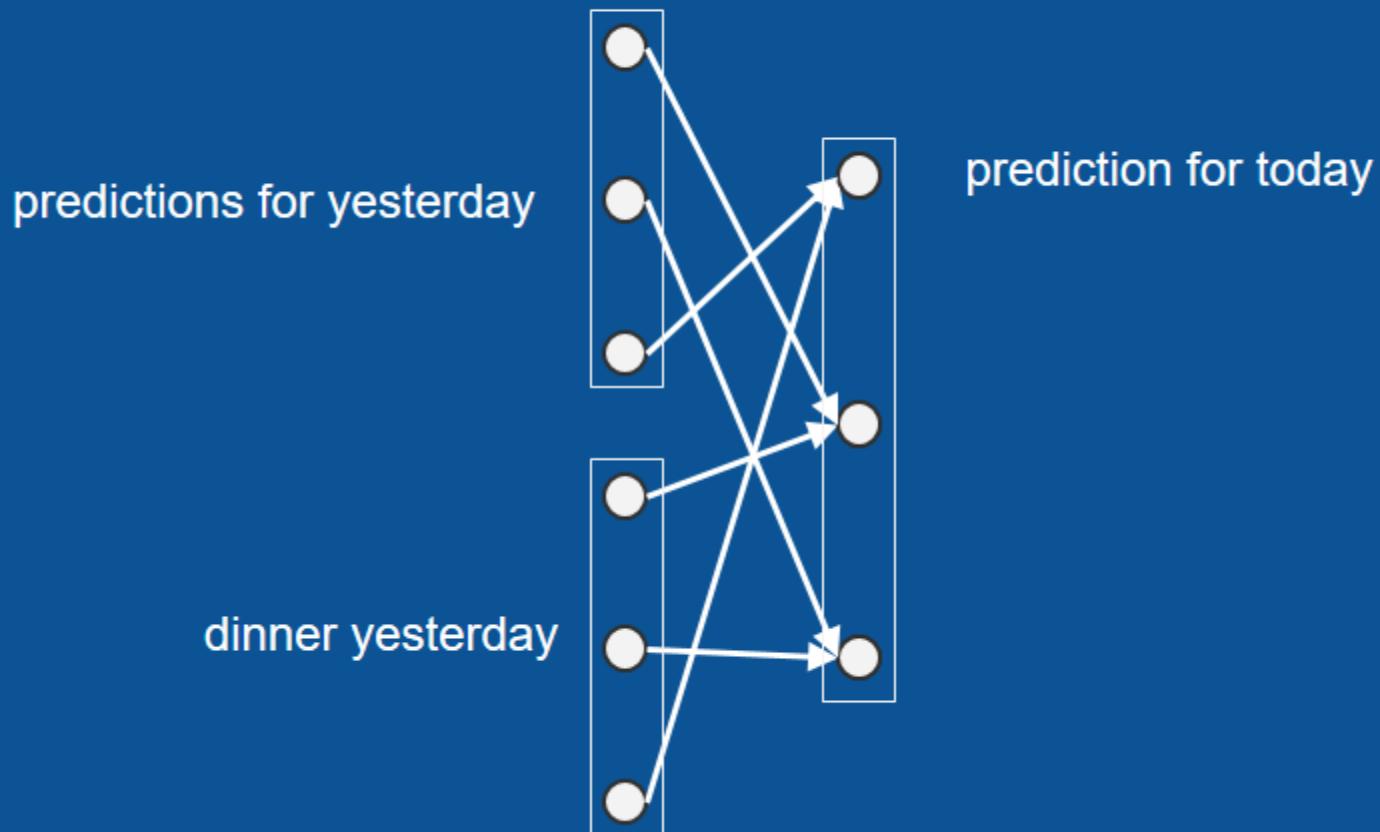
Element-by-Element Multiplication

$$\begin{array}{c} \begin{array}{|c|} \hline 3 \\ \hline 4 \\ \hline 5 \\ \hline \end{array} \\ \otimes \\ \begin{array}{|c|} \hline 6 \\ \hline 7 \\ \hline 8 \\ \hline \end{array} \end{array} = \begin{array}{|c|} \hline 3 \times 6 \\ \hline 4 \times 7 \\ \hline 5 \times 8 \\ \hline \end{array} = \begin{array}{|c|} \hline 18 \\ \hline 28 \\ \hline 40 \\ \hline \end{array}$$

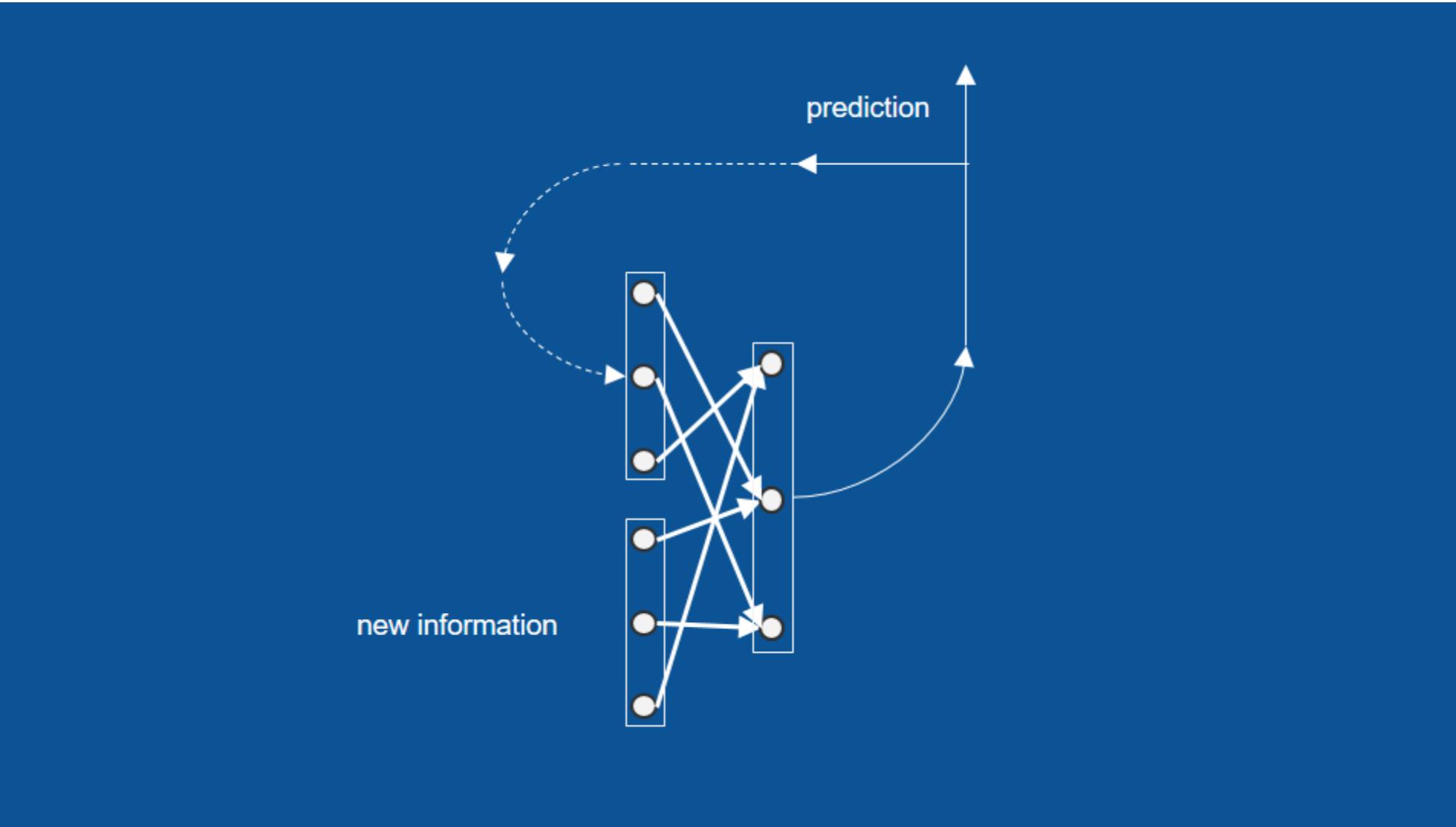
Gating



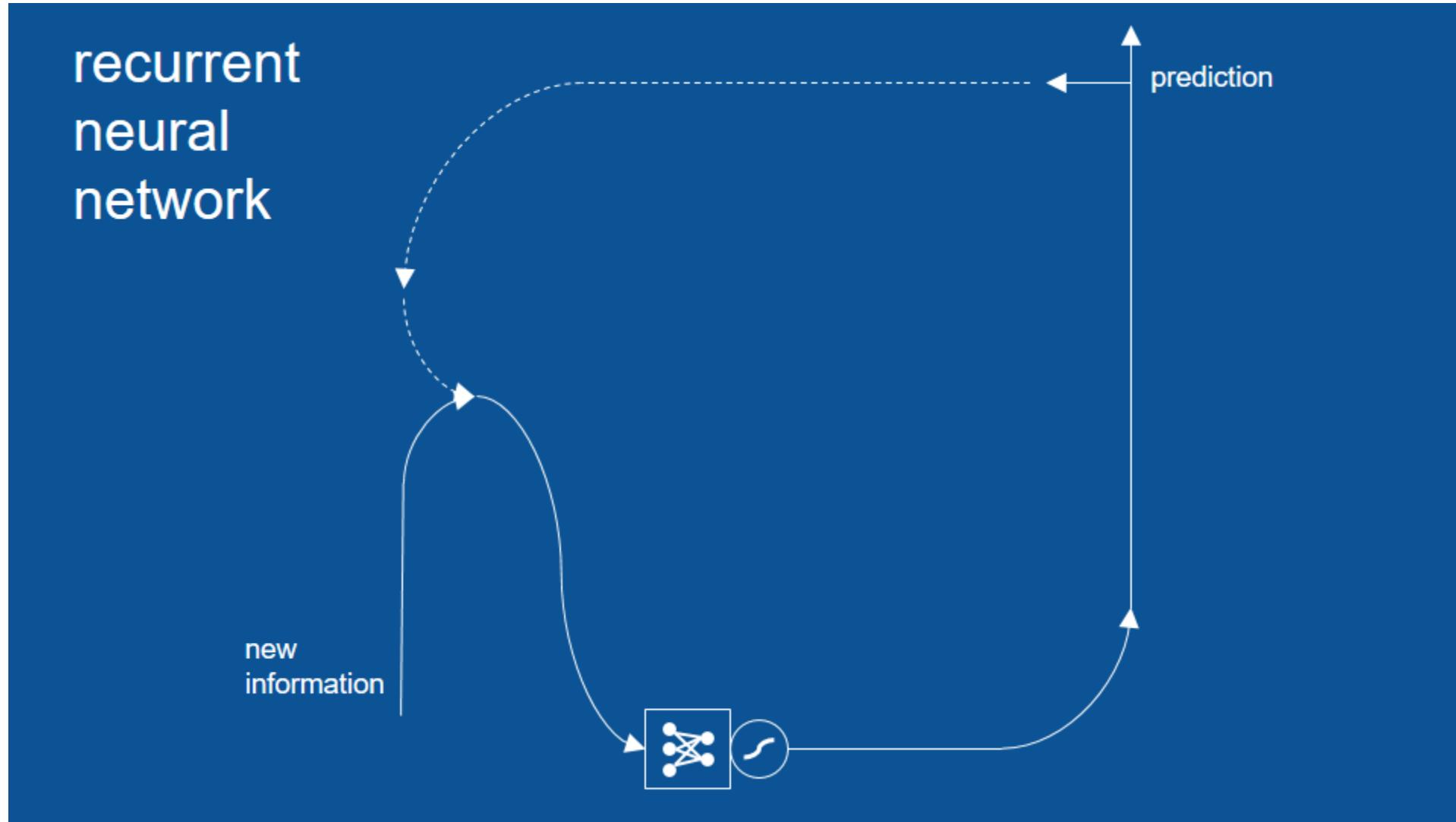
Long Short-Term Memory (LSTM)



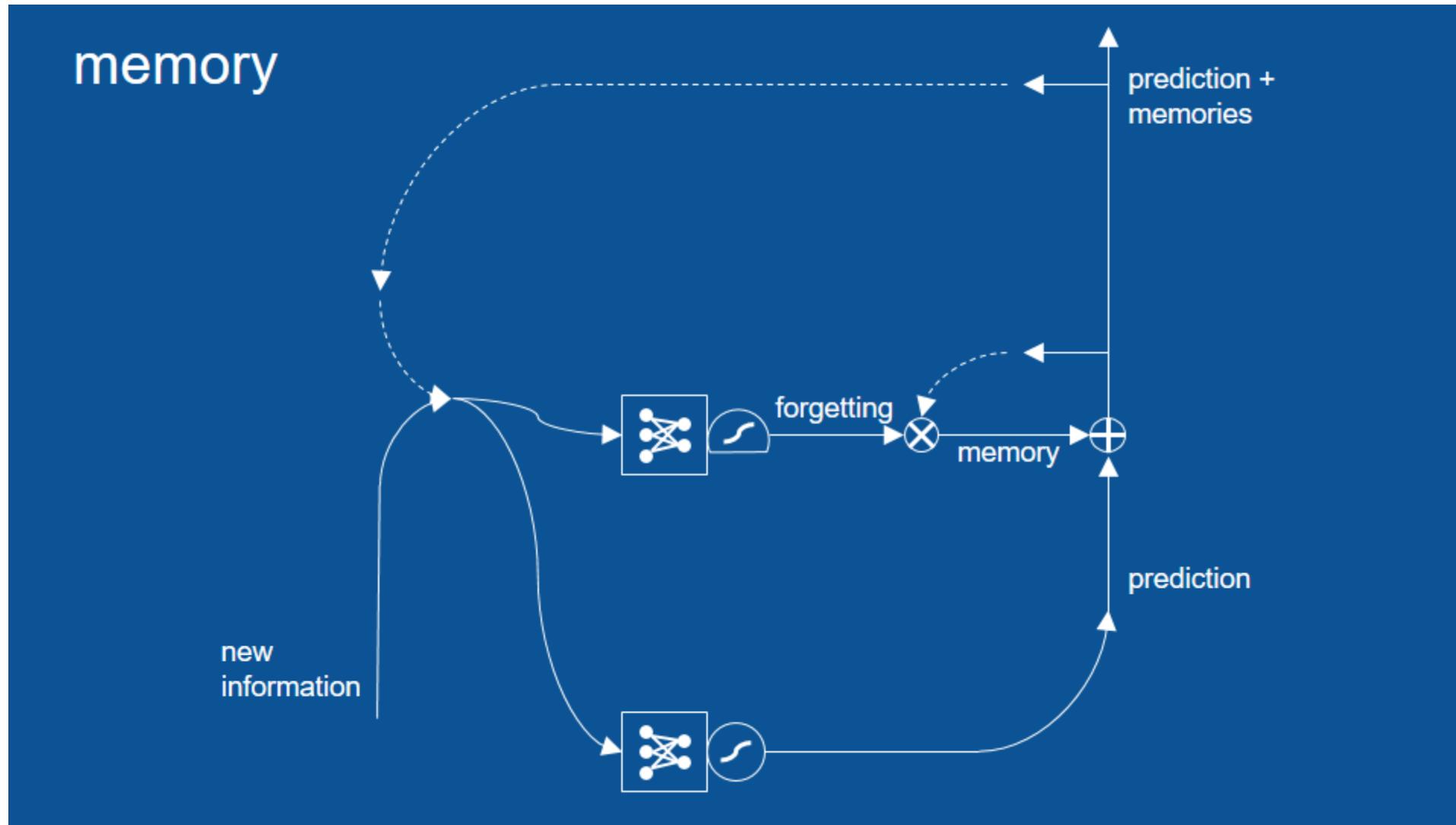
Long Short-Term Memory (LSTM)



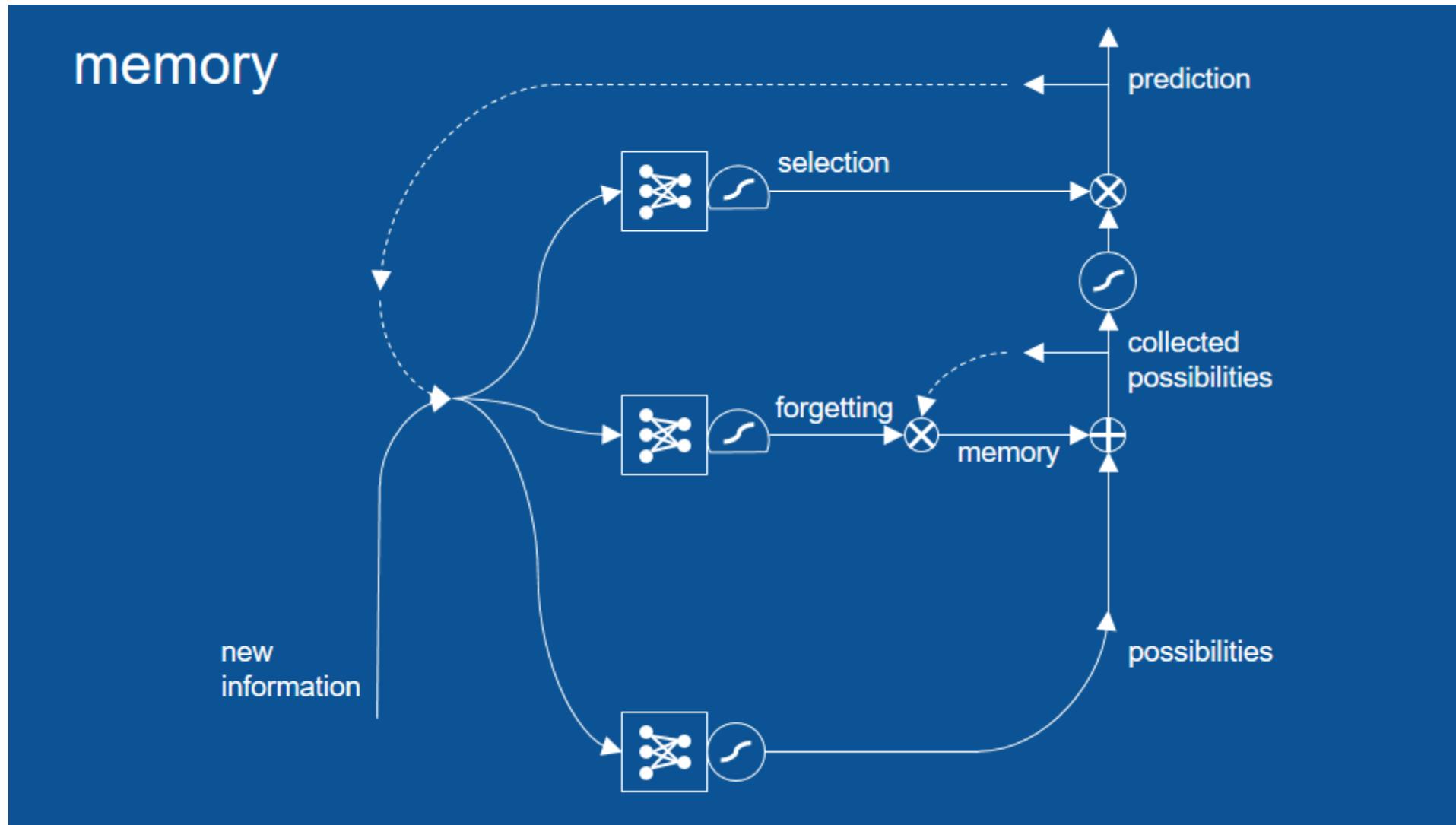
Long Short-Term Memory (LSTM)



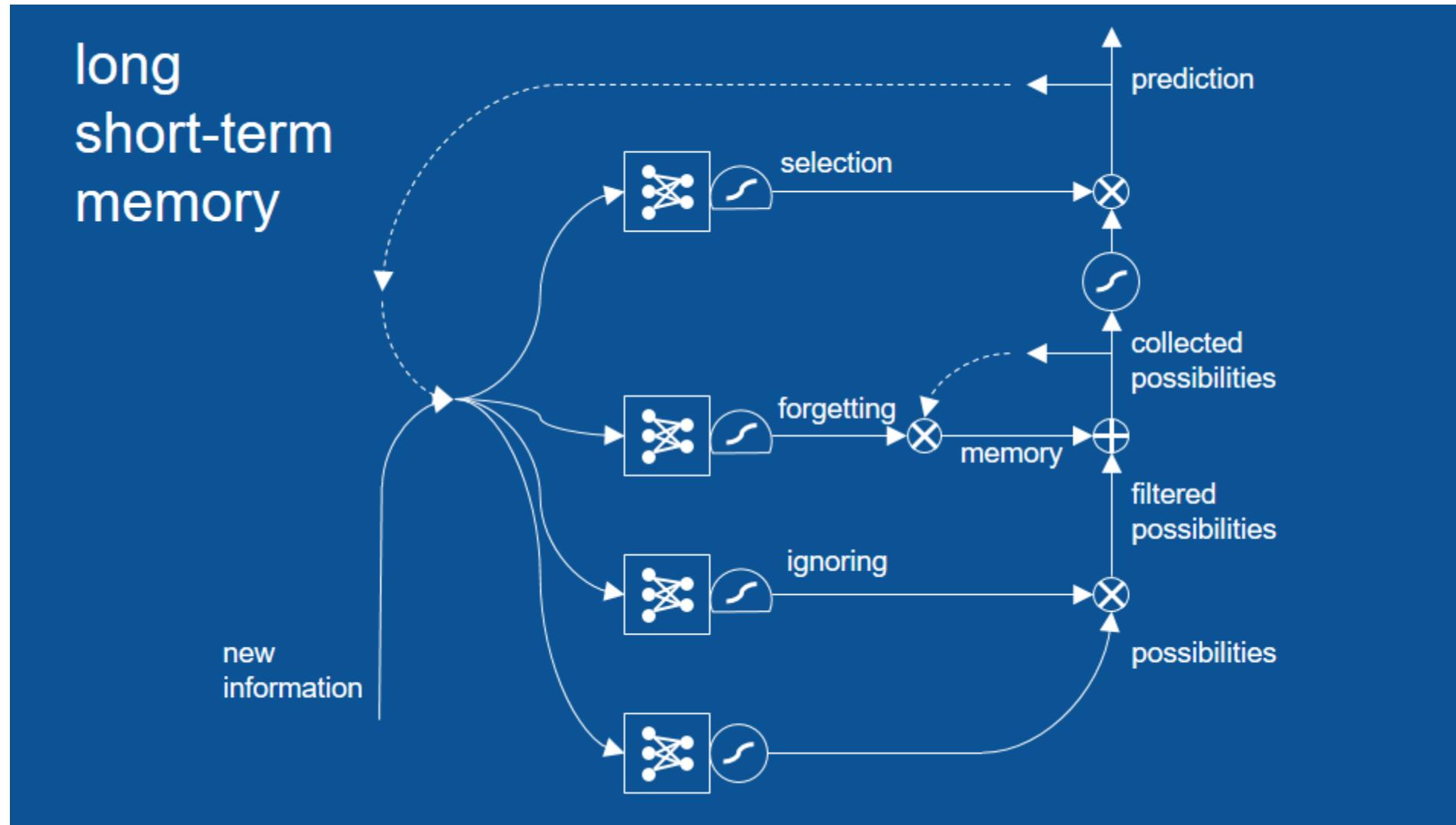
Long Short-Term Memory (LSTM)



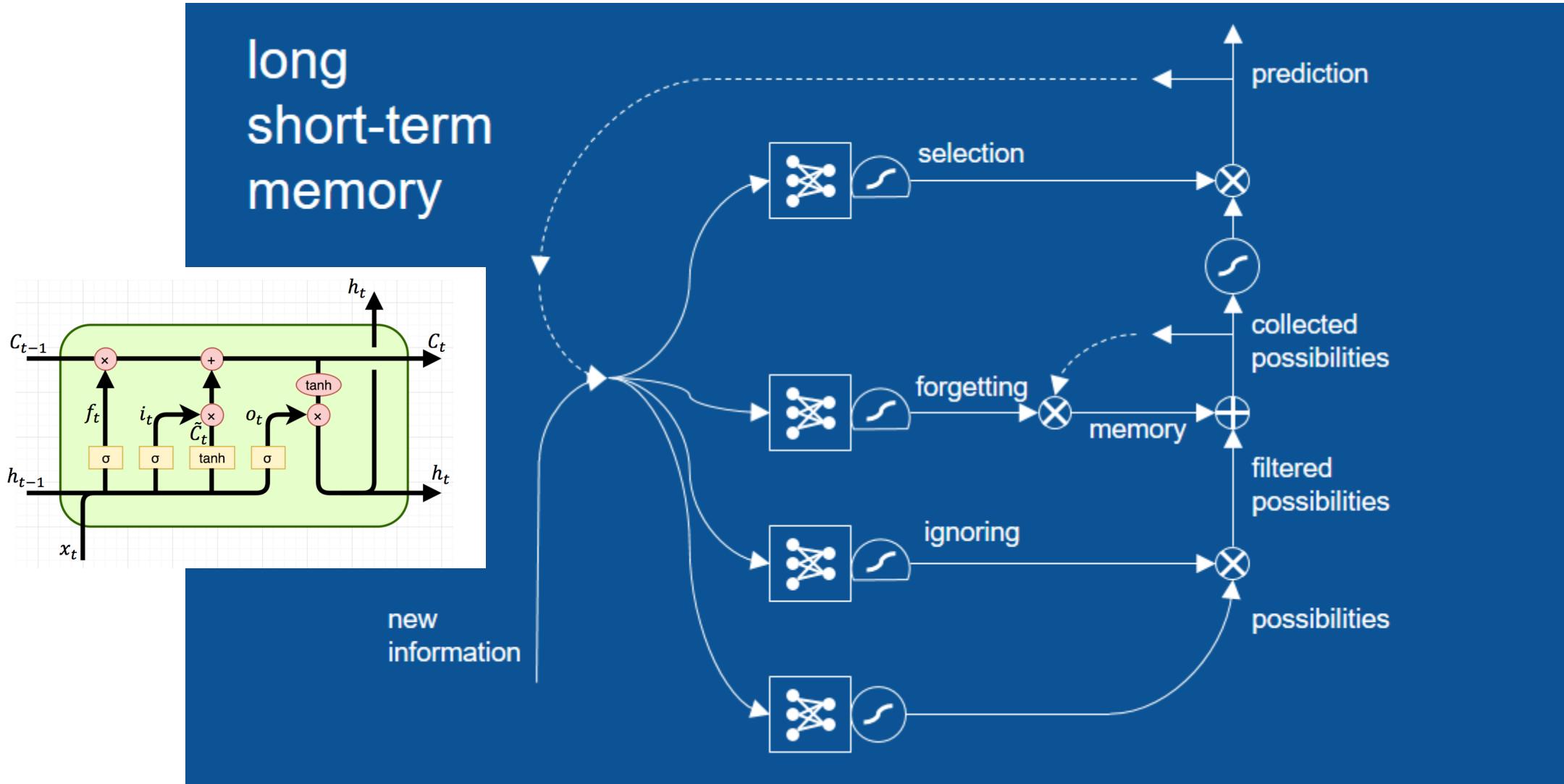
Long Short-Term Memory (LSTM)



Long Short-Term Memory (LSTM)

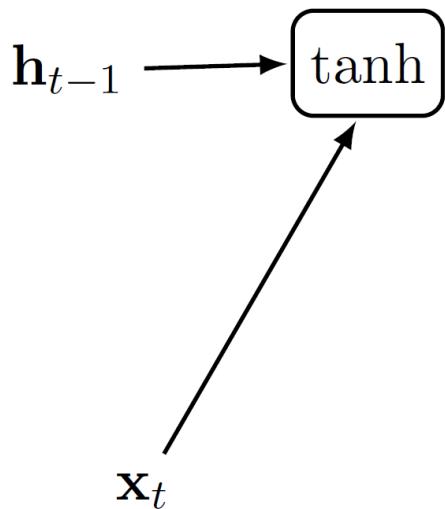
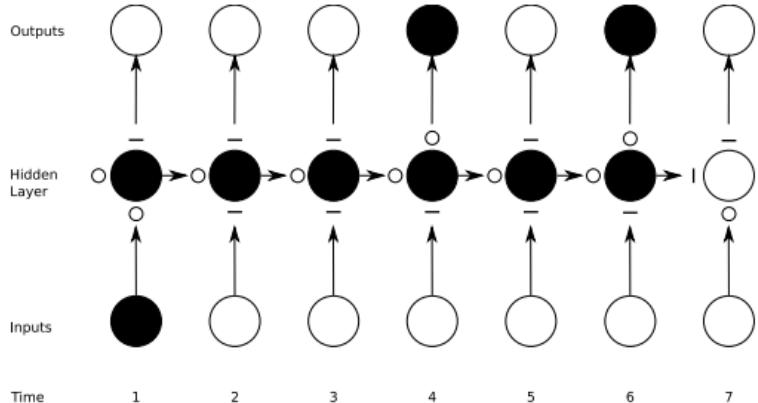


Long Short-Term Memory (LSTM)



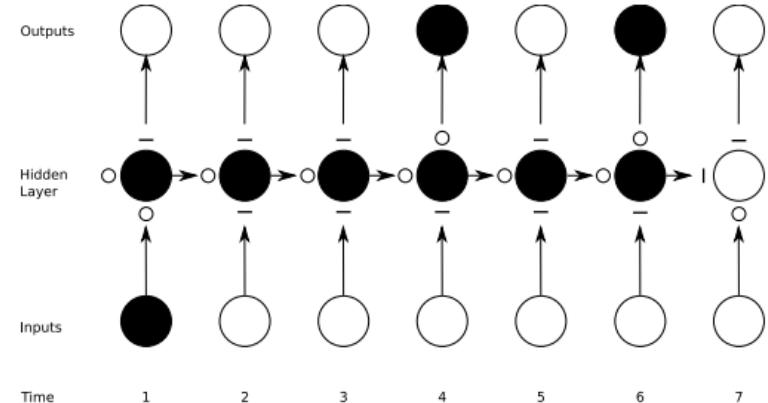
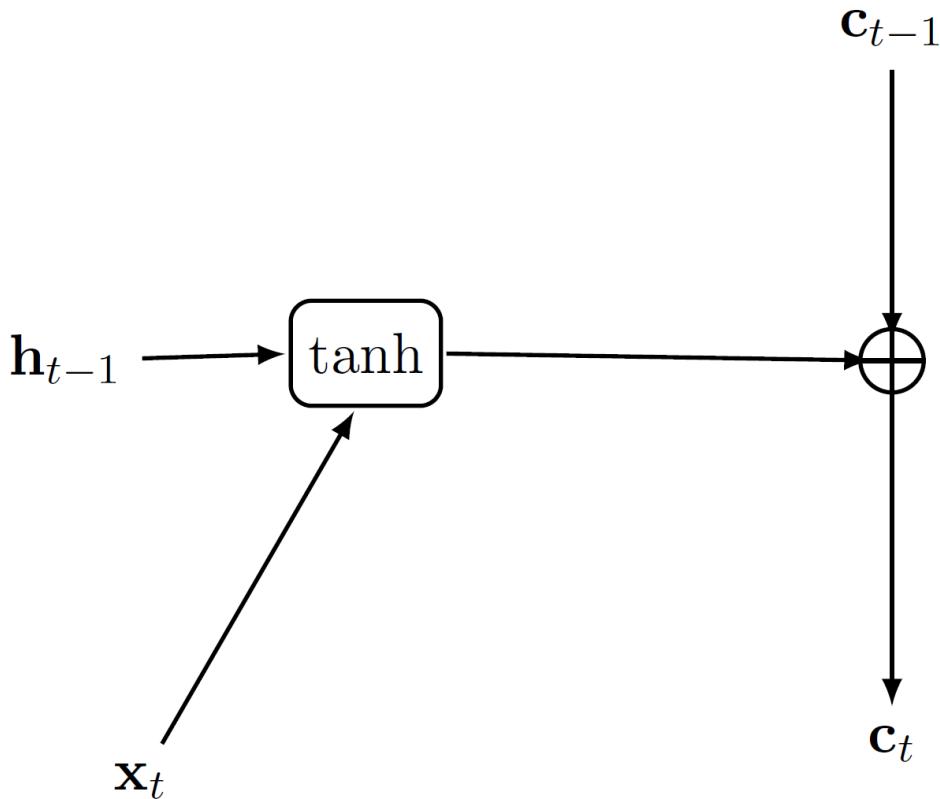
Long Short-Term Memory

- Forget gate controls erase operation
- Input gate controls write operation
- Output gate controls the read operation



Long Short-Term Memory

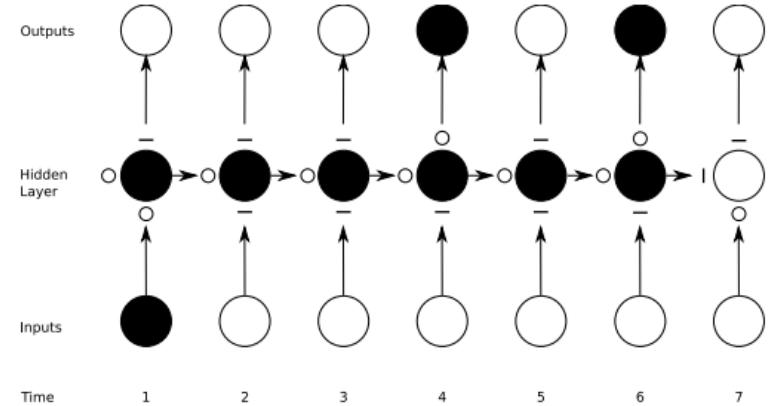
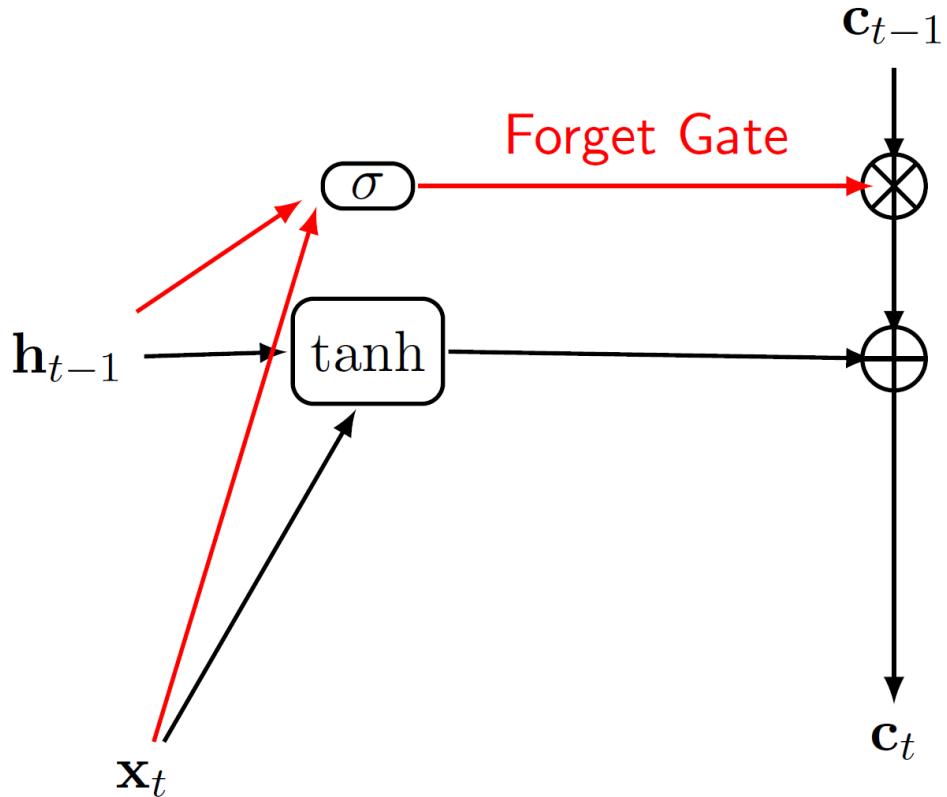
- Forget gate controls erase operation
- Input gate controls write operation
- Output gate controls the read operation



$$\begin{aligned}\tilde{\mathbf{c}}_t &= \tanh(W\mathbf{h}_{t-1} + U\mathbf{x}_t) \\ \mathbf{c}_t &= \mathbf{c}_{t-1} + \tilde{\mathbf{c}}_t\end{aligned}$$

Long Short-Term Memory

- Forget gate controls erase operation
- Input gate controls write operation
- Output gate controls the read operation



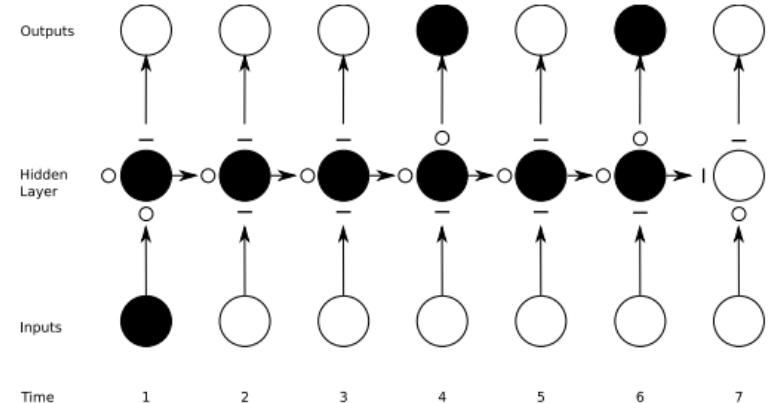
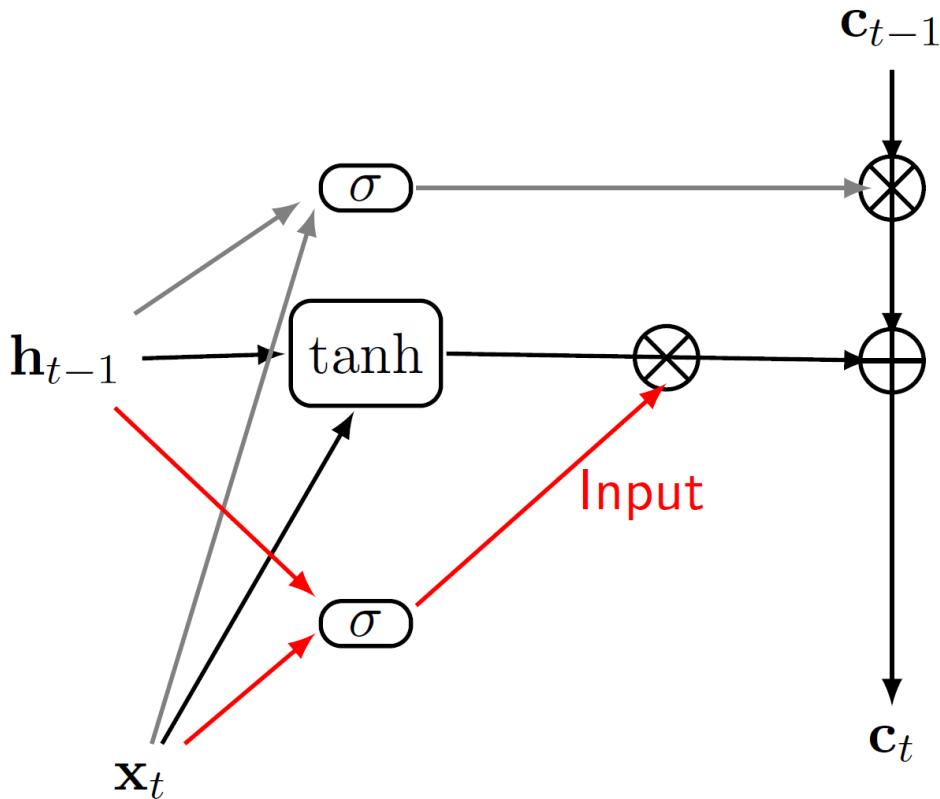
$$f_t = \sigma(W_f \mathbf{h}_{t-1} + U_f \mathbf{x}_t)$$

$$\tilde{\mathbf{c}}_t = \tanh(W \mathbf{h}_{t-1} + U \mathbf{x}_t)$$

$$\mathbf{c}_t = f_t \odot \mathbf{c}_{t-1} + \tilde{\mathbf{c}}_t$$

Long Short-Term Memory

- Forget gate controls erase operation
- Input gate controls write operation
- Output gate controls the read operation

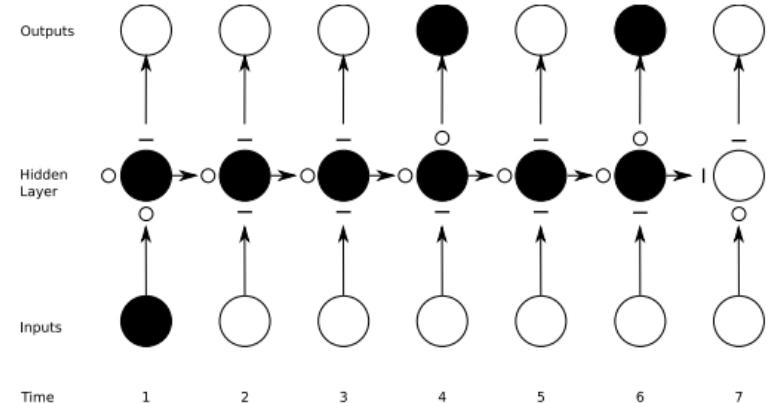
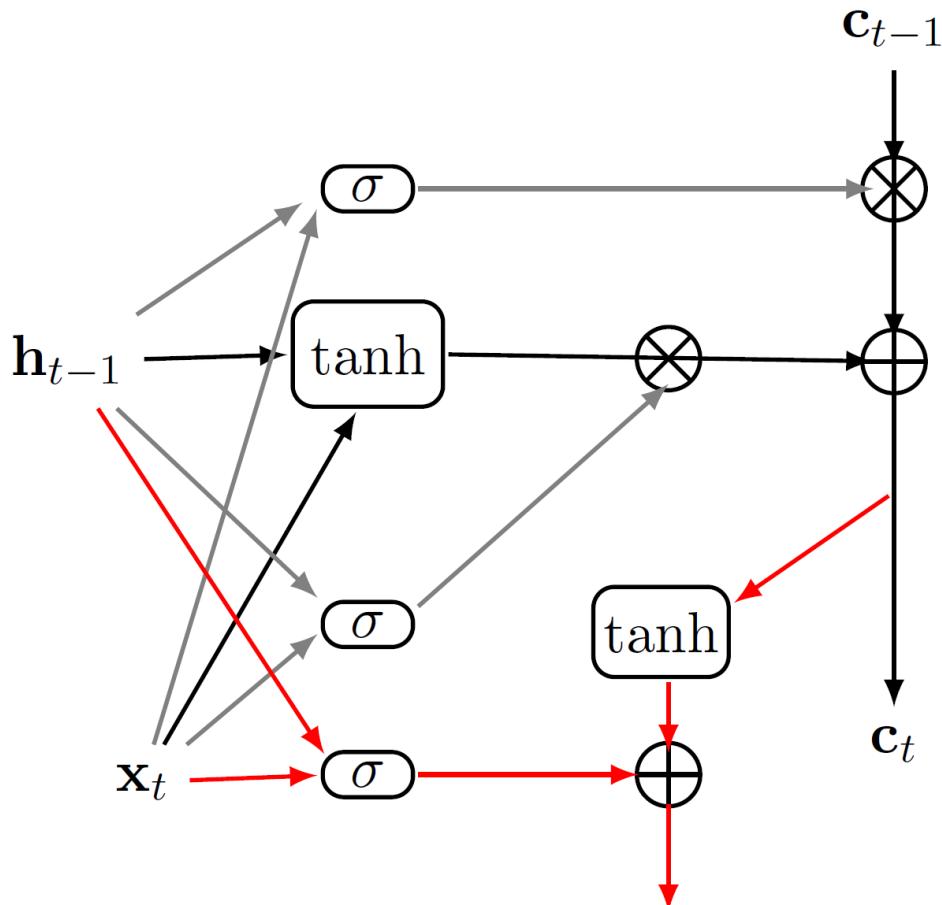


$$f_t = \sigma(W_f \mathbf{h}_{t-1} + U_f \mathbf{x}_t)$$
$$i_t = \sigma(W_i \mathbf{h}_{t-1} + U_i \mathbf{x}_t)$$

$$\tilde{\mathbf{c}}_t = \tanh(W \mathbf{h}_{t-1} + U \mathbf{x}_t)$$
$$\mathbf{c}_t = f_t \odot \mathbf{c}_{t-1} + i_t \odot \tilde{\mathbf{c}}_t$$

Long Short-Term Memory

- Forget gate controls erase operation
- Input gate controls write operation
- Output gate controls the read operation



$$f_t = \sigma(W_f \mathbf{h}_{t-1} + U_f \mathbf{x}_t)$$

$$i_t = \sigma(W_i \mathbf{h}_{t-1} + U_i \mathbf{x}_t)$$

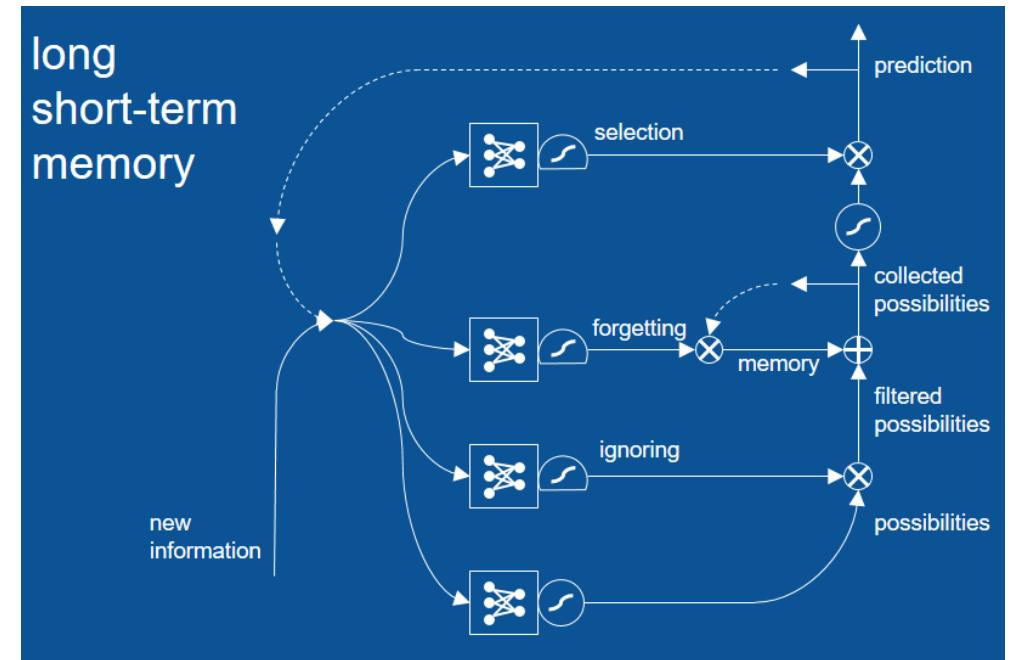
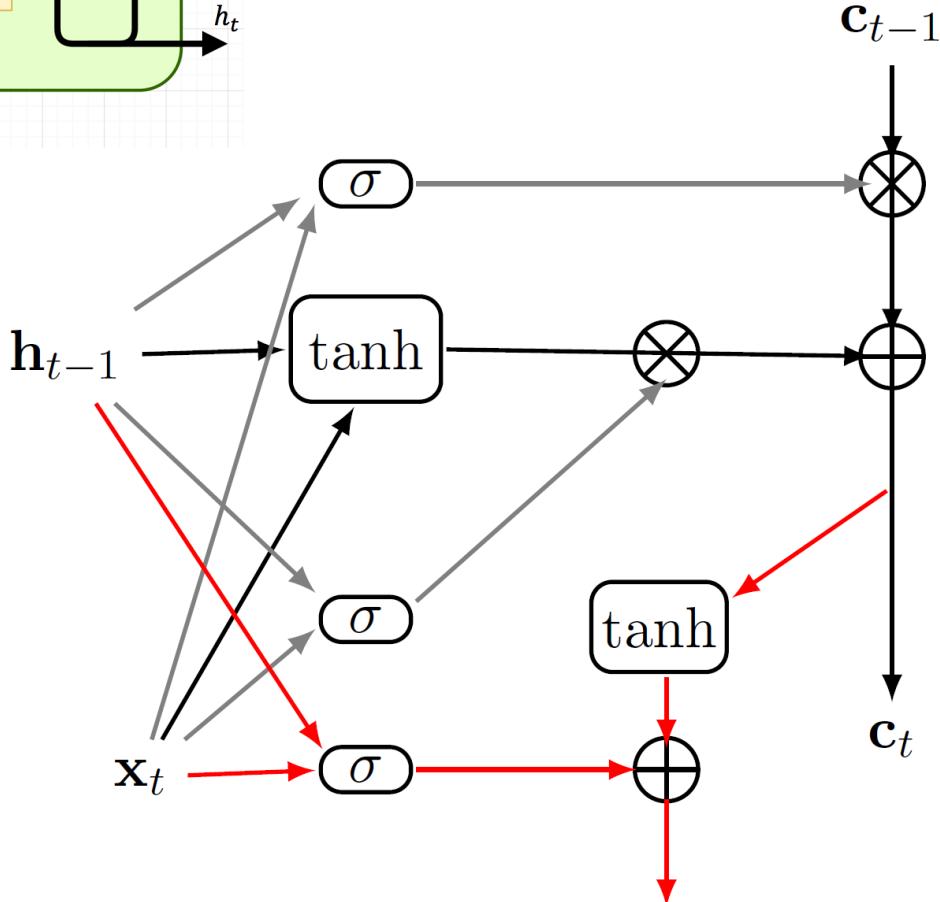
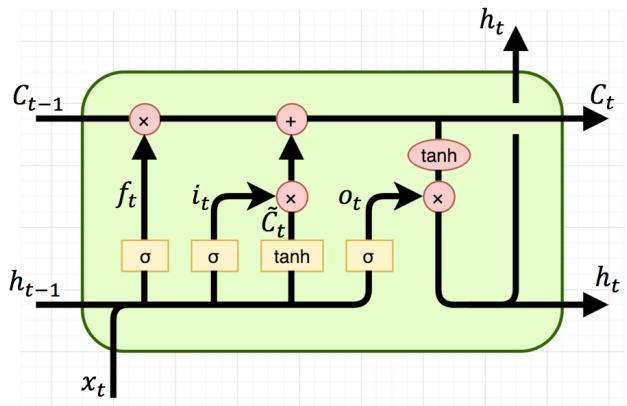
$$o_t = \sigma(W_o \mathbf{h}_{t-1} + U_o \mathbf{x}_t)$$

$$\tilde{\mathbf{c}}_t = \tanh(W \mathbf{h}_{t-1} + U \mathbf{x}_t)$$

$$\mathbf{c}_t = f_t \odot \mathbf{c}_{t-1} + i_t \odot \tilde{\mathbf{c}}_t$$

$$\mathbf{h}_t = o_t \odot \tanh(\mathbf{c}_t)$$

Long Short-Term Memory

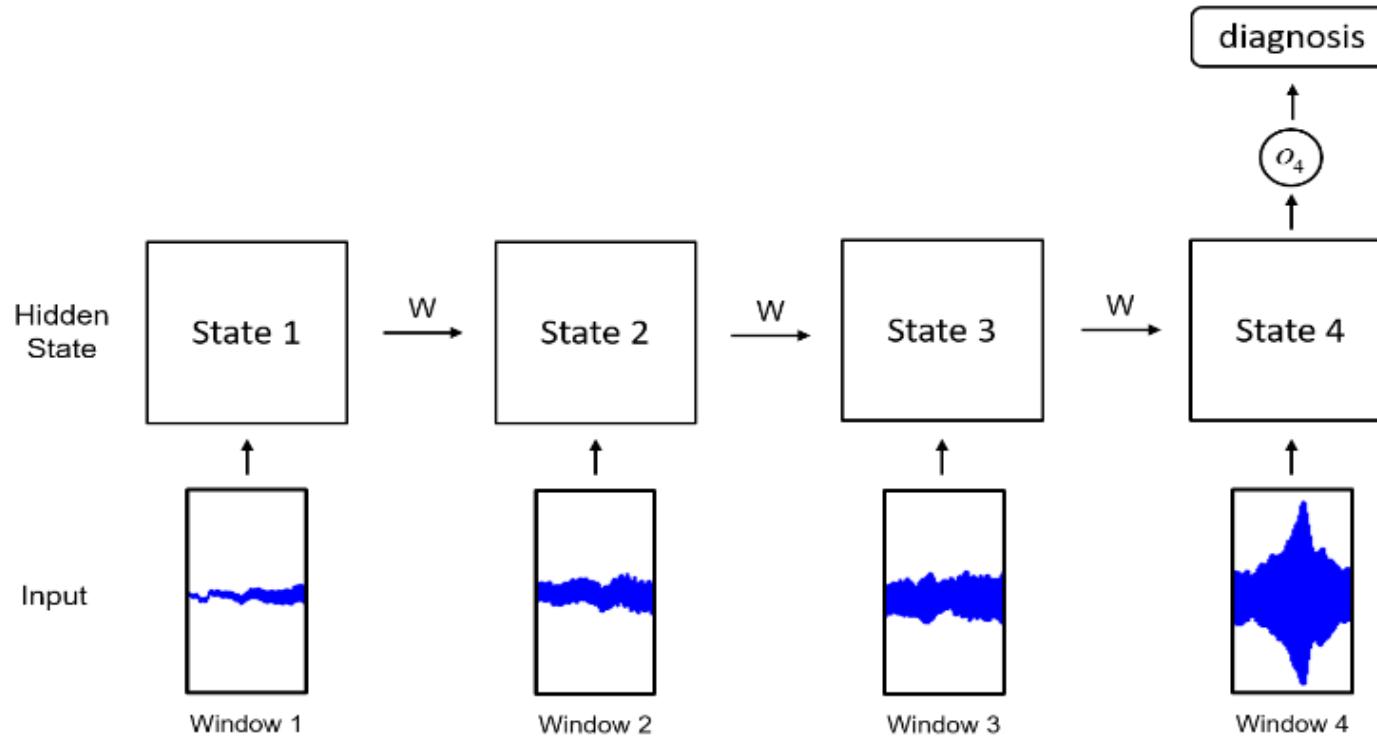


Weakness of RNN and LSTM

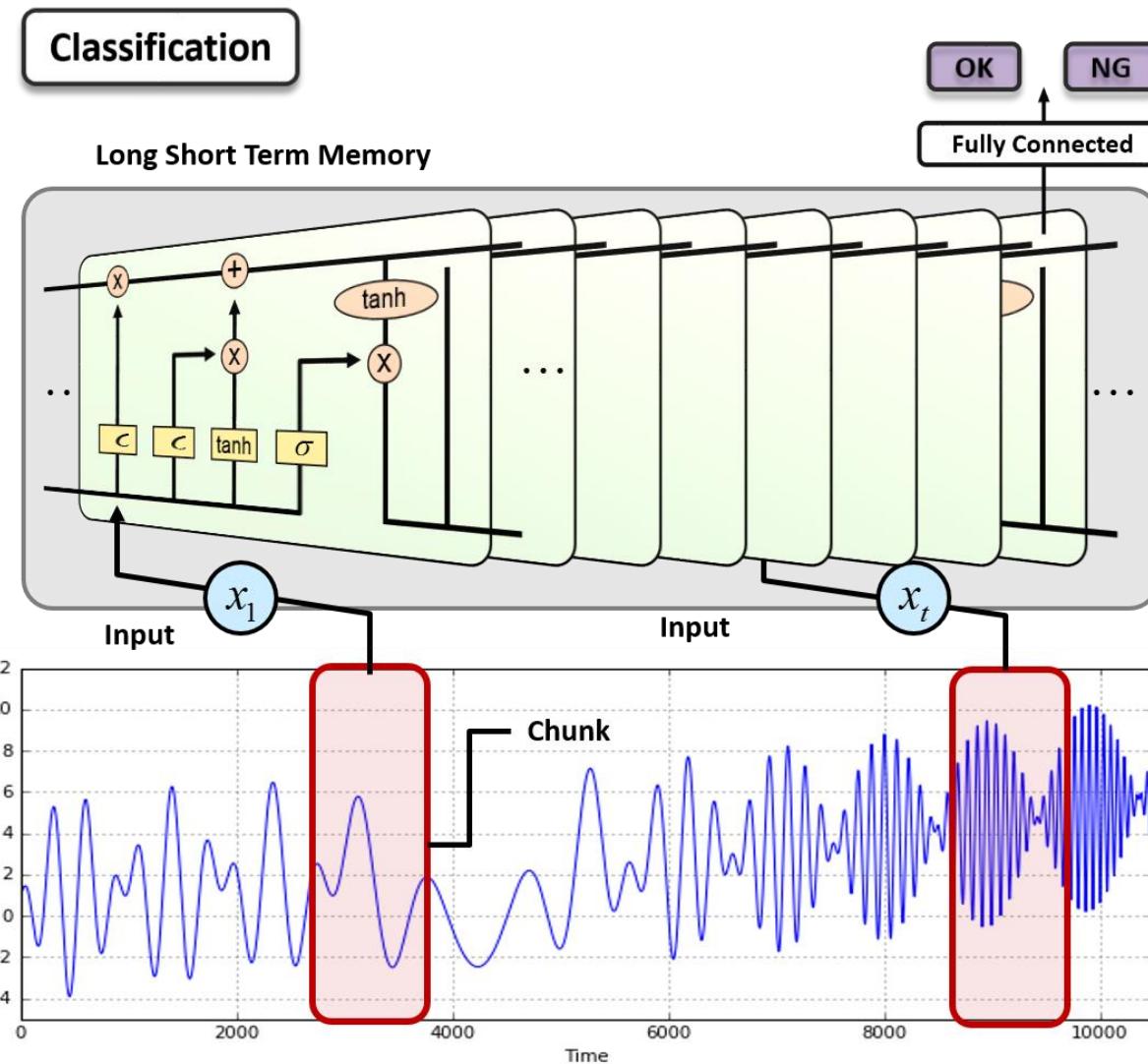
- Sequential computation is slow
- Vanishing and exploding gradients are still problematic
- Long-term credit assignment is difficult

Tutorial 1: RNN for Signal Prediction

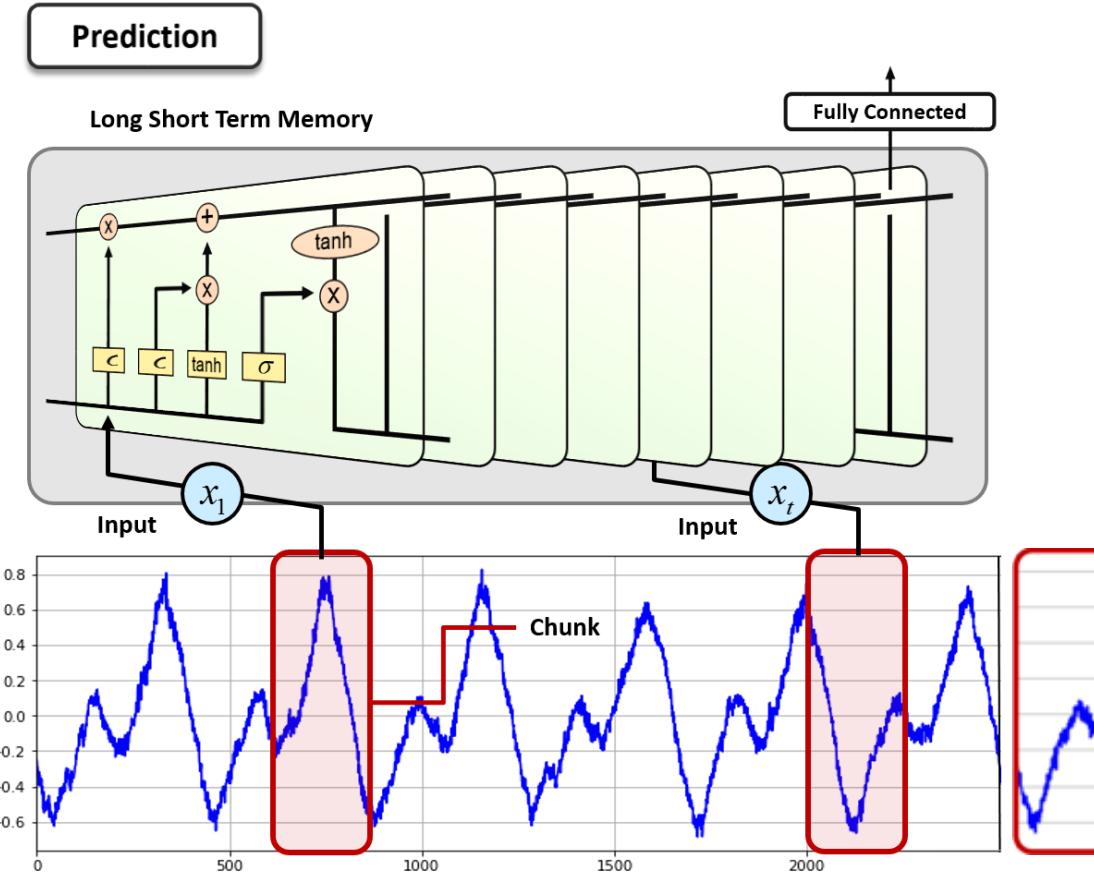
Time Series Data and LSTM



LSTM for Signal Prediction

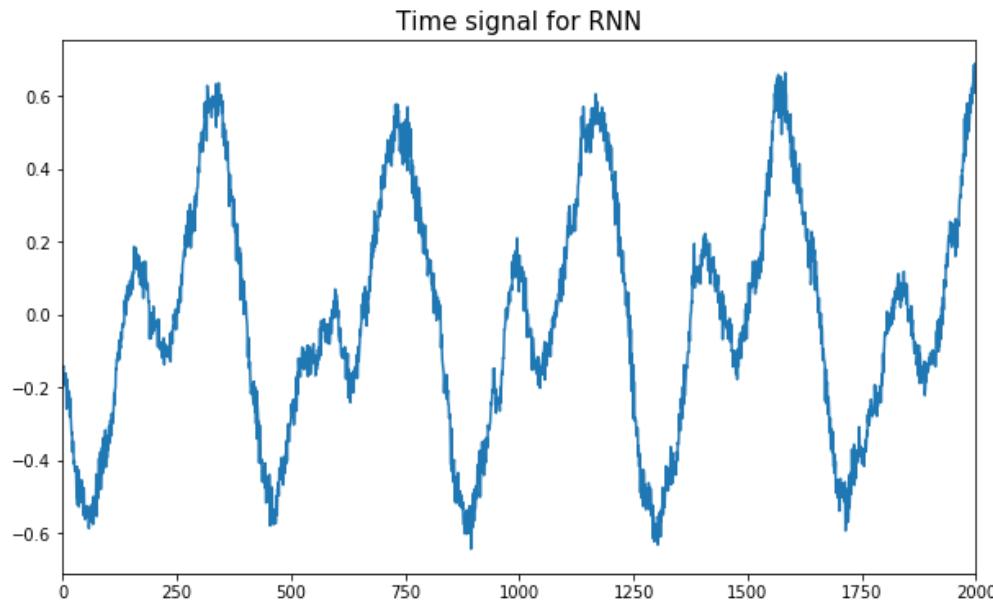


LSTM for Signal Prediction



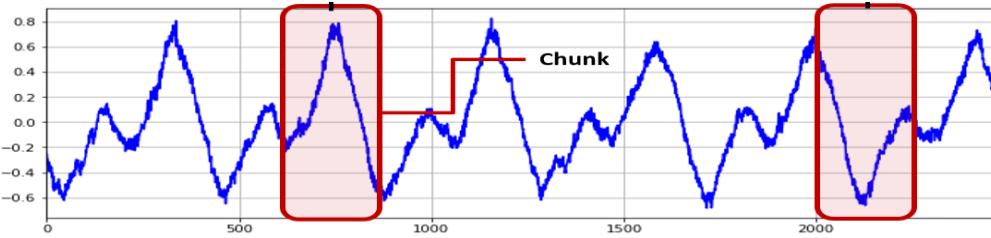
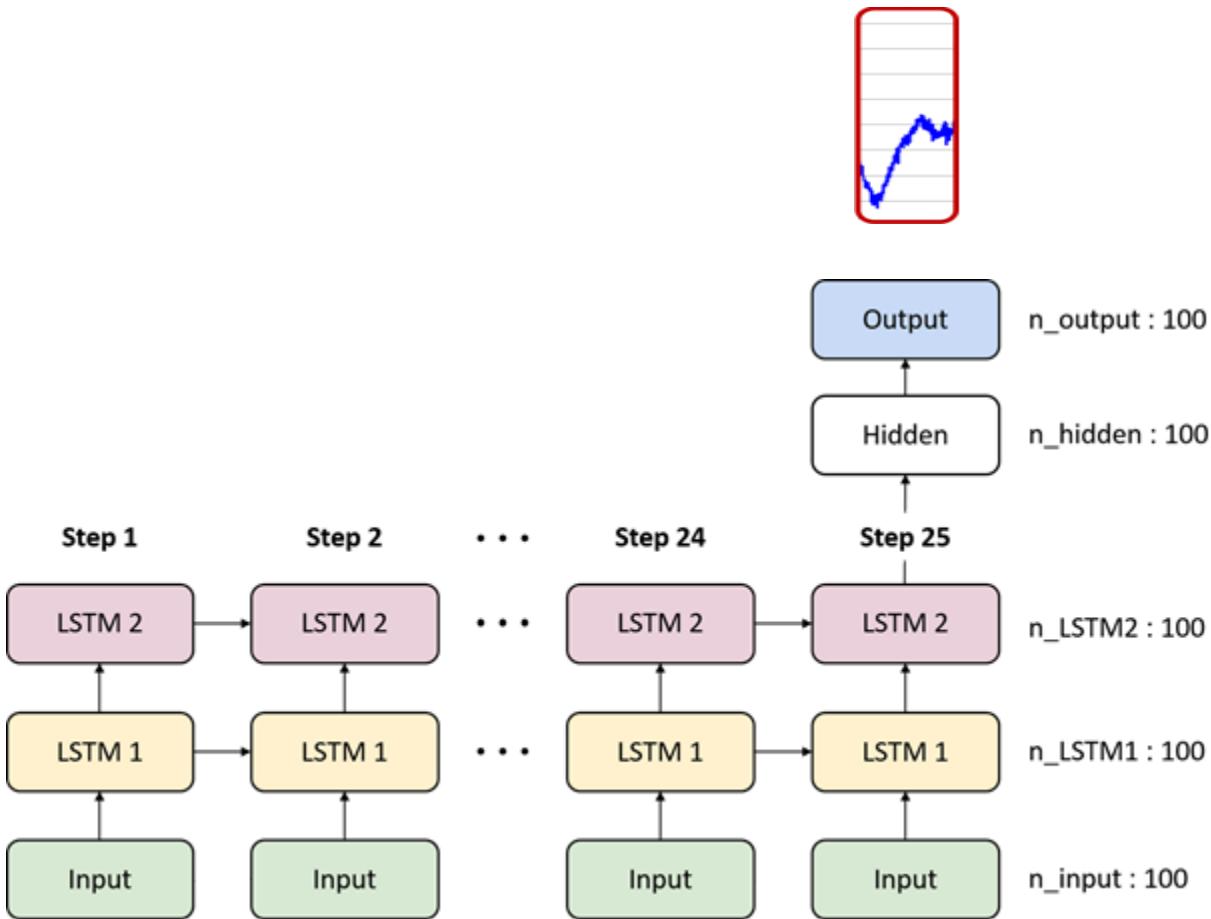
LSTM with TensorFlow

- An example for predicting a next piece of an acceleration signal
- Regression problem



LSTM Structure

```
n_step = 25  
n_input = 100  
  
## LSTM shape  
n_lstm1 = 100  
n_lstm2 = 100  
  
## Fully connected  
n_hidden = 100  
n_output = 100
```



LSTM, Weights and Biases

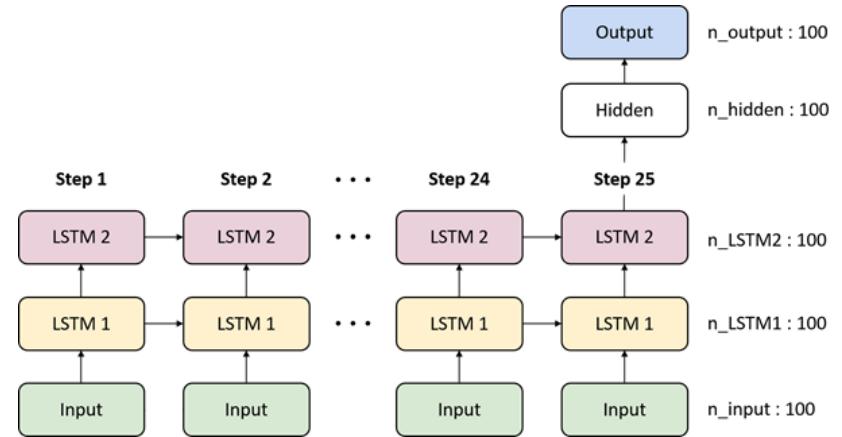
- LSTM Cell
 - Do not need to define weights and biases of LSTM cells

- Fully connected
 - Define parameters based on the predefined layer size
 - Initialize with a normal distribution with $\mu = 0$ and $\sigma = 0.01$

```
weights = {
    'hidden' : tf.Variable(tf.random_normal([n_lstm2, n_hidden], stddev = 0.01)),
    'output' : tf.Variable(tf.random_normal([n_hidden, n_output], stddev = 0.01))
}

biases = {
    'hidden' : tf.Variable(tf.random_normal([n_hidden], stddev = 0.01)),
    'output' : tf.Variable(tf.random_normal([n_output], stddev = 0.01))
}

x = tf.placeholder(tf.float32, [None, n_step, n_input])
y = tf.placeholder(tf.float32, [None, n_output])
```

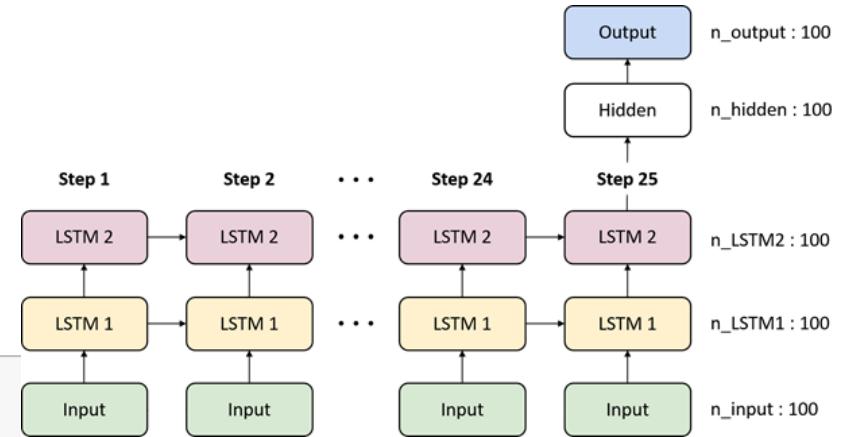


Build a Model

- First, define the LSTM cells
- Second, compute hidden state (h) and LSTM cell (c) with the predefined LSTM cell and input

```
def build_model(x, weights, biases):
    with tf.variable_scope('rnn'):
        # Build RNN network
        with tf.variable_scope('lstm1'):
            lstm1 = tf.nn.rnn_cell.LSTMCell(n_lstm1)
            h1, c1 = tf.nn.dynamic_rnn(lstm1, x, dtype = tf.float32)
        with tf.variable_scope('lstm2'):
            lstm2 = tf.nn.rnn_cell.LSTMCell(n_lstm2)
            h2, c2 = tf.nn.dynamic_rnn(lstm2, h1, dtype = tf.float32)

        # Build classifier
        hidden = tf.add(tf.matmul(h2[:, -1, :], weights['hidden']), biases['hidden'])
        hidden = tf.nn.relu(hidden)
        output = tf.add(tf.matmul(hidden, weights['output']), biases['output'])
    return output
```



Cost, Initializer and Optimizer

- Loss
 - Regression: Squared loss
- Initializer
 - Initialize all the empty variables
- Optimizer
 - AdamOptimizer: the most popular optimize

$$\frac{1}{N} \sum_{i=1}^N (\hat{y}^{(i)} - y^{(i)})^2$$

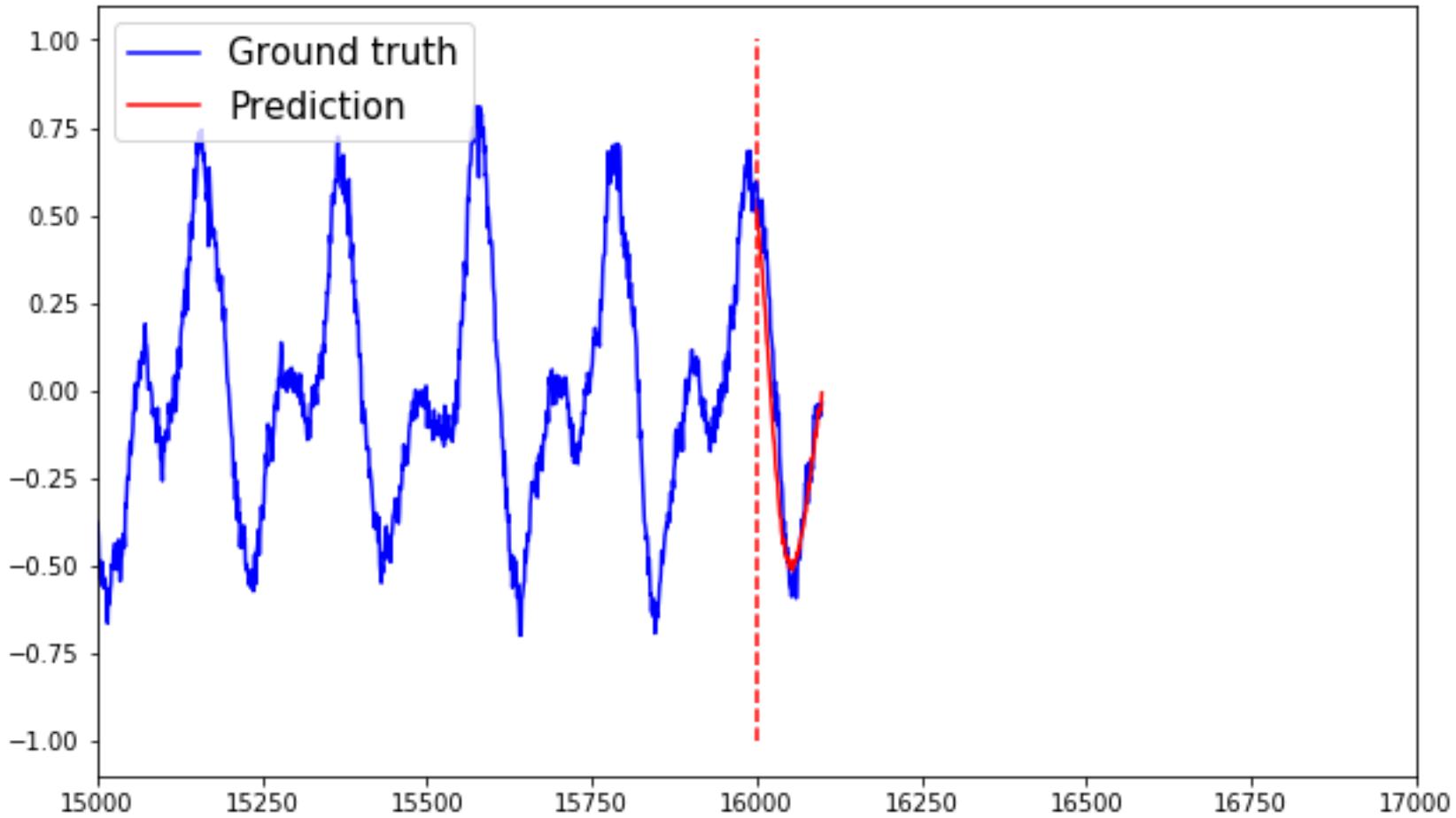
```
LR = 0.0001

pred = build_model(x, weights, biases)
loss = tf.square(tf.subtract(y, pred))
loss = tf.reduce_mean(loss)

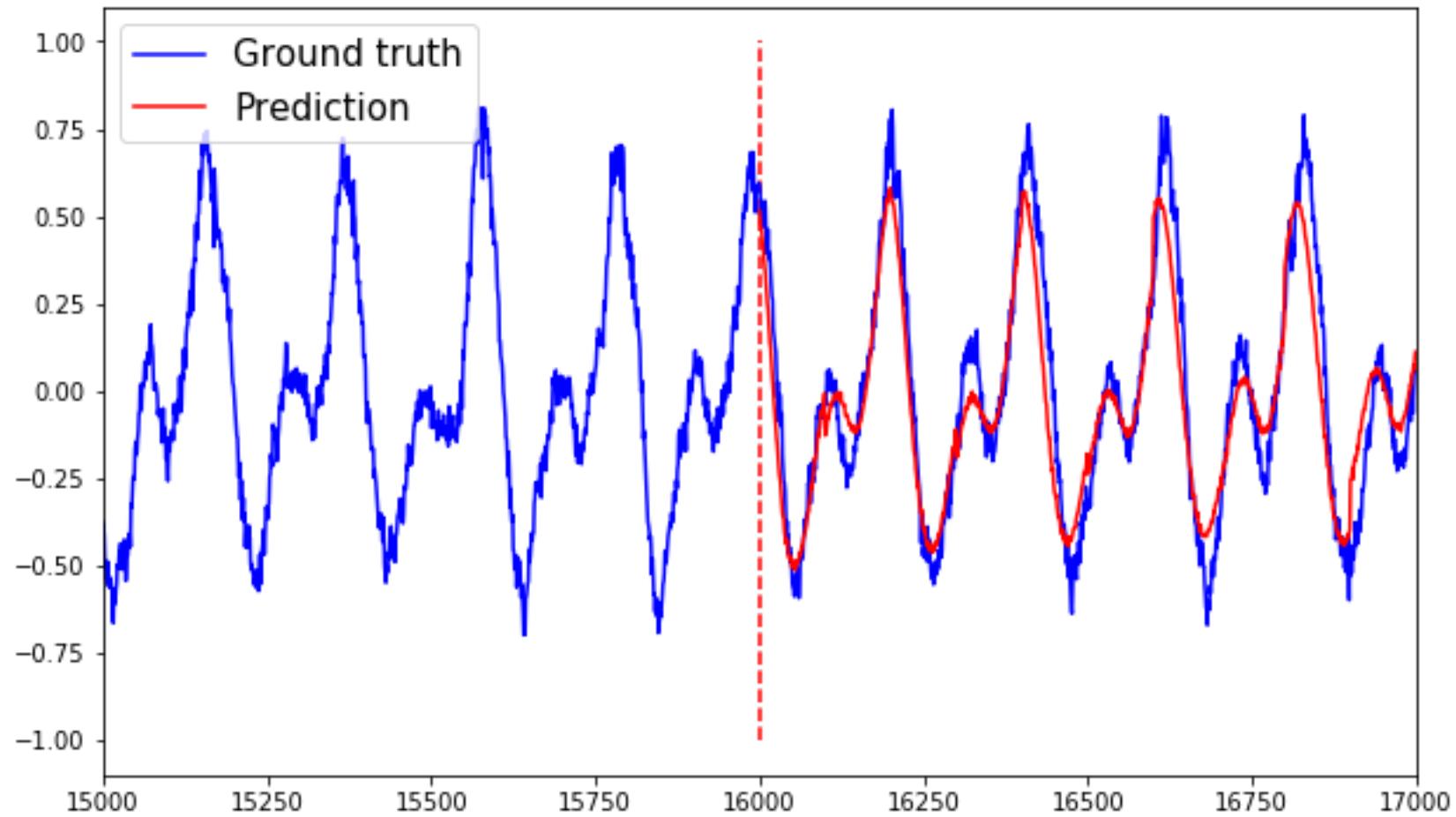
optm = tf.train.AdamOptimizer(LR).minimize(loss)
init = tf.global_variables_initializer()

sess = tf.Session()
```

Prediction Example



Prediction Example



Tutorial 2: RNN for Image Classification

MNIST

Label : 7



Label=5



Label=9



Label=8



Label=6



Label=7



Label=3



Label=8



Label=3



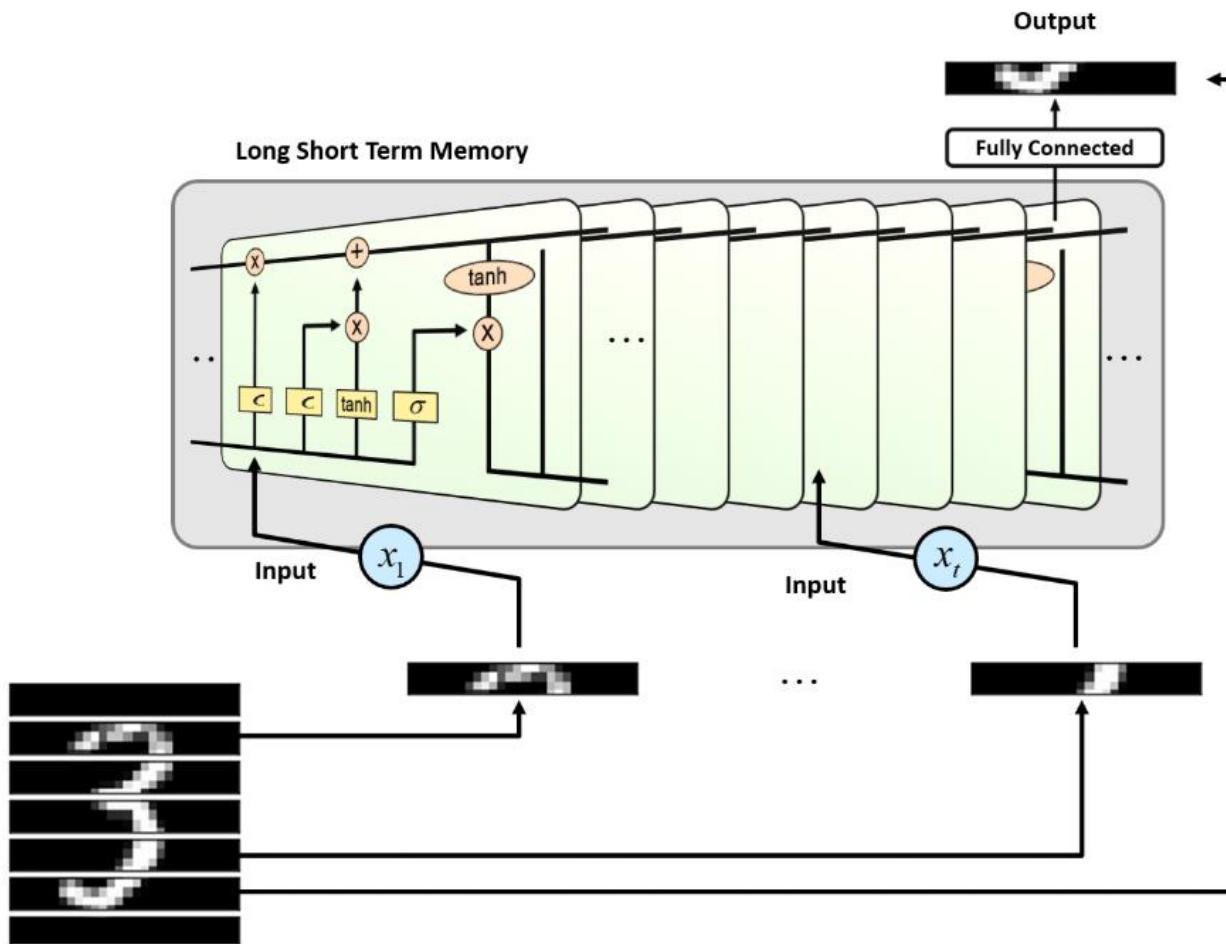
Label=5



Label=5



RNN for Image Prediction



RNN for Image Classification

