(Artificial) Neural Networks: Training

Industrial AI

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Training Neural Networks: Optimization

- Learning or estimating weights and biases of multi-layer perceptron from training data

- 3 key components
  - objective function $f(\cdot)$
  - decision variable or unknown ω
  - constraints $g(\cdot)$

- In mathematical expression

$$\min_{\omega} \quad f(\omega)$$

Training Neural Networks: Loss Function

- Measures error between target values and predictions

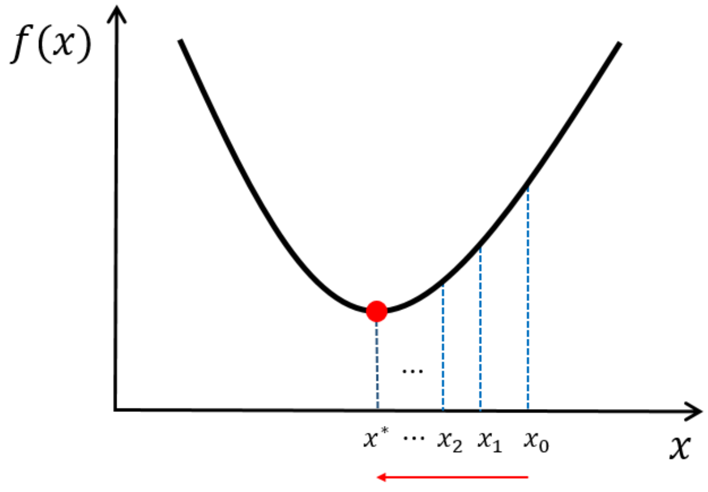
$$\min_{\omega} \sum_{i=1}^{m} \ell\left(h_{\omega}\left(x^{(i)}\right), y^{(i)}\right)$$

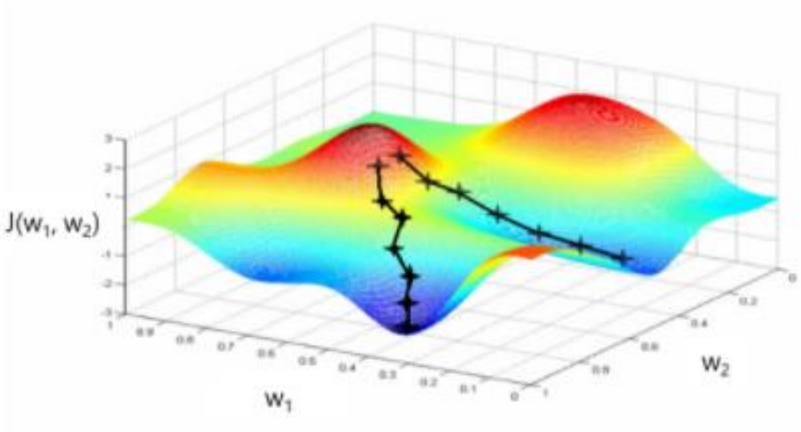
- Example
  - Squared loss (for regression):
  - Cross entropy (for classification):

Training Neural Networks: Gradient Descent

- Negative gradients points directly downhill of the cost function
- We can decrease the cost by moving in the direction of the negative gradient (α is a learning rate)

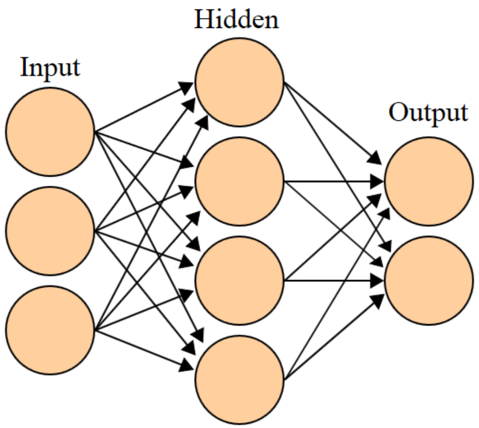
$$\omega \Leftarrow \omega - \alpha \nabla_{\omega} \ell \left(h_{\omega} \left(x^{(i)} \right), y^{(i)} \right)$$





Gradients in ANN

- Learning weights and biases from data using gradient descent
- $\frac{\partial \ell}{\partial \omega}$: too many computations are required for all ω



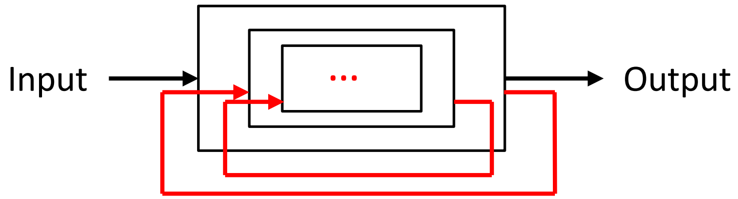
- Structural constraint of NN:
  - Composition of functions
  - Chain rule
  - Dynamic programming

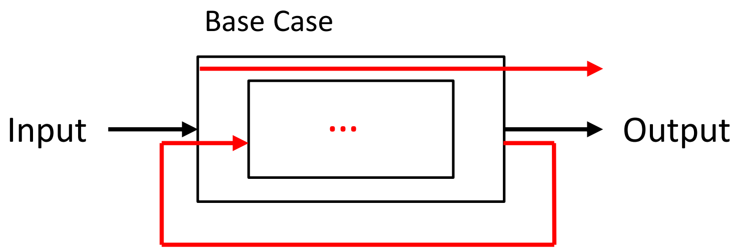
$$x \longrightarrow \boxed{\hat{y} = f_{\omega_1,\cdots,\omega_k}(x)} \longrightarrow y$$

Dynamic Programming

Recursive Algorithm

- One of the central ideas of computer science
- Depends on solutions to smaller instances of the same problem (= sub-problem)
- Function to call itself (it is impossible in the real world)
- Factorial example
  - $n! = n \cdot (n-1) \cdots 2 \cdot 1$





Dynamic Programming

- Dynamic Programming: general, powerful algorithm design technique

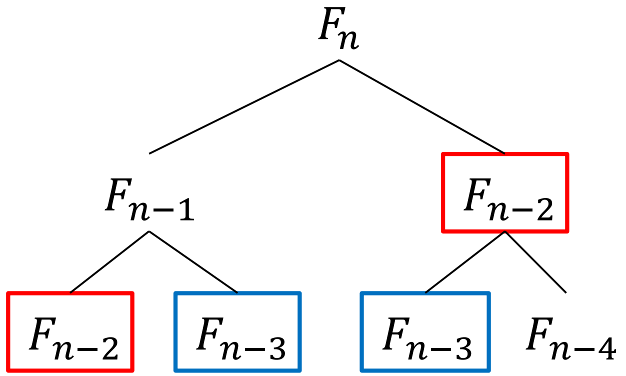
- Fibonacci numbers:

$$
\begin{aligned}
F_1 &= F_2 = 1 \\
F_n &= F_{n-1} + F_{n-2}
\end{aligned}
$$

Naïve Recursive Algorithm

fib(n) :
  if $n \leq 2$: $f = 1$
  else : $f = \text{fib}(n-1) + \text{fib}(n-2)$
  return f

- It works. Is it good?



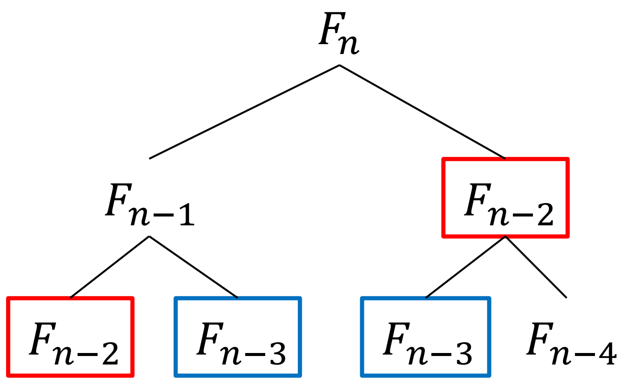
Memorized Recursive Algorithm

```
memo = [ ]
    fib(n) :
        if n in memo : return memo[n]

        if n ≤ 2 :  f = 1
        else :  f = fib(n − 1) + fib(n − 2)

        memo[n] = f
        return f
```

- Benefit?
  - fib(n) only recurses the first time it's called



Dynamic Programming Algorithm

- Memorize (remember) & re-use solutions to subproblems that helps solve the problem

- DP $\approx$ recursion + memorization

Backpropagation

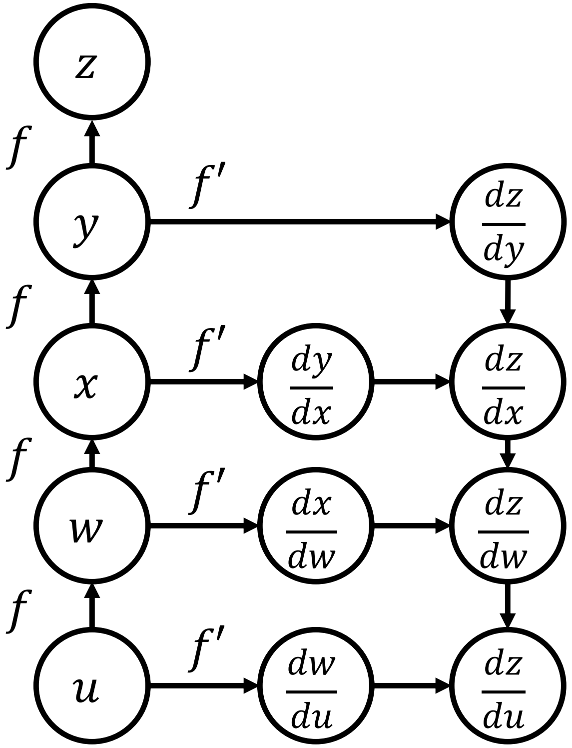
Training Neural Networks: Backpropagation Learning

- Forward propagation
  - the initial information propagates up to the hidden units at each layer and finally produces output

- Backpropagation
  - allows the information from the cost to flow backwards through the network in order to compute the gradients

Backpropagation

- Chain Rule
  - Computing the derivative of the composition of functions
    - $f\big(g(x)\big)' = f'\big(g(x)\big)g'(x)$
    - $\frac{dz}{dx} =$
    - $\frac{dz}{dw} =$
    - $\frac{dz}{du} =$



- Backpropagation
  - Update weights recursively

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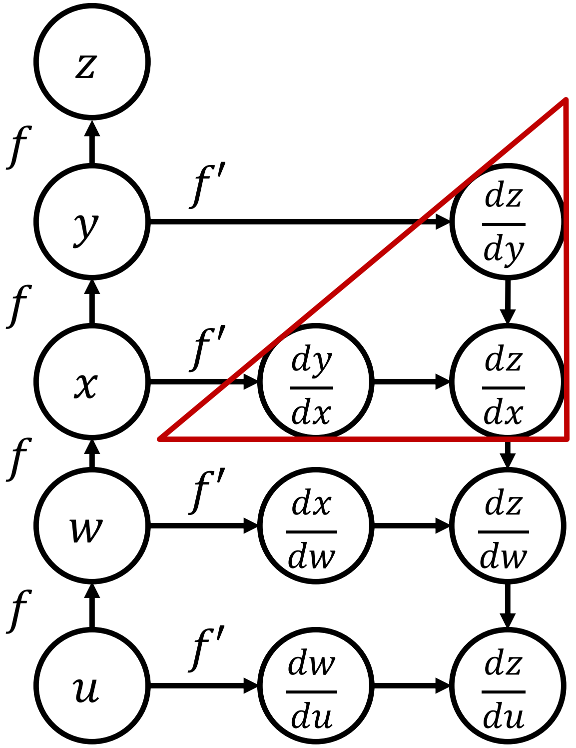

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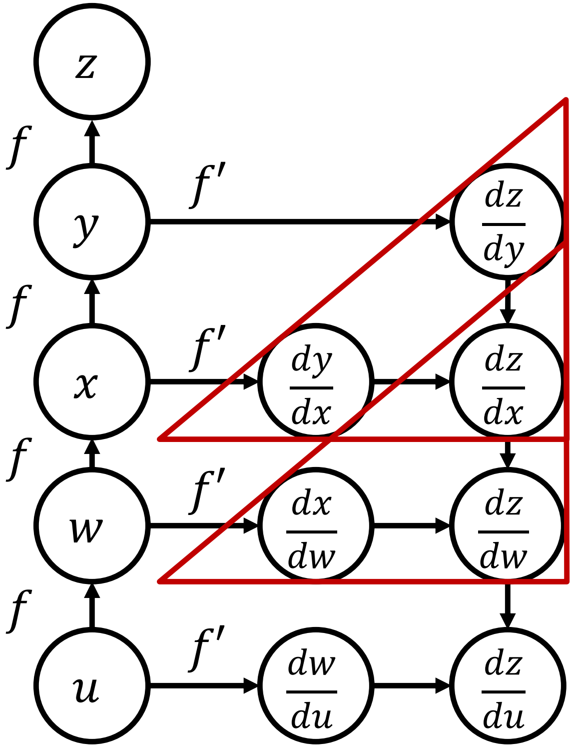

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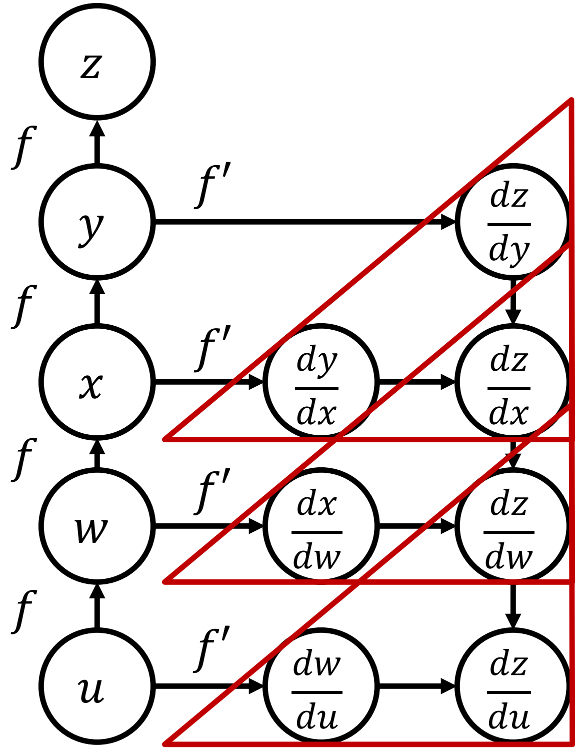
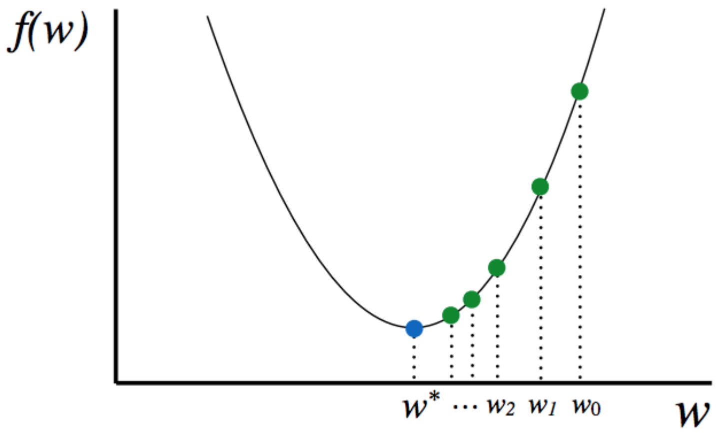


- Backpropagation
  - Update weights recursively with memory

Training Neural Networks with TensorFlow

- Optimization procedure

Start at a random point
Repeat
    Determine a descent direction
    Choose a step size
    Update
Until stopping criterion is satisfied

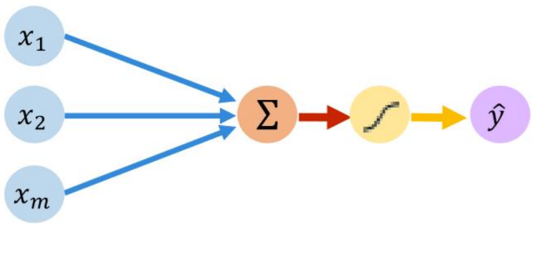


- It is not easy to numerically compute gradients in network in general.
  - The good news: people have already done all the "hard work" of developing numerical solvers (or libraries)
  - There are a wide range of tools → We will use the TensorFlow

Core Foundation Review

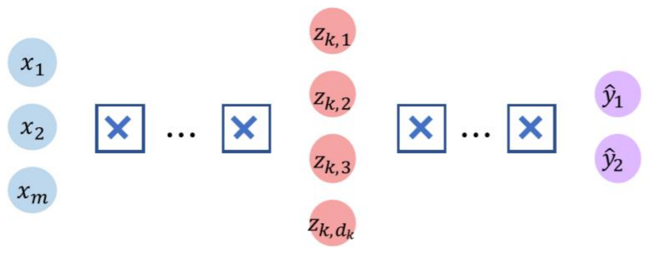
The Perceptron

- Structural building blocks
- Nonlinear activation functions



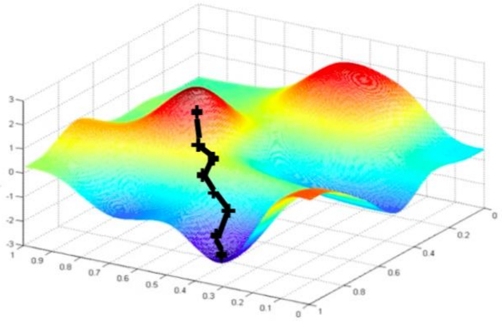
Neural Networks

- Stacking Perceptrons to form neural networks
- Optimization through backpropagation



Training in Practice

- Adaptive learning
- Batching
- Regularization



Source: 6.S191 Intro. to Deep Learning at MIT