

Dynamic Programming

Industrial AI Lab.

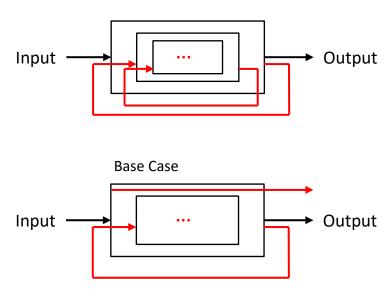
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Recursive Algorithm

- One of the central ideas of computer science
- Depends on solutions to smaller instances of the same problem (= sub-problem)
- Function to call itself (it is impossible in the real world)
- Factorial example

$$-n! = n \cdot (n-1) \cdots 2 \cdot 1$$



Dynamic Programming

- Dynamic Programming: general, powerful algorithm design technique
- Fibonacci numbers:

$$F_1 = F_2 = 1 \ F_n = F_{n-1} + F_{n-2}$$

Naïve Recursive Algorithm

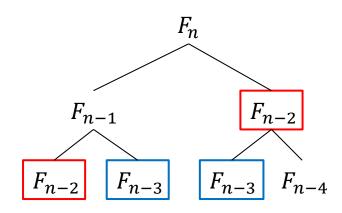
```
fib(n):

if n \le 2: f = 1

else: f = fib(n-1) + fib(n-2)

return f
```

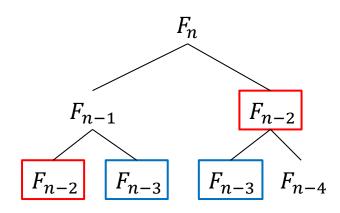
• It works. Is it good?



Memorized Recursive Algorithm

```
memo = []
fib(n):
if n in memo : return memo[n]
if n \le 2 : f = 1
else : f = fib(n - 1) + fib(n - 2)
memo[n] = f
return f
```

- Benefit?
 - fib(n) only recurses the first time it's called



Dynamic Programming Algorithm

 Memorize (remember) & re-use solutions to subproblems that helps solve the problem

• DP ≈ recursion + memorization

Example 1: Climbing a Stair

• You are climbing a stair case. Each time you can either make 1 step, 2 steps, or 3 steps. How many distinct ways can you climb if the stairs has n=30 steps?



Example 2: Knapsack Problem

- Burglar (or thief) can carry at most 20 kg (= maximum capacity = 20)
- Quickly decide which item to carry

	items	1	2	3	4	5	6
	weight	10	9	4	2	1	20
	value	175	90	20	50	10	200

- Approaches
 - Guess
 - Exhaustive search if possible
 - "smarter way" → recursive or dynamic programming

$$key\ ideas = original\ problem \rightarrow \left\{ \begin{array}{l} subproblem \rightarrow \\ subproblem \rightarrow \\ subproblem \rightarrow \\ subproblem \rightarrow \\ subproblem \rightarrow \end{array} \right. \left. \left\{ \begin{array}{l} subproblem \rightarrow \\ subproblem \rightarrow \\ subproblem \rightarrow \\ \end{array} \right. \right.$$

Example 2: Knapsack Problem

• "smarter way" → recursive or dynamic programming

```
Suppose we have the following function:
   [value, taken] = chooseBest(items(1:6),maxWeight)
1) item 1 is not taken
   [v 1,t 1] = chooseBest(items(2:6),maxWeight)
2) item 1 is taken
   [v_2,t_2] = chooseBest(items(2:6),maxWeight - weights(1))
          v 2 = v 2 + values(1)
          t_2 = [items(1), t_2]
```