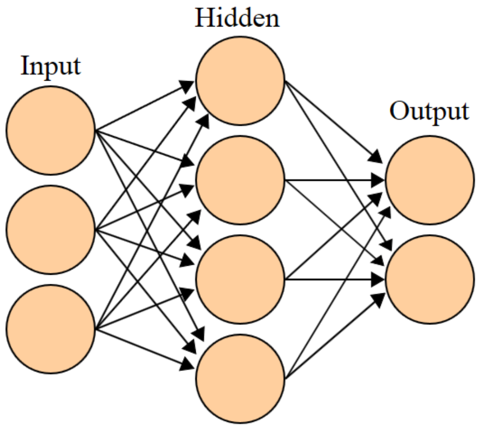
# ANN Training

Industrial AI

Prof. Seungchul Lee

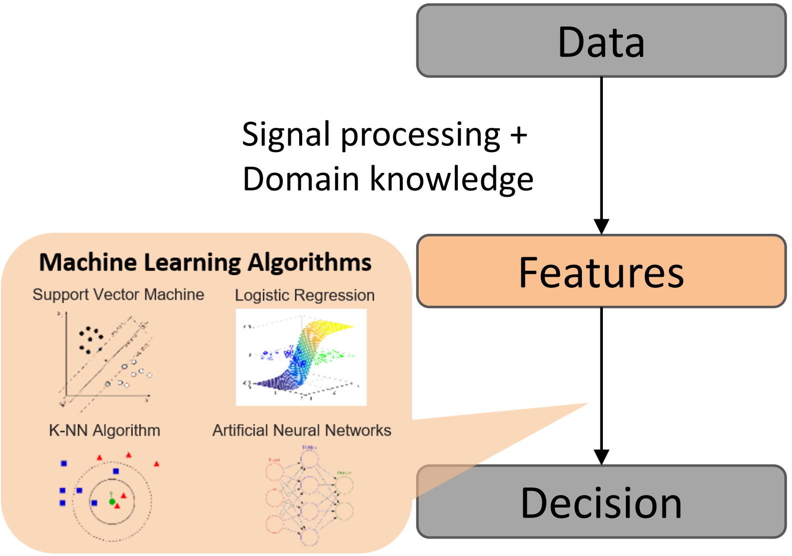
# Summary

- Learning weights and biases from data using gradient descent




$$x \longrightarrow \boxed{\hat{y} = f_{\omega_1,\cdots,\omega_k}(x)} \longrightarrow y$$

# *Machine* Learning

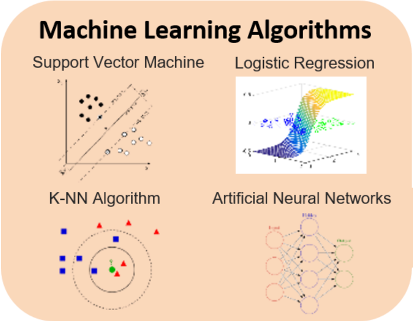


# *Machine* Learning

Data

Signal processing +
Domain knowledge

Features




Decision

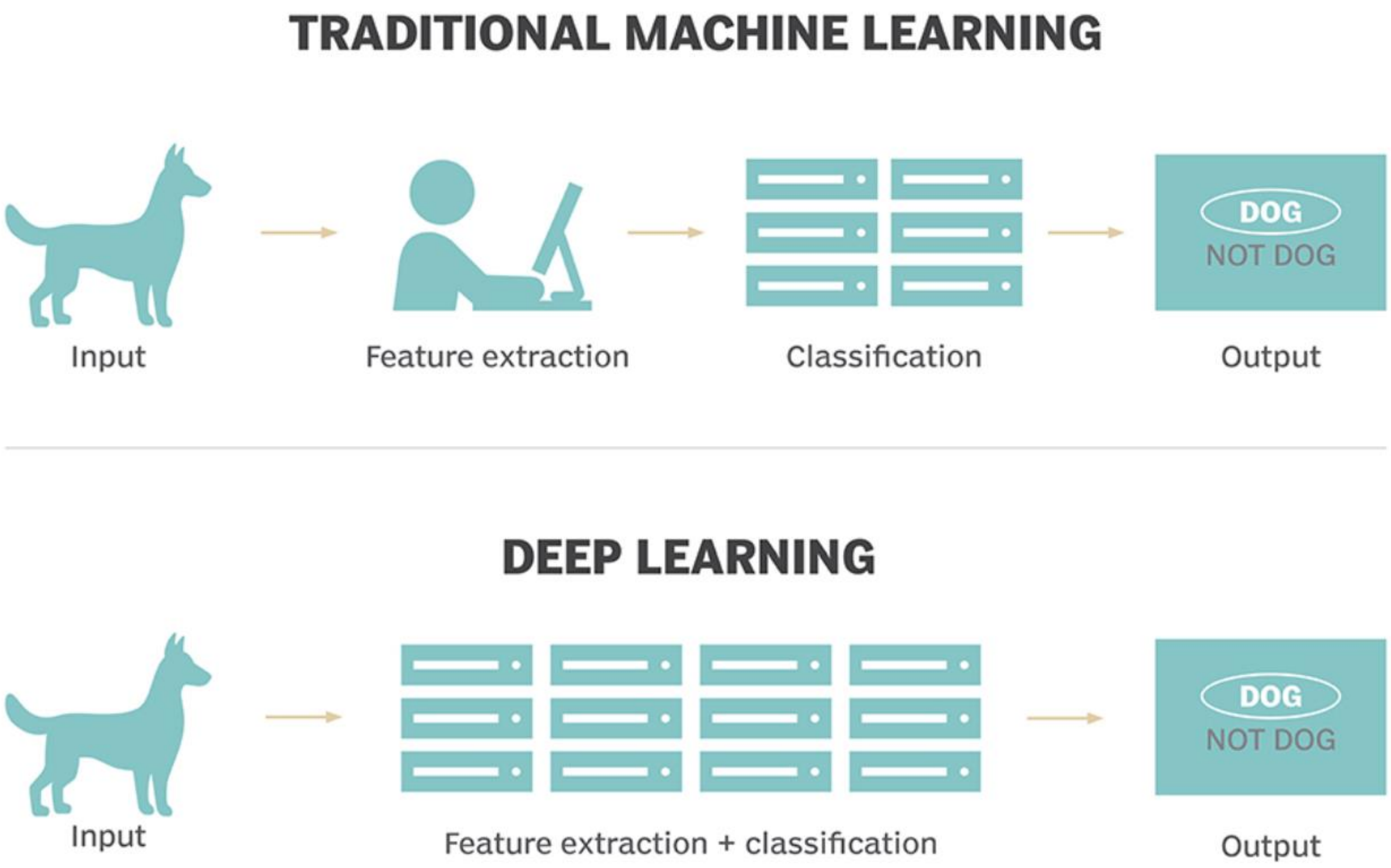
# *Deep* Learning

Data

Less depends on
domain knowledge

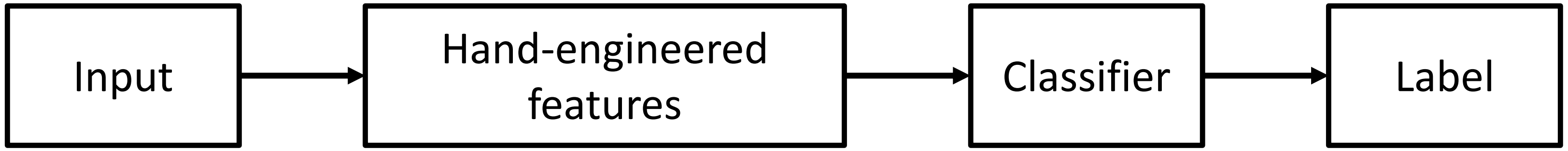
Decision

# Recall Supervised Learning Setup

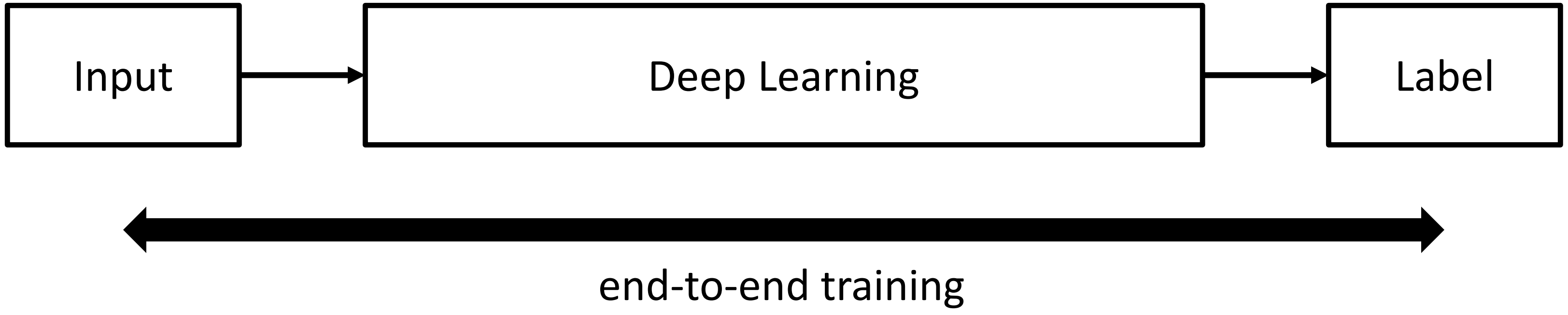


# Machine Learning and Deep Learning

- Machine Learning



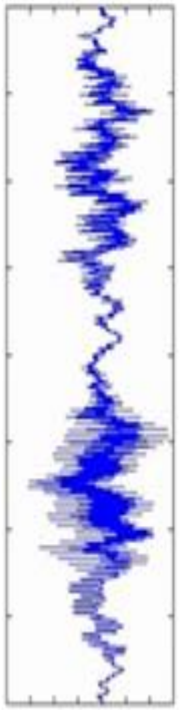

- Deep supervised learning



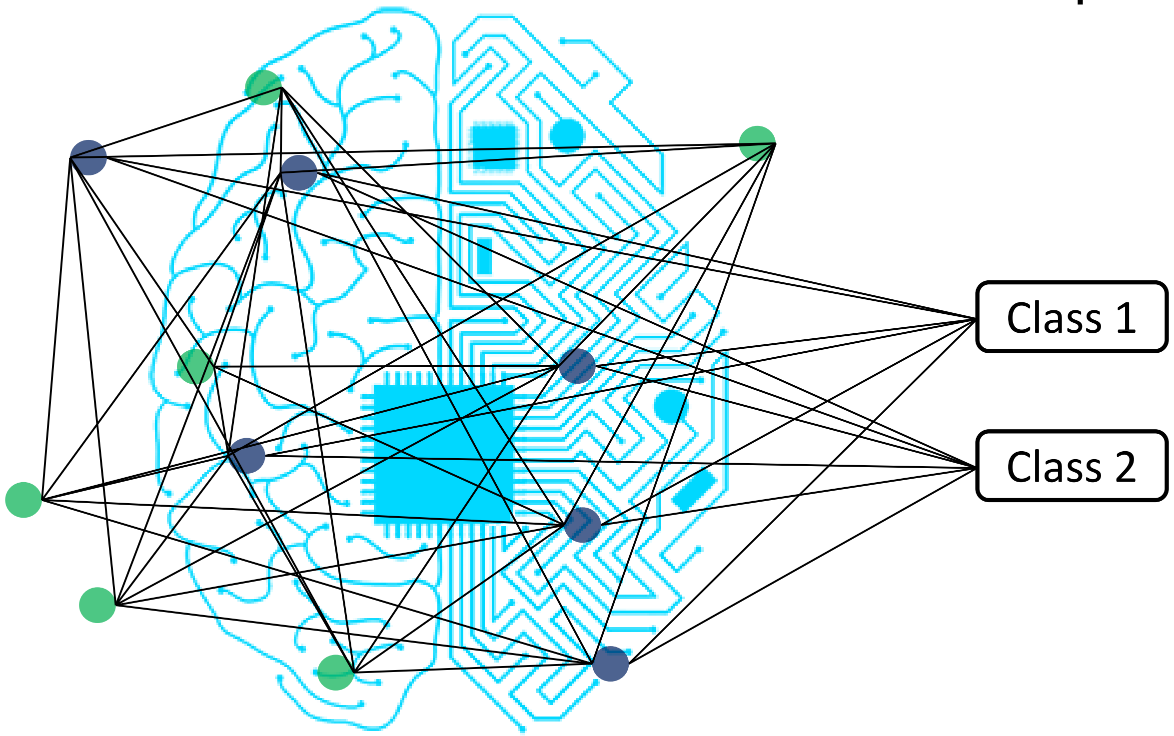
# Artificial Neural Networks

- Complex/Nonlinear universal function approximator
  - Linearly connected networks
  - Simple nonlinear neurons

**Input**



**Output**



# Artificial Neural Networks

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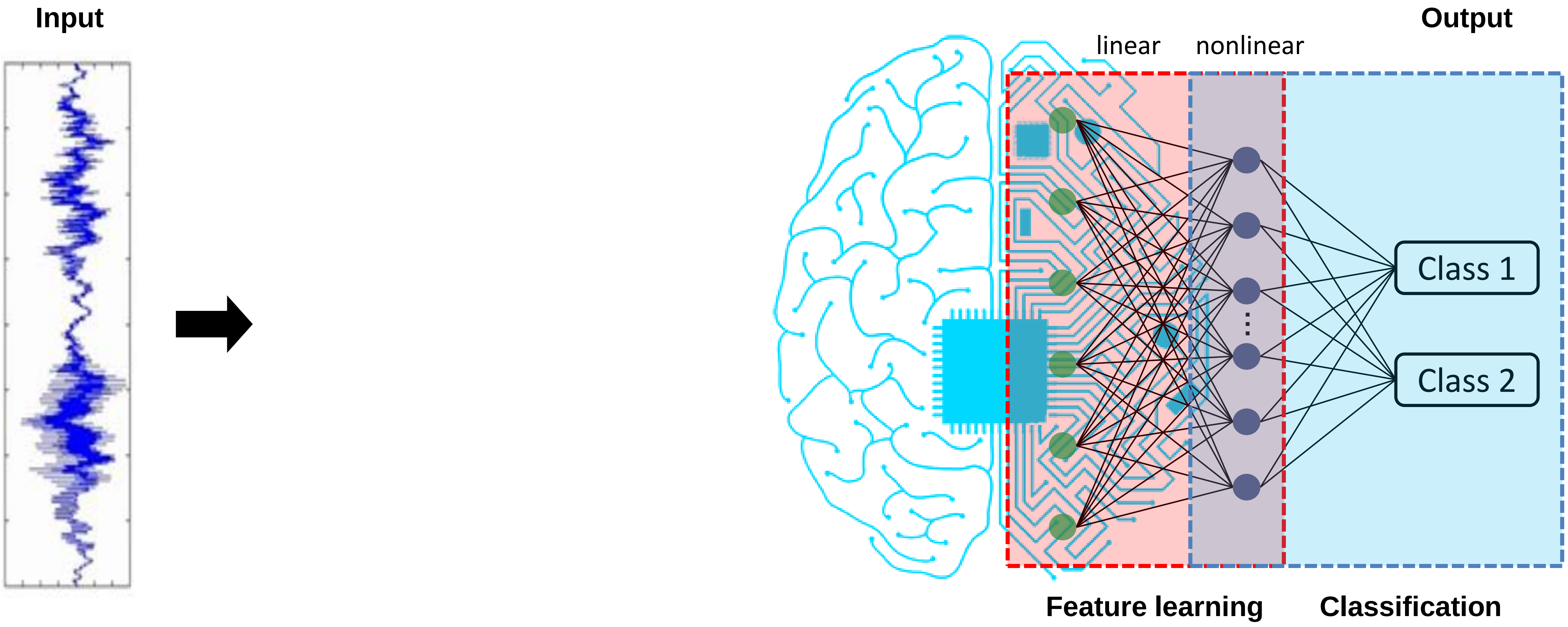
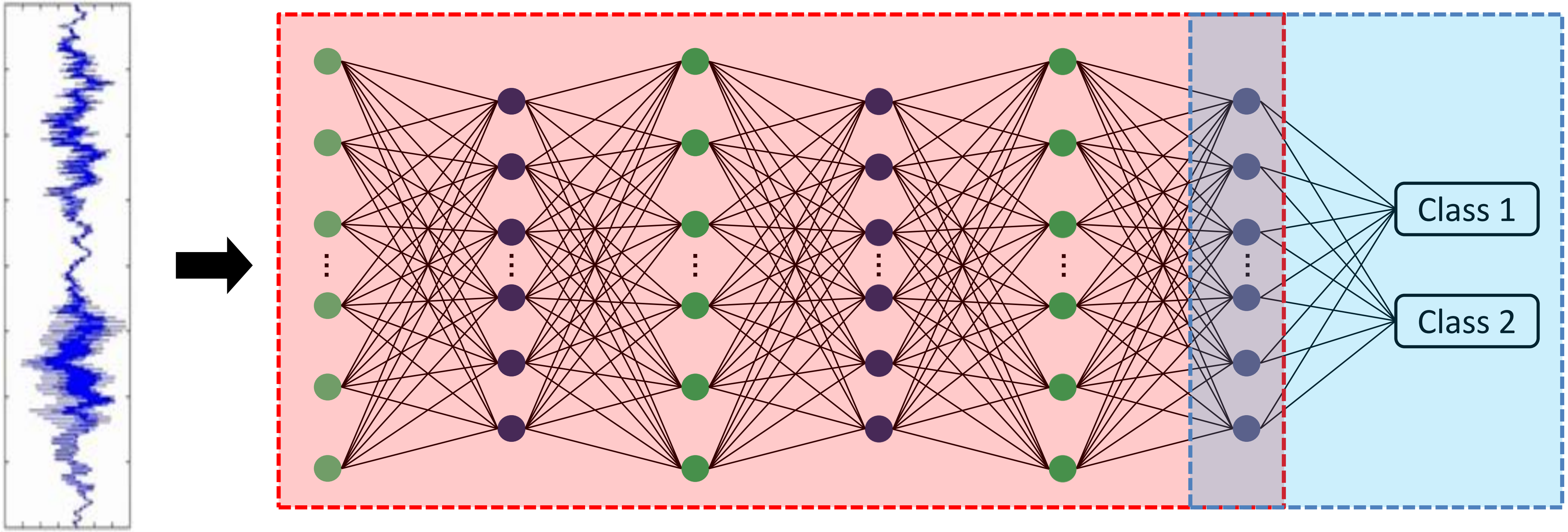
  - Simple nonlinear neurons



# Deep Artificial Neural Networks

- Complex/Nonlinear universal function approximator
  - Linearly connected networks
  - Simple nonlinear neurons



# Training: Backpropagation

# Training Neural Networks: Optimization

- Learning or estimating weights and biases of multi-layer perceptron from training data

- 3 key components
  - objective function $f(\cdot)$
  - decision variable or unknown $\omega$
  - constraints $g(\cdot)$

- In mathematical expression

$$\min_{\omega} \quad f(\omega)$$

# Training Neural Networks: Loss Function

- Measures error between target values and predictions

$$\min_{\omega} \sum_{i=1}^{m} \ell\left(h_{\omega}\left(x^{(i)}\right), y^{(i)}\right)$$

- Example
  - Squared loss (for regression):

$$\frac{1}{m} \sum_{i=1}^{m} \left(h_{\omega}\left(x^{(i)}\right) - y^{(i)}\right)^2$$

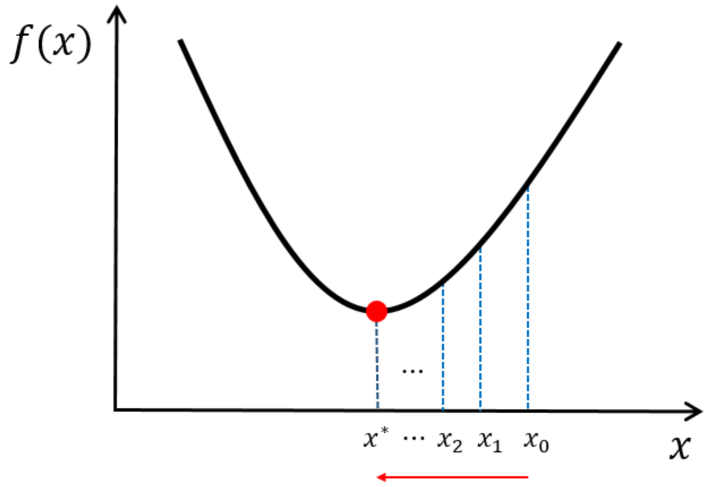
  - Cross entropy (for classification):

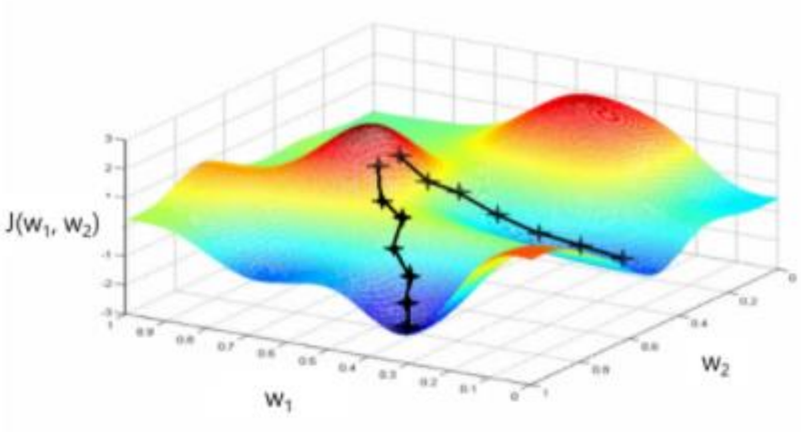
$$-\frac{1}{m} \sum_{i=1}^{m} y^{(i)} \log\left(h_{\omega}\left(x^{(i)}\right)\right) + \left(1 - y^{(i)}\right) \log\left(1 - h_{\omega}\left(x^{(i)}\right)\right)$$

# Training Neural Networks: Gradient Descent

- Negative gradients points directly downhill of the cost function
- We can decrease the cost by moving in the direction of the negative gradient ($\alpha$ is a learning rate)

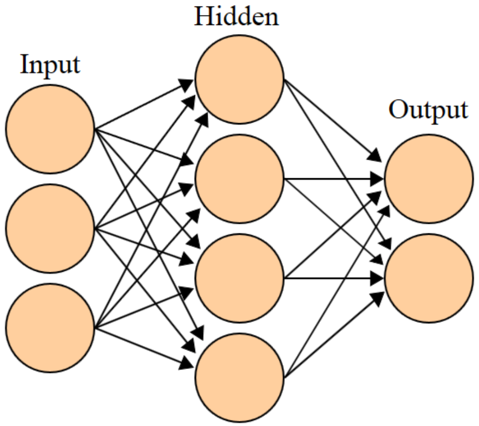
$$\omega \Leftarrow \omega - \alpha \nabla_{\omega} \ell \left( h_{\omega} \left( x^{(i)} \right), y^{(i)} \right)$$



# Gradients in ANN

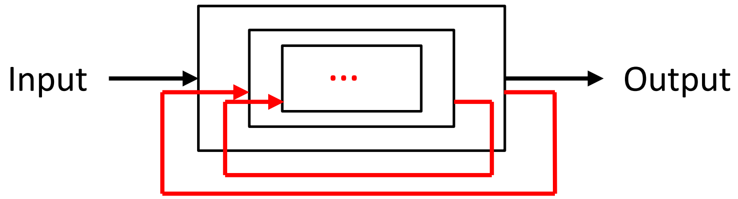
- Learning weights and biases from data using gradient descent
- $\frac{\partial \ell}{\partial \omega}$: too many computations are required for all $\omega$
- Structural constraint of NN:
  - Composition of functions
  - Chain rule
  - Dynamic programming

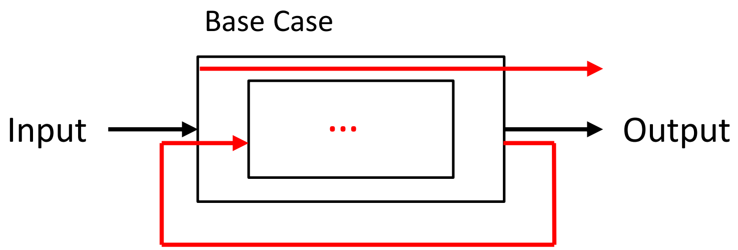



$$x \longrightarrow \boxed{\hat{y} = f_{\omega_1,\cdots,\omega_k}(x)} \longrightarrow y$$

# Recursive Algorithm

- One of the central ideas of computer science
- Depends on solutions to smaller instances of the same problem ( = sub-problem)
- Function to call itself (it is impossible in the real world)
- Factorial example
  - $n! = n \cdot (n-1) \cdots 2 \cdot 1$



# Dynamic Programming

- Dynamic Programming: general, powerful algorithm design technique

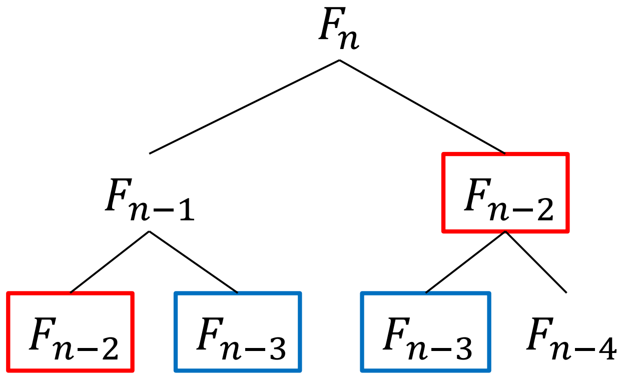
- Fibonacci numbers:

$$
\begin{aligned}
F_1 &= F_2 = 1 \\
F_n &= F_{n-1} + F_{n-2}
\end{aligned}
$$

# Naïve Recursive Algorithm

fib($n$) :
  if $n \leq 2$ : $f = 1$
  else : $f = \text{fib}(n-1) + \text{fib}(n-2)$
  return $f$

- It works. Is it good?



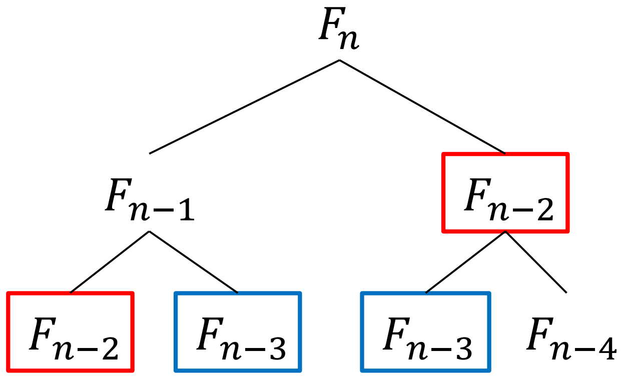
# Memorized Recursive Algorithm

```
memo = [ ]
    fib(n) :
        if n in memo : return memo[n]

        if n ≤ 2 :  f = 1
        else :  f = fib(n − 1) + fib(n − 2)

        memo[n] = f
        return f
```

- Benefit?
  - fib(n) only recurses the first time it's called



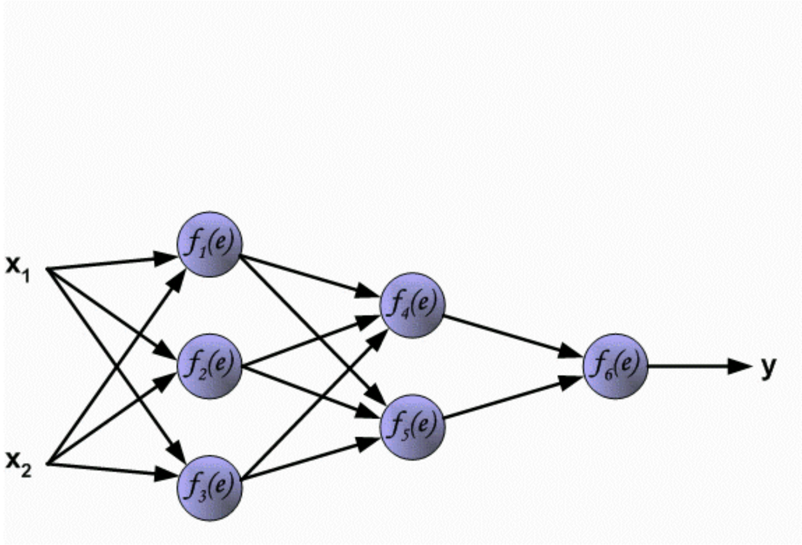
# Dynamic Programming Algorithm

- Memorize (remember) & re-use solutions to subproblems that helps solve the problem

- DP $\approx$ recursion + memorization

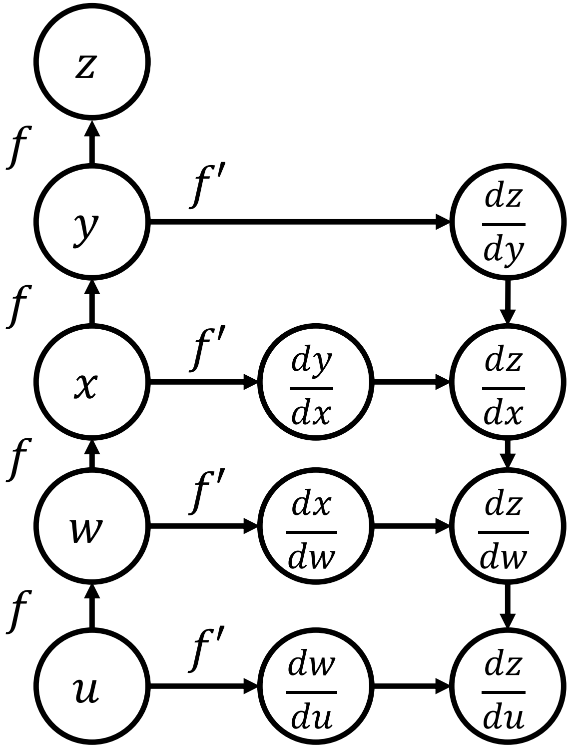
# Training Neural Networks: Backpropagation Learning

- Forward propagation
  - the initial information propagates up to the hidden units at each layer and finally produces output
- Backpropagation
  - allows the information from the cost to flow backwards through the network in order to compute the gradients



# Backpropagation

- Chain Rule
  - Computing the derivative of the composition of functions
    - $f\left(g(x)\right)' = f'\left(g(x)\right)g'(x)$
    - $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$
    - $\frac{dz}{dw} = \left(\frac{dz}{dy} \cdot \frac{dy}{dx}\right) \cdot \frac{dx}{dw}$
    - $\frac{dz}{du} = \left(\frac{dz}{dy} \cdot \frac{dy}{dx} \cdot \frac{dx}{dw}\right) \cdot \frac{dw}{du}$




- Backpropagation
  - Update weights recursively

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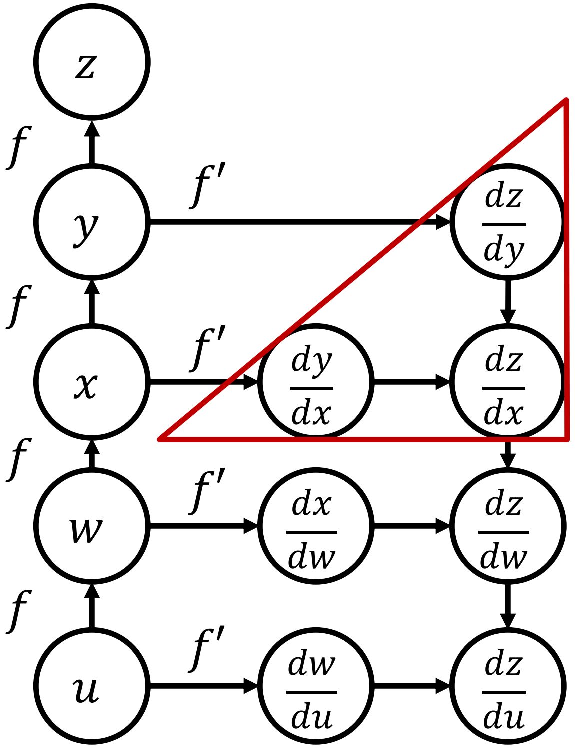


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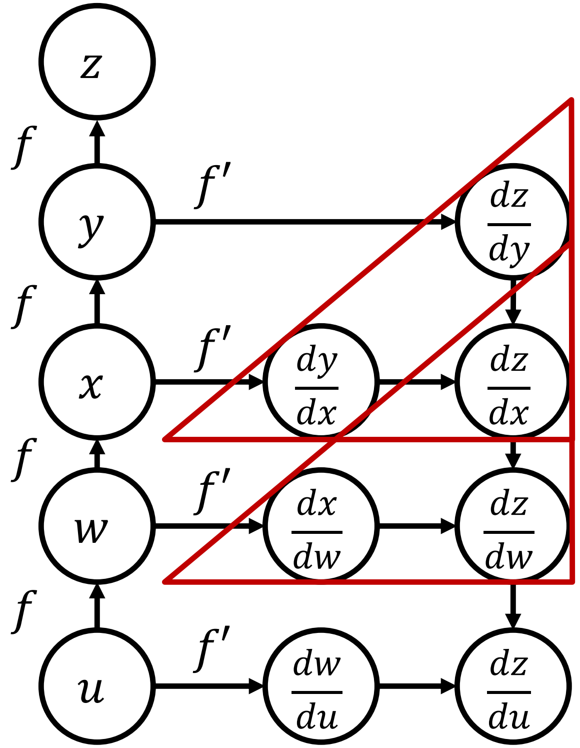


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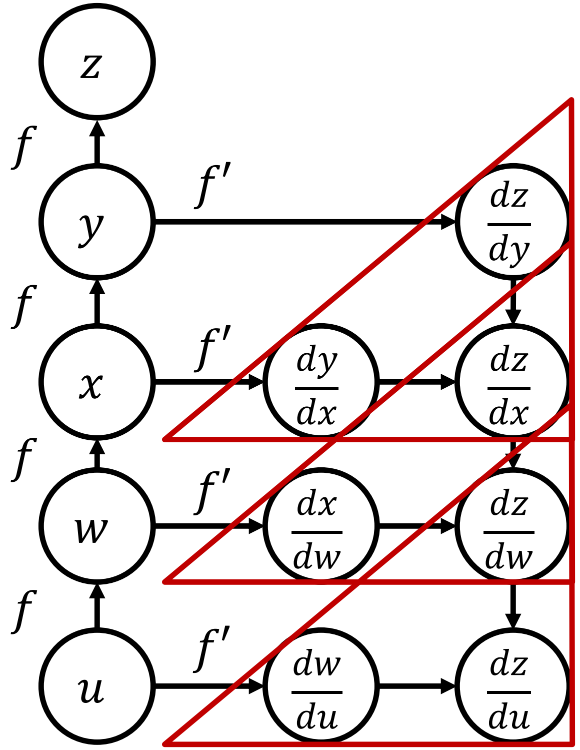
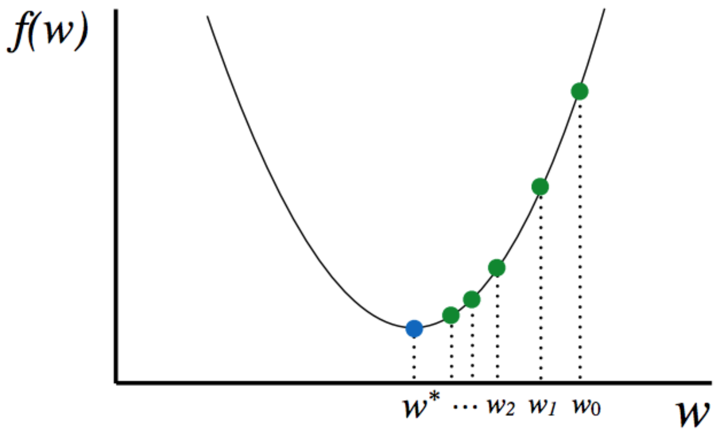



- Backpropagation
  - Update weights recursively with memory

# Training Neural Networks

- Optimization procedure

Start at a random point
**Repeat**
    Determine a descent direction
    Choose a step size
    Update
**Until** stopping criterion is satisfied

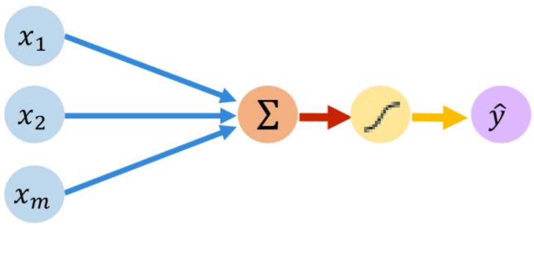



- It is not easy to numerically compute gradients in network in general.
  - The good news: people have already done all the "hard work" of developing numerical solvers (or libraries)
  - There are a wide range of tools → We will use the TensorFlow

# Core Foundation Review

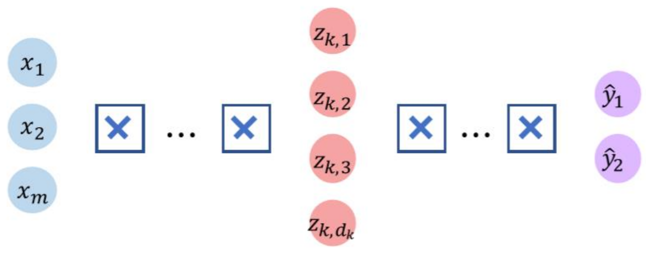
## The Perceptron

- Structural building blocks
- Nonlinear activation functions




## Neural Networks

- Stacking Perceptrons to form neural networks
- Optimization through backpropagation




## Training in Practice

- Adaptive learning
- Batching
- Regularization