Neural Network Architectures for Time-Series

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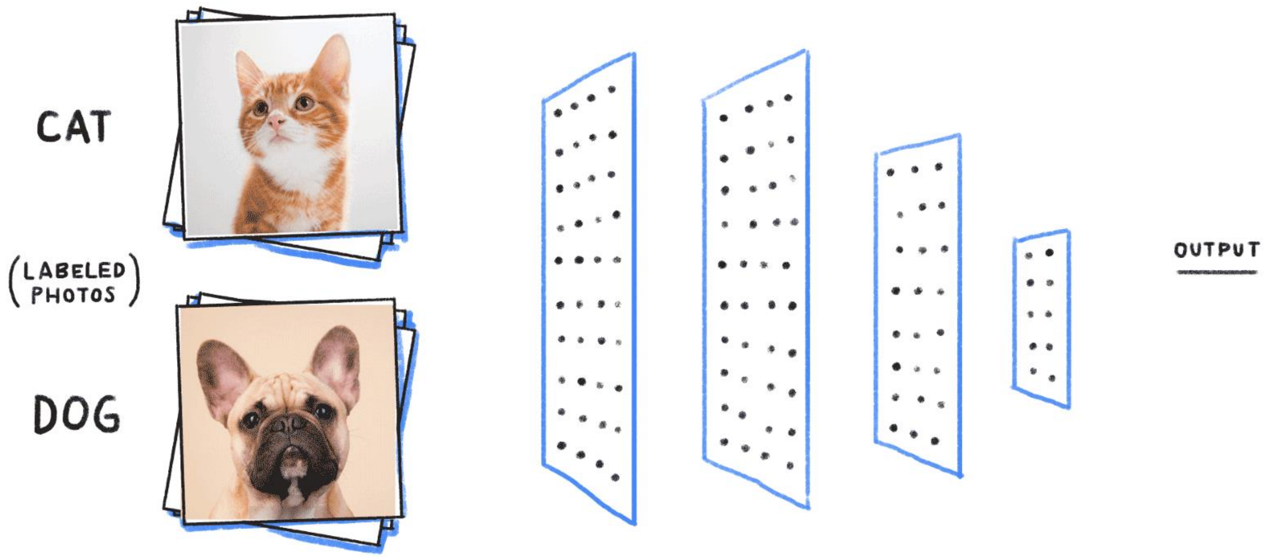
So Far

- Regression, Classification, Dimension Reduction,
- Based on snapshot-type data

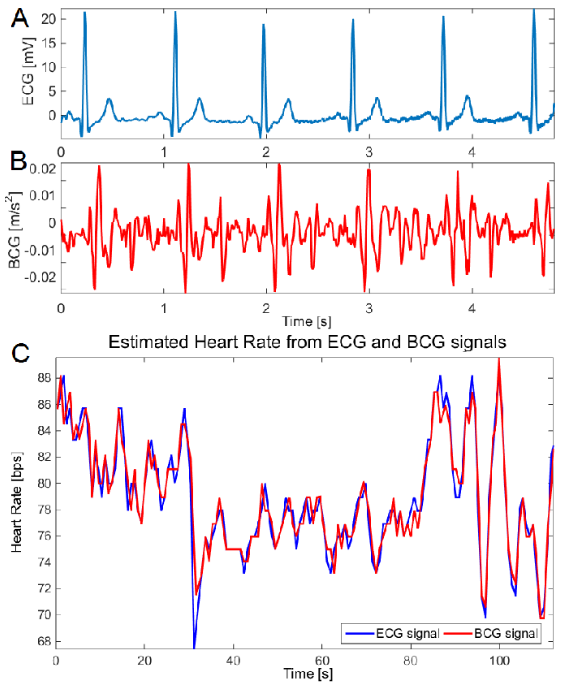


Sequence Matters

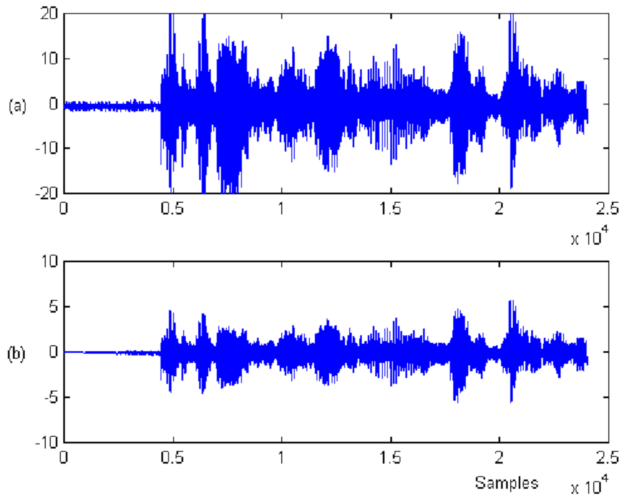
What is a Sequence ?

- Sentence
  - "This morning I took the dog for a walk."

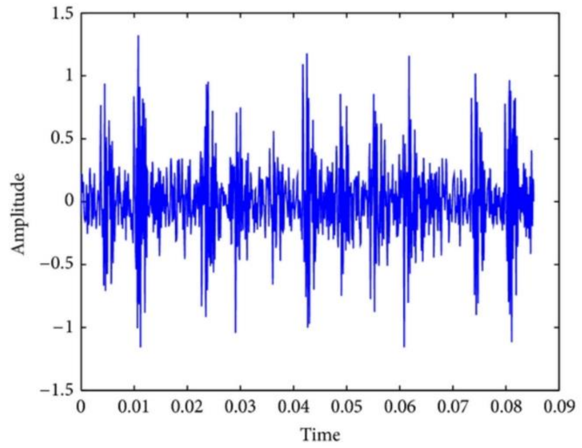
- Medical signals



- Speech waveform



- Vibration measurement



Sequence Modeling

- Most of the real-world data is time-series
- There are important bits to be considered
  - Past events
  - Relationship between events
- Use the past and present observations to predict the future

(Deterministic) Time Series Data

- For example

$$y[0] = 1, \quad y[1] = \frac{1}{2}, \quad y[2] = \frac{1}{4}, \quad \cdots$$

- Closed-form

$$y[n] = \left(\frac{1}{2}\right)^n, \quad n \geq 0$$

- Linear difference equation (LDE) and initial condition

$$y[n] = \frac{1}{2} y[n-1], \quad y[0] = 1$$

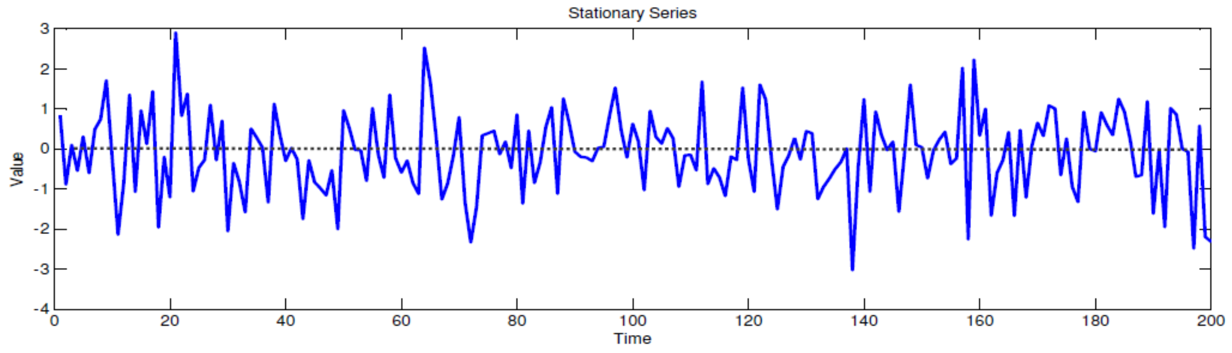
- High order LDEs

$$y[n] = \alpha_1 y[n-1] + \alpha_2 y[n-2]$$

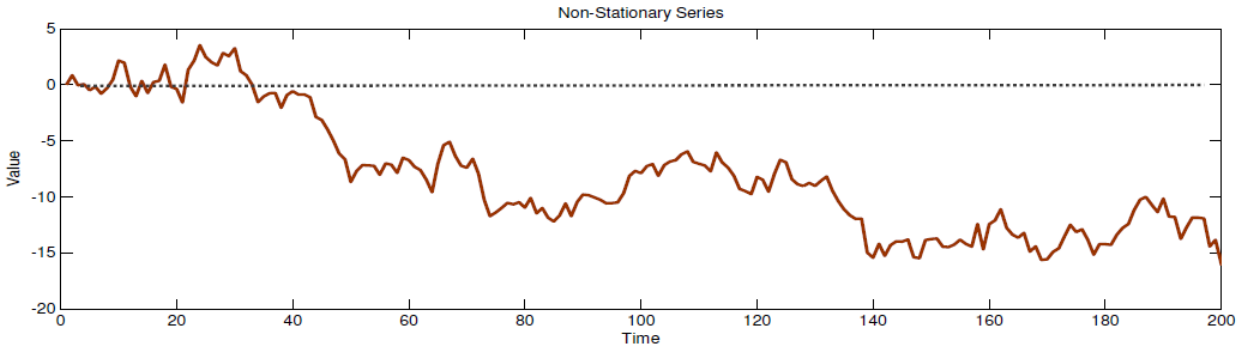
$$y[n] = \alpha_1 y[n-1] + \alpha_2 y[n-2] + \cdots + \alpha_k y[n-k]$$

(Stochastic) Time Series Data

- Stationary

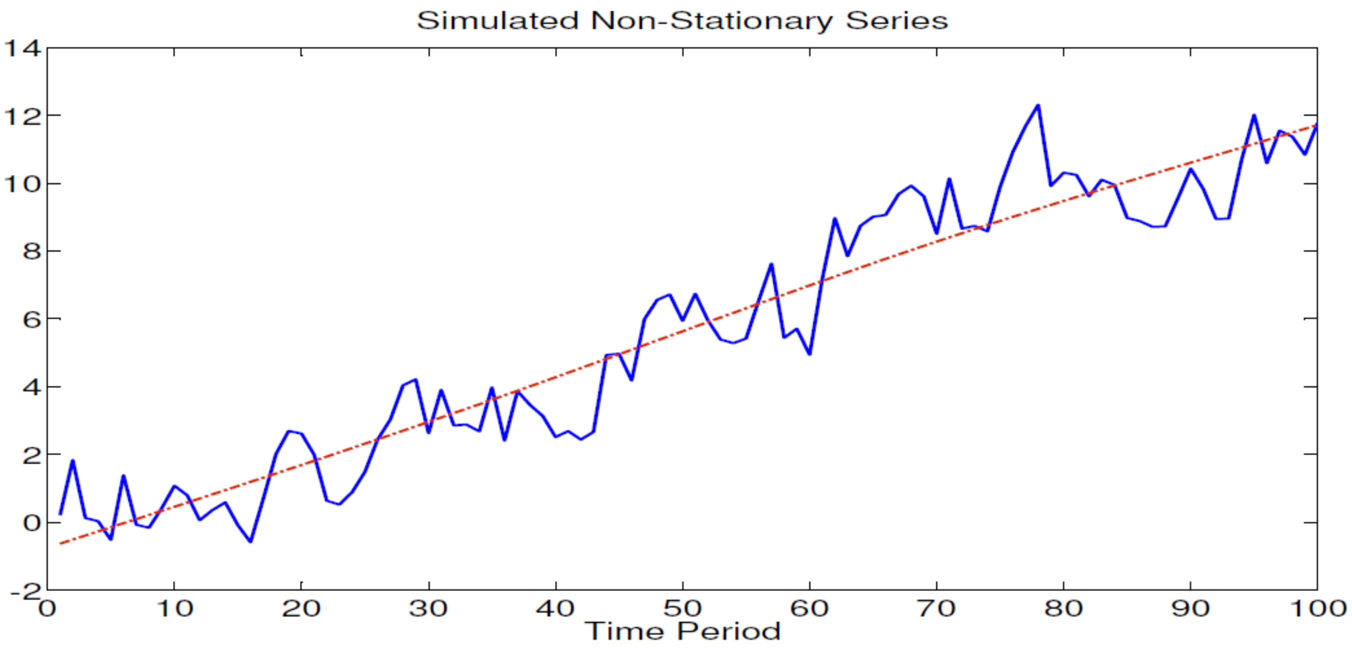


- Non-stationary
  - Mean and variance change over time



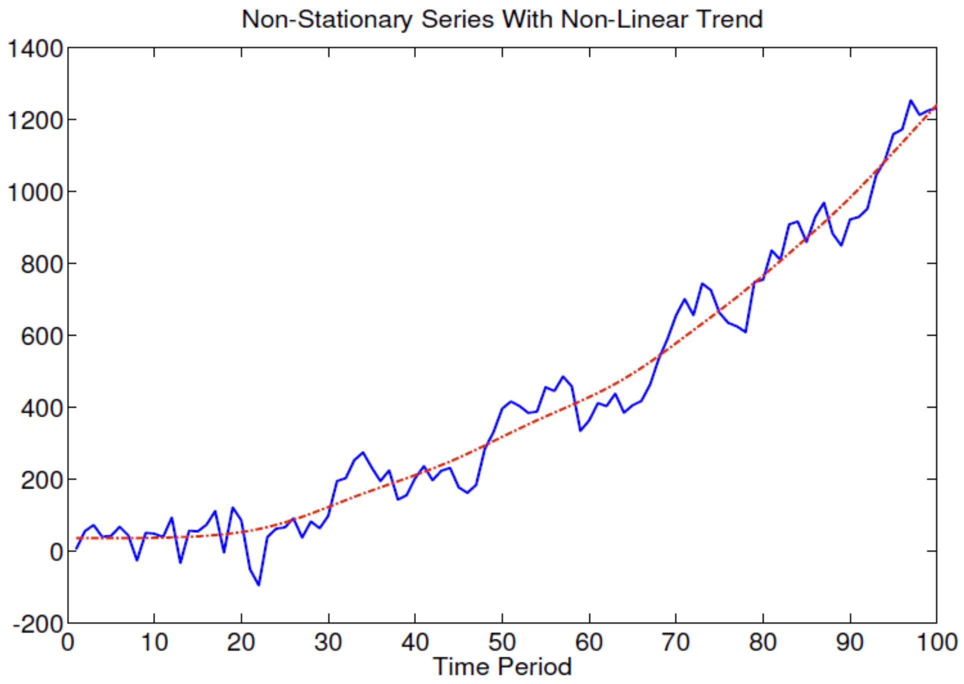
Dealing with Non-Stationarity

- Linear trends



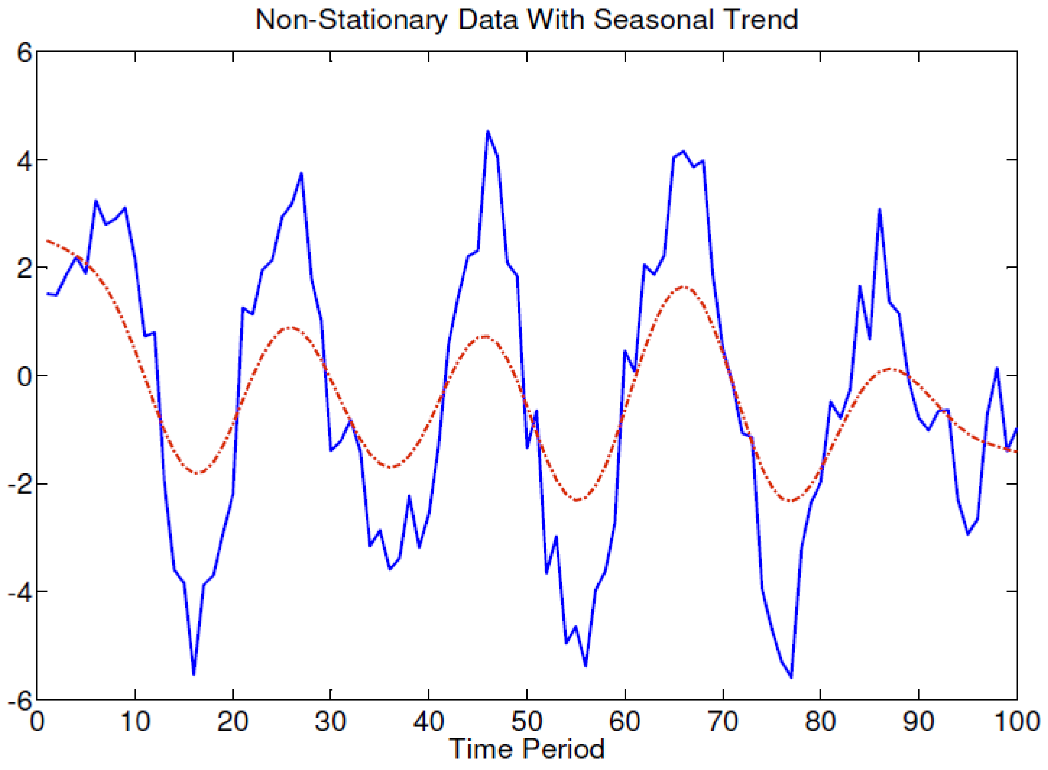
Dealing with Non-Stationarity

- Non-linear trends



Dealing with Non-Stationarity

- Seasonal trends



Sequential Process

Sequential Process

- Most classifiers ignored the sequential aspects of data

- Consider a system which can occupy one of *N* discrete states or categories

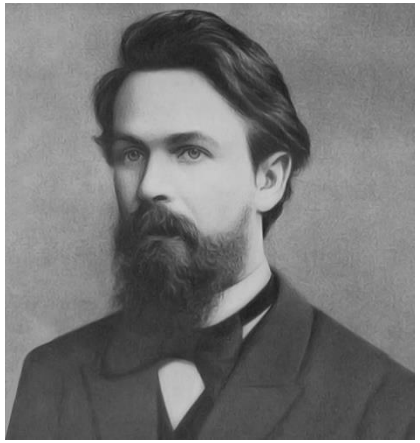
$$q_t \in \{S_1, S_2, \cdots, S_N\}$$

- We are interested in stochastic systems, in which state evolution is random
- Any joint distribution can be factored into a series of conditional distributions

$$p(q_0, q_1, \cdots, q_T) = p(q_0)\, p(q_1 \mid q_0)\, p(q_2 \mid q_1 q_0)\, p(q_3 \mid q_2 q_1 q_0) \cdots$$

Almost impossible to compute !

Markov Chain



- Markovian property (assumption)
  - Information state: sufficient statistic of history

$$p(q_{t+1} \mid q_t, \cdots, q_0) = p(q_{t+1} \mid q_t)$$

- Tractable in computation of joint distribution

$$\begin{aligned} p(q_0, q_1, \cdots, q_T) &= p(q_0)\, p(q_1 \mid q_0)\, p(q_2 \mid q_1, q_0)\, p(q_3 \mid q_2, q_1, q_0) \cdots \\ &= p(q_0)\, p(q_1 \mid q_0)\, p(q_2 \mid q_1)\, p(q_3 \mid q_2) \cdots \end{aligned}$$

- Future is independent of past given present
  - Rain → snow → sunny → sunny → sunny → rain → snow → ??
  - ~~Rain → snow → sunny → sunny → sunny → rain →~~ snow → ??

State Transition Matrix

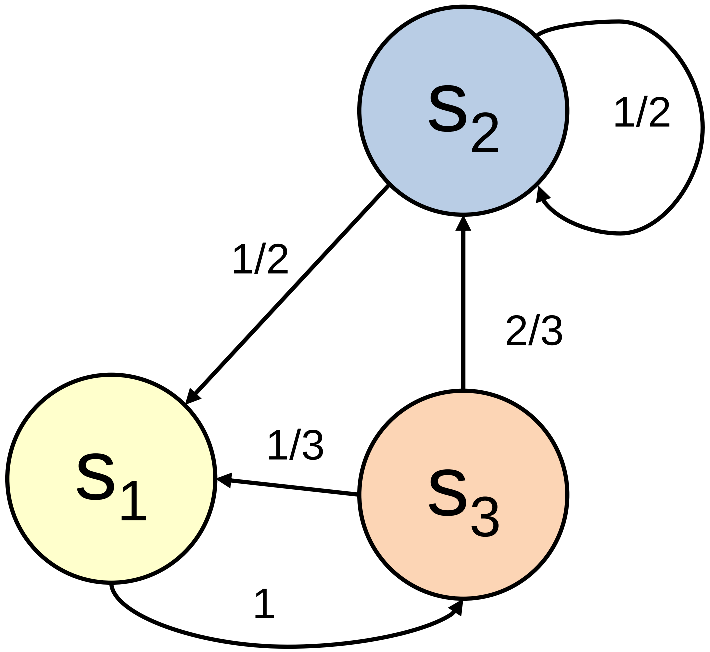
- For a Markov state s and successor state s', the state transition probability is defined by

$$\mathcal{P}_{ss'} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s\right]$$

- State transition matrix P defines transition probabilities from all states s to all successor states s',

$$\mathcal{P} = \textit{from} \begin{array}{c} \textit{to} \\ \begin{bmatrix} \mathcal{P}_{11} & \dots & \mathcal{P}_{1n} \\ \vdots & & \\ \mathcal{P}_{n1} & \dots & \mathcal{P}_{nn} \end{bmatrix} \end{array}$$

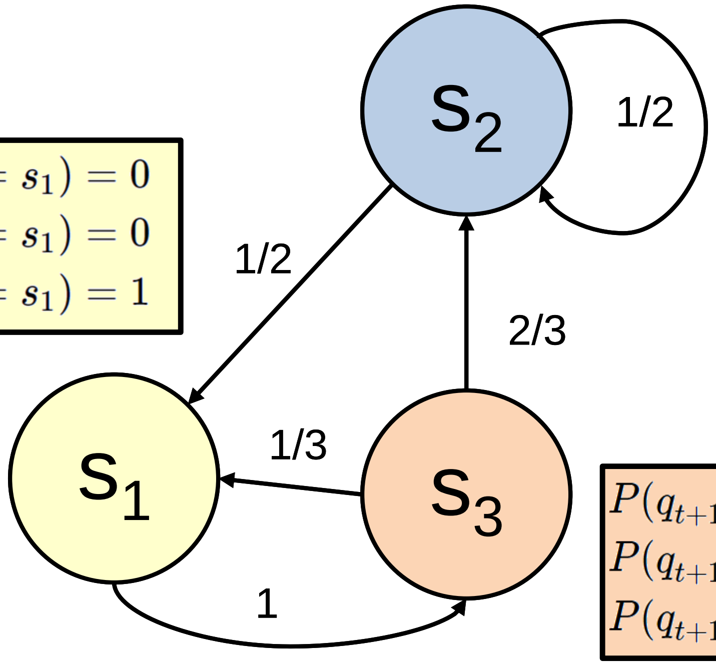
State Transition Matrix



State Transition Matrix

$$P(q_{t+1}=s_1|q_t=s_2)=1/2$$
$$P(q_{t+1}=s_2|q_t=s_2)=1/2$$
$$P(q_{t+1}=s_3|q_t=s_2)=0$$

$$P(q_{t+1}=s_1|q_t=s_1)=0$$
$$P(q_{t+1}=s_2|q_t=s_1)=0$$
$$P(q_{t+1}=s_3|q_t=s_1)=1$$

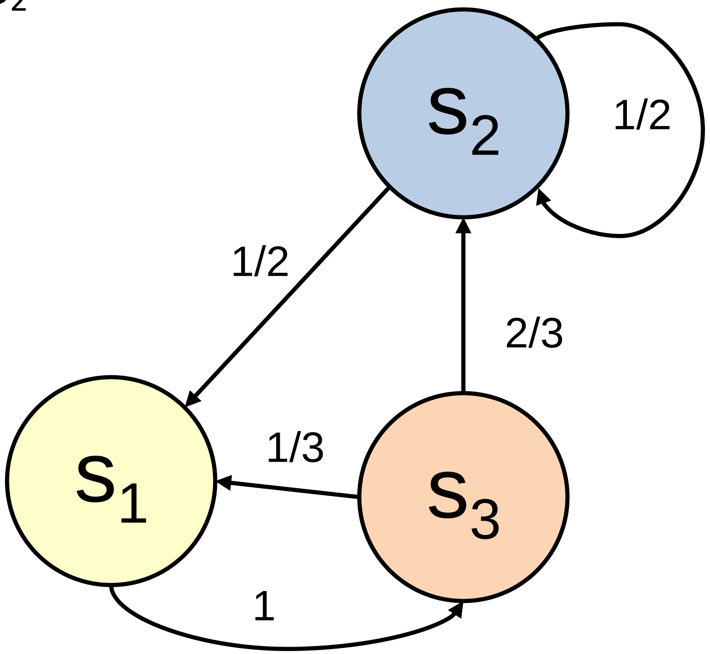


$$P(q_{t+1}=s_1|q_t=s_3)=1/3$$
$$P(q_{t+1}=s_2|q_t=s_3)=2/3$$
$$P(q_{t+1}=s_3|q_t=s_3)=0$$

$$P=\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}=\begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \\ 1/3 & 2/3 & 0 \end{bmatrix}$$

Example: MC Episodes

- Sample episodes starting from S_1
  - $S_1 \rightarrow S_3 \rightarrow S_2 \rightarrow S_2 \rightarrow S_1 \rightarrow \cdots$
  - $S_1 \rightarrow S_3 \rightarrow S_1 \rightarrow S_3 \rightarrow S_2 \rightarrow \cdots$
  - $S_1 \rightarrow S_3 \rightarrow S_2 \rightarrow S_2 \rightarrow S_2 \rightarrow \cdots$

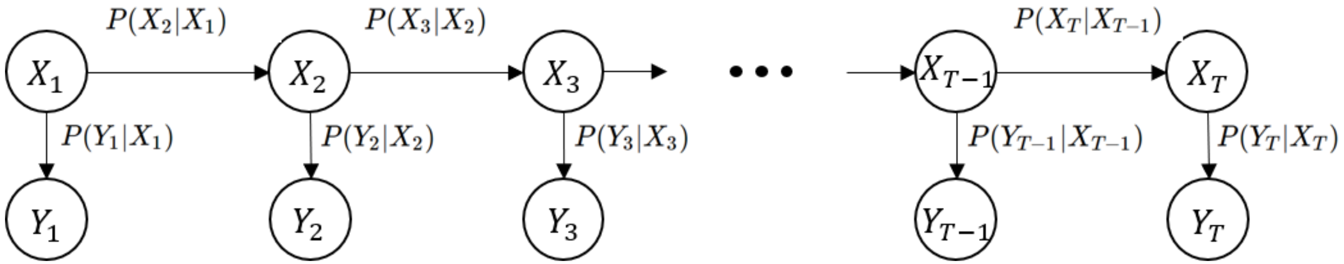


- Generate passive stochastic sequence

Modeling of Sequential Data

Hidden Markov Models

- Discrete state-space model

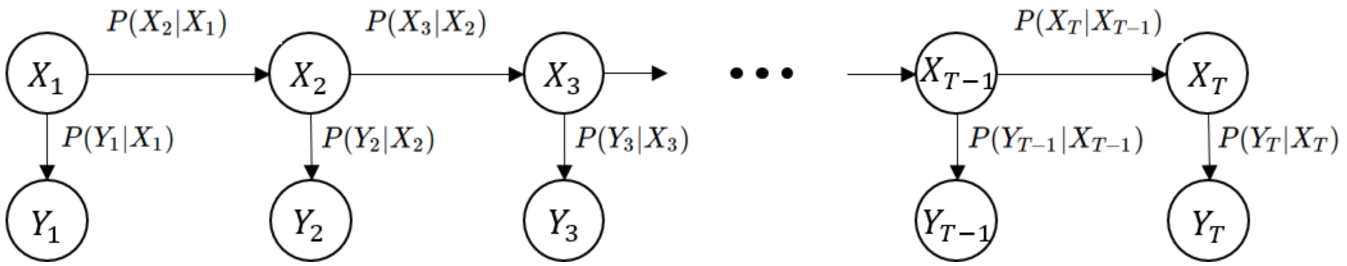


- Assumption
  - We can observe something that's affected by the true state
  - Natural way of thinking

- Noisy sensors
  - Unreliable

Hidden Markov Model (HMM)

- True state (or hidden variable) follows Markov chain
- Observation emitted from state
  - Y_t is noisily determined depending on the current state X_t



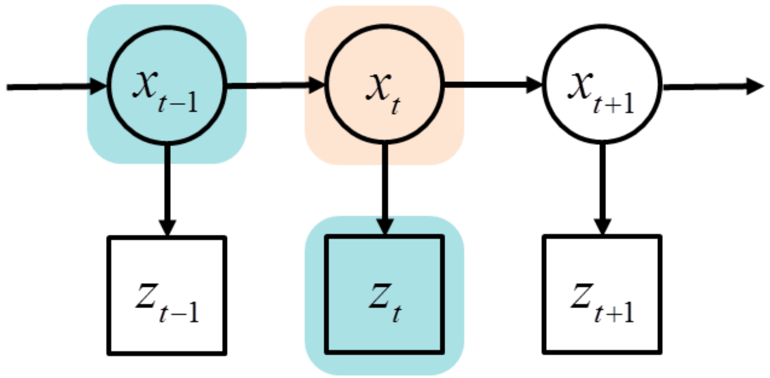
- Forward: sequence of observations can be generated
- Question: state estimation

$$P(X_T = s_i \mid Y_1 Y_2 \cdots Y_T)$$

- HMM can do this, but with many difficulties

Kalman Filter

- Linear dynamical system of motion
- Continuous State space model
  - Widely used in many applications:
    - GPS, weather systems, etc.
- Weakness
  - Linear state space model assumed
  - Difficult to apply to highly non-linear domains



$$x_{t+1} = Ax_t + Bu_t$$
$$z_t = Cx_t$$