Neural Network Architectures for Time-Series

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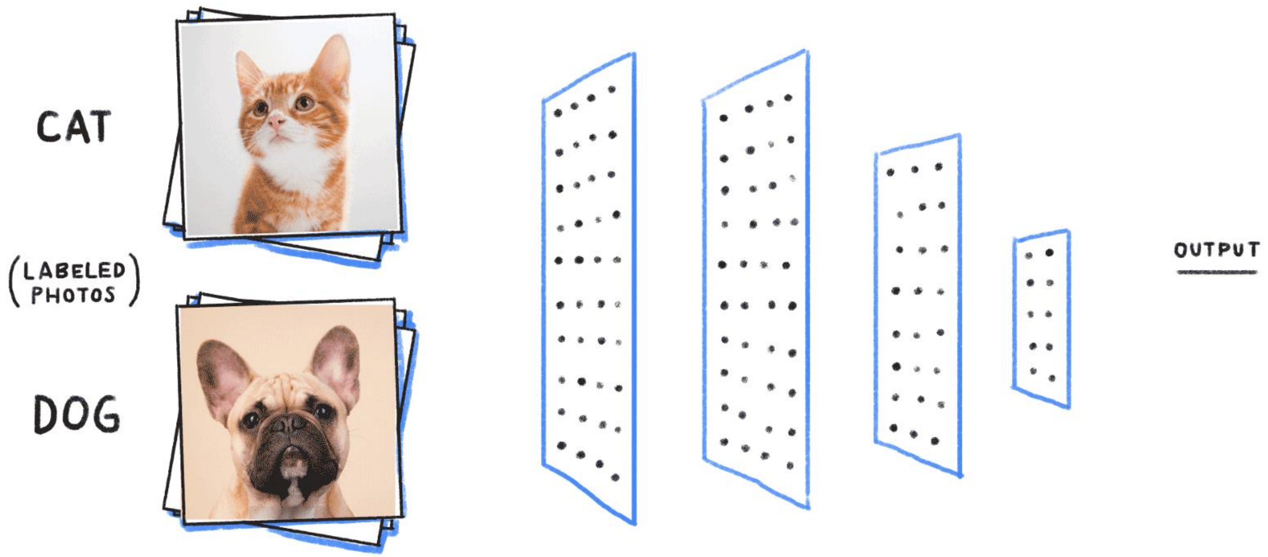
So Far

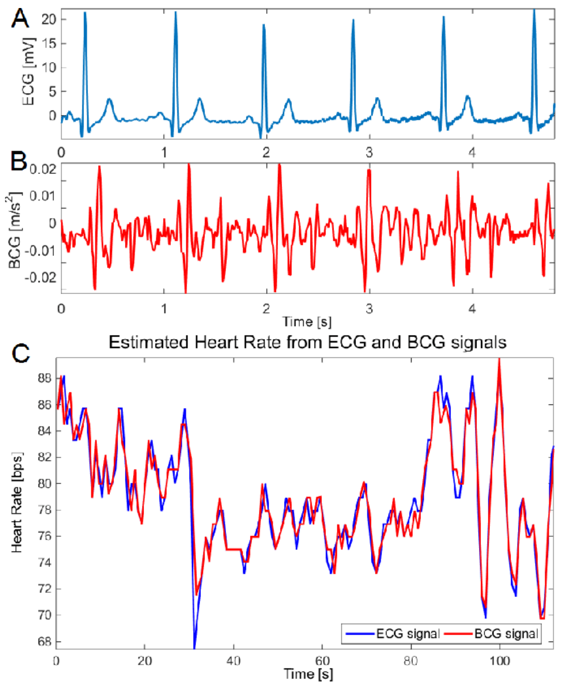
- Regression, Classification, Dimension Reduction,
- Based on snapshot-type data



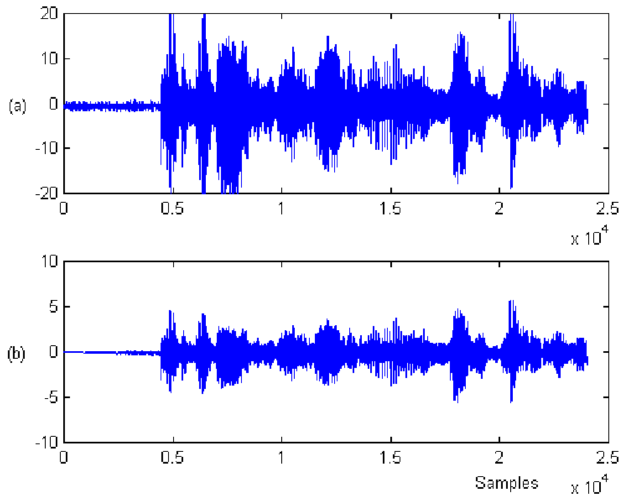
What is a Sequence ?

- Sentence
  - "This morning I took the dog for a walk."

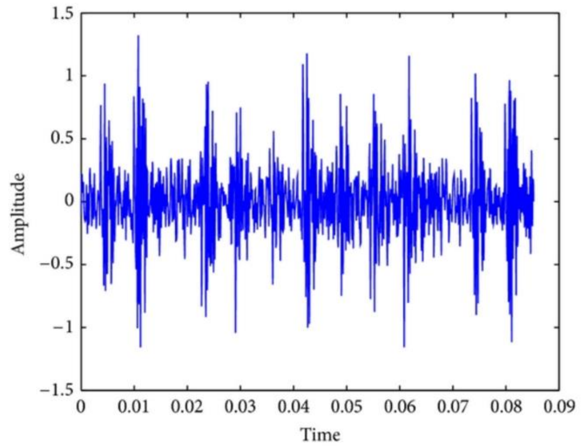
- Medical signals



- Speech waveform



- Vibration measurement



Sequence Modeling

- Most of the real-world data is time-series

- There are important bits to be considered
  - Past events
  - Relationship between events
    - Causality
    - Credit assignment
  - Learning the structure and hierarchy

- Use the past and present observations to predict the future

(Deterministic) Time Series Data

- For example

$$y[0] = 1, \quad y[1] = \frac{1}{2}, \quad y[2] = \frac{1}{4}, \quad \cdots$$

- Closed-form

$$y[n] = \left(\frac{1}{2}\right)^n, \quad n \geq 0$$

- Linear difference equation (LDE) and initial condition

$$y[n] = \frac{1}{2} y[n-1], \quad y[0] = 1$$

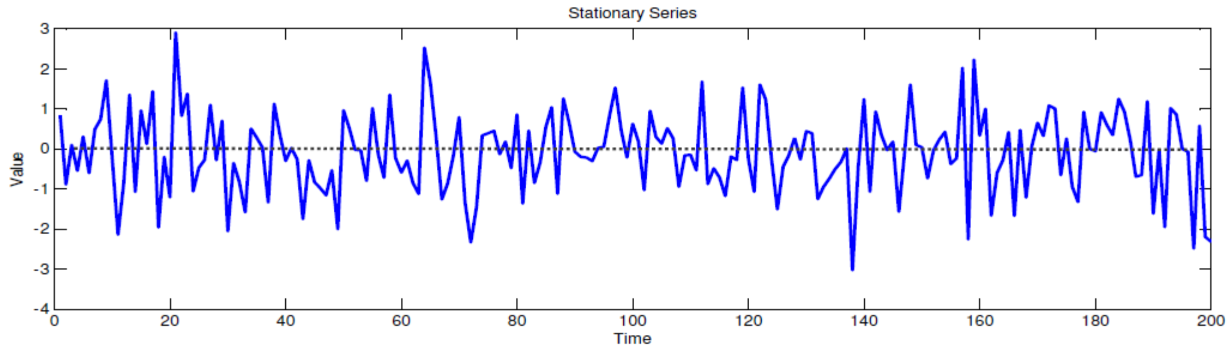
- High order LDEs

$$y[n] = \alpha_1 y[n-1] + \alpha_2 y[n-2]$$

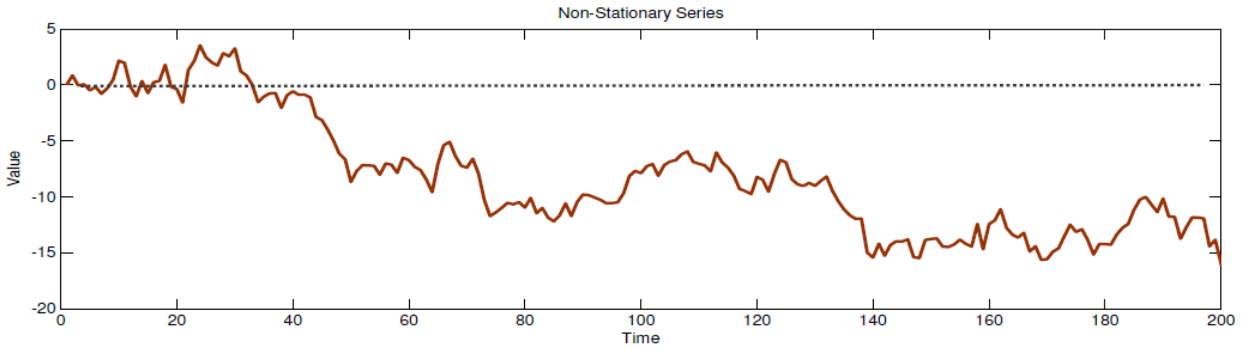
$$y[n] = \alpha_1 y[n-1] + \alpha_2 y[n-2] + \cdots + \alpha_k y[n-k]$$

(Stochastic) Time Series Data

- Stationary

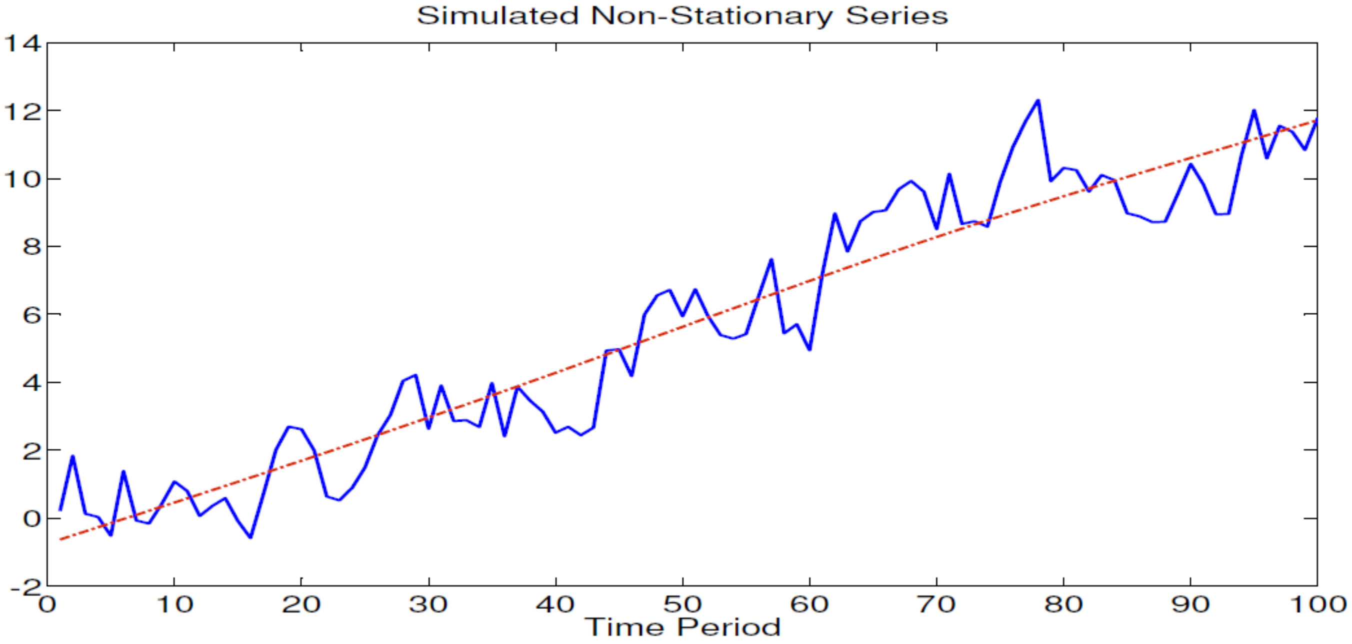


- Non-stationary
  - Mean and variance change over time



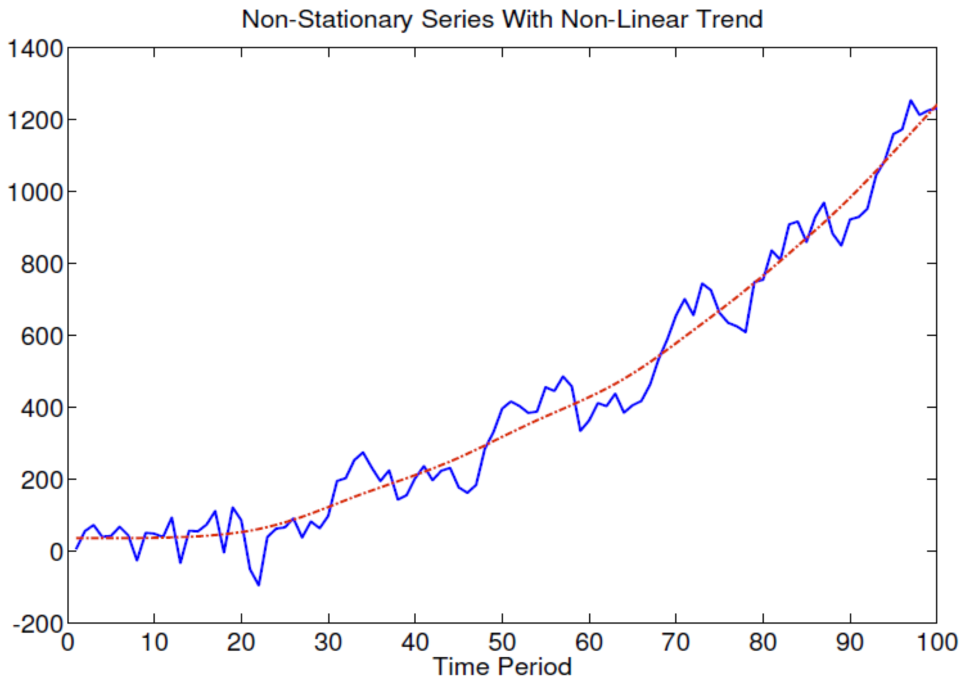
Dealing with Non-Stationarity

- Linear trends



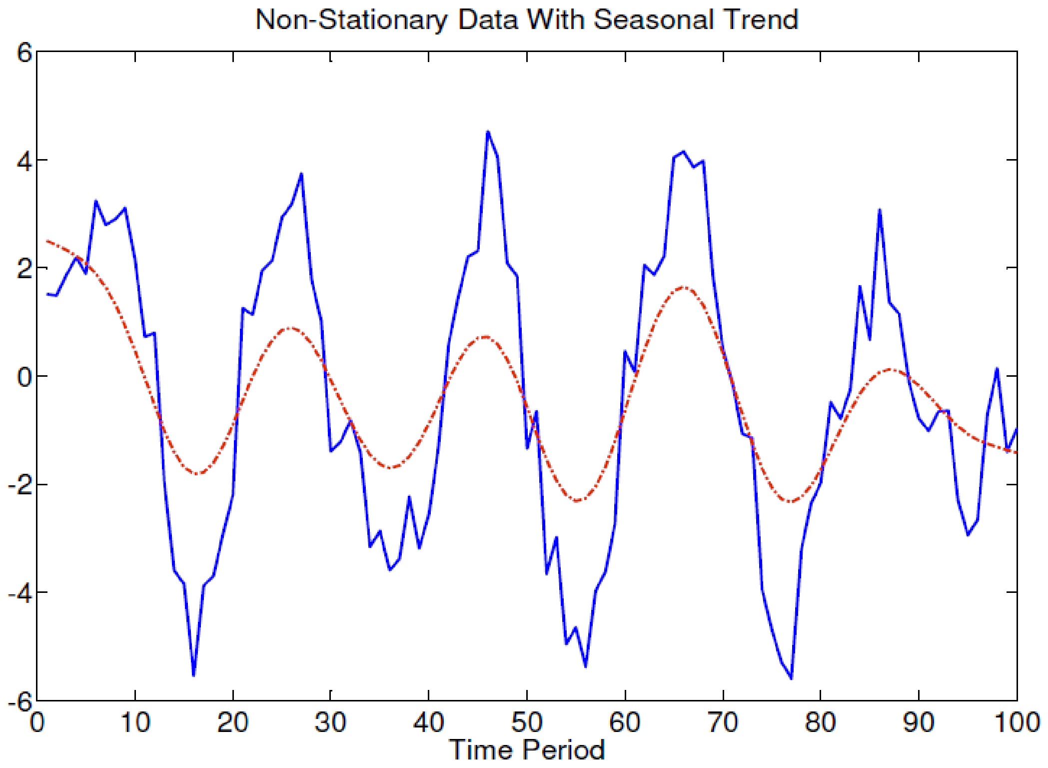
Dealing with Non-Stationarity

- Non-linear trends



Dealing with Non-Stationarity

- Seasonal trends



Dealing with Non-Stationarity

- Model assumption

$$
\begin{aligned}
Y_t = {} & \beta_1 + \beta_2 Y_{t-1} \\
& + \beta_3 t + \beta_4 t^{\beta_5} \\
& + \beta_6 \sin \frac{2\pi}{s} t + \beta_7 \cos \frac{2\pi}{s} t \\
& + u_t
\end{aligned}
$$

Markov Process

Sequential Processes

- Most classifiers ignored the sequential aspects of data

- Consider a system which can occupy one of *N* discrete states or categories

$$q_t \in \{S_1, S_2, \dots, S_N\}$$

- We are interested in stochastic systems, in which state evolution is random
- Any joint distribution can be factored into a series of conditional distributions

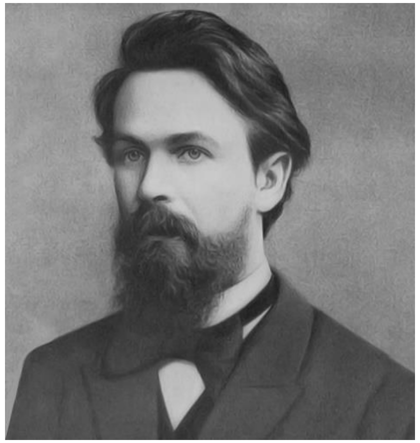
$$p(q_0, q_1, \cdots, q_T) = p(q_0)\, p(q_1 \mid q_0)\, p(q_2 \mid q_1, q_0) \cdots$$

Almost impossible to compute !

Markov Chain

- Markovian property (assumption)
  - Information state: sufficient statistic of history

$$p(q_{t+1} \mid q_t , \cdots , q_0) = \boxed{\qquad\qquad}$$



- Tractable in computation of joint distribution

$$\begin{aligned} p(q_0 , q_1 , \cdots , q_T) &= p(q_0)\, p(q_1 \mid q_0)\, p(q_2 \mid q_1 , q_0)\, p(q_3 \mid q_2 , q_1 , q_0) \cdots \\ &= p(q_0)\, p(q_1 \mid q_0)\, p(q_2 \mid q_1)\, p(q_3 \mid q_2) \cdots \end{aligned}$$

- Future is independent of past given present
  - Rain → snow → sunny → sunny → sunny → rain → snow → ??
  - ~~Rain → snow → sunny → sunny → sunny → rain →~~ snow → ??

State Transition Matrix

- For a Markov state s and successor state s', the state transition probability is defined by

$$\mathcal{P}_{ss'} = \mathbb{P}\left[S_{t+1} = s' \mid S_t = s\right]$$

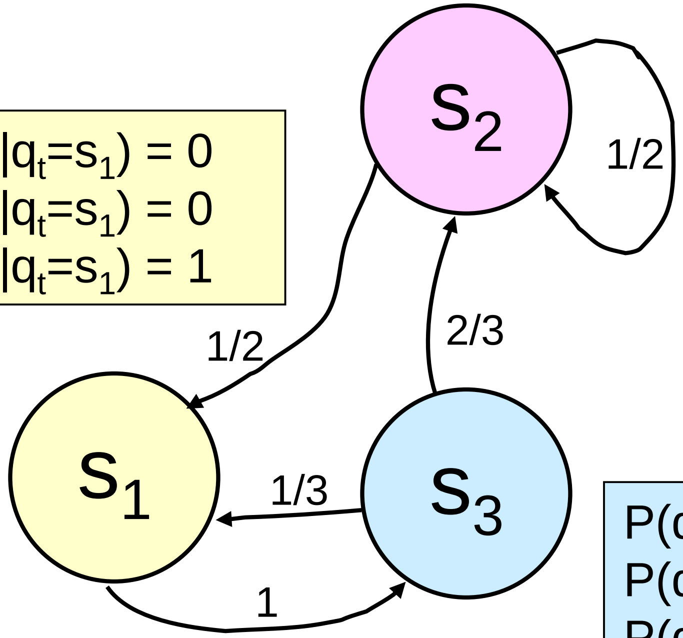
- State transition matrix P defines transition probabilities from all states s to all successor states s',

$$\mathcal{P} = \textit{from} \begin{array}{c} \textit{to} \\ \begin{bmatrix} \mathcal{P}_{11} & \dots & \mathcal{P}_{1n} \\ \vdots & & \\ \mathcal{P}_{n1} & \dots & \mathcal{P}_{nn} \end{bmatrix} \end{array}$$

State Transition Matrix

$P(q_{t+1}=s_1|q_t=s_2) = 1/2$
$P(q_{t+1}=s_2|q_t=s_2) = 1/2$
$P(q_{t+1}=s_3|q_t=s_2) = 0$

$P(q_{t+1}=s_1|q_t=s_1) = 0$
$P(q_{t+1}=s_2|q_t=s_1) = 0$
$P(q_{t+1}=s_3|q_t=s_1) = 1$

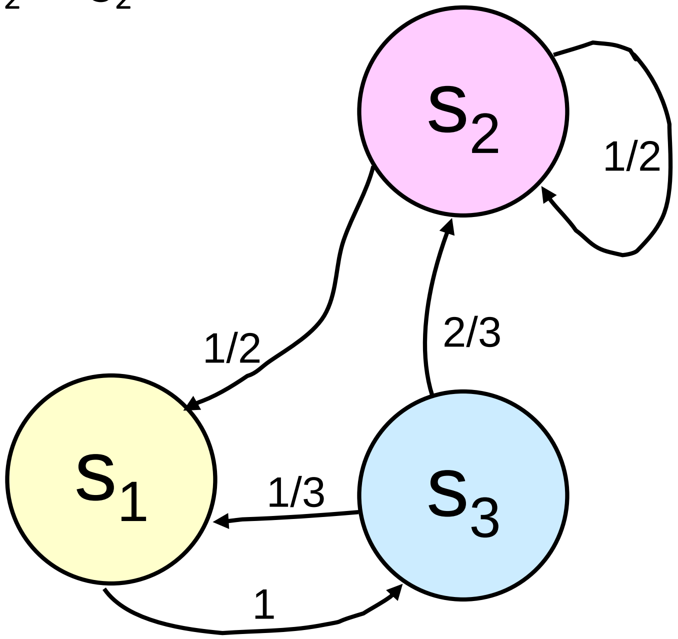


$P(q_{t+1}=s_1|q_t=s_3) = 1/3$
$P(q_{t+1}=s_2|q_t=s_3) = 2/3$
$P(q_{t+1}=s_3|q_t=s_3) = 0$

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} =$$

Example: MC Episodes

- Sample episodes starting from S_1
  - $S_1 \rightarrow S_3 \rightarrow S_2 \rightarrow S_2 \rightarrow S_1 \rightarrow \cdots$
  - $S_1 \rightarrow S_3 \rightarrow S_1 \rightarrow S_3 \rightarrow S_2 \rightarrow \cdots$
  - $S_1 \rightarrow S_3 \rightarrow S_2 \rightarrow S_2 \rightarrow S_2 \rightarrow \cdots$

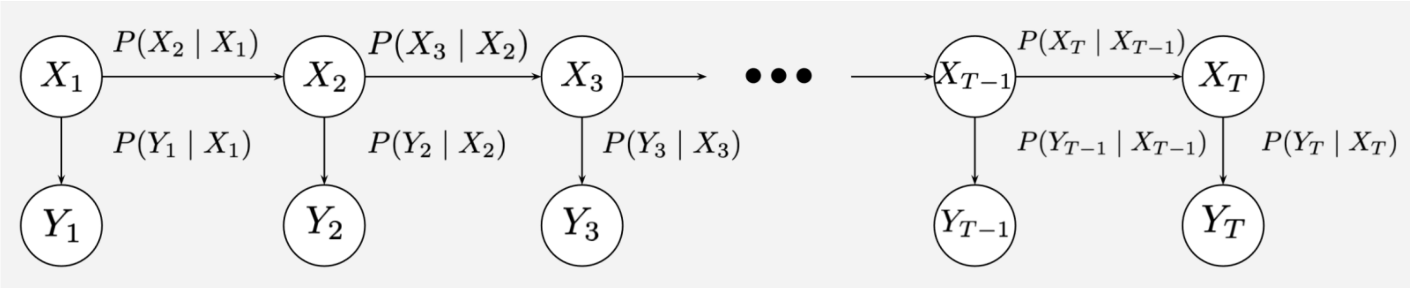


- Generate passive stochastic sequence

Hidden Markov Model

Hidden Markov Models

- Discrete state-space model
  - Used in speech recognition
  - State representation is simple
  - Hard to scale-up the training



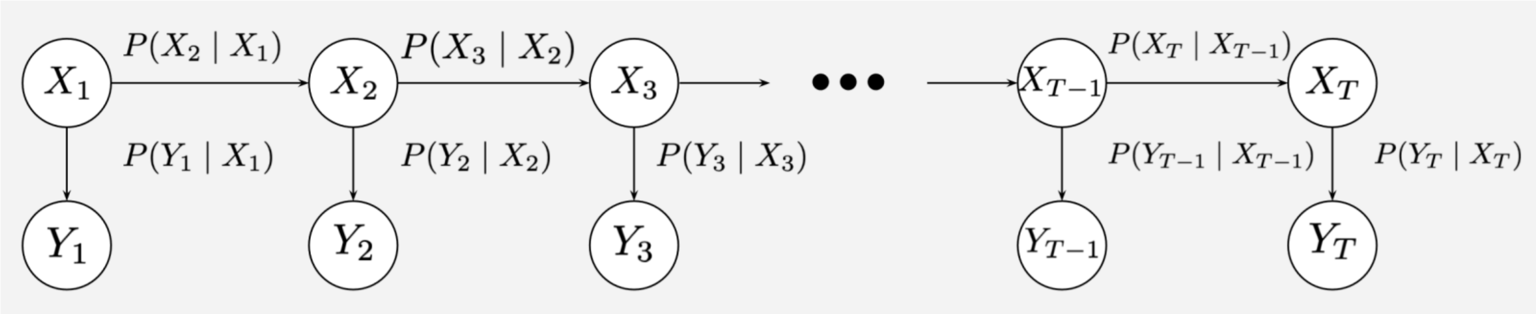
- Assumption
  - We can observe something that's affected by the true state
  - Natural way of thinking

- Limited sensors (incomplete state information)
  - But still partially related

- Noisy sensors
  - Unreliable

Hidden Markov Model (HMM)

- True state (or hidden variable) follows Markov chain
- Observation emitted from state
  - Y_t is noisily determined depending on the current state X_t



- Forward: sequence of observations can be generated
- Question: state estimation

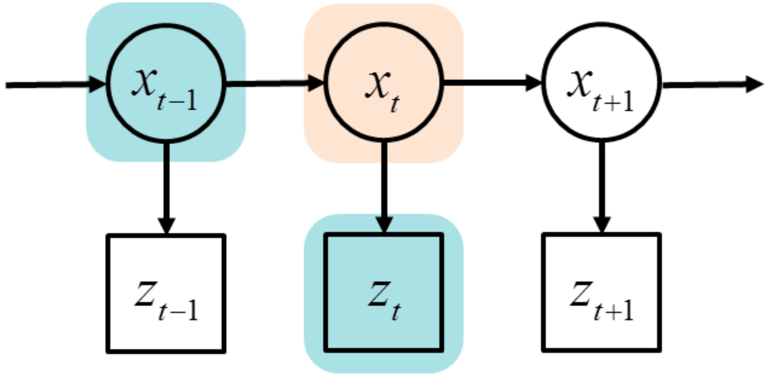
$$p(X_t = s_i \mid Y_1, Y_2, \cdots, Y_T)$$

- HMM can do this, but with many difficulties

Kalman Filter

Kalman Filter

- Linear dynamical system of motion
- *A, B, C* ?
- Continuous State space model
  - For filtering and control applications
  - Linear-Gaussian state space model
  - Widely used in many applications:
    - GPS, weather systems, etc.
- Weakness
  - Linear state space model assumed
  - Difficult to apply to highly non-linear domains



$$x_{t+1} = Ax_t + Bu_t$$
$$z_t = Cx_t$$