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ABSTRACT

Key words:

1 INTRODUCTION

2 THE MAIN SYSTEM OF EQUATIONS

The crucial part of all the dynamic equations is the non-linear term $(\mathbf{v}\nabla)\mathbf{v}$, containing all the inertial terms. In spherical coordinates:

$$(\mathbf{v}\nabla)\mathbf{v} = \left(v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\varphi}{r \sin \theta} \frac{\partial v_r}{\partial \varphi} - \frac{v_\theta^2 + v_\varphi^2}{r}\right) \mathbf{e}^r + \left(v_r \frac{\partial v_\theta}{\partial r} + \frac{\theta}{v_\theta} + \frac{v_\varphi}{r \sin \theta} \frac{\partial v_\theta}{\partial \varphi} + \frac{v_\theta v_r}{r} - \frac{v_\varphi^2}{r} \cot^2 \theta\right) \mathbf{e}^\theta + \left(v_r \frac{\partial v_\varphi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\varphi}{\partial \theta} + \frac{v_\varphi}{r \sin \theta} \frac{\partial \varphi}{\partial v_\varphi} + \frac{v_\varphi v_r}{r}\right) \mathbf{e}^\varphi,$$

$$(1)$$

where e are unit vectors along different coordinate directions, r is radius, θ is polar angle, φ is azimuthal angle. Directly, this expression will be used for the vertical structure equations. On the sphere, the velocities may be converted to vorticity and divergence:

$$\omega = [\nabla \times \mathbf{v}], \tag{2}$$

$$\delta = (\nabla \cdot \mathbf{v}) = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 v_r \right) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta v_\theta \right) + \frac{1}{r \sin \theta} \frac{\partial v_\varphi}{\partial \varphi}. \tag{3}$$

Taking curl of Euler equation allows to directly derive the equation for vorticity

$$\frac{\partial \left[\nabla \mathbf{v}\right]}{\partial t} + (\mathbf{v}\nabla) \left[\nabla \mathbf{v}\right] = \frac{1}{\rho^2} \left[\nabla p \times \nabla \rho\right]. \tag{4}$$

The right-hand-side term is important if motion is baroclinic. Baroclinic term allows to generate vorticity from non-axisymmetric perturbations of density and temperature, hence it is important for any realistic calculations.