Pablo Saëtard García 24-4-2020

1

a) Sum blue + red = 3 $\begin{cases} 1 \text{ blue } 2 \text{ red} \\ \text{or} \\ 2 \text{ blue } 1 \text{ red} \end{cases}$

$P(A) = P((1 \text{ blue} \cap 2 \text{ red}) \cup (2 \text{ blue} \cap 1 \text{ red})) = \left(\frac{1}{6} \cdot \frac{1}{6}\right) + \left(\frac{1}{6} \cdot \frac{1}{6}\right) = \frac{2}{36} = 0{,}0\overline{5}$

b) $\Omega = \{1, 2, 3, 4, 5, \underbrace{6, 6}_{\text{Double prob.}}\}$

$P(B) = \frac{2}{7} = 0{,}283$

#6

total #

2

a) $P(A) = P((\text{Suffer Diabetes} \cap \text{Positive Diagnose}) \cup (\text{Not Suffer Diabetes} \cap \text{Positive Diagnose})) =$

$0{,}04 \cdot 0{,}95 + \underbrace{0{,}96}_{1-0{,}04} \cdot 0{,}02 = 0{,}057$

b)

$P(\text{Diabetic} \mid \text{Positive Diagnose}) = \frac{P(\text{Diabetic} \cap \text{Positive Diagnose})}{P(\text{Positive Diagnose})} = \frac{0{,}04 \cdot 0{,}95}{0{,}057} = 0{,}664$

③

a)

		Marginal Prob
X	-1	0,2
	0	0,35
	1	0,45

$\sum$ Marg P. $= 1$ ✓

$$P_X(x_i) = P(X = x_i) = \sum_{j \geq 1} P_{X,Y}(x_i, y_j)$$

$$E(X) = \sum_{i=1}^{\infty} x_i p_i =$$

$$-1 \cdot 0{,}2 + 0 \cdot 0{,}35 + 1 \cdot 0{,}45 = 0{,}25$$

b)

Marginal Prob Y: 1 2 3 4
0,25 0,2 0,2 0,35 $\sum$ Marg Prop $= 1$ ✓

$$E(Y) = 1 \cdot 0{,}25 + 2 \cdot 0{,}2 + 3 \cdot 0{,}2 + 4 \cdot 0{,}35 = 2{,}65$$

$$\sigma^2 = V(Y) = E((Y - E(Y))^2) = (1 - 2{,}65)^2 \cdot 0{,}25 + (2 - 2{,}65)^2 \cdot 0{,}2 + (3 - 2{,}65)^2 \cdot 0{,}2$$
$$+ (4 - 2{,}65)^2 \cdot 0{,}35 = 1{,}4275$$

c)

$$P(X \leq 0 \mid Y \leq 3) = \frac{P(X \leq 0, Y \leq 3)}{P(Y \leq 3)} = \frac{0{,}3}{0{,}65} = 0{,}462$$

d) X and Y are independent if $P(X = x_i, Y = y_j) = P(X = x_i) \cdot P(Y = y_j)$ $\forall\, i, j \geq 1$

We can see that for example $P(X = -1, Y = 2) \neq P(X = -1) \cdot P(Y = 2)$
$0 \neq 0{,}2 \cdot 0{,}2$, thus X and Y are dependent.

④

a) $P(29 \leq X \leq 31) = P\left(\frac{29-30}{1} \leq Z \leq \frac{31-30}{1}\right) = P(-1 \leq Z \leq 1) = 1 - 2P(Z \geq 1) = 0{,}683$

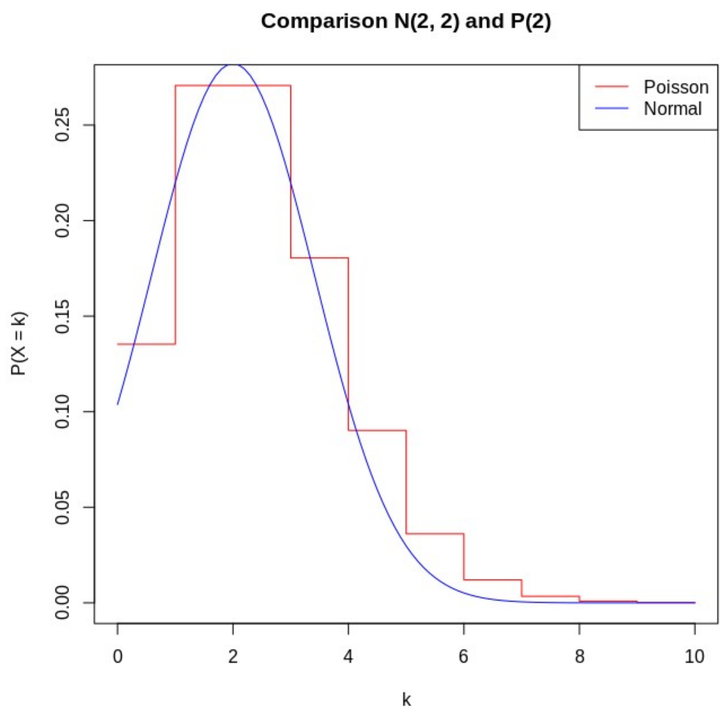
b) This follows a binomial distribution $B(80, p)$, being $p = 0{,}683$ (prob. that amount of fertiliser ≤ 31 and ≥ 29). As $p \cdot n = 0{,}683 \cdot 80 > 5$ and $(1-p) \cdot n = 0{,}317 \cdot 80 > 5$ and $p = 0{,}683 > 0{,}05$ and $1 - p = 0{,}317 > 0{,}05$ we can approximate $B(n, p) \approx N(np, np(1-p))$

yates correction

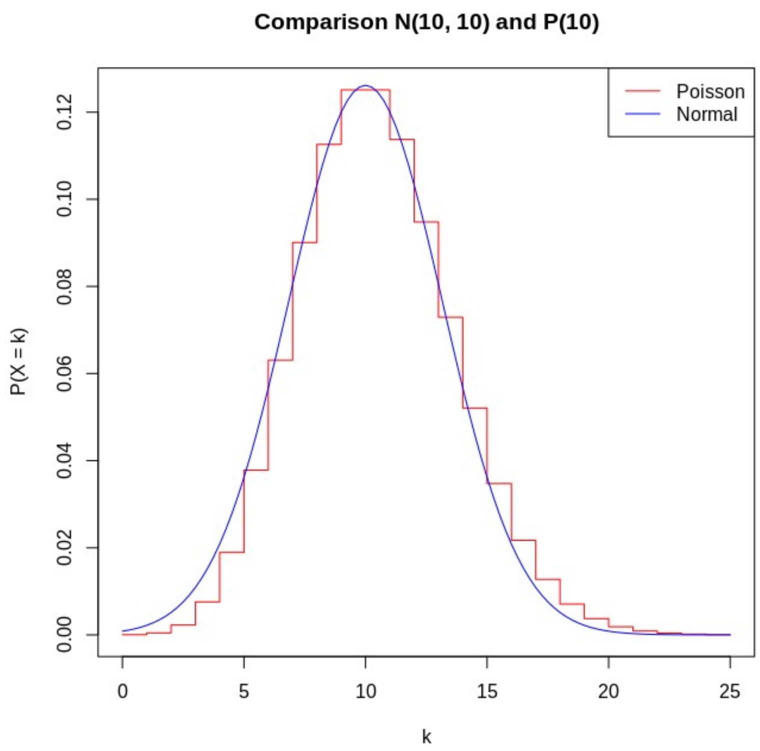
$$P(X > 50) = P\left(Z > \frac{50 + 0{,}5 - np}{\sqrt{np(1-p)}}\right) = P(Z > -0{,}994) = 1 - P(Z > 0{,}994) = 0{,}84$$

5

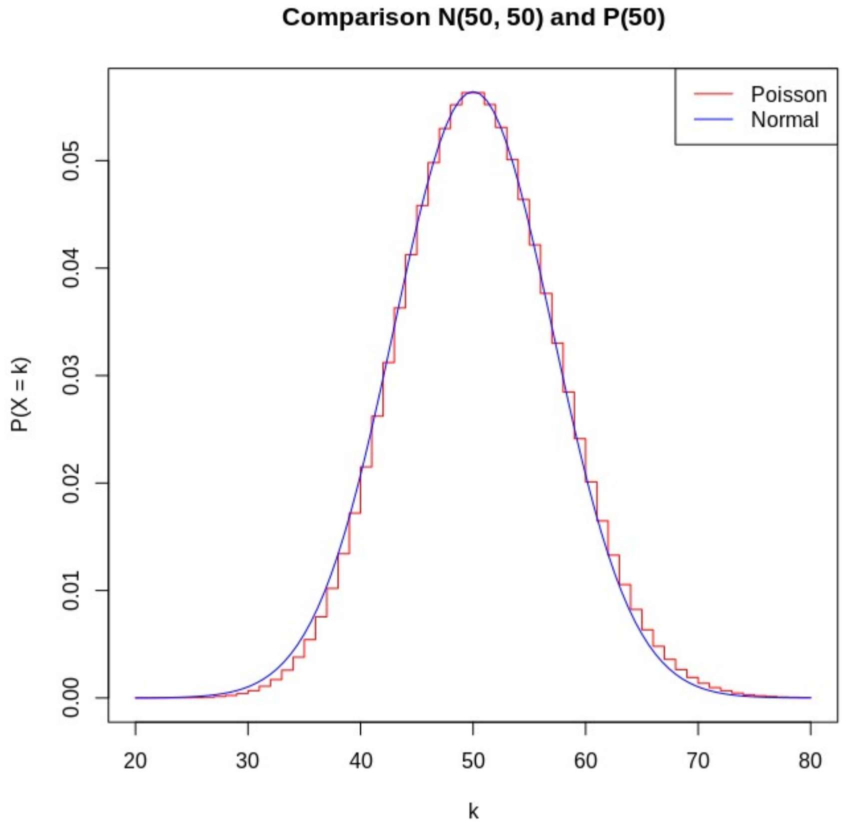
a)


Comparison N(2, 2) and P(2)
Poisson
Normal
P(X = k)
0.00
0.05
0.10
0.15
0.20
0.25
0
2
4
6
8
10
k

b)


Comparison N(10, 10) and P(10)
Poisson
Normal
P(X = k)
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0
5
10
15
20
25
k

c)



d)

In each of the above graphs, we can observe a probability mass function of a Poisson distribution (red) and a density function of a Normal distribution (blue) plotted.
The input parameters λ (Poisson), μ and σ^2 (Normal), have been changed to 2, 10 and 50 respectively for each graph.

We can observe that as we increase the parameters the maximum of the Poisson distribution decreases along with the Normal, starting at 0,27 on graph a, then 0,125 for b, and finally 0,056 for graph c. Also we can see that those maximum values are always situated at x = λ, that point is also the mean of the Normal(λ, λ) distribution.

Given the results shown above we can suggest that, as we increase λ, Poisson(λ) distribution approximates Normal(λ, λ) distribution. This can be proved using the Central Limit Theorem.