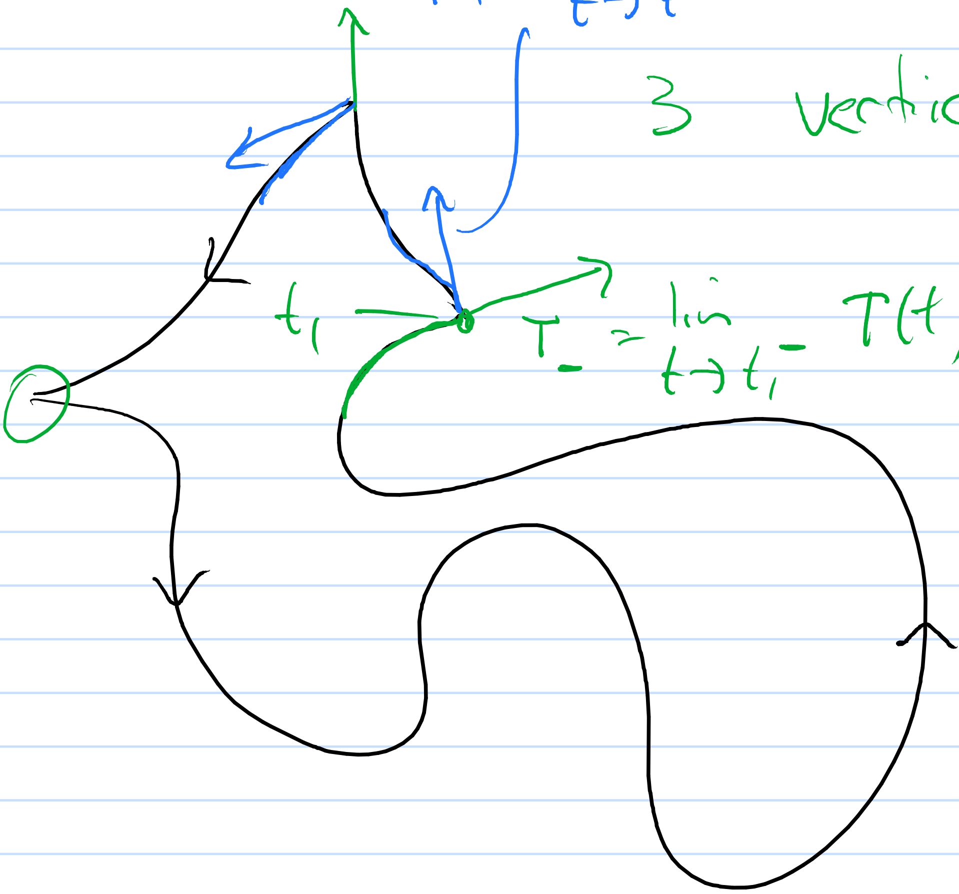
Piecewise Regular

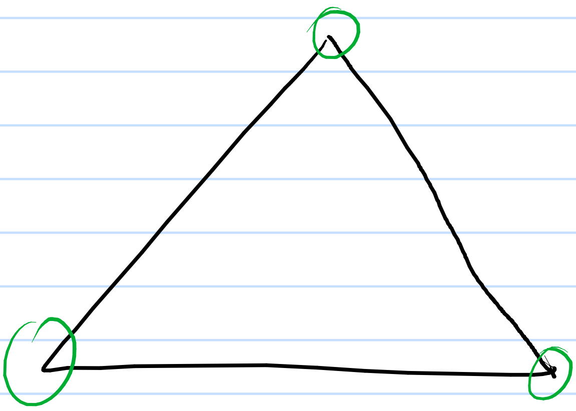
$$T_+ = \lim_{t \to t^+} T(t)$$

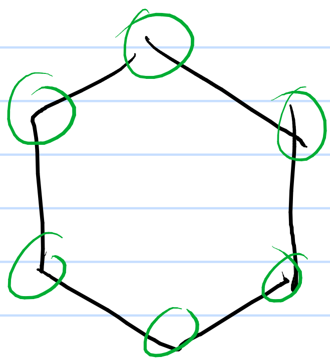
3 vertices

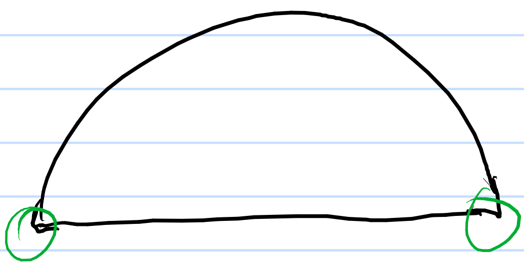


t_1

$$T_- = \lim_{t \to t_1^-} T(t)$$





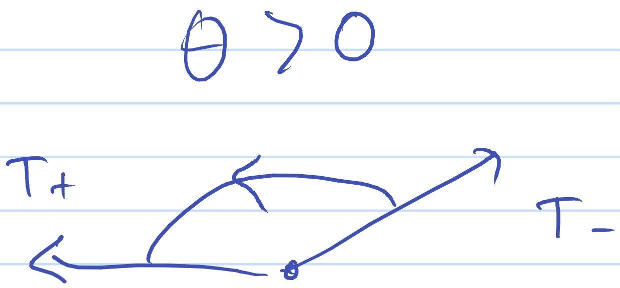


Angle on an oriented surface S

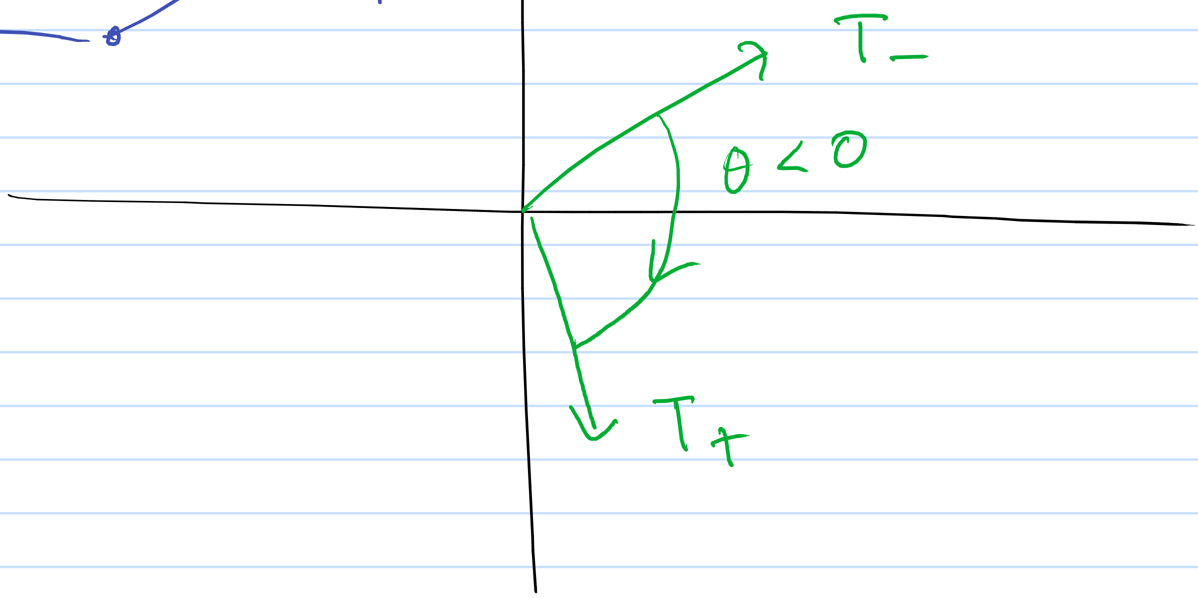
If (T_-, T_+) is lin indp.

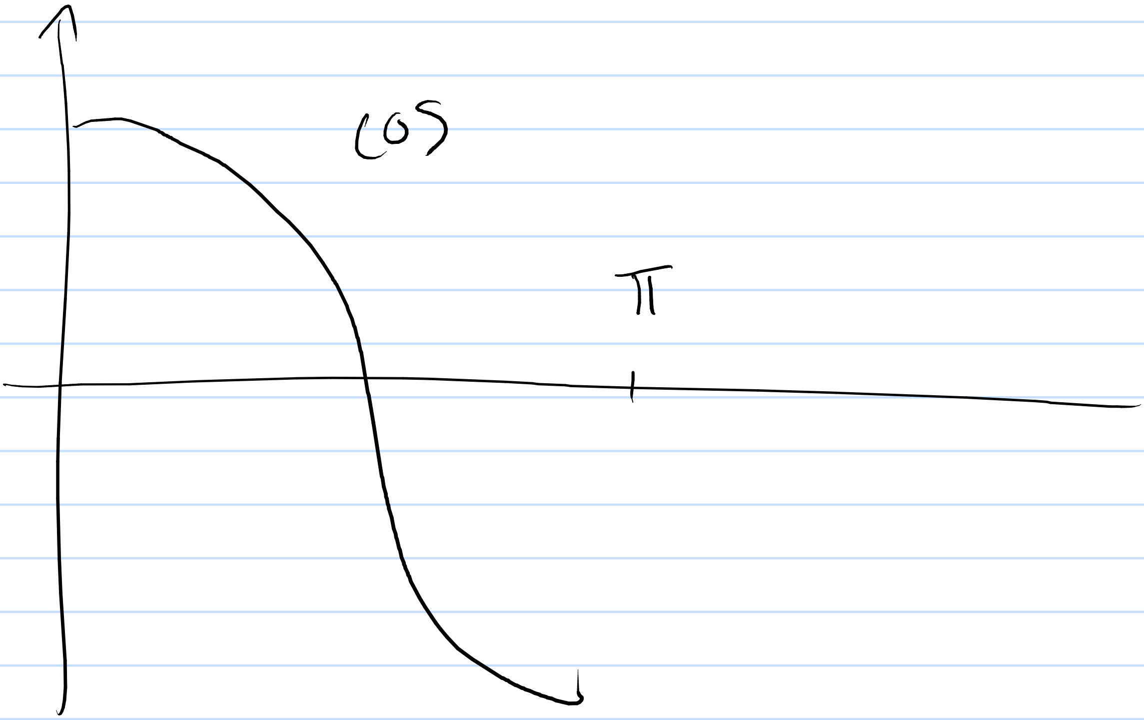
then (T_-, T_+) is either

pos. oriented or neg oriented.

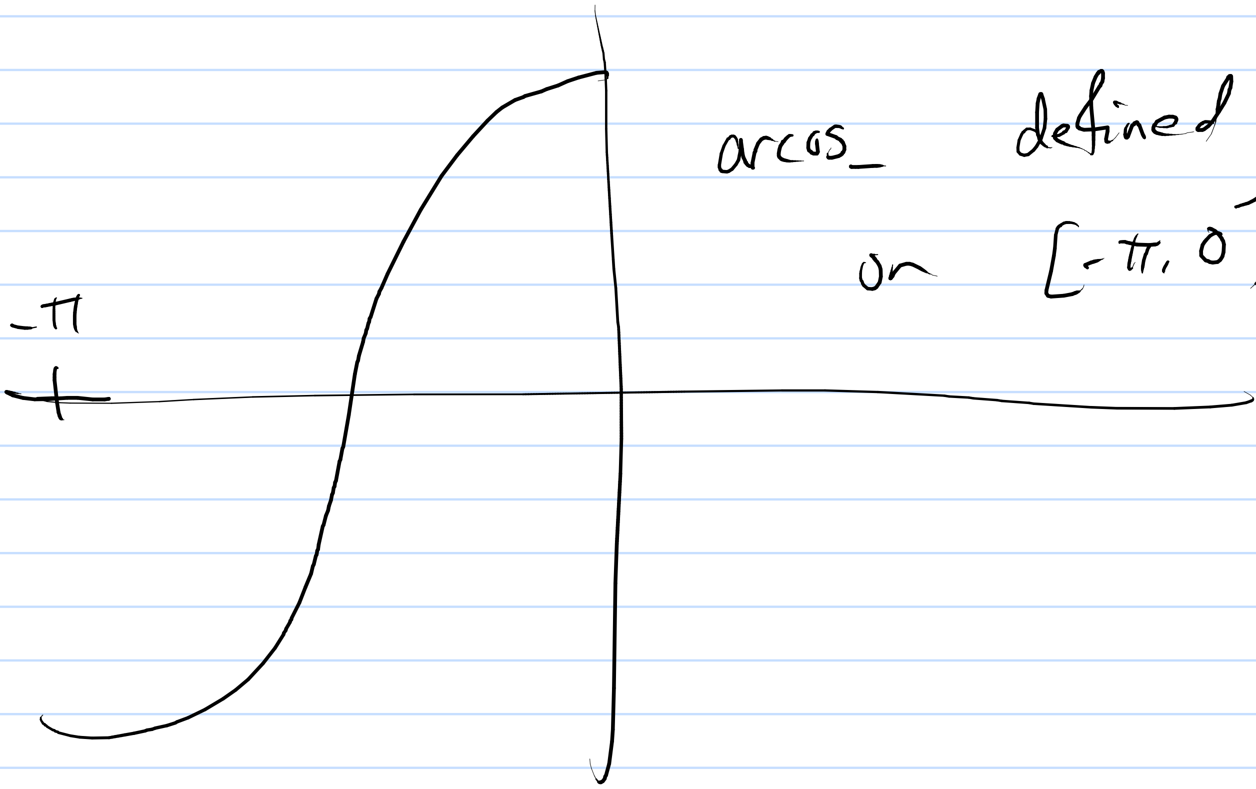


(T_-, T_+) neg. oriented



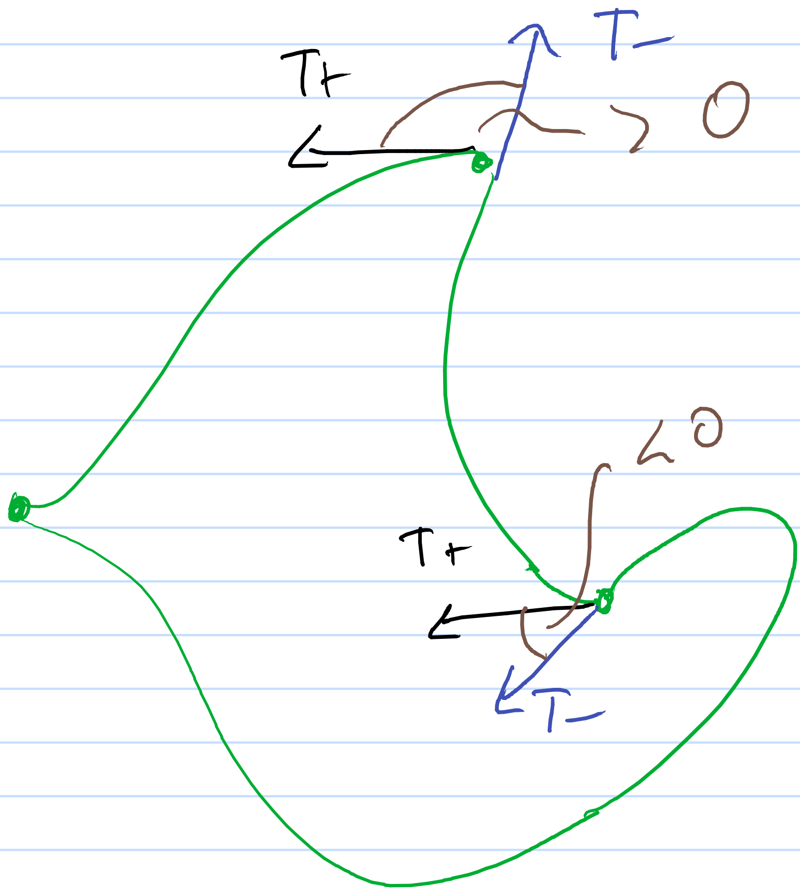


$\arccos_+$ defined on $[0, \pi]$

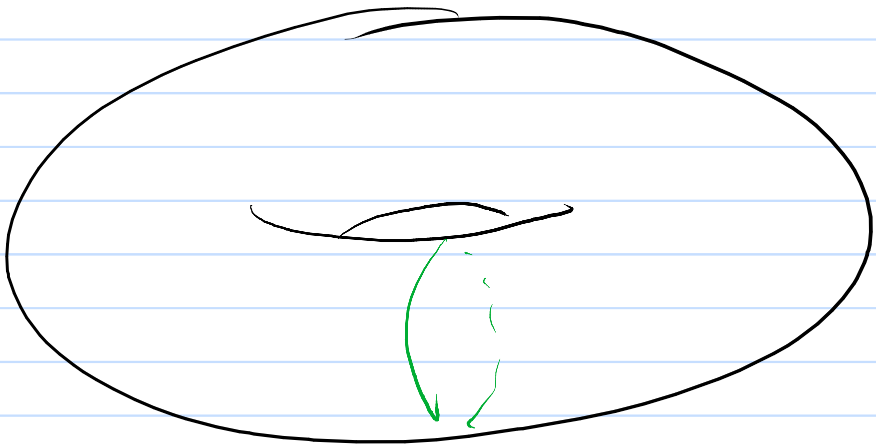


arcos_- defined on $[-\pi, 0]$

Angle

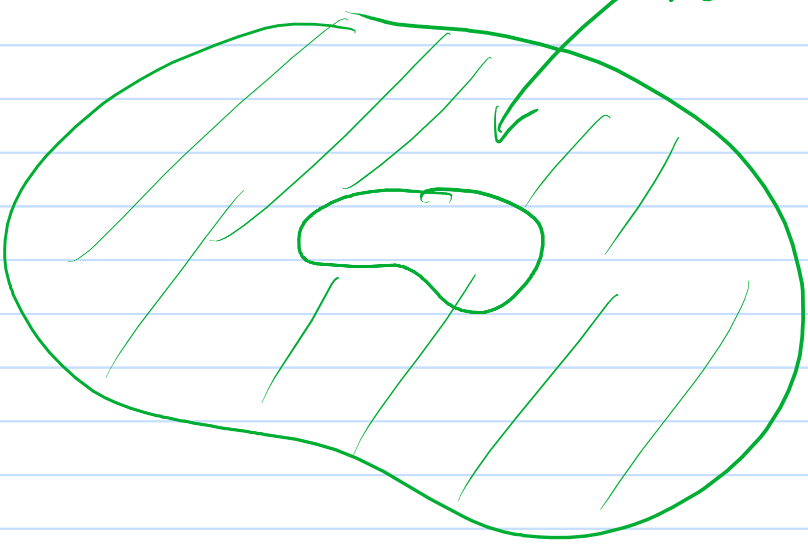


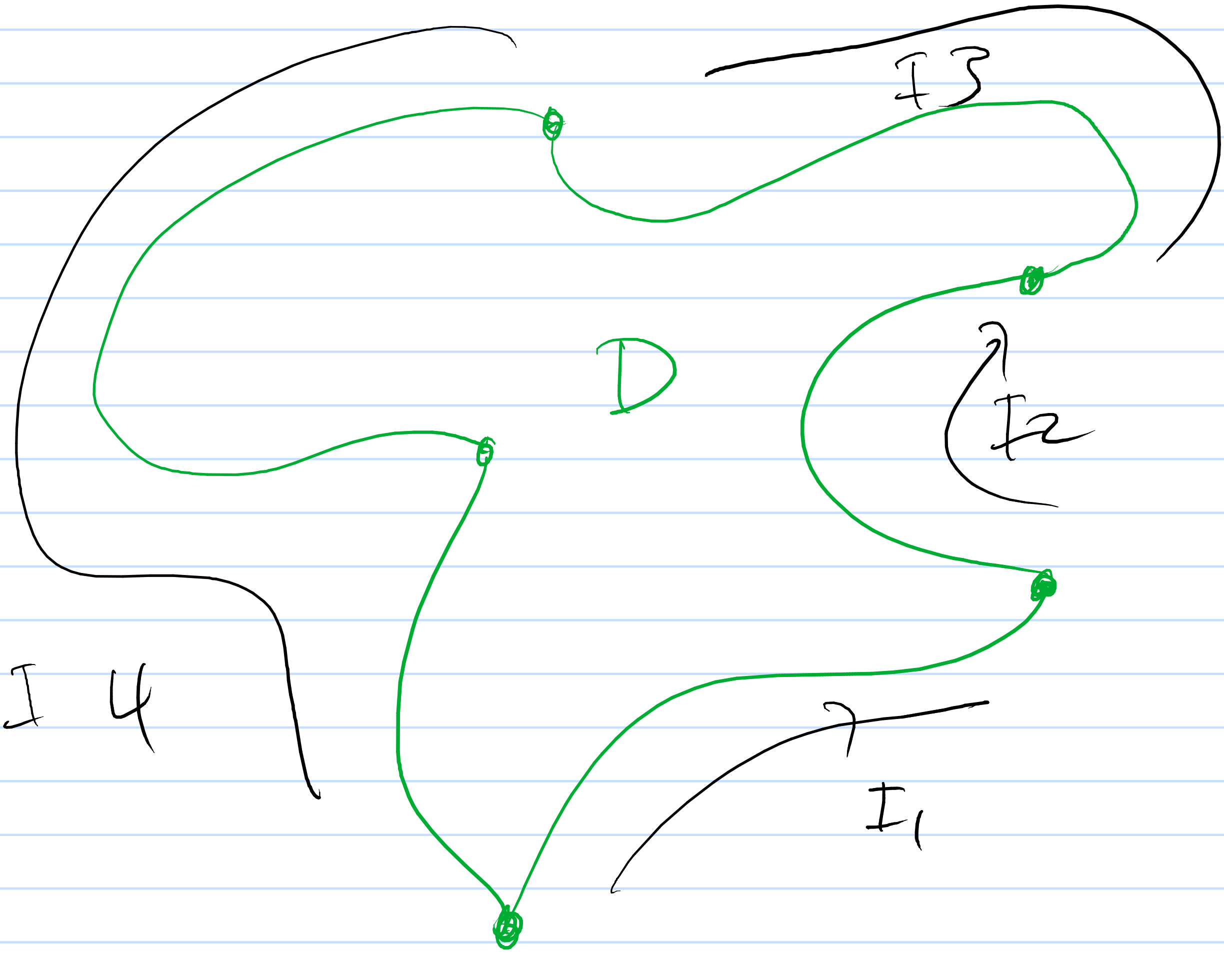
Boundary



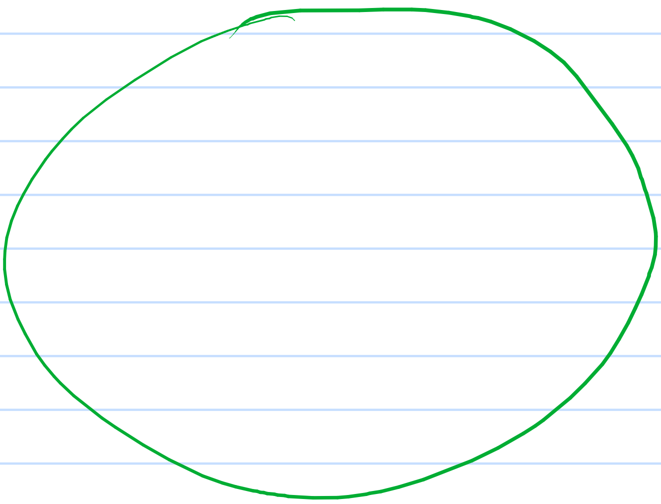
Not boundary of any domain

not $\simeq$ Disc





S^1 homeo



$$\iint_D K\, dA = -\int_{\partial D} \kappa\, ds + \underbrace{2\pi - \sum \theta_i}_{\text{turning angle}}$$

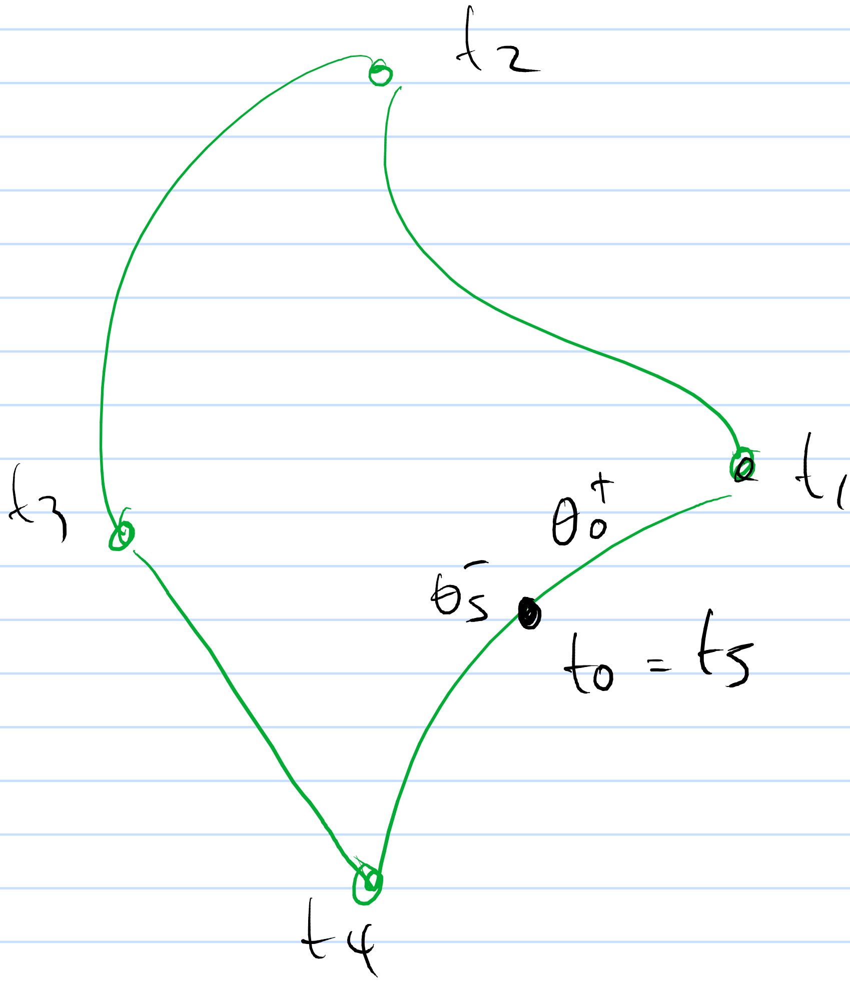
Basically

$$\iint_D \operatorname{div}(X)\, dA = \int_{\partial D} \langle X, N \rangle\, ds$$

Divergence ~ Green's thm

~ Stokes' thm

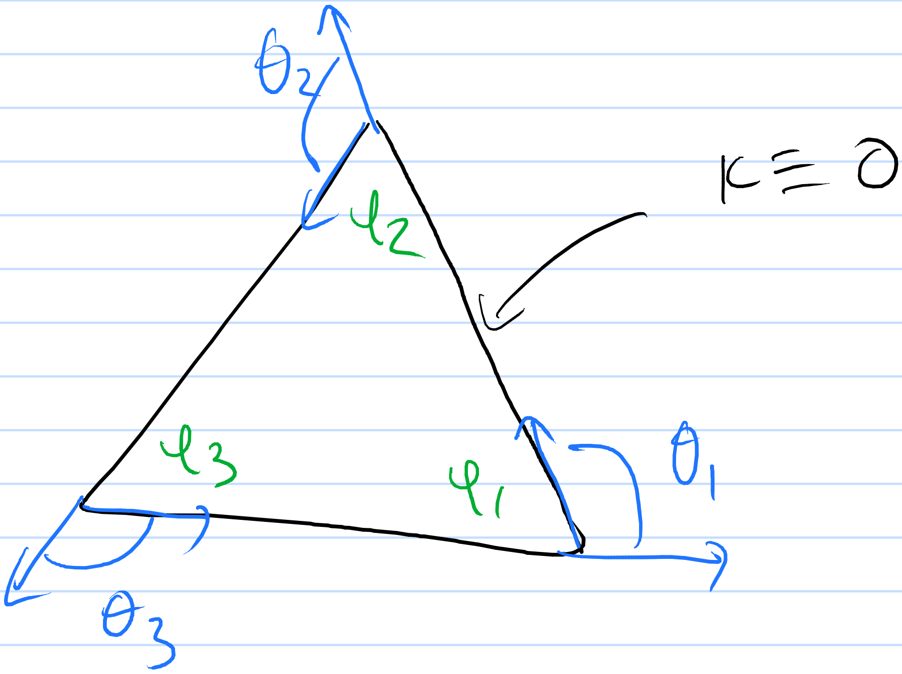
FTC



$$\theta_5^- - \theta_0^+ = 2\pi$$

Turning tangents

Triangles in Plane



$$\varphi_i = \pi - \theta_i$$

GB:

$$0 = \int_T K\,ds = 2\pi - \sum_{i=1}^{3} \theta_i$$

$$= 2\pi - \sum \pi - \varphi_i$$

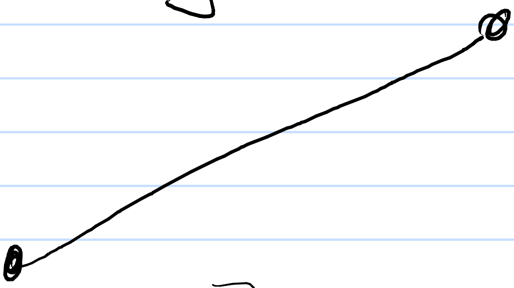
$$= \underbrace{2\pi - 3\pi}_{-\pi} + \sum \varphi_i$$

$$\therefore \quad \sum_{i=1}^{3} \varphi_i = \pi$$

$$Q = \frac{\partial_v g_{vv}}{2\sqrt{g_{uu} g_{vv}}}$$

$dv = \partial_s v \, ds$ change of vars

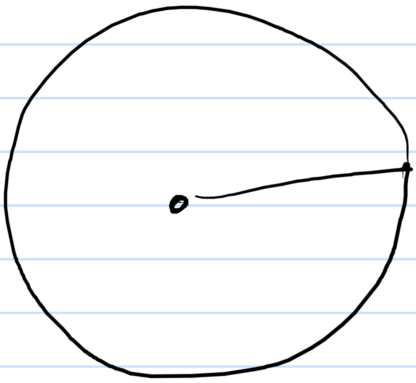
$\exists$



Euclid

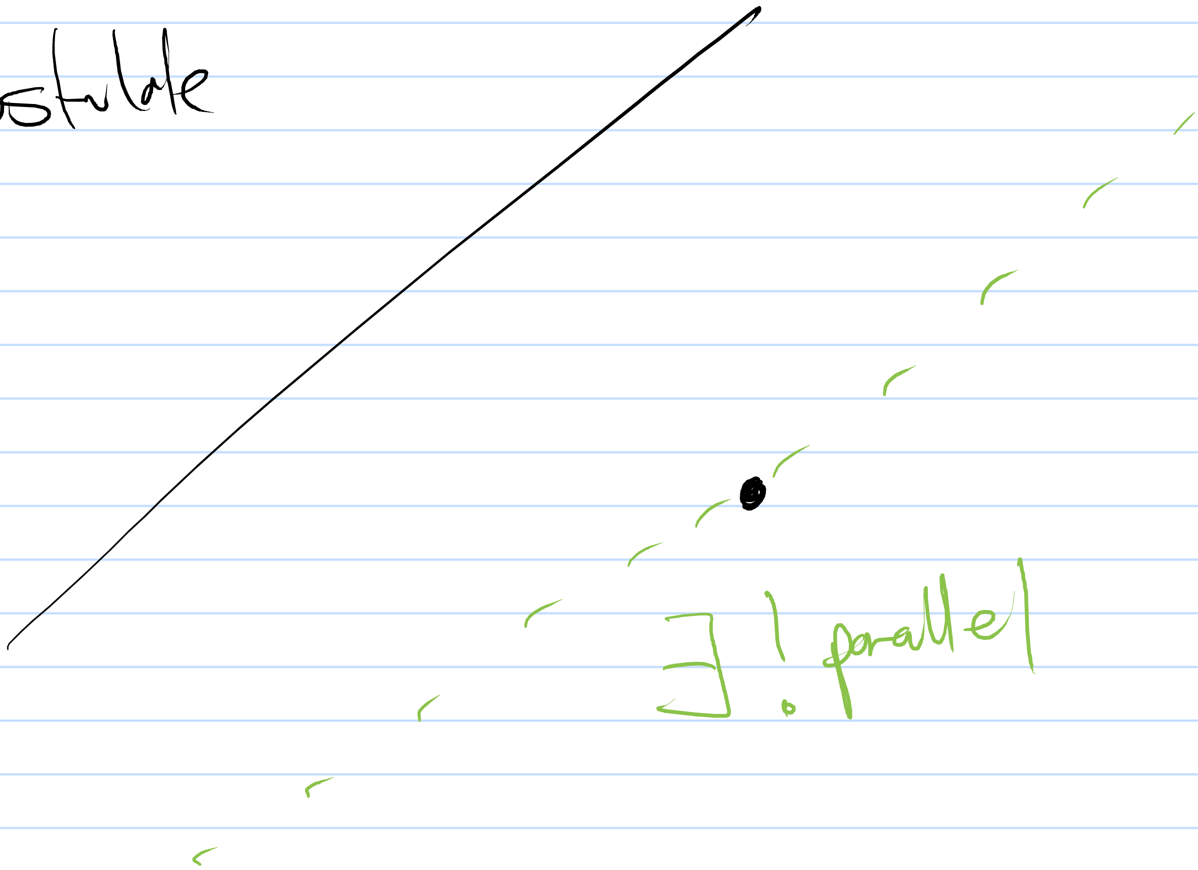
Axiom

$\exists$

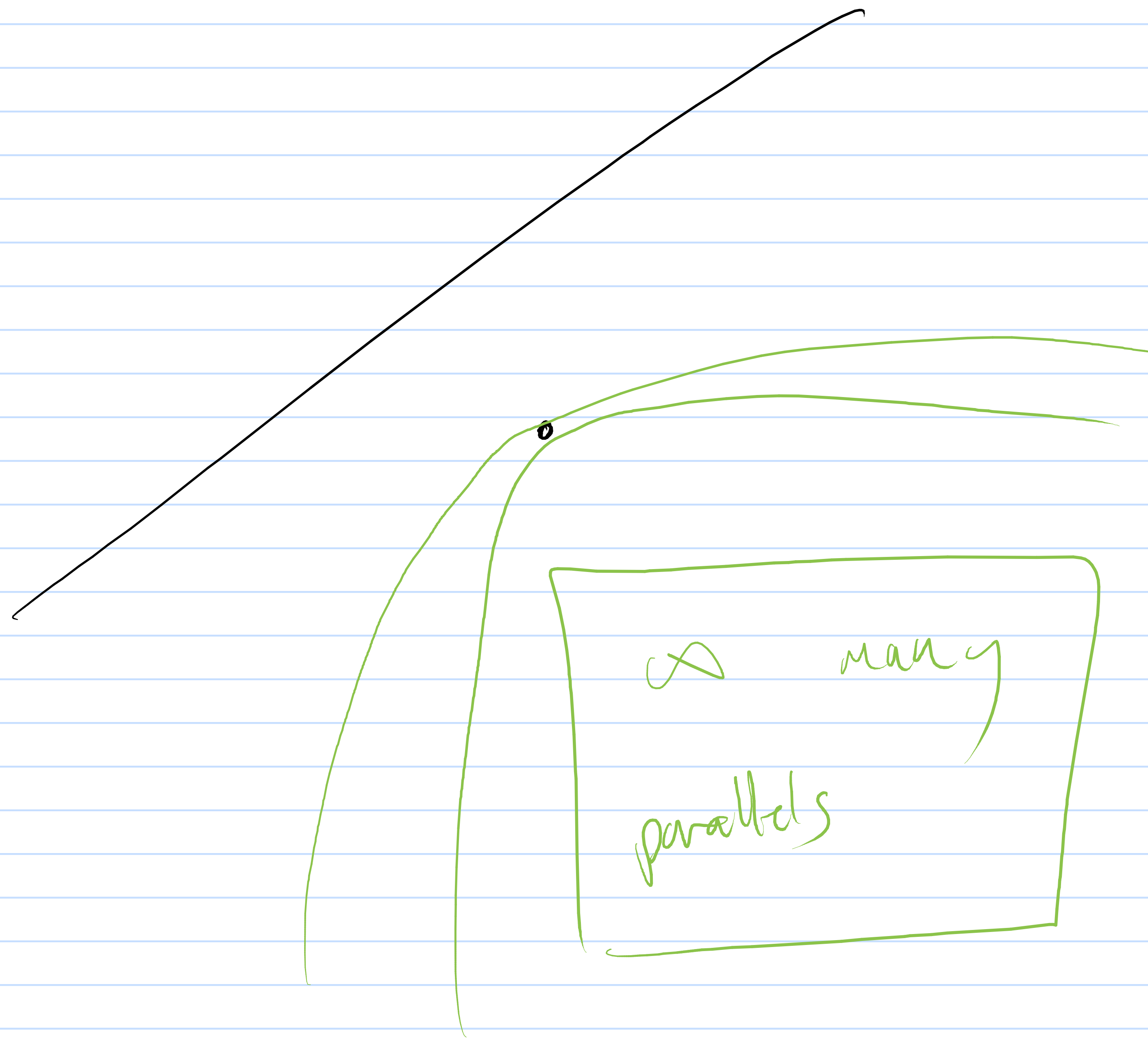


circle w/
prescribed
center &
radius

Postulate



$\exists!$ parallel



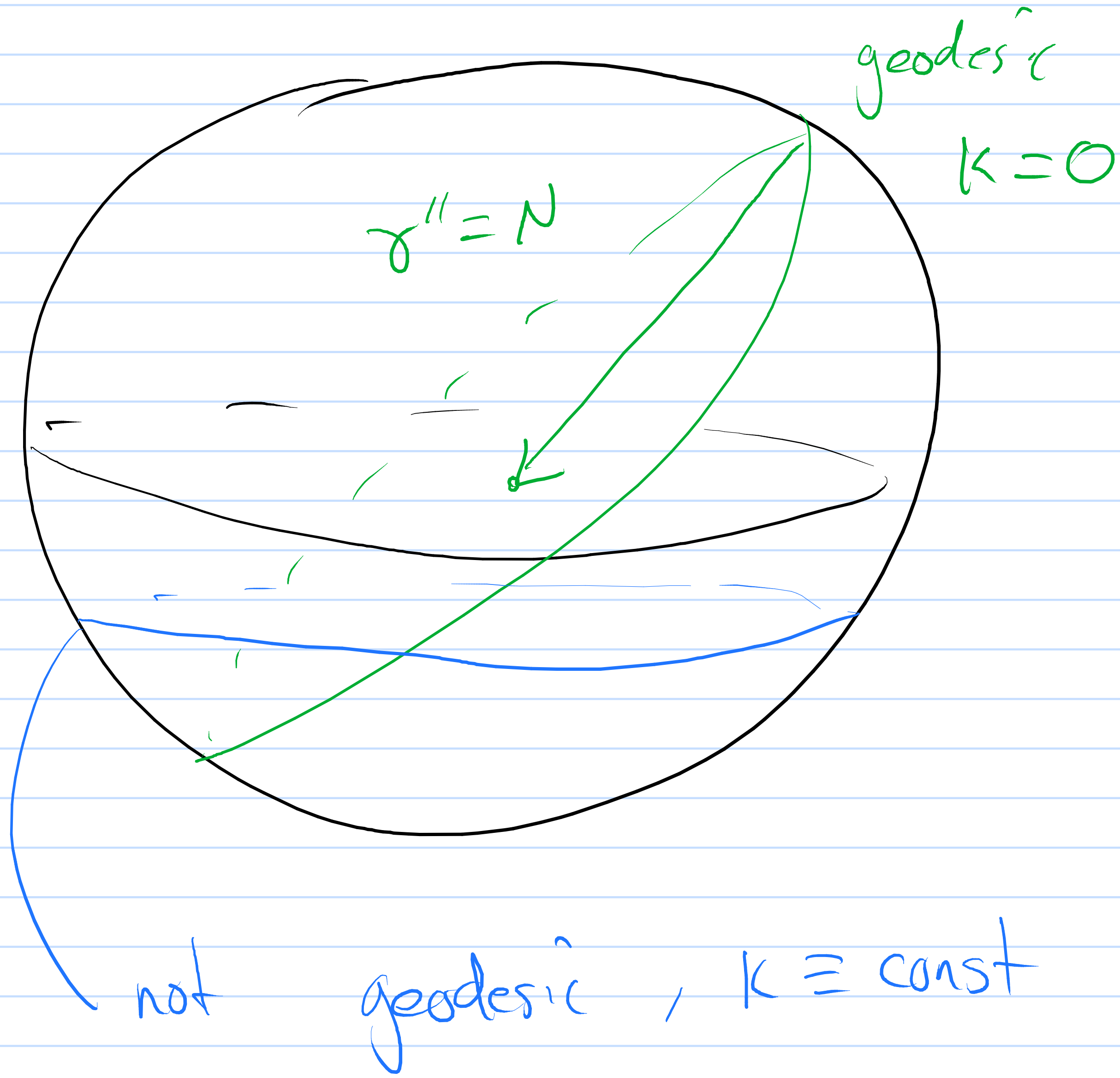
Projective Space

All lines intersect!

no parallels

$$\gamma'' = \nabla_{\gamma'} \gamma'$$

$$= 0 \qquad \text{no acceleration}$$



$$\nabla_T T = 0 \qquad \kappa = \langle \nabla_T T, \underset{\substack{\Uparrow \\ \text{normal to } \gamma \\ \in TS}}{n} \rangle$$

$$\Updownarrow$$

$$\kappa = 0 \Leftrightarrow D^{\mathbb{R}^3}_T T \perp S$$

$$\nabla_T T = \pi_{TM}(\underbrace{D_T T}_{\gamma''})$$

$$\therefore \quad \nabla_T T = 0$$

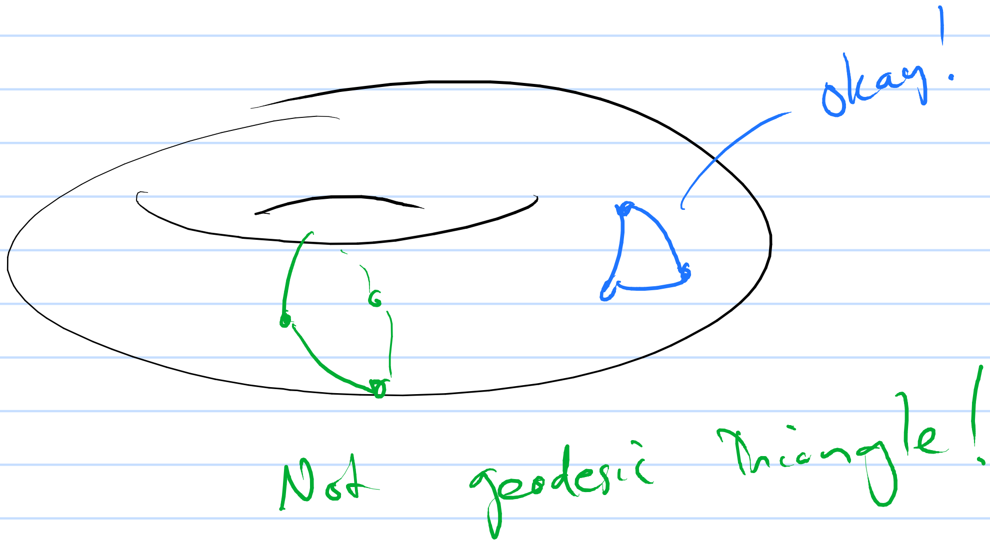
$$\Updownarrow$$

$$\pi_{TM}(D_T T) = 0$$

$$\Updownarrow$$

$$D_T T \perp TM$$

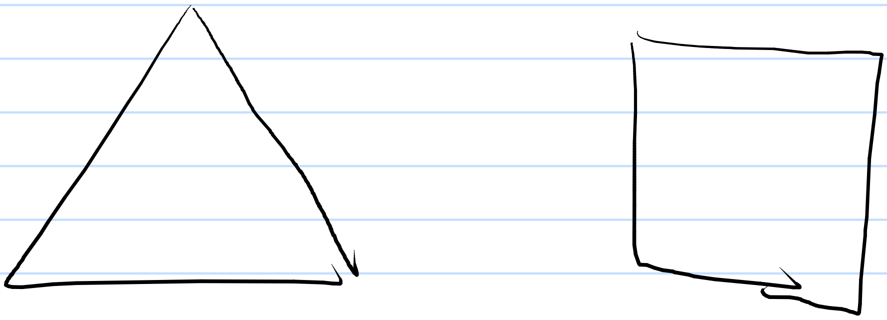
$$D_T = \frac{d}{dt}$$



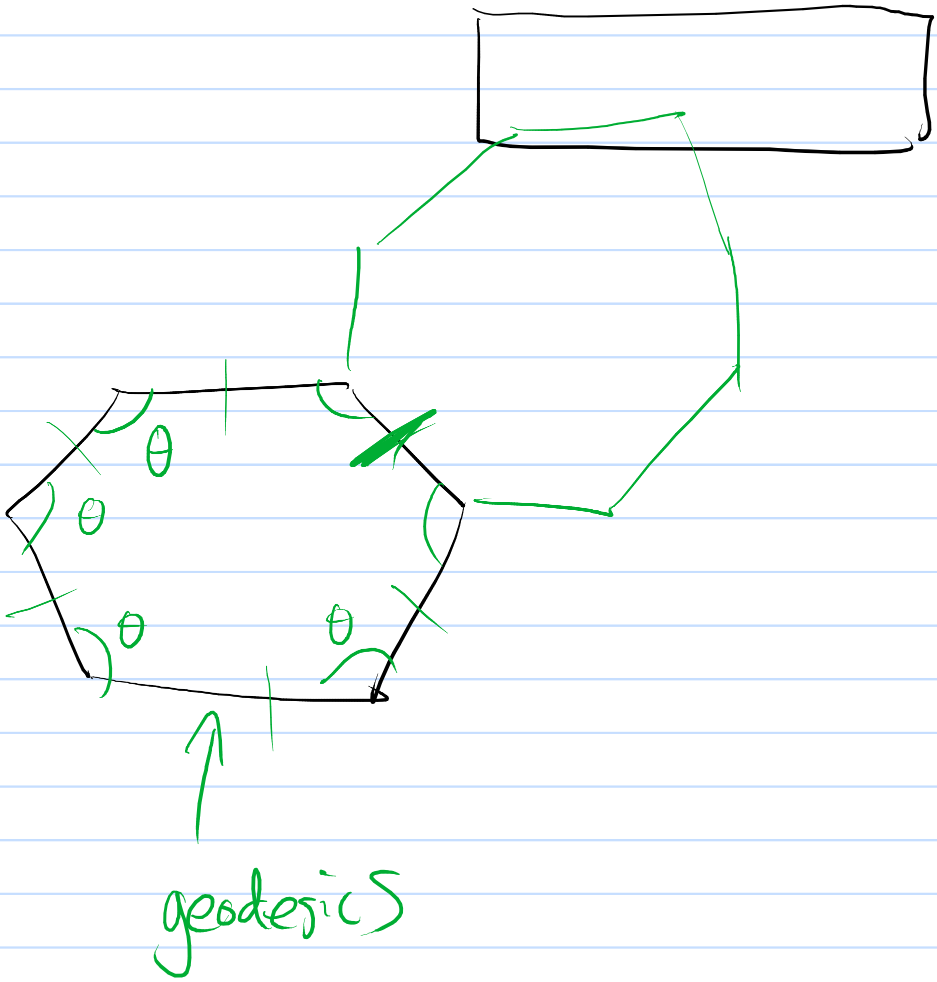
$$\theta_i \in (-\pi, \pi)$$

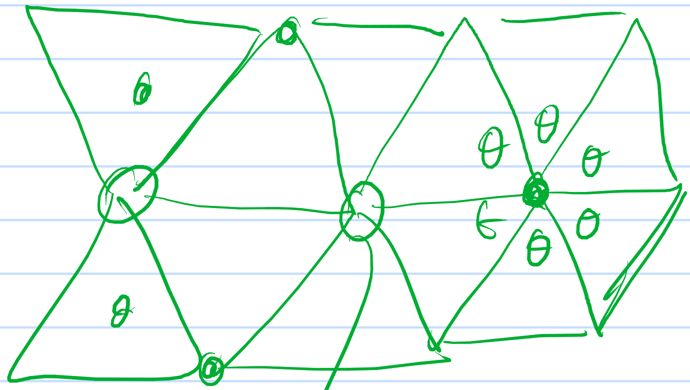
$$\varphi_i = \pi - \theta_i \in (0, 2\pi)$$

regular



not regular





$6\theta = 2\pi$

$k = 6$

$$\underbrace{(k-2)}_{>0}\underbrace{(n-2)}_{>0} = 4$$

$n \geq 3$

$k \geq 3$

$a = k-2 > 0$

$b = n-2 > 0$

$ab = 4$

$$2\pi = \frac{kn - 2k}{n}\,\pi + \underbrace{\overset{\text{Const}}{\int_D K\,dA}}_{\text{Area}(D)}$$

$$\sum \overset{\text{exterior}}{\varphi_i} = 2\pi - \text{Area}(D)$$

equal

$$\underbrace{\int_D K\,dA}_{\text{Area}(D)} + \int_{\partial D} \overset{0}{\cancel{k\,ds}} = 2\pi - \sum \varphi_i$$

$$K = 1$$

Global Gauss-Bonnet

Closed surface

compact, no boundary

$$\int_S K\,dA = 2\pi\chi(S)$$

topological invariant

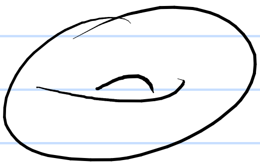
Euler char.

$$\chi(S) = 2(1-\lambda)$$

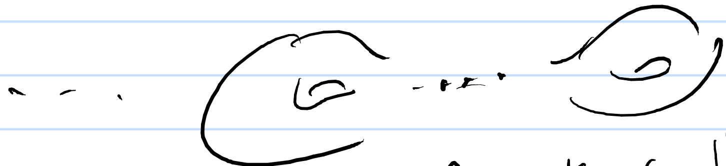
λ = genus = # holes

$\lambda = 0$

$\lambda = 1$

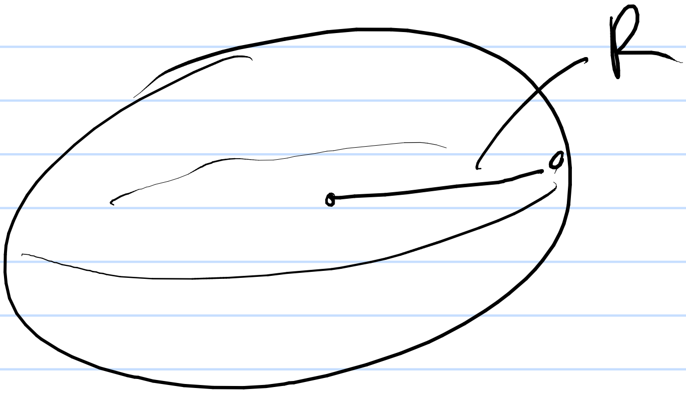


$\lambda = 2$

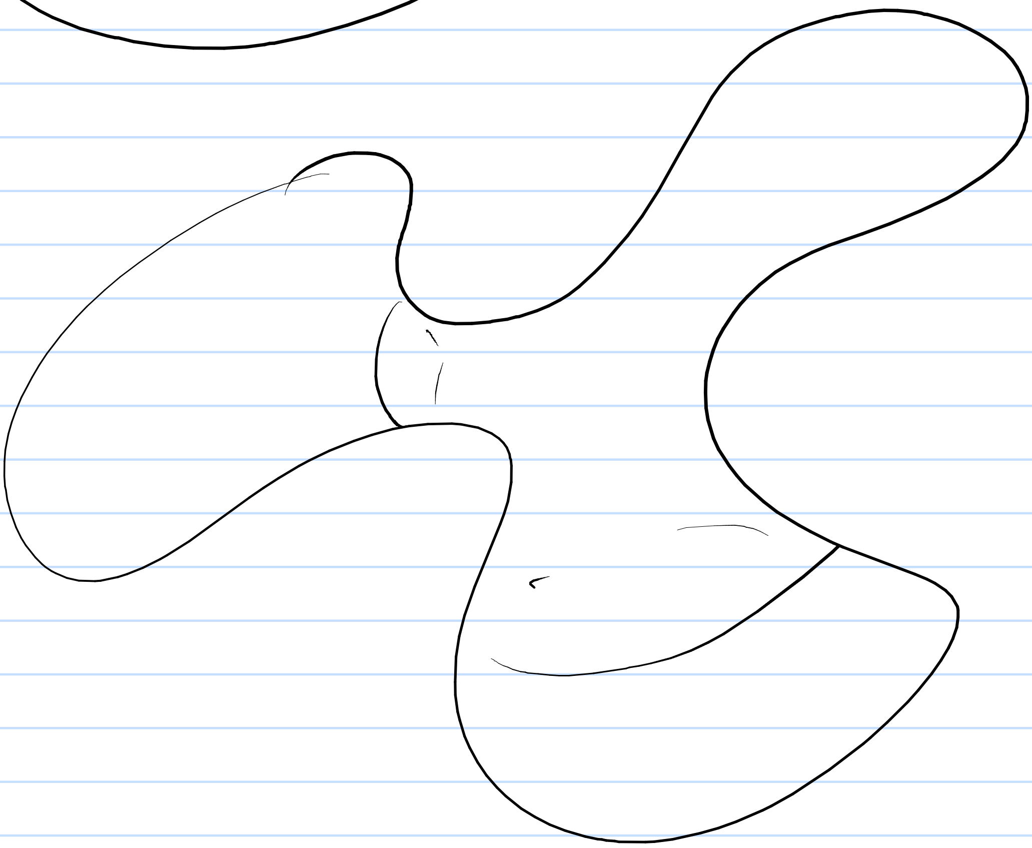


$\lambda = k \in \mathbb{N}$

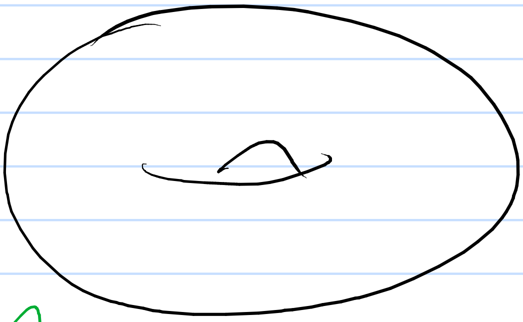
$$\frac{1}{2\pi}\int_S K\,dA = \chi(S)$$



$= 2$



$$\frac{1}{2\pi}\int_S K\,dA = 2$$



$$\int_S K\,dA = 0$$

$\therefore$ K must change sign!

i.e. points w/ $K > 0$

w/ $K < 0$

$$\int_S K\,dA = 0$$

same topology but very different geometry!