Math 20D Lecture A Midterm 2 Version Q FALL 2013

5 Questions; Each problem is worth 10 points; 50 points total.

Instructions:

1. Write your Name, PID, Section, and Exam Version (Q, R, S or T) on the front of your Blue Book.
2. The only things you are allowed to use are writing instruments and erasers and one page, double-sided and handwritten, of notes. (NO calculators, electronic devices, or book.)
3. Write your solutions clearly in your Blue Book and indicate the number and letter of each question.
4. Start each answer on a new page, in the same order they appear in the exam.
5. Show all of your work. No credit will be given for unsupported answers.

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1. (a) Using the Method of Undetermined Coefficients, **write the correct form of a particular solution** to $y'' - 2y' - 3y = 5e^{4t}\sin t$ (but do NOT solve).

   (b) Using the Method of Undetermined Coefficients, **write the correct form of a particular solution** to $y'' + y' - 12y = 3te^{-4t}$ (but do NOT solve).

2. (a) Compute the Wronskian of $y_1 = e^{3t}$ and $y_2 = te^{3t}$.

   (b) Find a particular solution of $y'' - 6y' + 9y = t^{-2}e^{3t}$, $t > 0$, using the Method of Variation of Parameters.

3. Consider the linear system: $\begin{aligned} x_1' &= 2x_1 + 3x_2 \\ x_2' &= -x_1 - 2x_2. \end{aligned}$

   (a) Writing this as $\vec{x}\,' = \mathbb{A}\,\vec{x}$, where $\vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, what is $\mathbb{A}$? Find the eigenvalues and corresponding eigenvectors of $\mathbb{A}$.

   (b) Write the general solution to the linear system.

   (c) Find the solution with $x_1(0) = 4$ and $x_2(0) = -2$.

4. Consider the equation $\vec{x}\,' = \mathbb{A}\,\vec{x}$, where $\mathbb{A} = \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}$. There is a repeated eigenvalue $\lambda = -1$ with associated eigenvector $\vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, so that $\vec{x}^{(1)} = e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is a solution (do **not** prove).

   (a) Using this, find a second independent solution $\vec{x}^{(2)}$.

   (b) Compute the Wronskian $W\left(\vec{x}^{(1)}, \vec{x}^{(2)}\right)$. Why are $\vec{x}^{(1)}$ and $\vec{x}^{(2)}$ independent?

5. The following two parts are unrelated.

   (a) Using that $\mathbb{B} = \begin{pmatrix} 1 & 5 \\ -1 & -1 \end{pmatrix}$ has an eigenvalue $2i$ with an associated eigenvector $\begin{pmatrix} 1+2i \\ -1 \end{pmatrix}$, find the general solution to the equation $\vec{x}\,' = \mathbb{B}\,\vec{x}$.

   (b) Consider the ODE $\vec{x}\,'(t) = \mathbb{A}\,\vec{x}(t) + \vec{g}(t)$, where $\mathbb{A} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\vec{g}(t) = \begin{pmatrix} t^{10} \\ \sin t \end{pmatrix}$. The eigenvalue 1 has eigenvector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and the eigenvalue -1 has eigenvector $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$. Let $\mathbb{T} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$, so that $\mathbb{T}^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Write down the ODE solved by $\vec{y} = \mathbb{T}^{-1}\vec{x}$ (but do NOT solve).