Name: _	PID:

Math 142A Final Examination March 19, 2008

Turn off and put away your cell phone.

No calculators or any other electronic devices are allowed during this exam.

You may use one page of notes, but no books or other assistance on this exam.

Read each question carefully, answer each question completely, and show all of your work.

Write your solutions clearly and legibly; no credit will be given for illegible solutions.

If any question is not clear, ask for clarification.

#	Points	Score
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	·
Σ	36	

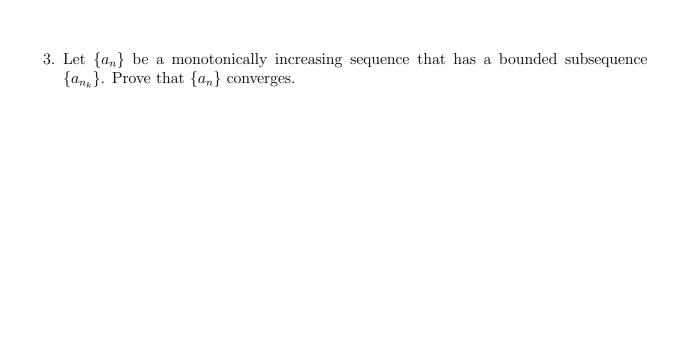
1. Define the sequence $\{x_n\}$ by

$$\begin{cases} x_1 = 1\\ x_{n+1} = \sqrt{x_n + 5} \text{ for } n \ge 1 \end{cases}$$

(a) Show that $\{x_n\}$ increases monotonically.

(b) Show that $\{x_n\}$ converges.

2. Let a be a real number and let $S = \{x \in \mathbb{Q} \mid x < a\}$, where \mathbb{Q} is the set of rational numbers. Prove that $a = \sup S$.



4. Let I be a neighborhood of x_0 and suppose that the function $g:I\to\mathbb{R}$ is differentiable. Define

$$h(x) = \begin{cases} \frac{g(x) - g(x_0)}{x - x_0} & \text{if } x \neq x_0, \\ g'(x_0) & \text{if } x = x_0. \end{cases}$$

Prove that $h: I \to \mathbb{R}$ is continuous.

5. Let I be a neighborhood of x_0 and let $f:I\to\mathbb{R}$ be differentiable. Prove that if $f(x)\leq f(x_0)$ for all $x\in I$, then $f'(x_0)=0$.

6. A function $f: D \to \mathbb{R}$ is **C-Lipschitz** provided $|f(x) - f(y)| \le C|x - y|$ for all $x, y \in D$. Prove that if a C-Lipschitz function $f: \mathbb{R} \to \mathbb{R}$ is differentiable, then $|f'(x)| \le C$ for all $x \in \mathbb{R}$.