Name: _____ PID: _____

Math 142A
Midterm Exam 2
February 29, 2008

Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance on this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	6	
2	6	
3	6	
4	6	
Σ	24	

1. For each of the following *false* statements, exhibit a counterexample. Be sure to *briefly* state why each counterexample shows the corresponding statement to be false.

   (a) A monotone function $f : [0, 1] \to \mathbb{R}$ is one-to-one.

   (b) A strictly increasing function $f : [0, 1] \to \mathbb{R}$ is continuous.

   (c) A one-to-one function $f : [0, 1] \to \mathbb{R}$ is monotone.

2. Recall that $f : D \to \mathbb{R}$ is a *Lipschitz function* if and only if there is a nonnegative number C such that $|f(u) - f(v)| \leq C|u - v|$ for all $u, v \in D$. Prove that a Lipschitz function is uniformly continuous.

3. Let S be a set that is bounded above but has no maximum. Prove that the supremum of S is a limit point of S.

4. Let S be a sequentially compact set and let $f : S \to \mathbb{R}$ be continuous. Prove that the image $f(S)$ is sequentially compact.