Name: _____ **PID:** _____

TA: _____ **Sec. No:** _____ **Sec. Time:** _____

Math 20C.
Midterm Exam 2
November 19, 2010

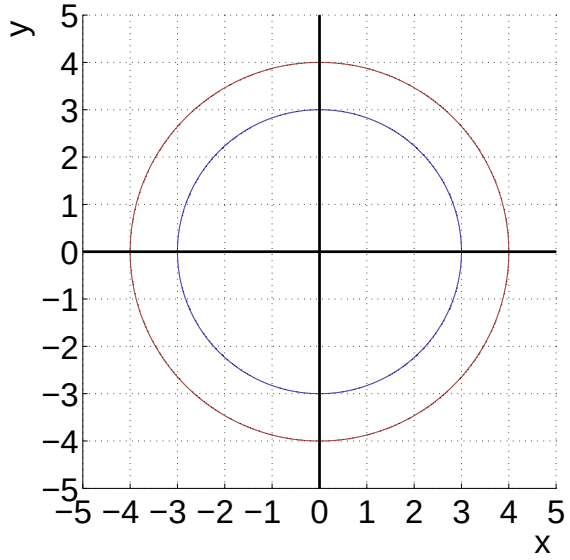
Turn off and put away your cell phone.
No calculators or any other electronic devices are allowed during this exam.
You may use one page of notes, but no books or other assistance during this exam.
Read each question carefully, answer each question completely, and show all of your work.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

#	Points	Score
1	6	
2	9	
3	6	
4	6	
Σ	27	

1. Let $z = f(x, y) = x^2 + y^2 + 800$.

   (a) (3 points) Compute $\dfrac{\partial}{\partial x}\left(f\,(x, y)^{100}\right)$.

   (b) (3 points) Two contours $f(x, y) = k$ are shown on this plot. Label each contour with its value of k. The grid lines are shown to help you read off coordinates that may be useful in doing this computation.



2. Let $f(x, y) = x^2 - xy - 6y + 40$.

(a) (3 points) Find the directional derivative of f at the point $(2, 4)$ in the direction towards the point $(6, 1)$.

(b) (6 points) Find all critical points of f. Classify each critical point as a local maximum, local minimum, or saddle point. Compute the value of f at each critical point.

3. Let $f(x, y) = x^2y^3 + 20$.

(a) (4 points) Find an equation for the tangent plane to the surface $z = f(x, y)$ at the point where $x = 3$ and $y = -1$.

(b) (2 points) Use the linear approximation to $f(x, y)$ at $(3, -1)$ to estimate the value of $f(3.1, -1.2)$. *Hint: This makes use of your answer to (b) and is not the exact value of* $f(3.1, -1.2)$.

4. The temperature at a point (x, y) is $T(x, y)$, measured in degrees Celsius, and satisfies $T_x(2, 3) = 8$ and $T_y(2, 3) = 6$. A scarab beetle crawls so that its position after t seconds is given by $x = \sqrt{2 + t}$, $y = 2 + \frac{1}{2}t$, where x and y are measured in centimeters.

   (a) (3 points) How fast is the temperature rising on the beetle's path after 2 seconds?

   (b) (3 points) In what direction from the point $(2, 3)$ does the temperature *decrease* the fastest? Express your answer as a unit vector.