MATH142B Practice Midterm 2

Instructions

1. You may use any type of calculator, but no other electronic devices during thi.s exam.
2. You may use one page of notes (written on both sides), but no books or other assistance during this exam.
3. Write your Name, PID, and Section on the front of your Blue Book.
4. Write your solutions clearly in your Blue Book
   (a) Carefully indicate the number and letter of each question and question part.
   (b) Present your answers in the same order they appear in the exam.
   (c) Start each question on a new page.
5. Read each question carefully, and answer each question completely.
6. Show all of your work; no credit will be given for unsupported answers.
7. The bonus questions only count if you answer all the other questions.

Questions

1. Let f be a 3 times differentiable function f satisfying $f'(x_0) = 0$ and $f''(x_0) = -1$. Using the Lagrange Remainder Theorem, prove that f has a local maximum at $x = x_0$ (i.e. there is an open interval I containing x_0 such that $f(x) \leq f(x_0)$ for all $x \in I$).
2. Estimate $\sin(1/2)$. Use the n'th Taylor polynomial for $f(x) = \sin(x)$ and the Lagrange remainder theorem to find a n such that $|p_n(1/2) - \sin(1/2)| < 1/25$.
3. Let g, h be continuous functions with $h \geq 0$.
   (a) Using the extreme value theorem, monotonicity of integrals and the intermediate value theorem, prove that there exists a $c \in (a, b)$ such that

   $$\int_a^b g(x)h(x)dx = g(c)\int_a^b h(x)dx.$$

   (b) Use the first part to prove that the Cauchy Integral Remainder Theorem implies the Lagrange Remainder Theorem. That is, assuming

   $$R_n(x) = \frac{1}{n!}\int_{x_0}^{x} f^{(n+1)}(t)(x-t)^n dt,$$

   then there exists a c strictly between x and x_0 such that

   $$R_n(x) = \frac{1}{(n+1)!} f^{(n+1)}(c)(x-x_0)^{n+1}.$$

1. Let $f : \mathbb{R} \to \mathbb{R}$ be an infinitely differentiable, odd function, i.e. $f(-x) = -f(x)$. In particular, this implies $f(0) = 0$. Prove that the Taylor expansion of f at $x_0 = 0$ has no terms of even powers. For example, $\sin x = x - x^3/3! + x^5/5! + \dots$.