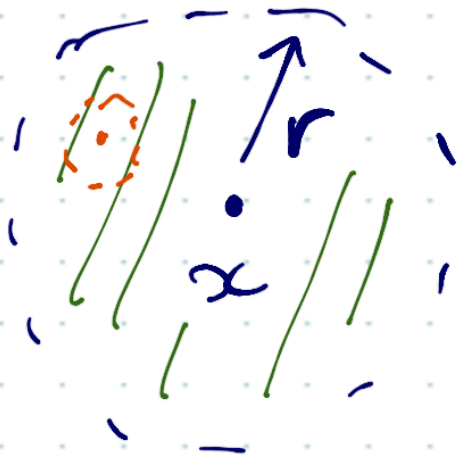
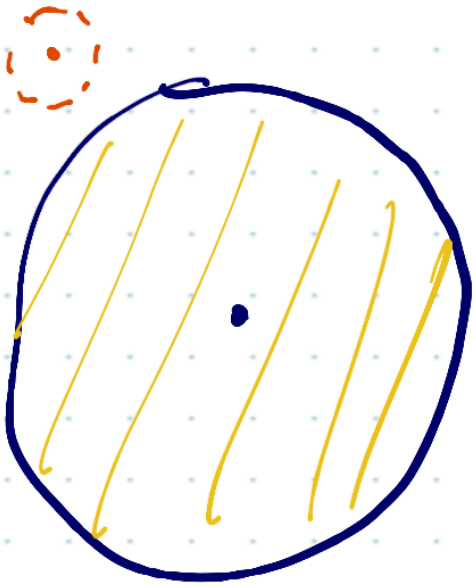
Open & Closed Balls

In 2d

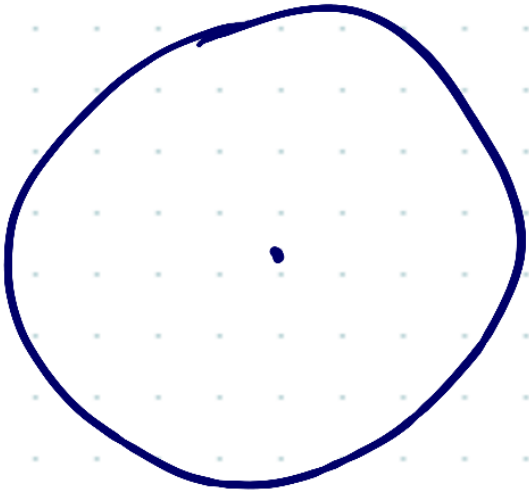


OPEN

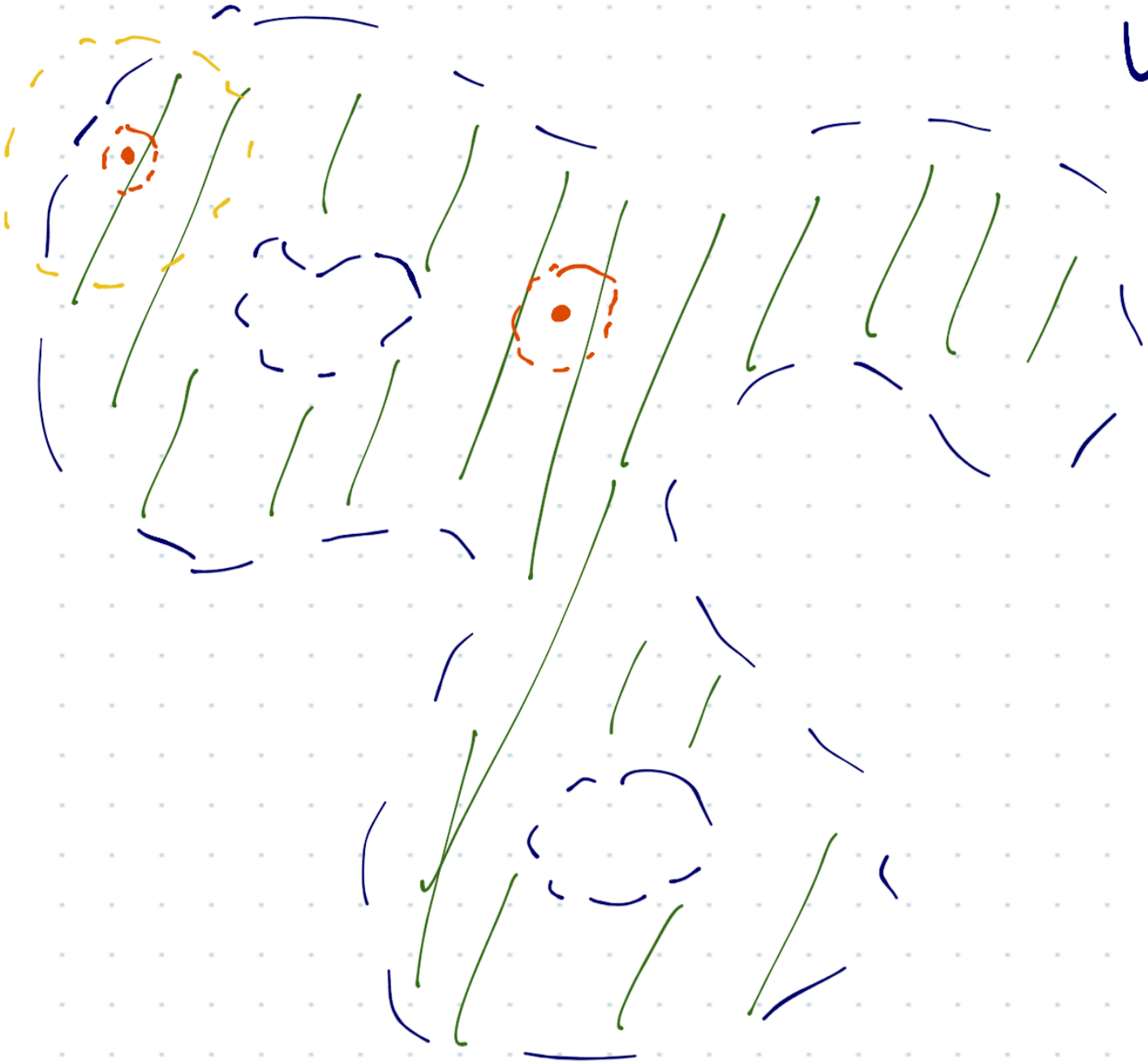


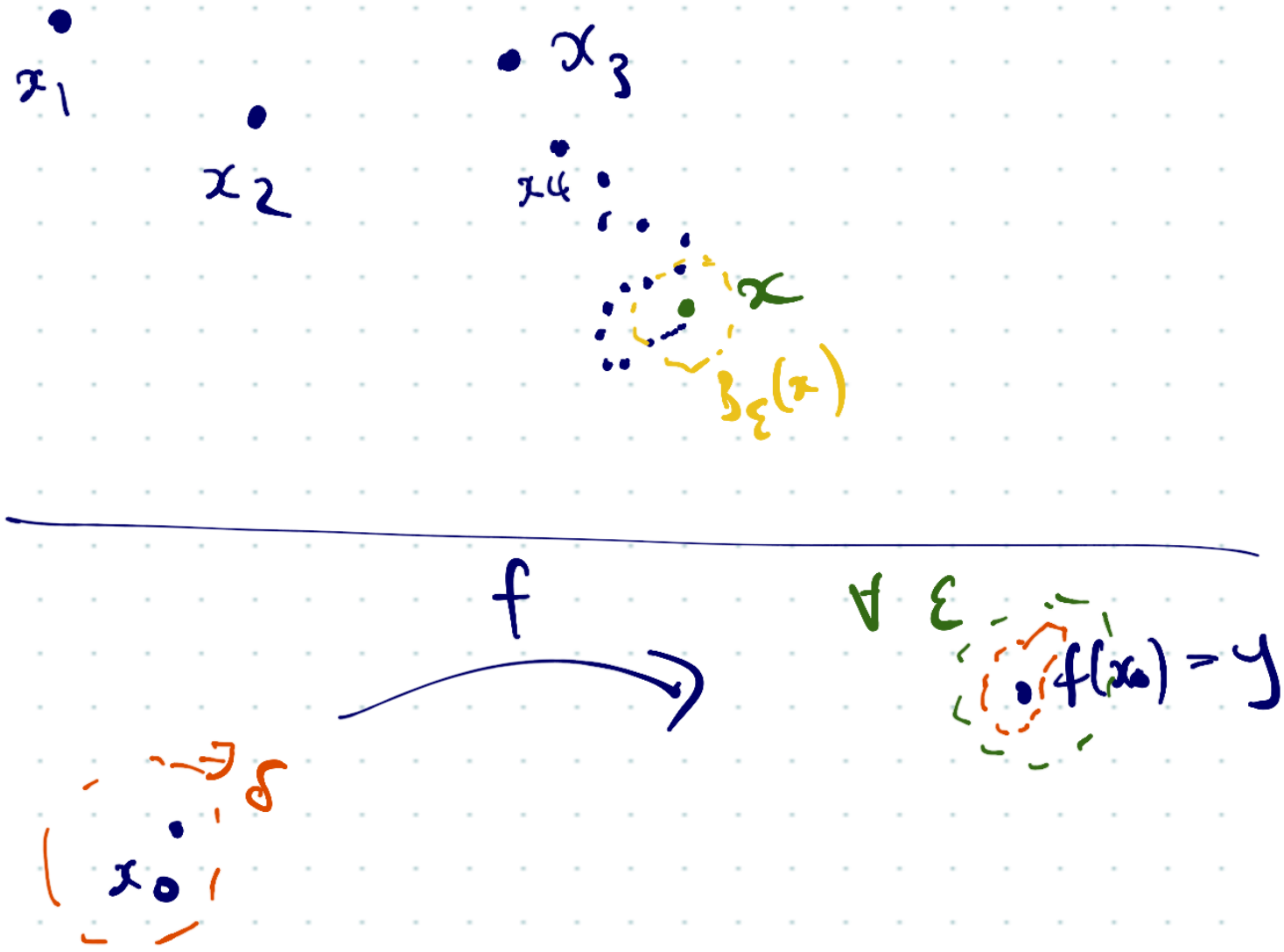
CLOSED

In 3d



SPHERE




x_1
x_2
x_3
x_4
x
$B_\varepsilon(x)$
f
$\forall\ \varepsilon$
$\exists$
$f(x_0) = y$
x_0
δ

$$f^{-1}(y) = f^{-1}(y_0) + (df_{x_0})^{-1} \cdot (y - y_0)$$

$$+ o(|y - y_0|)$$

Mean Val thm

$$f(x_1) - f(x_2) = f'(c)(x_1 - x_2)$$

for some c between x_1 & x_2

$$\therefore |f(x_1) - f(x_2)| = |f'(c)| |x_1 - x_2|$$

$$\leq \sup |f'| \, |x_1 - x_2|$$