Millikan-Fletcher Oil drop experiment

Reference: `https://en.wikipedia.org/wiki/Oil_drop_experiment`

This problem is based on Boyce-DePrima question 27 from section 2.3.

A body falling in a relatively dense fluid, oil for example, is acted on by three forces: a resistive force R, a buoyant force B, and its weight W due to gravity.

Archimedes' principal states that the buoyant force is equal to the weight of the fluid displaced by the object. **Warning**: Weight is a force (not the mass!) of magnitude equal to the mass times the gravitational constant, $g \simeq 9.8m/s^2$.

For a slowly moving spherical body of radius a, the resistive force is given by Stokes' law, $R = -6\pi\mu a v$, where v is the velocity of the body, and μ is the coefficient of viscosity of the surrounding fluid.

Let then, a solid sphere of radius a and constant density ρ fall through a dense fluid with constant density ρ' and viscosity coefficient μ. Let v denote the velocity with down the positive direction.

1. Draw a free body diagram, and show that the forces are given the following expressions:

$$\begin{aligned} R &= -6\pi\mu a v \quad \text{(given in Stokes' law above)} \\ B &= -\frac{4}{3}\pi g a^3 \rho' \quad \text{(from Archimedes' principle)} \\ W &= \frac{4}{3}\pi g a^3 \rho \end{aligned}$$

   Recall the volume of a sphere of radius a is given by $V = 4/3\pi a^3$.

2. Assuming (somewhat naively) that Newton's second law holds for solid bodies, show that the equation of motion is,

$$v' = g\frac{\rho - \rho'}{\rho} - \frac{9\mu}{2a^2\rho}v$$

3. Show that the equilibrium velocity (known as terminal velocity) is given by,

$$V_L = \frac{2a^2 g}{9\mu}(\rho - \rho')$$

and draw a phase diagram indicating stability.

Notice that terminal velocity is attained precisely when the all the forces balance, and that a body travelling a terminal velocity will continue to do so indefinitely.

4. The equation of motion may be written more succinctly as

$$v' + av = b$$

with

$$a = \frac{9\mu}{2a^2\rho}, \quad \text{and} \quad b = g\frac{\rho - \rho'}{\rho}.$$

Notice that $v_L = b/a$.

Solve this equation to obtain

$$v = (v_0 - v_L)\exp\left(-\frac{9\mu}{2a^2\rho}t\right) + v_L$$

confirming the conclusions in part 3. Here v_0 is the initial velocity.

5. The Millikan-Fletcher oil drop experiment aims to determine the charge of an electron using the above calculations. Charged droplets (which we assume to be approximately spherical) of oil are allowed to fall between a pair of parallel metal plates of distance d apart. The velocity of the droplets approaches v_L exponentially fast as can be seen from the solution obtained in the previous question. Then a voltage V is applied across the parallel plates inducing an electric field E of magnitude V/d pointing upwards. This causes an upward force of magnitude Vq/d where q is the charge of a droplet. By adjusting the voltage V to just the right value, the electric force will balance out the other forces and the droplets become stationary (i.e. $v = 0$). Show that,

$$q = \frac{4\pi a^3 gd}{3V}(\rho - \rho').$$

By repeating the experiment with many drops, the charge of an electron may be estimated. The estimate obtained by Millikan and Fletcher was $1.5924(17)10^{-19}$ Coulombs, within 1% of the currently accepted value of $1.602176487(40)10^{-19}$ Coulombs, although there is some controversy related to the experiment which you can read about at the Wikipedia article linked to above.