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1/ a) Let $x(u,v) = (u, v, f(u,v))$
for $(u,v) \in U$

Then (i) x is differentiable since f is differentiable.

(ii) $x^{-1}(x,y,z) = (x,y)$ is continuous for $(x,y,z) \in$ Graph f.

(iii) $$dx = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ f_u & f_v \end{pmatrix}$$

is injective (since rank $dx = 2$)

b) $x_u = (1, 0, f_u)$
$x_v = (0, 1, f_v)$

$E = \langle x_u, x_u \rangle = 1 + (f_u)^2$

$F = \langle x_u, x_v \rangle = f_u f_v$

$G = \langle x_v, x_v \rangle = 1 + (f_v)^2$

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2/ a) $\phi = F|_{S^2}$ where $F : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$
$v \longmapsto -v$
is differentiable

$\therefore$ ϕ is differentiable

$\phi^{-1} = \phi$ since $\phi \circ \phi(v) = \phi(-v) = -(-v) = v$

$\therefore$ ϕ^{-1} is differentiable and
ϕ is a diffeomorphism

b) Let $\alpha(0) = p$
$\alpha'(0) = v$

Then $d\phi \cdot v = \frac{d}{dt}\Big|_{t=0} \phi(\alpha(t))$

$= \frac{d}{dt}\Big|_{t=0} (-\alpha(t))$

$= -\alpha'(0) = -v$

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3 a) $x_z = (1, f'\cos\theta, f'\sin\theta)$

$x_\theta = (0, -f\sin\theta, f\cos\theta)$

$E = \langle x_z, x_z \rangle = 1 + (f')^2\cos^2\theta + (f')^2\sin^2\theta$

$= 1 + (f')^2$

$F = \langle x_z, x_\theta \rangle = -ff'\sin\theta\cos\theta + ff'\sin\theta\cos\theta$

$= 0$

$G = \langle x_\theta, x_\theta \rangle = f^2\sin^2\theta + f^2\cos^2\theta$

$= f^2$

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3| b)

$$L[\alpha_z] = \int_a^b |\alpha_z'(t)|\, dt$$

$$= \int_a^b |x_z(t, \theta_0)|\, dt$$

$$= \int_a^b \sqrt{E(t, \theta_0)}\, dt$$

$$= \int_a^b \sqrt{1 + f'(t)^2}\, dt$$

$$L[\alpha_\theta) = \int_0^{2\pi} |\alpha_\theta'(t)|\, dt$$

$$= \int_\theta^{2\pi} |x_\theta(z_0, t)|\, dt$$

$$= \int_0^{2\pi} \sqrt{G(z_0, t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{f^2(z_0)}\, dt$$

$$= 2\pi f(z_0)$$