POTENTIAL FUNCTIONS

- Test for gradients
- Simply connected domains
- Determining potential functions

TEST FOR GRADIENTS

TEST FOR GRADIENTS

LEMMA

$F = (P, Q) = \nabla f = (\partial_x f, \partial_y f)$ satisfies

$$\partial_y P = \partial_x Q$$

EXAMPLE

EXAMPLE

$$F = \left(2xe^{x^2-y}, -e^{x^2-y}\right)$$

EXAMPLE

EXAMPLE

$$F = (\cos y, x^2)$$

SIMPLY CONNECTED DOMAINS

SIMPLY CONNECTED DOMAINS

DEFINITION

A **simply connected domain** is a connected open set with no holes.

- Disc - simply connected
- Annulus - not simply connected

VECTOR FIELDS ON SIMPLY CONNECTED DOMAINS

THEOREM

Let $F = (P, Q)$ be a vector field on a simply connected domain. Then F is a gradient field if and only $\partial_y P = \partial_x Q$.

EXAMPLE

EXAMPLE

$$F = (P, Q) = \frac{1}{x^2 + y^2}(-y, x), \qquad (x, y) \neq (0, 0)$$

- $\partial_y P = \partial_x Q$
- Not a gradient

POTENTIAL FUNCTIONS

DETERMINING POTENTIAL FUNCTIONS

- $\partial_y P = \partial_x Q$
- Solve $\nabla f = (\partial_x f, \partial_y f) = (P, Q)$
- $\partial_x f = P \Rightarrow f = \int P dx + h(y)$
- Sub into $\partial_y f = Q$ and solve for h

EXAMPLE

EXAMPLE

$$F(x, y) = (2xy, x^2 + e^y)$$

EXAMPLE

EXAMPLE

$$F(x, y) = (2xy, x^2 + xe^y)$$