GAUSS' LAW

- Gauss' Law
- Electric Field

GAUSS' LAW

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THEOREM

Let
$$\overrightarrow{\mathbf{F}}(p)=rac{p}{r^3}$$

Then for any closed surface S enclosing the region Ω

$$\iint_{S} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA = egin{cases} 4\pi & 0 \in \Omega \ 0 & 0
otin \Omega \end{cases}$$

• If $0
otin \overline{\Omega}$ then $\overrightarrow{\mathbf{F}}$ is defined on Ω

$$ullet \operatorname{div} \overrightarrow{\mathbf{F}} = 0$$
 on Ω

Divergence theorem

$$\iint_{S} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA = \iiint_{\Omega} \operatorname{div} \overrightarrow{\mathbf{F}} dV = 0$$

ullet $0\in \overline{\Omega} : \overrightarrow{\mathbf{F}}$ defined on $\Omega_\epsilon = \Omega ackslash \mathbb{B}_\epsilon(0)$

$$ullet \operatorname{div} \overrightarrow{\mathbf{F}} = 0$$
 on Ω_{ϵ}

Divergence theorem

$$0 = \iiint_{\Omega_\epsilon} ext{div} \overrightarrow{\mathbf{F}} dV = \iint_{\partial \Omega_\epsilon} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA$$

$$ullet 0 = \iiint_{\Omega_\epsilon} ext{div} \overrightarrow{\mathbf{F}} dV = \iint_{\partial \Omega_\epsilon} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA$$

$$ullet$$
 $\partial\Omega_{\epsilon}=S-\mathbb{S}_{\epsilon}$

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$$ullet$$
 $\mathbf{N}(p)=rac{p}{\epsilon}$

• Then

$$egin{aligned} \iint_{\mathbb{S}_{\epsilon}} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA &= \iint_{\mathbb{S}_{\epsilon}} rac{p}{\epsilon^3} \cdot rac{p}{\epsilon} dA \ &= \iint_{\mathbb{S}_{\epsilon}} rac{1}{\epsilon^2} dA \ &= 4\pi \end{aligned}$$

ELECTRIC FIELD

ELECTRIC FIELD

THEOREM

The flux of the electric field through a surface S is proportional to the enclosed charge.

• point charge:
$$\overrightarrow{\mathbf{E}} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$

 multiple point charges: superposition (linearity)