THE DERIVATIVE

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DEFINITION

The derivative of f at x equals

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

DERIVATIVE NOTATION

The derivative may be written as any of

$\frac{df}{dx}$
$\frac{d}{dx}f$
f'
$\dot{f}$

EXAMPLE

EXAMPLE

Show that

$$\frac{d}{dx}x = 1$$

EXAMPLE

EXAMPLE

Show that

$$\frac{d}{dx}x^2 = 2x$$

SECANT LINE

SECANT LINE

DEFINITION

The *secant line* for $f(x)$ between x_1, x_2 is the straight line though the points in the plane, $(x_1, f(x_1))$ and $(x_2, f(x_2))$.

DIFFERENCES

DEFINITION

$$\Delta x = x_2 - x_1.$$

$$\Delta f = f(x_2) - f(x_1).$$

DIFFERENCE QUOTIENT

DEFINITION

The quantity $\frac{\Delta f}{\Delta x}$ is called the *difference quotient*.

SECANT LINE SLOPE

LEMMA

The slope of the secant line is

$$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

EXAMPLE

EXAMPLE

Let $f(x) = x^2$ and let $x_1 = 2$, $x_2 = 3$. The secant line has equation

$$y = 5x - 6$$

TANGENT LINE

TANGENT LINE

DEFINITION

The *tangent line* at x is the line with slope $f'(x)$ and passing through the point $(x, f(x))$ in the plane.

TANGENT LINE SLOPE

LEMMA

$$f'(x_1) = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}.$$

TANGENT LINE AND SECANT LINE

THEOREM

The tangent line at x is the limit of the secant lines as $x_2 \to x_1$.

EXAMPLE

EXAMPLE

Let $f(x) = x^2$ and let $x_1 = 2$.

Secant line has slope

$$4 + \Delta x.$$

Tangent line has slope 4

DIFFERENTIABILITY IMPLIES CONTINUITY

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THEOREM

If f is differentiable at x_0, then f is also continuous at x_0.

PROOF

$$f(x) - f(x_0) = \frac{\Delta f}{\Delta x}\Delta x \to 0.$$