INCREASING AND DECREASING FUNCTIONS

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DEFINITION

A function f is said to be **increasing** if $f(x_2) \geq f(x_1)$ whenever $x_2 \geq x_1$.

A function f is said to be **decreasing** if $f(x_2) \leq f(x_1)$ whenever $x_2 \geq x_1$.

EXAMPLE

EXAMPLE

$$f(x) = x^2$$

FIRST DERIVATIVE

THEOREM

Let f be a differentiable function.

Increasing $f' \geq 0$
Decreasing: $f' \leq 0$

EXAMPLE

EXAMPLE

$$f(x) = \cos x.$$

MINIMUM AND MAXIMUM

MINIMUM AND MAXIMUM

DEFINITION

Minimum: $f(x) \geq f(x_{\text{min}})$
Maximum: $f(x) \leq f(x_{\text{max}})$

EXTREME VALUE THEOREM

THEOREM

A continuous function defined on a closed, bounded interval $[a, b]$ attains both a maximum and minimum.

EXAMPLE

EXAMPLE

$f(x) = x^2 + 1$ for $x \in [-2, 1]$.

LOCAL MIN AND MAX

DEFINITION

A function f has a **local minimum** at x_0 if $f(x) \geq f(x_0)$ for every x in some open interval containing x_0.

A function f has a **local maximum** at x_0 if $f(x) \leq f(x_0)$ for every x in some open interval containing x_0.

EXAMPLE

EXAMPLE

$$f(x) = x^3 - x = x(x-1)(x+1).$$

CRITICAL POINTS

CRITICAL POINTS

DEFINITION

A **critical point** for a function f is point x where $f'(x) = 0$ or $f'(x)$ is not defined.

EXAMPLE

EXAMPLE

Let $f(x) = x^3 - x = x(x-1)(x+1)$.

FIRST DERIVATIVE TEST

LEMMA

If a function f has a local minimum or maximum at the point x, then x is a critical point.

EXAMPLE

EXAMPLE

$$f(x) = x^3 - x$$

EXAMPLE

EXAMPLE

Let $f(x) = |x|$.

CONCAVITY

SECOND DERIVATIVE

DEFINITION

Let f be a differentiable function. If f' is also differentiable, we say that f is twice differentiable and write f'' for the derivative of f'.

SECOND DERIVATIVE TEST

THEOREM

local minimum: $f'' \geq 0$
local maximum: $f'' \leq 0$

EXAMPLE

EXAMPLE

$$f(x) = x^2$$

EXAMPLE

EXAMPLE

$$f(x) = x^4$$

EXAMPLE

EXAMPLE

$$f(x) = x^3$$

ASYMPTOTES

VERTICAL ASYMPTOTE

DEFINITION

Vertical asymptote: $\lim_{x \to x_0^{\pm}} = \pm\infty$

EXAMPLE

EXAMPLE

$$f(x) = \frac{1}{x^2}$$

EXAMPLE

EXAMPLE

$$f(x) = \frac{1}{x-1}$$

LIMITS AT INFINITY

DEFINITION

If $f(x)$ approaches L as x becomes arbitrarily large we write

$$\lim_{x \to \infty} f(x) = L.$$

Similarly for $\lim_{x \to -\infty} f(x) = L$.

EXAMPLE

EXAMPLE

$$\lim_{x \to \infty} x^2 = \infty$$

EXAMPLE

EXAMPLE

$$f(x) = \frac{x}{x+1}$$

EXAMPLE

EXAMPLE

$$f(x) = \frac{x}{x^2+1}$$

HORIZONTAL ASYMPTOTE

DEFINITION

If $\lim_{x \to \pm\infty} f(x) = L$ then f has a horizontal asymptote L at $\pm\infty$.

EXAMPLE

EXAMPLE

$$f(x) = \frac{x}{x+1}$$