INCREASING AND DECREASING FUNCTIONS

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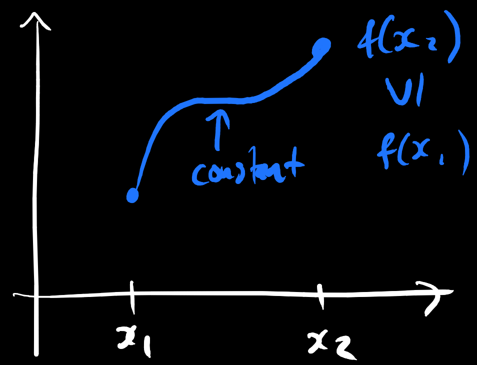
DEFINITION

A function f is said to be **increasing** if $f(x_2) \geq f(x_1)$ whenever $x_2 \geq x_1$.

A function f is said to be **decreasing** if $f(x_2) \leq f(x_1)$ whenever $x_2 \geq x_1$.

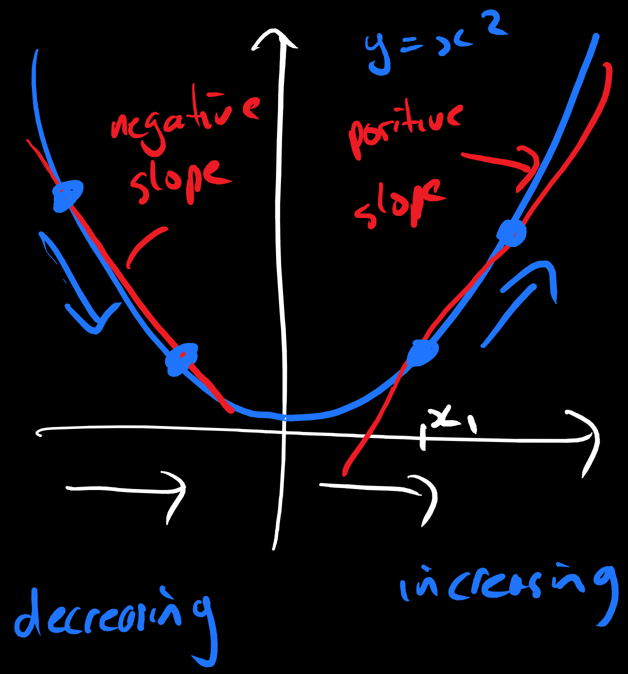


Increasing

EXAMPLE

EXAMPLE

$$f(x) = x^2$$



$f' = \lim$ slope of secant lines

$= \lim$ " ≥ 0 "

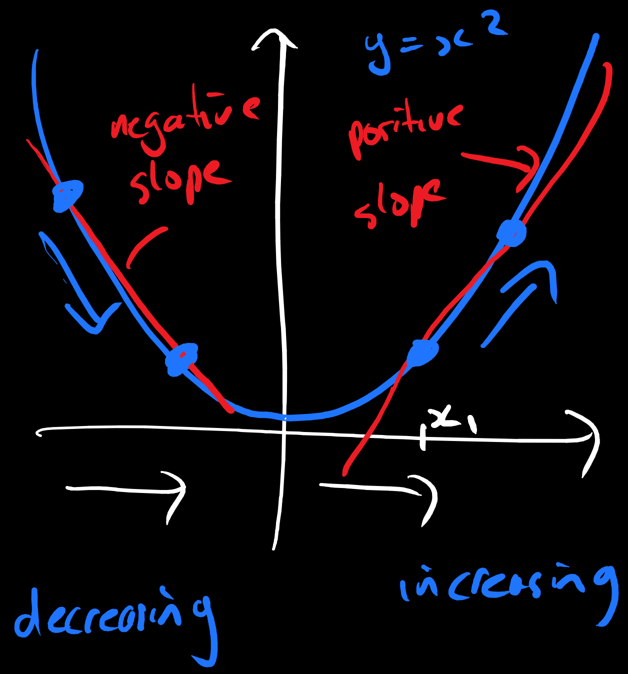
≥ 0

FIRST DERIVATIVE

THEOREM

Let f be a differentiable function.

Increasing	$f' \geq 0$
Decreasing:	$f' \leq 0$



$f' = \lim$ slope of secant lines

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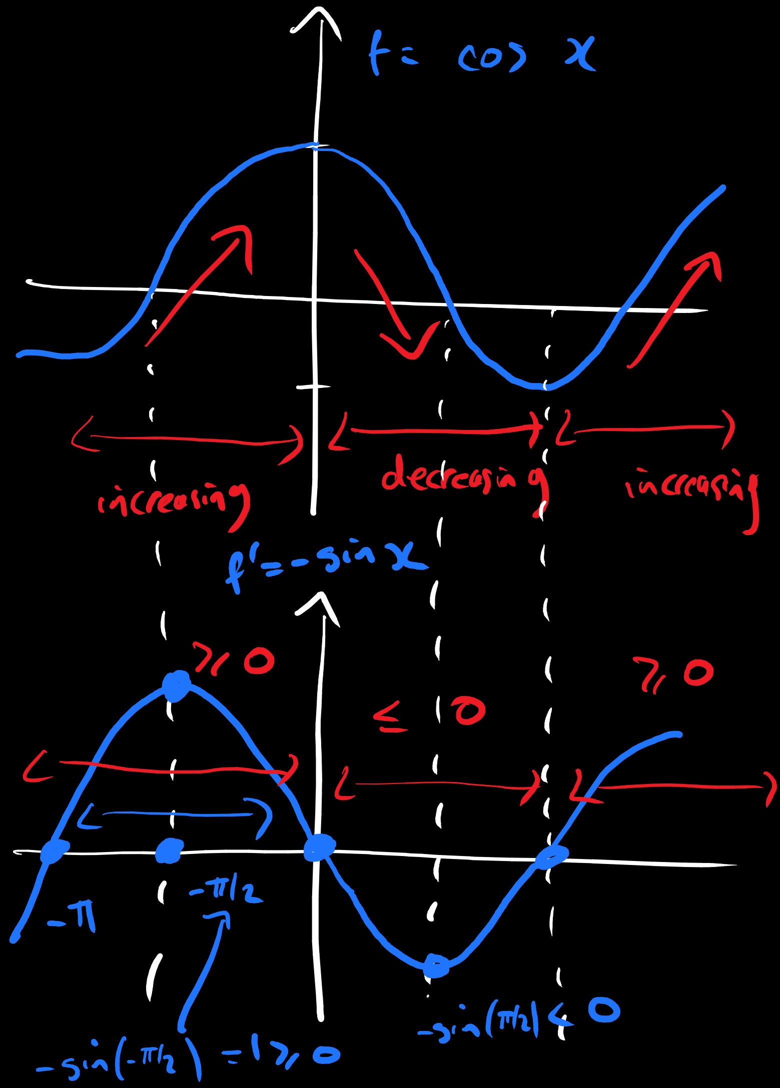
≥ 0

EXAMPLE

EXAMPLE

$$f(x) = \cos x.$$

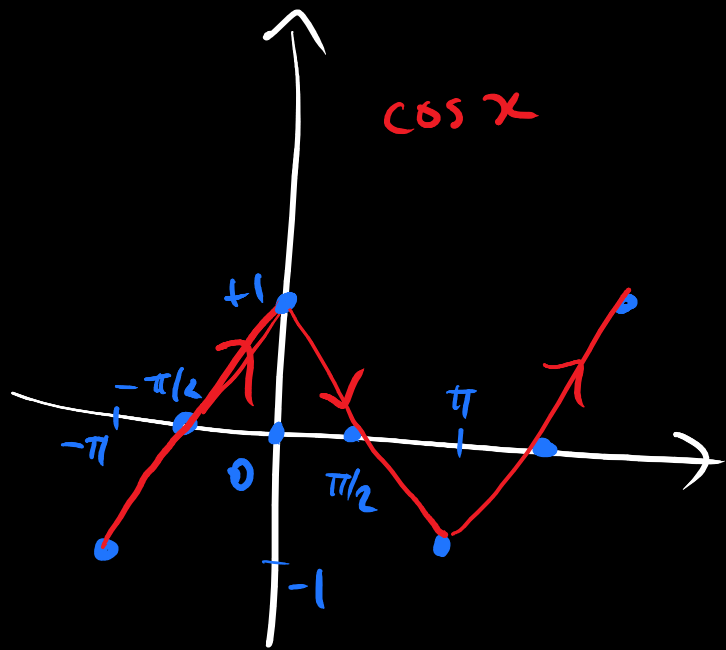


EXAMPLE

EXAMPLE

$$f(x) = \cos x.$$

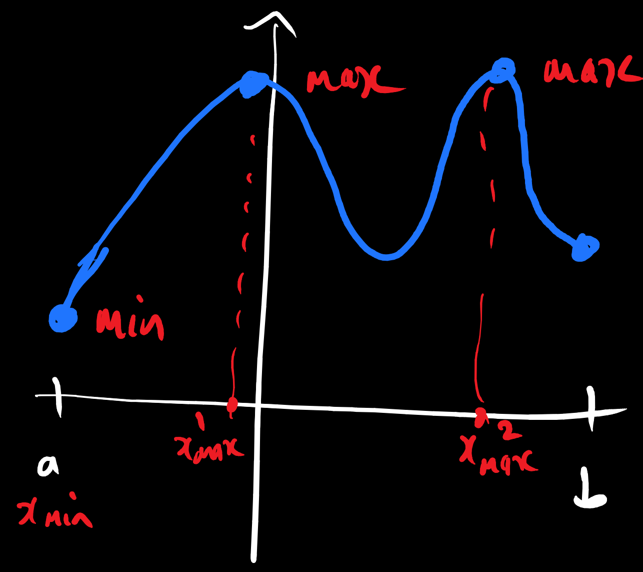
Rough Sketch



MINIMUM AND MAXIMUM

MINIMUM AND MAXIMUM

DEFINITION
Minimum: $f(x) \geq f(x_{\min})$
Maximum: $f(x) \leq f(x_{\max})$



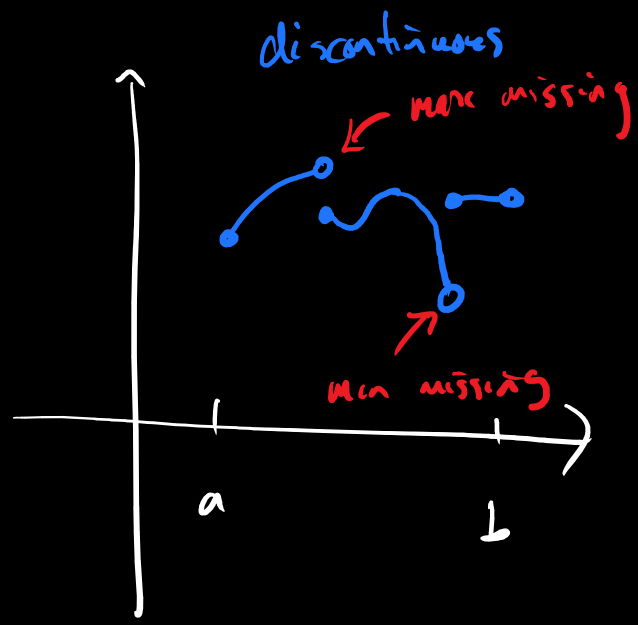
$f(x_{min}) \leq f(x)$ all x

$f(x_{max}) \geq f(x)$ all x

EXTREME VALUE THEOREM

THEOREM

A continuous function defined on a closed, bounded interval $[a, b]$ attains both a maximum and minimum.

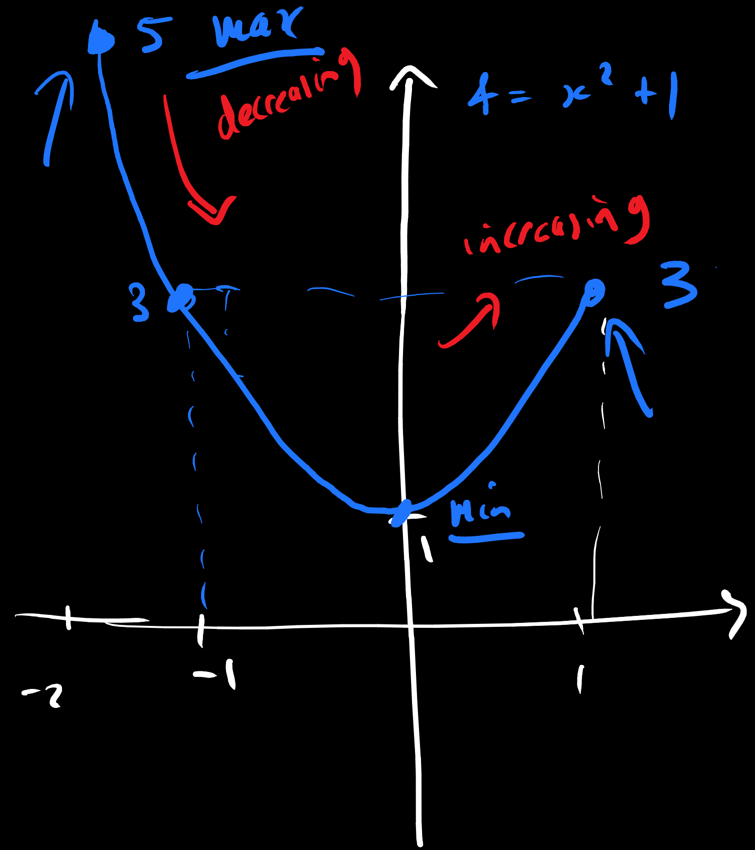


No max or min!

EXAMPLE

EXAMPLE

$f(x) = x^2 + 1$ for $x \in [-2, 1]$.



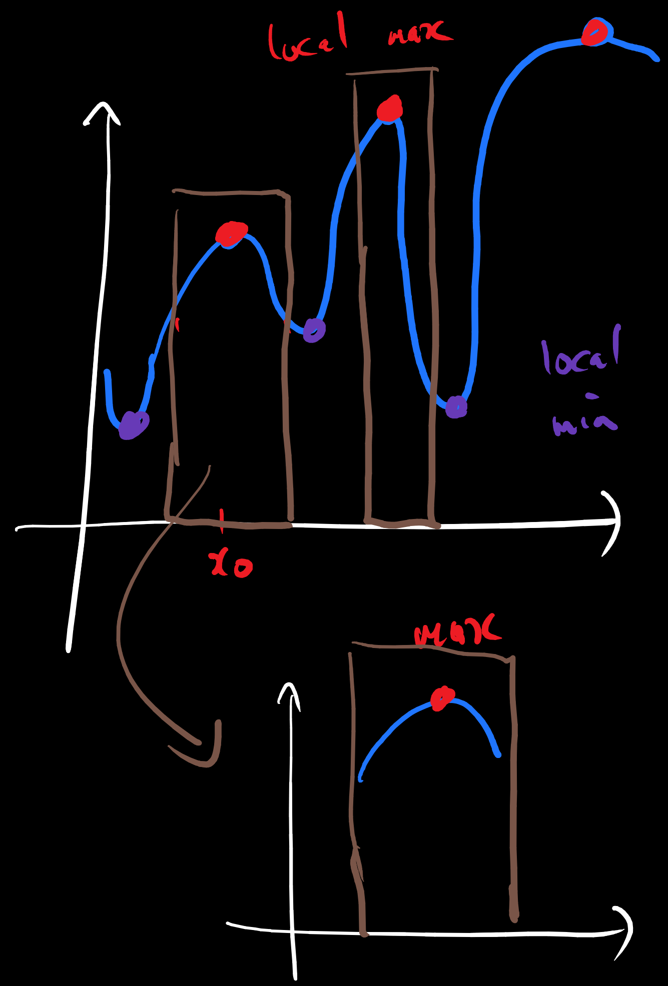
$$f' = 2x \begin{cases} \geq 0 & x \geq 0 \\ \leq 0 & x \leq 0 \end{cases}$$

LOCAL MIN AND MAX

DEFINITION

A function f has a **local minimum** at x_0 if $f(x) \geq f(x_0)$ for every x in some open interval containing x_0.

A function f has a **local maximum** at x_0 if $f(x) \leq f(x_0)$ for every x in some open interval containing x_0.

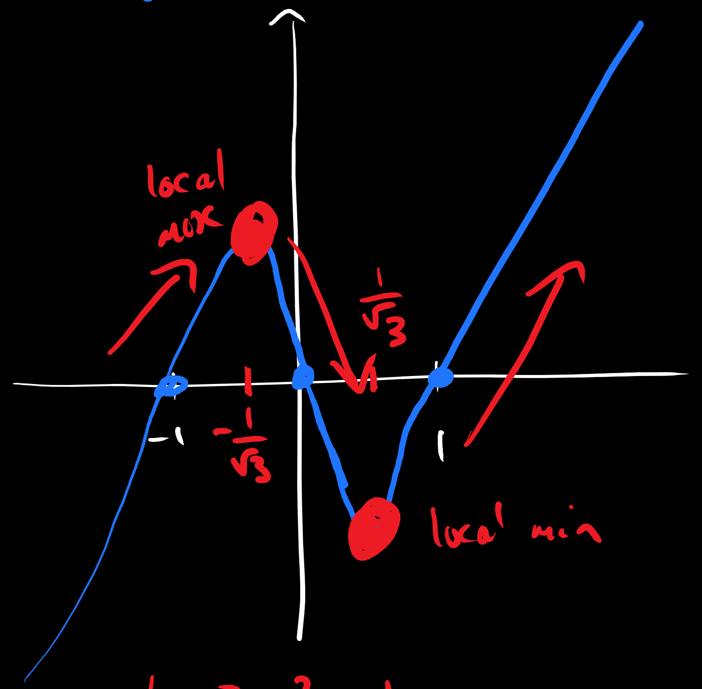


EXAMPLE

EXAMPLE

$$f(x) = x^3 - x = x(x-1)(x+1).$$

$y = x(x-1)(x+1) = x^3 - x$



$y' = 3x^2 - 1$

$y' > 0$ — y increasing

$y' < 0$ — y decreasing

$y' > 0$ — increasing

EXAMPLE

EXAMPLE

$$f(x) = x^3 - x = x(x-1)(x+1).$$

$y' = \underline{3x^2 - 1}$

set $y'(x) = 0$

i.e. $3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = 1/3$

$x = \pm \frac{1}{\sqrt{3}}$

>0 $\quad$ $\underline{<0}$ $\quad$ $\underline{>0}$

$-1 \quad -\frac{1}{\sqrt{3}} \quad 0 \quad \frac{1}{\sqrt{3}} \quad 1$

$y'(-1) = 3-1 \geq 0$ $\qquad$ $y'(1) = 3-1 \geq 0$

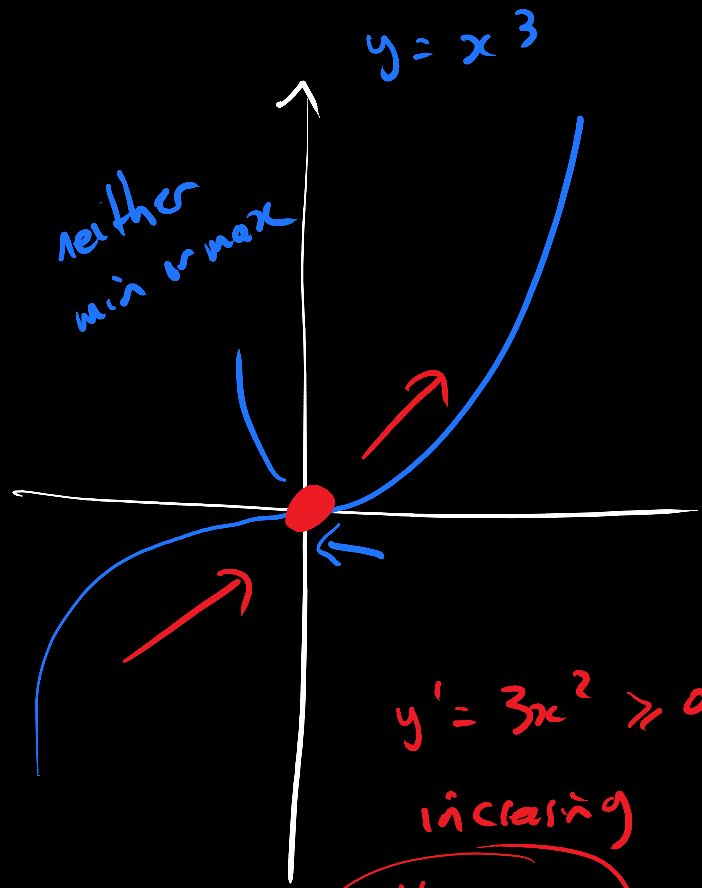
$y'(0) = 0-1 \leq 0$

CRITICAL POINTS

CRITICAL POINTS

DEFINITION

A **critical point** for a function f is point x where $f'(x) = 0$ or $f'(x)$ is not defined.



$y' = 3x^2 \geq 0$

increasing

$y'(0) = 0$

EXAMPLE

EXAMPLE

Let $f(x) = x^3 - x = x(x-1)(x+1)$.

$y = x^3 - x$, $y' = 3x^2 - 1$

crit points: $x = \frac{\pm 1}{\sqrt{3}}$

$y = x^3$ $y' = 3x^2$

crit points $x = 0$

$y = \cos x$ $y' = -\sin x$

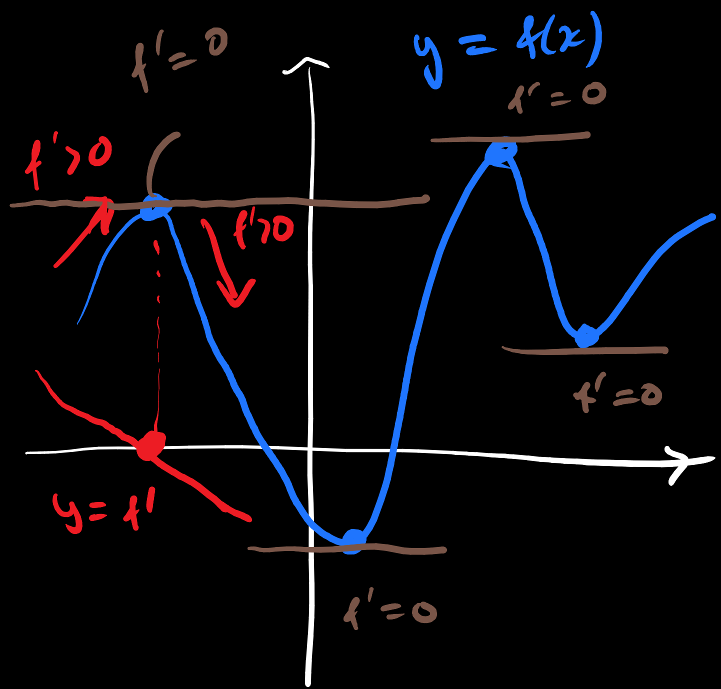
crit points: $x = 2\pi n$

$n \in \mathbb{Z}$

FIRST DERIVATIVE TEST

LEMMA

If a function f has a local minimum or maximum at the point x, then x is a critical point.

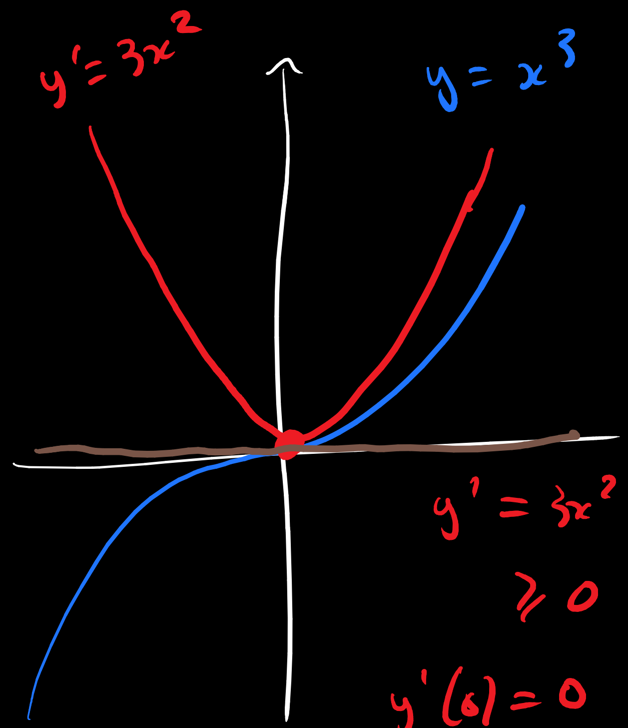


- local max or local min

EXAMPLE

EXAMPLE

$$f(x) = x^3 - x$$



$y' = 3x^2$

≥ 0

$y'(0) = 0$

0 is a critical point

but no local min/max

EXAMPLE

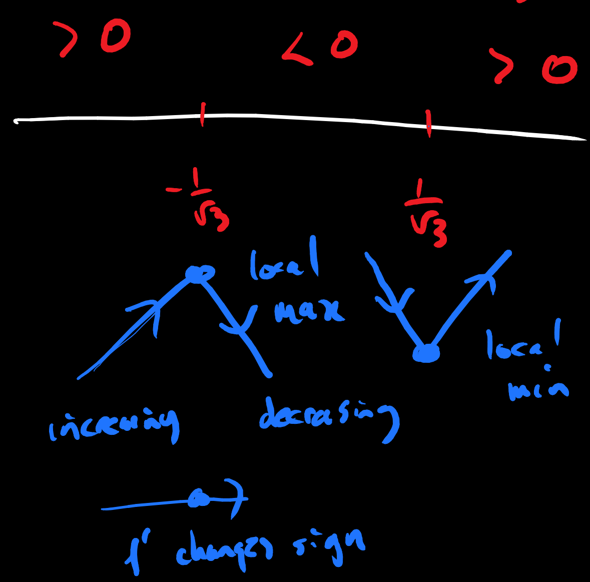
EXAMPLE

$$f(x) = x^3 - x$$

$y = x^3 - x$

$y' = 3x^2 - 1$

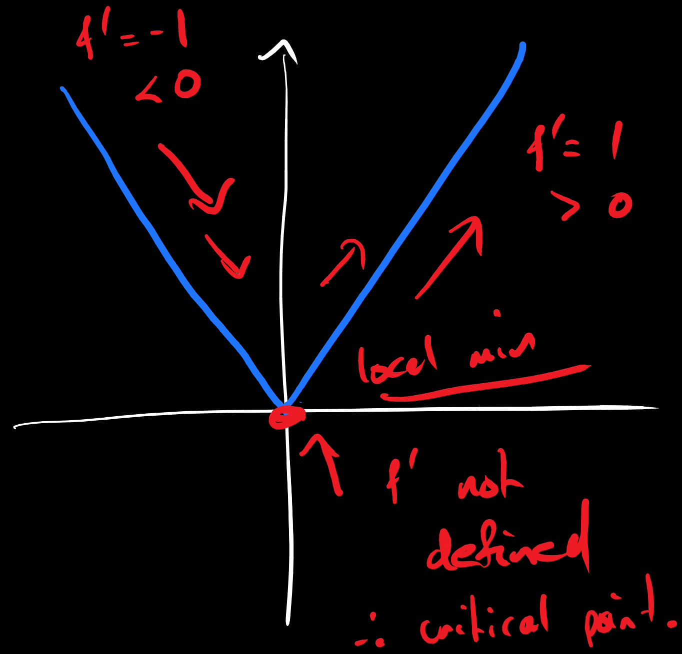
crit points : $x = \pm \frac{1}{\sqrt{3}}$



EXAMPLE

EXAMPLE

Let $f(x) = |x|$.



$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x \leq 0 \end{cases}$$

EXAMPLE

EXAMPLE

Let $f(x) = |x|$.