VECTOR FIELDS

- Vector Fields
- Gradient Fields
- Potential Functions

VECTOR FIELDS

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DEFINITION

A vector field is a function

$$\overrightarrow{\mathbf{F}} = (F_1, \ldots, F_n) : \mathbb{R}^n \to \mathbb{R}^n$$

VECTOR FIELDS EXAMPLES

- Position Vector: $\overrightarrow{\mathbf{F}}(x, y, z) = (x, y, z) = r$
- Rotation Field: $\overrightarrow{\mathbf{F}}(x, y) = (-y, x)$
- Inverse Square Law: $\overrightarrow{\mathbf{F}}(r) = \frac{C}{|r|^2} \frac{r}{|r|}$

GRADIENT FIELDS

GRADIENT FIELDS

DEFINITION

A vector field of the form $\overrightarrow{\mathbf{F}}(r) = \nabla f(r)$ is called a *gradient vector field*.

Here $r = (x_1, \ldots, x_n)$

GRADIENT FIELDS EXAMPLES

- $f(r) = \frac{|r|^2}{2}$
- $f(r) = x^2y^2$

UNIQUENESS OF GRADIENT FIELDS

LEMMA

$\nabla f = \nabla g$ if and only if $g(r) = f(r) + C$.

LEVEL SETS

THEOREM

Let f be a function with $\nabla f \neq 0$. Then ∇f is perpendicular to the level sets of f.