STOKES' THEOREM

- Stokes' Theorem
- Calculating Flux Integrals
- Calculating Line Integrals

STOKES' THEOREM

CURL FORM OF GREEN'S THEOREM

THEOREM

Let $U \subseteq \mathbb{R}^2$ have regular boundary curve C. For any vector field on $\mathbb{R}^3$ of the form $\overrightarrow{\mathbf{F}} = (P, Q, 0)$ we have

$$\iint_U \operatorname{curl} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} = \int_C \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{T}} ds$$

where $\overrightarrow{\mathbf{N}} = \overrightarrow{\mathbf{e}}_3$.

STOKES' THEOREM

THEOREM

Let $S \subseteq \mathbb{R}^3$ be a regular surface with unit normal $\overrightarrow{\mathbf{N}}$ and regular boundary curve $\partial S = C$.

For any vector field $\overrightarrow{\mathbf{F}}$ on $\mathbb{R}^3$ we have

$$\iint_S \operatorname{curl} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} = \int_C \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{T}} ds$$

EXAMPLE

- $S = \{x^2 + y^2 + z^2 = 1, z \geq 0\}$
- $C = \{x^2 + y^2 = 1, z = 0\}$
- $\overrightarrow{\mathbf{F}} = (-y, 0, x)$

CALCULATING FLUX INTEGRALS

EXAMPLE

$$F = (x, 2xy, x + y)$$

- hemisphere $\{x^2 + y^2 + (z-1)^2 = 1, z \geq 1\}$
- capped cylinder $C \cup D$:
  - $C = \{x^2 + y^2 = 1, 1 \leq z \leq 2\}$
  - $D = \{x^2 + y^2 \leq 1, z = 2\}$
- boundary S the unit circle in the $z = 1$ plane

EXAMPLE

THEOREM

The flux of a curl is independent of the surface. It depends only on the boundary curve.

CALCULATING LINE INTEGRALS

EXAMPLE

Calculate $\int_C z dx + x dy + y dz$

where C is the triangle with vertices $(0, 0, 1)$, $(3, 0, -2)$, $(0, 1, 2)$.