# REGULAR PARAMETRISED SURFACES

- Regular Parametrised Surfaces
- Examples

# REGULAR PARAMETRISED SURFACES

# REGULAR PARAMETRISION

## DEFINITION

A *regular parametrisation* for a surface is a $C^1$ map,

$$\overrightarrow{\mathbf{r}}(u,v) = \big(x(u,v), y(u,v), z(u,v)\big)$$

such that the differential $d\overrightarrow{\mathbf{r}}$ is injective.

# REGULAR PARAMETRISION

- Domain: $(u, v) \in U$ an open set of $\mathbb{R}^2$
- Injectivity: columns $\partial_u \overrightarrow{\mathbf{r}}, \partial_v \overrightarrow{\mathbf{r}}$ linearly independent

# REGULAR PARAMETRISED SURFACES

**DEFINITION**

A regular (parametrised) surface $S$ is the image of a regular parametrisation.

# EXAMPLES

# PARABOLOID

## EXAMPLE

$$\overrightarrow{\mathbf{r}}(u, v) = (u, v, u^2 + v^2)$$

# SPHERE

## EXAMPLE

$$\overrightarrow{\mathbf{r}}(\theta, \varphi) = (\sin\varphi\cos\theta, \sin\varphi\sin\theta, \cos\varphi)$$

$$0 < \theta < 2\pi, \quad 0 < \varphi < \pi$$

# TORUS

## EXAMPLE

$$
\begin{aligned}
x(\theta, \varphi) &= (R_{\text{out}} + R_{\text{in}} \cos\theta) \cos\varphi \\
y(\theta, \varphi) &= (R_{\text{out}} + R_{\text{in}} \cos\theta) \sin\varphi \\
z(\theta, \varphi) &= R_{\text{in}} \sin\theta
\end{aligned}
$$

# SURFACES OF REVOLUTION

**DEFINITION**

Let $f : (a, b) \to \mathbb{R}$ be a positive function.

*Surface of revolution* of $f$ around the $z$ axis:

$$\overrightarrow{\mathbf{r}}(t, \theta) = (f(t)\cos\theta, f(t)\sin\theta, t)$$

# SURFACES OF REVOLUTION: EXAMPLES

- Sphere: $f = \sqrt{1 - t^2}$
- Cylinder: $f = 1$
- Paraboloid $f = t^2$

# HYPERBOLOIDS

- Upper sheet of two sheeted hyperboloid: $f = \sqrt{1+t^2}$
  - $x^2 + y^2 - z^2 = -1$
  - $\overrightarrow{\mathbf{r}}(\theta, \varphi) = (\cosh\varphi\cos\theta, \cosh\varphi\sin\theta, \sinh\varphi)$ (hyperbolic polar coords)
- One sheeted hyperboloid: $f = \sqrt{t^2 - 1}$