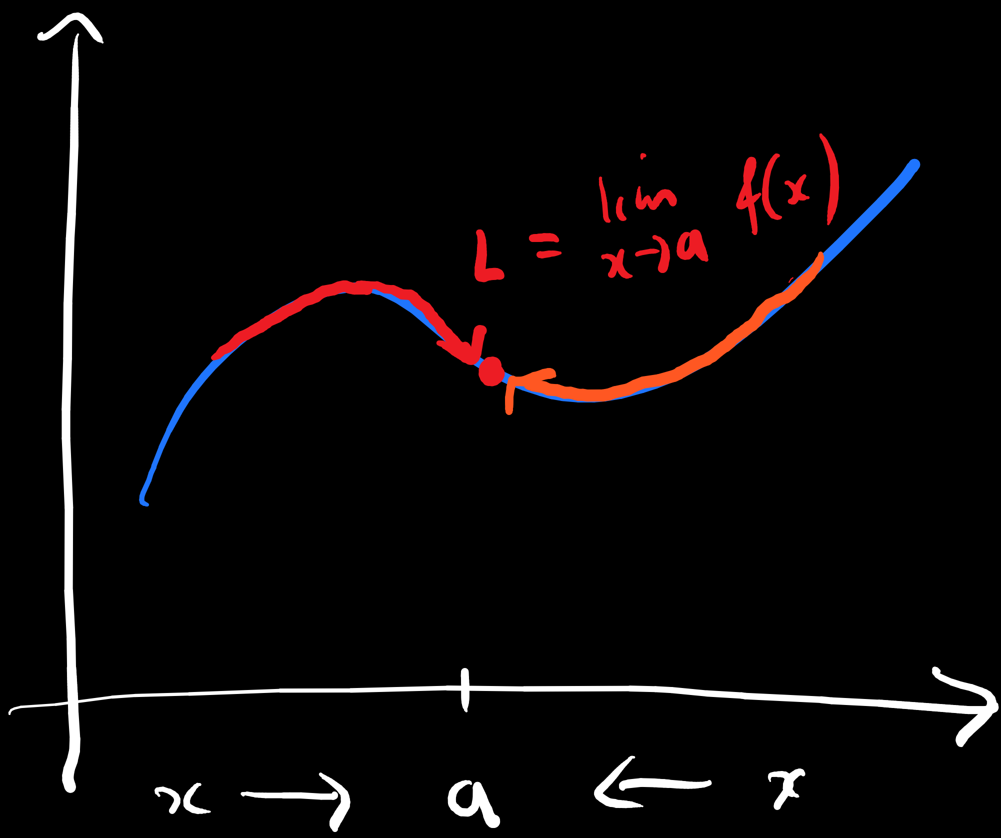
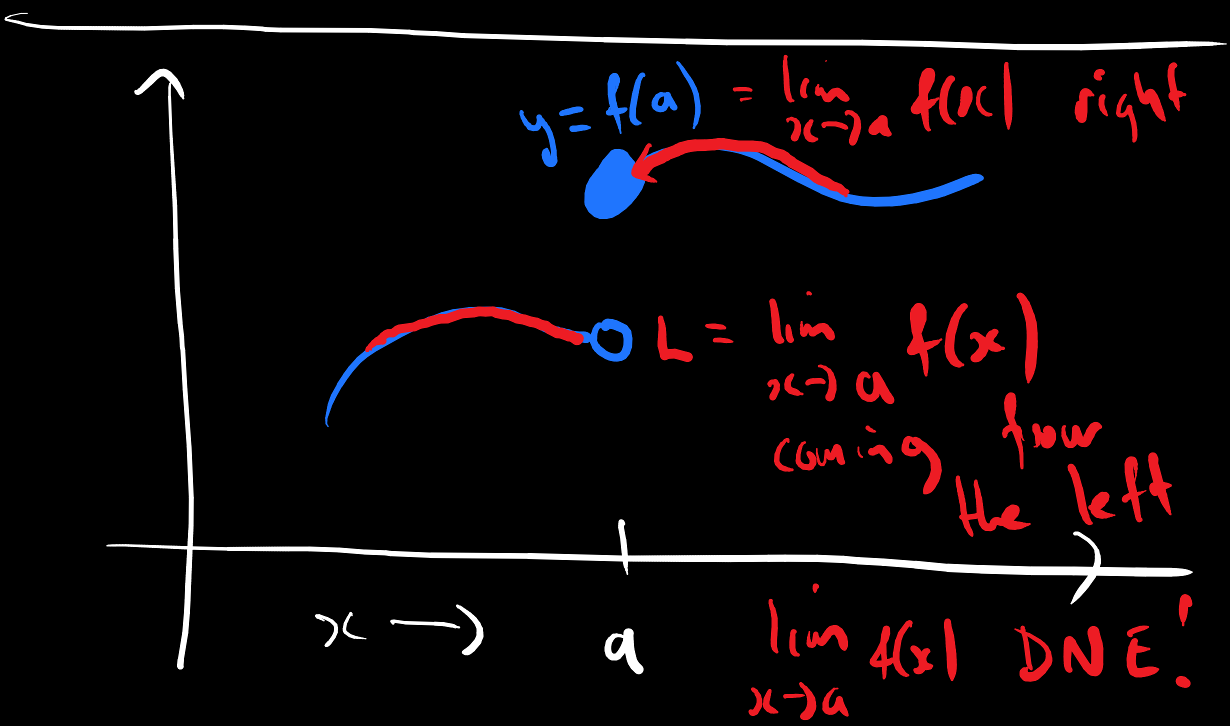

$L = \lim_{x \to a} f(x)$
$x \to a \leftarrow x$


$y = f(a) = \lim_{x \to a} f(x)$ right
$L = \lim_{x \to a} f(x)$ coming from the left
$x \to a$
$\lim_{x \to a} f(x)$ DNE!

$$\lim_{x \to 2} (\underbrace{x}_{} + \underbrace{4}_{})$$

$f(x) = x \quad g(x) = 4$

$\boxed{\text{By Thm}}$

$$= \lim_{x \to 2} x + \lim_{x \to 2} 4$$

$$= 2 + 4$$

$$= 6$$

$$\lim_{x\to 3} x^2 = \lim_{x\to 3} (\underbrace{x}_{f(x)} \cdot \underbrace{x}_{g(x)})$$

$$= \left(\lim_{x\to 3} x\right) \cdot \left(\lim_{x\to 3} x\right) \quad \text{THM}$$

$$= \left(\lim_{x\to 3} x\right)^2$$

$$= 3^2 = 9$$

$$\lim_{x \to 4} \frac{x+1}{x} \qquad \frac{= f(x)}{= g(x)}$$

$$= \frac{\lim\limits_{x \to 4} x+1}{\lim\limits_{x \to 4} x} \qquad \text{THM}$$

$$= \frac{5}{4}$$

$$\lim_{x \to 3} \left(2x^2 + 5x - 7\right)$$

$$= \lim_{x \to 3} \left(2x^2\right) + \lim_{x \to 3} (5x) + \lim_{x \to 3} (-7)$$

SUM LAW

$$= \lim_{x \to 3} (2) \lim_{x \to 3} \left(x^2\right) + \lim_{x \to 3} (5) \lim_{x \to 3} (x) - 7$$

PRODUCT LAW

$$= 2 \cdot 3^2 + 5 \cdot 3 - 7$$

$$= 18 + 15 - 7$$

$$= 26$$

$$\lim_{x\to 2}\left(\frac{x^2-4}{x-2}\right) = \frac{\lim_{x\to 2}(x^2-4)}{\lim_{x\to 2}(x-2)}$$

$$= \frac{2^2-4}{2-2} = \frac{0}{0}$$

DIVIDE BY ZERO ERROR!

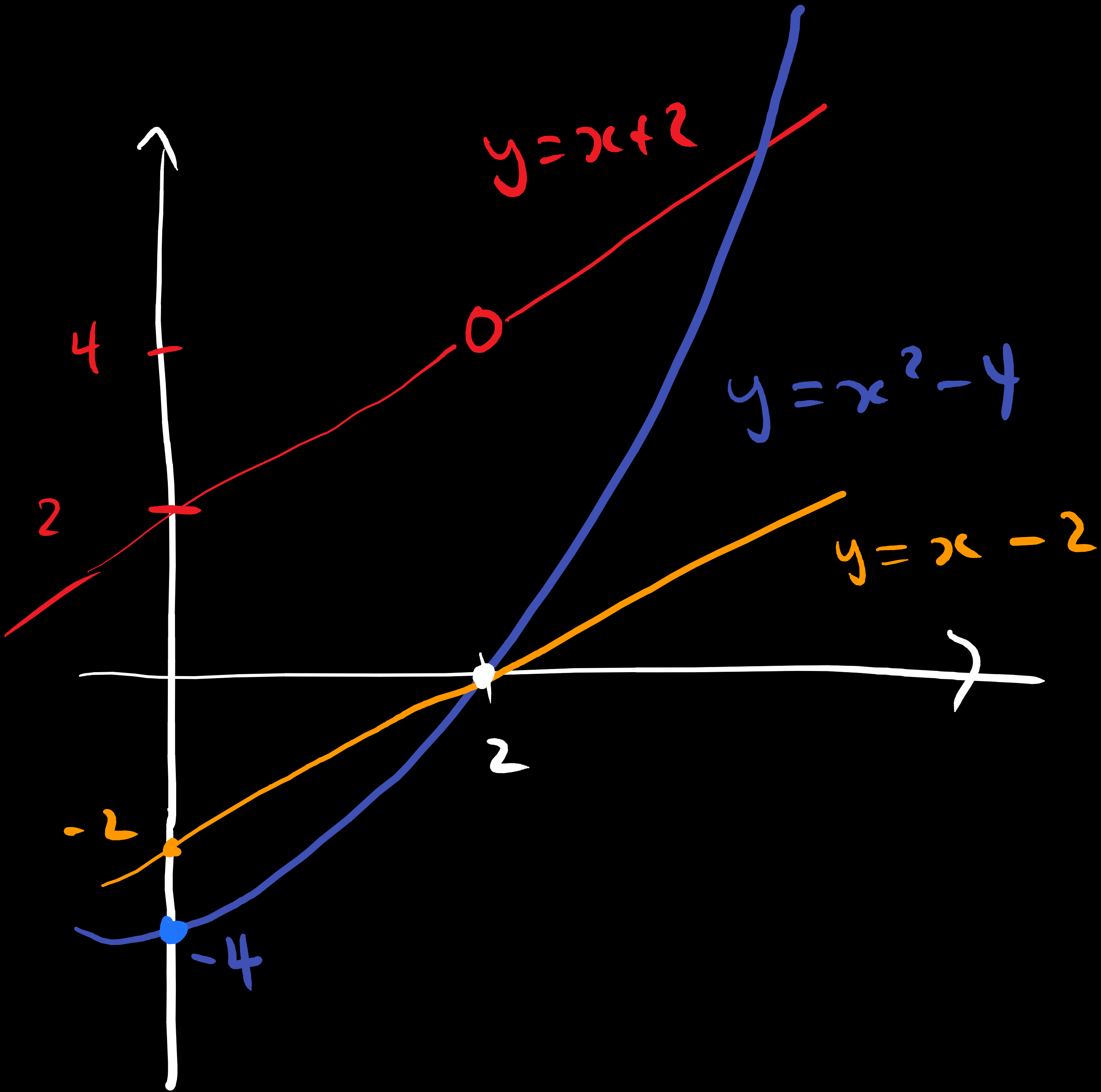
INSTEAD: FOR $x \neq 2$!

$$\frac{x^2-4}{x-2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$$

$$= x+2$$

$$\lim_{x\to 2}\frac{x^2-4}{x-2} = \lim_{x\to 2} x+2$$

$$= 4$$

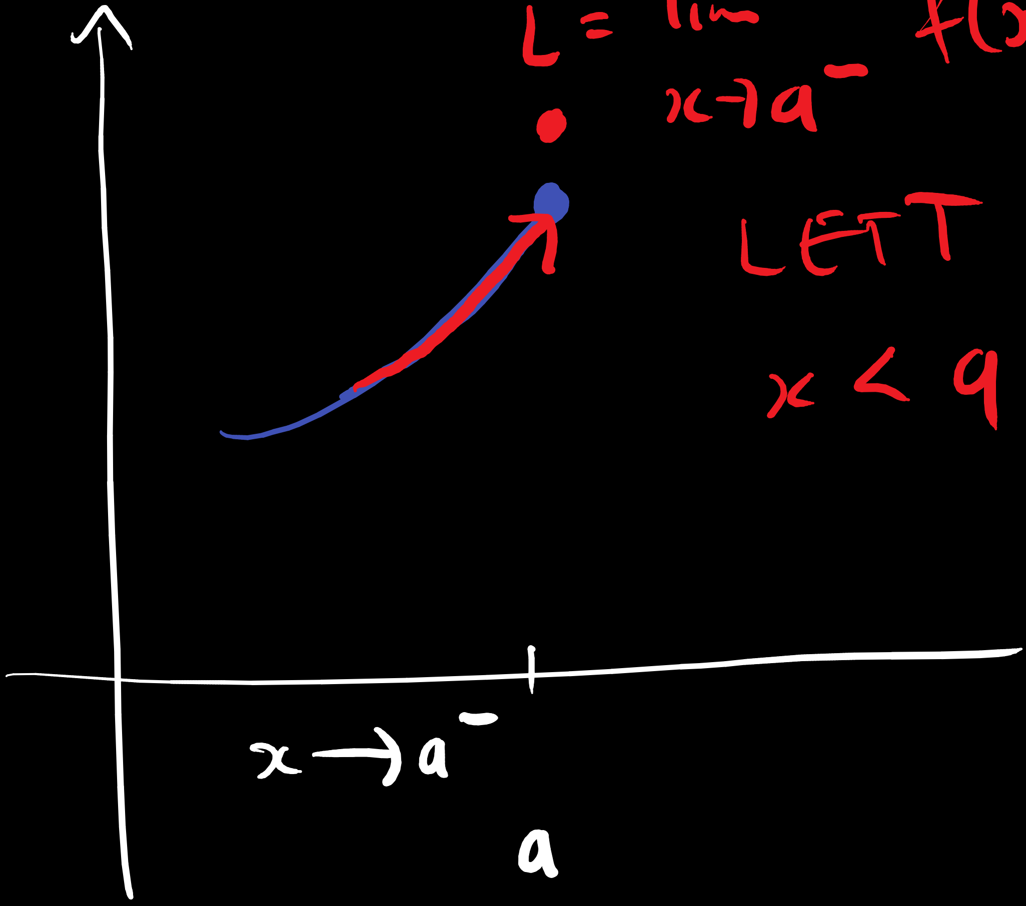


$$\frac{x^2-4}{x-2} = x+2$$

$$L = \lim_{x \to a^-} f(x)$$

LEFT

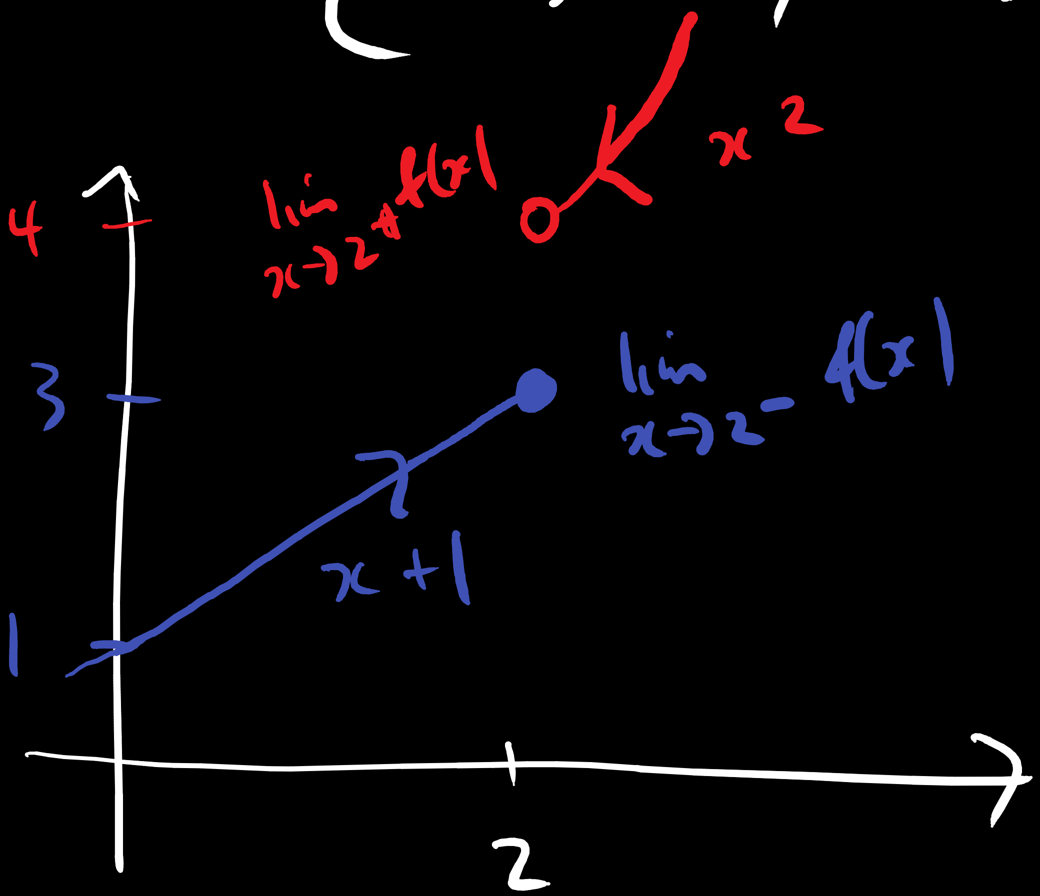
$x < a$



RIGHT: $x > a$

$$\lim_{x \to a^+} f(x)$$

$$f(x) = \begin{cases} x+1, & x \le 2 \\ x^2, & x > 2 \end{cases}$$



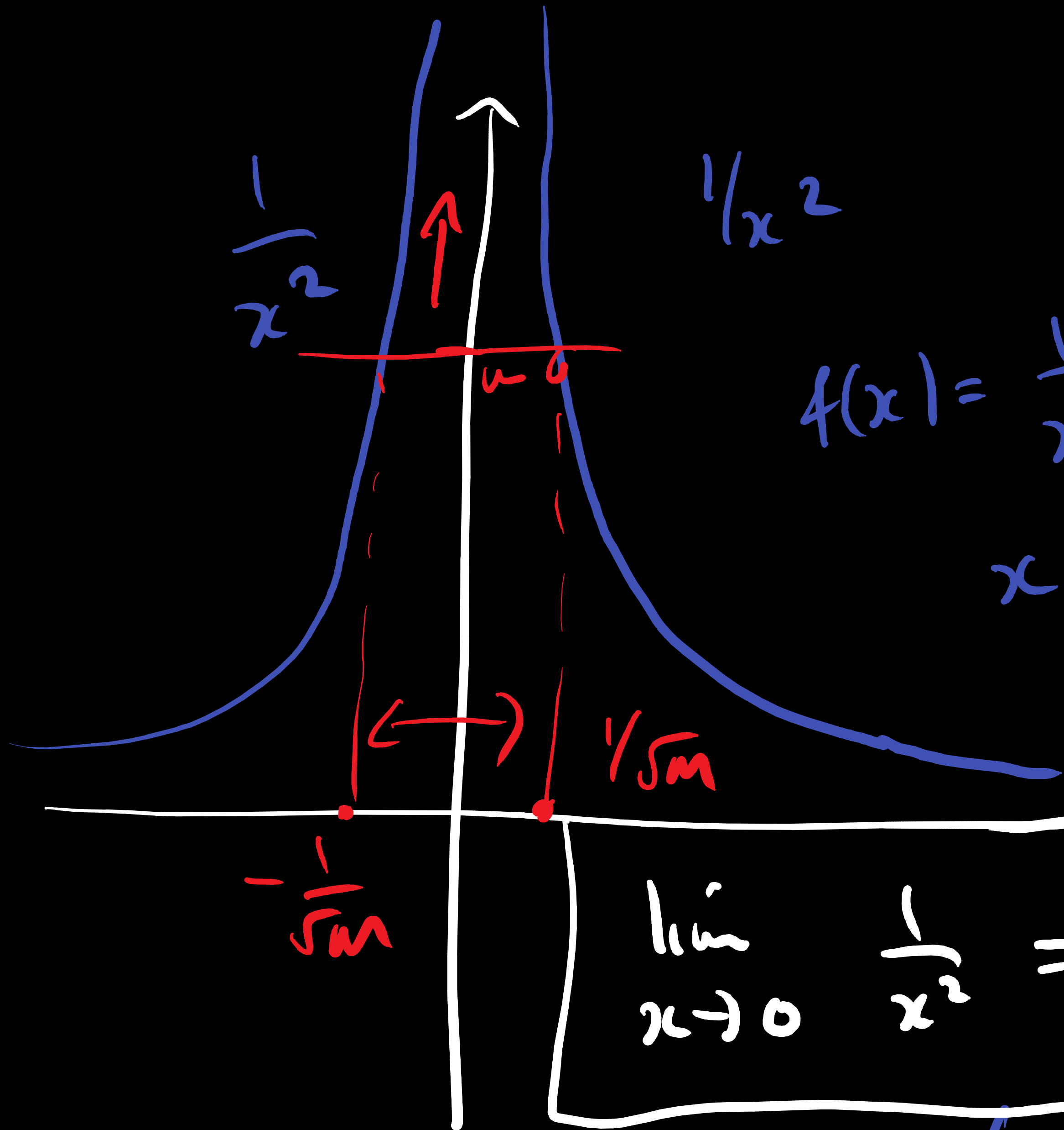
$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x+1$$

$(x<2)$ $= \lim_{x\to 2} x+1$ (THM)

$= 3$ $\neq$ $\therefore \lim$ DNE

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2} x^2 = 4$$

$(x>2)$



$$f(x) = \frac{1}{x^2}$$

$$x \neq 0$$

$$\boxed{\lim_{x \to 0} \frac{1}{x^2} = \infty}$$

consider $x = 10^{-23}$ (small)

$$x^2 = \left(10^{-23}\right)^2 = 10^{-46}$$

$$\frac{1}{x^2} = \frac{1}{10^{-46}} = \frac{1}{1/10^{46}}$$

$$= 10^{46}$$

very large

as $x \to 0$ $\frac{1}{x^2}$ becomes larger than any number!

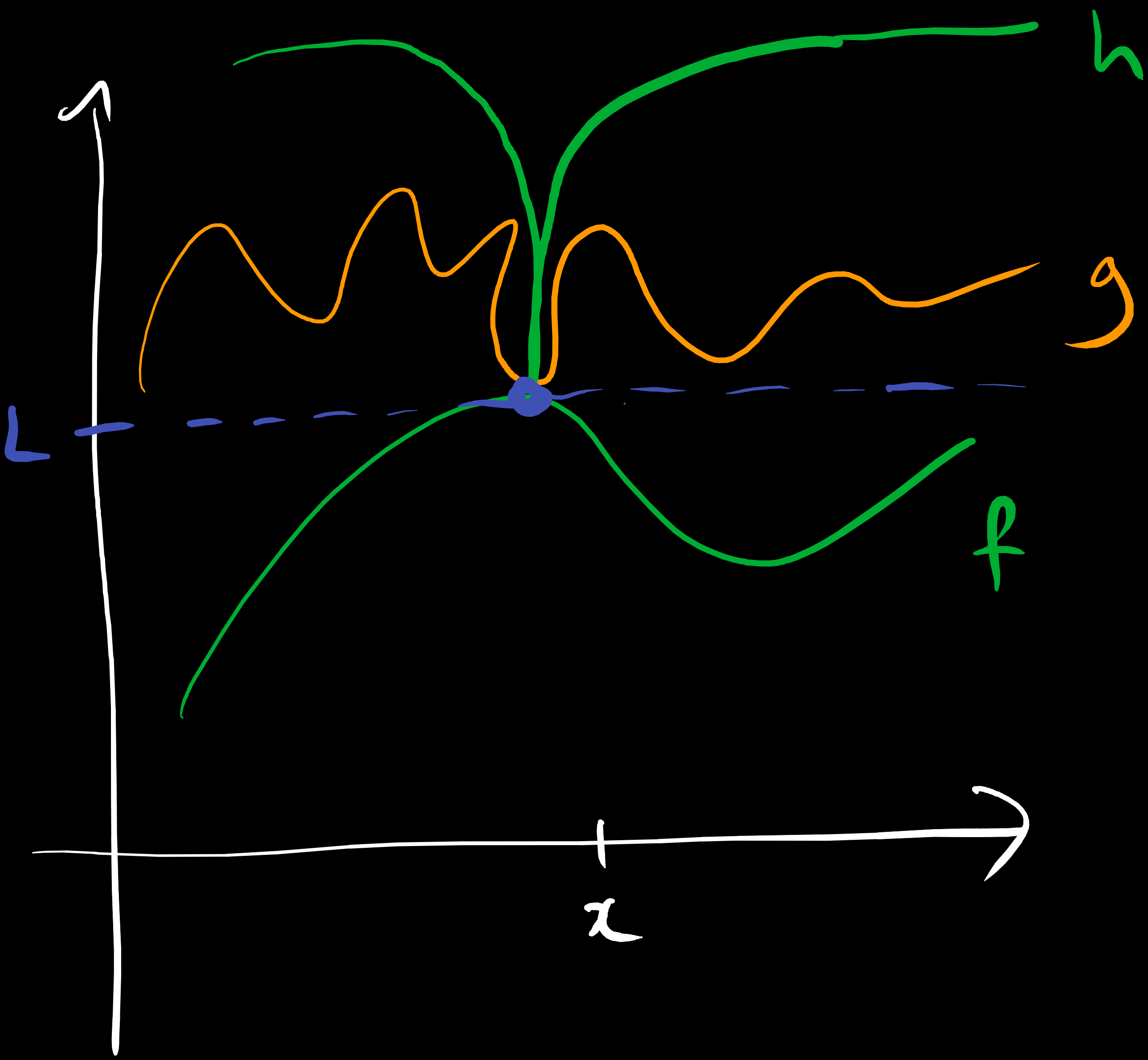
For any $M > 0$

let $x \in \left(-\frac{1}{\sqrt{M}}, \frac{1}{\sqrt{M}}\right)$

$x \neq 0$

then $\frac{1}{x^2} > M$.

$\therefore \lim_{x \to 0} \frac{1}{x^2} > M$



$$f \le g \le h$$

$$\lim_{x \to a} f = \lim_{x \to a} h$$

$$\therefore \lim_{x \to a} g = L$$

$f(x) = -x^2$, $h(x) = x^2$

$g(x) = x^2 \sin x$

since $-1 \leq \sin x \leq 1$

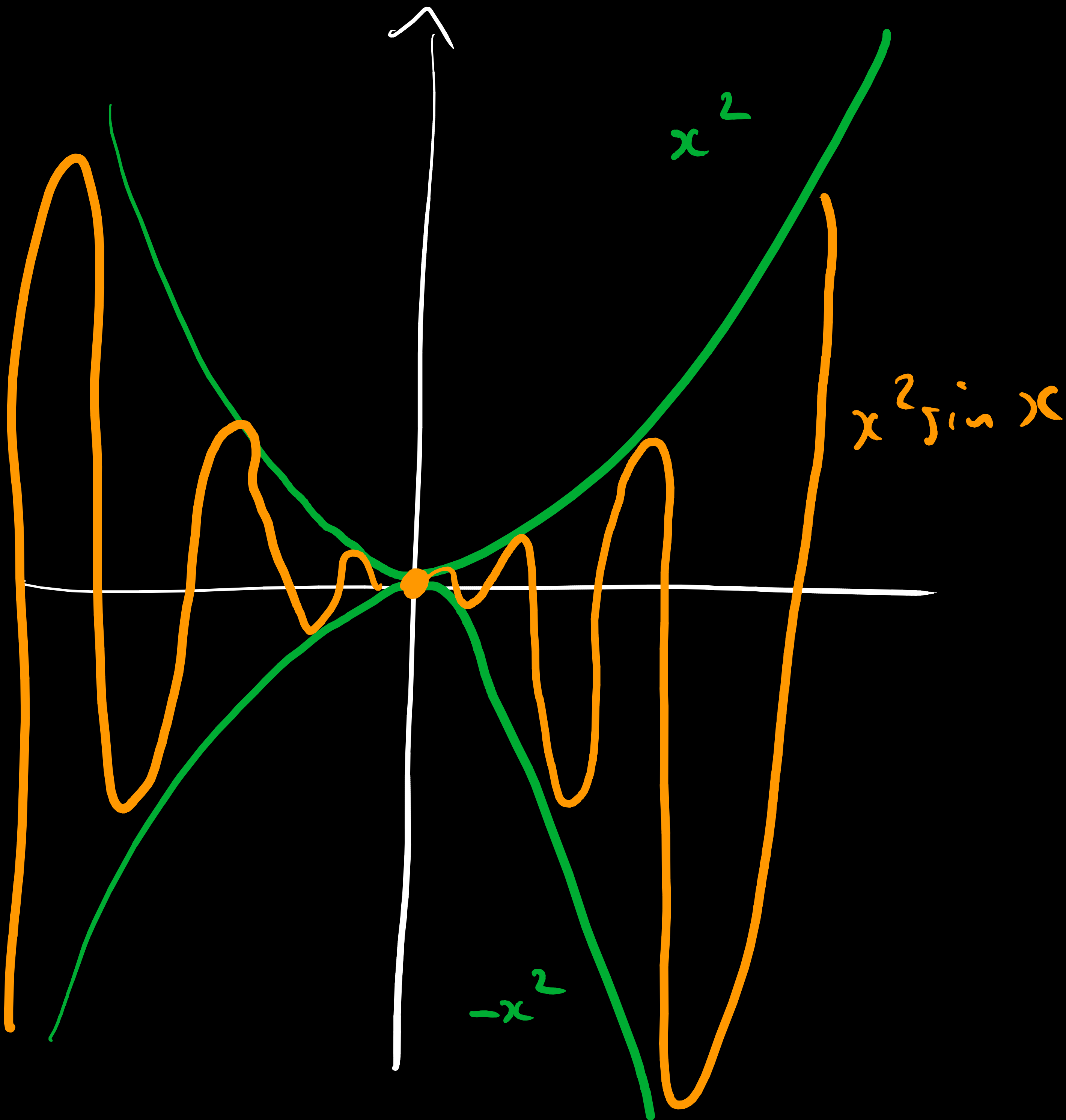
$$\Rightarrow \quad -x^2 \leq x^2 \sin x \leq x^2$$

$$f(x) \leq g(x) \leq h(x)$$

$$\begin{array}{ccccc} \downarrow & & \downarrow & & \downarrow \\ 0 & \boxed{\text{S.THM}} & 0 & & 0 \end{array}$$

as $x \to 0$

i.e. $\lim\limits_{x \to 0} (-x^2) = 0$


x^2
$x^2 \sin x$
$-x^2$