CURVES

- Regular Curves
- Arc Length
- Scalar Line Integrals

REGULAR CURVES

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DEFINITION

A C^1 *regular, parametrised curve* is a C^1 function

$$c(t) = (x_1(t), \ldots, x_n(t))$$

with $c'(t) \neq 0$ and $t \in (a, b)$.

EXAMPLES

- **Circle:** $\vec{c}(t) = (\cos(t), \sin(t)), 0 \le t \le 2\pi$
- **Helix:** $\vec{c}(t) = (\cos(t), \sin(t), t), t \in \mathbb{R}$
- **Parabola:** $\vec{c}(t) = (t, t^2), t \in \mathbb{R}$
- **Cardiod:** $\vec{c}(t) = \big((1 - \cos(t))\cos(t), (1 - \cos(t))\sin(t)\big)$

ARC LENGTH

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DEFINITION

The length, L (or arc length) of a C^1 regular, parametrised curve $c : [a, b] \to \mathbb{R}^n$ is

$$L = \int_C ds = \int_a^b |c'(t)| dt$$

where $ds = |c'(t)| dt$.

ARC LENGTH MOTIVATION

Partition $[a, b]$ as $a = t_0 < t_1 \cdots < t_{n-1} < t_n = b$.

$$\begin{aligned} L &\simeq \sum_{i=1}^{n} |c(t_i) - c(t_{i-1})| \\ &= \sum_{i=1}^{n} \left| \frac{c(t_i) - c(t_{i-1})}{t_i - t_{i-1}} \right| (t_i - t_{i-1}) \\ &= \sum_{i=1}^{n} \left| \frac{\Delta c}{\Delta t} \right| \Delta t \to \int_a^b |c'(t)| dt \text{ as } n \to \infty. \end{aligned}$$

ARC LENGTH EXAMPLES

- Straight line: $c(t) = p + tV$
- Circle: $c(t) = (R\cos t, R\sin t)$
- Parabola: $c(t) = (t, t^2)$

SCALAR LINE INTEGRALS

SCALAR LINE INTEGRALS

DEFINITION

The integral of f along C is

$$\int_C f ds = \int_a^b f(c(t)) \left|c'(t)\right| dt$$

SCALAR LINE EXAMPLES

EXAMPLE

$$c(t) = (\cos t, \sin t, t)$$

$$f(x, y, z) = x^2 + y^2 + z^2.$$