STOKES' THEOREM

- Stokes' Theorem
- Calculating Flux and Line Integrals
- Irrotational Vector Fields

STOKES' THEOREM

CURL FORM OF GREEN'S THEOREM

THEOREM

Let $U \subseteq \mathbb{R}^2$ have regular boundary curve C. For any vector field on $\mathbb{R}^3$ of the form $\overrightarrow{\mathbf{F}} = (P, Q, 0)$ we have

$$\iint_U \operatorname{curl} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA = \int_C \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{T}} ds$$

STOKES' THEOREM

THEOREM

Let $S \subseteq \mathbb{R}^3$ be a regular surface with unit normal $\overrightarrow{\mathbf{N}}$ and regular boundary curve $\partial S = C$.

For any vector field $\overrightarrow{\mathbf{F}}$ on $\mathbb{R}^3$ we have

$$\iint_S \operatorname{curl} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA = \int_C \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{T}} ds$$

EXAMPLE

- $S = \{x^2 + y^2 + z^2 = 1, z \geq 0\}$
- $C = \{x^2 + y^2 = 1, z = 0\}$
- $\overrightarrow{\mathbf{F}} = (-y, 0, x)$

CALCULATING FLUX AND LINE INTEGRALS

SURFACE INDEPENDENCE

THEOREM

The flux of a curl is independent of the surface. It depends only on the boundary curve.

EXAMPLE

$$F = (x, 2xy, x + y)$$

- hemisphere
  $\{x^2 + y^2 + (z-1)^2 = 1, z \geq 1\}$
- capped cylinder $C \cup D$:
  - $C = \{x^2 + y^2 = 1, 1 \leq z \leq 2\}$
  - $D = \{x^2 + y^2 \leq 1, z = 2\}$
- boundary S the unit circle in the $z = 1$ plane

EXAMPLE

Calculate $\int_C z\,dx + x\,dy + y\,dz$ where C is the triangle with vertices $(0, 0, 1)$, $(3, 0, -2)$, $(0, 1, 2)$.

IRROTATIONAL VECTOR FIELDS

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DEFINITION

A vector field $\overrightarrow{\mathbf{F}}$ is called **irrotational** if $\operatorname{curl} \overrightarrow{\mathbf{F}} = 0$.

IRROTATIONAL VECTOR FIELDS

THEOREM

The following are equivalent

1. $\overrightarrow{\mathbf{F}}$ is irrotational
2. $\overrightarrow{\mathbf{F}}$ is conservative: work around any loop is 0
3. On simply connected domains $\overrightarrow{\mathbf{F}} = \nabla f$

EXAMPLE

$\overrightarrow{\mathbf{F}} = (x, y, z)$ is the gradient of $f = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2}$