LIMITS

LIMITS

DEFINITION

If the function values $f(x)$ approach L as the values x approach a, then the limit exists and we write

$$\lim_{x \to a} f(x) = L.$$

Note: Here we let x approach a but we consider only $x \neq a$.

SUM LAW

THEOREM

If the limits $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

PRODUCT LAW

THEOREM

If the limits $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then

$$\lim_{x \to a} [f(x)g(x)] = \left[\lim_{x \to a} f(x)\right] \left[\lim_{x \to a} g(x)\right]$$

QUOTIENT LAW

THEOREM

If the limits $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist and if $\lim_{x \to a} g(x) \neq 0$, then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$$

EXAMPLE

EXAMPLE

Calculate the limit,

$$\lim_{x \to 3} 2x^2 + 5x - 7$$

EXAMPLE

EXAMPLE

Calculate the limit,

$$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

ONE SIDED LIMITS

ONE SIDED LIMITS (LEFT)

DEFINITION

If the function values $f(x)$ approach L as the values x approach a **from the left**, then the limit from the left exists and we write

$$\lim_{x \to a^-} f(x) = L.$$

Note: To say that x approaches a from the left means that we restrict to $x < a$.

ONE SIDED LIMITS (RIGHT)

DEFINITION

If the function values $f(x)$ approach L as the values x approach a **from the right**, then the limit from the right exists and we write

$$\lim_{x \to a^+} f(x) = L.$$

Note: To say that x approaches a from the right means that we restrict to $x > a$.

LIMITS AND ONE SIDE LIMITS

THEOREM

$$\lim_{x \to a} f(x) = L$$

if and only if

$$\lim_{x \to a^-} f(x) = L \text{ and } \lim_{x \to a^+} f(x) = L$$

EXAMPLE

EXAMPLE

Calculate the left and right limits of the function

$$f(x) = \begin{cases} x + 1, & x \leq 2 \\ x^2, & x > 2 \end{cases}$$

as $x \to 2$.

INFINITE LIMITS

POSITIVE INFINITE LIMITS

DEFINITION

If the functions values $f(x)$ become positive and unbounded as $x \to a$, then we write

$$\lim_{x \to a} f(x) = \infty.$$

NEGATIVE INFINITE LIMITS

DEFINITION

If the functions values $f(x)$ become negative and unbounded as $x \to a$, then we write

$$\lim_{x \to a} f(x) = -\infty.$$

EXAMPLE

EXAMPLE

Calculate the limit

$$\lim_{x \to 0} \frac{1}{x^2}$$

EXAMPLE

EXAMPLE

Calculate the limit

$$\lim_{x \to 1} \frac{x + 1}{x - 1}$$

SQUEEZE THEOREM

SQUEEZE THEOREM

THEOREM

Suppose that $f(x) \leq g(x) \leq h(x)$ and that

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L.$$

Then

$$\lim_{x \to a} g(x) = L.$$

EXAMPLE

EXAMPLE

Evaluate the limit

$$\lim_{x \to 0} x^2 \sin \frac{1}{x}.$$

EXAMPLE

EXAMPLE

Evaluate the limit

$$\lim_{\theta \to 0} \sin \theta$$