GAUSS' LAW

- Gauss' Law
- Electric Field
- Maxwell's Equations

GAUSS' LAW

GAUSS' LAW

THEOREM

Let
$$\overrightarrow{\mathbf{F}}(p)=rac{p}{r^3}$$

Then for any closed surface S enclosing the region Ω

$$\iint_{S} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA = egin{cases} 4\pi & 0 \in \Omega \ 0 & 0
otin \ \Omega \end{cases}$$

• If $0
otin \overline{\Omega}$ then $\overrightarrow{\mathbf{F}}$ is defined on Ω

$$ullet \operatorname{div} \overrightarrow{\mathbf{F}} = 0$$
 on Ω

Divergence theorem

$$\iint_{S} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA = \iiint_{\Omega} \operatorname{div} \overrightarrow{\mathbf{F}} dV = 0$$

ullet $0\in \overline{\Omega} : \overrightarrow{\mathbf{F}}$ defined on $\Omega_\epsilon = \Omega ackslash \mathbb{B}_\epsilon(0)$

$$ullet \operatorname{div} \overrightarrow{\mathbf{F}} = 0$$
 on Ω_{ϵ}

Divergence theorem

$$0 = \iiint_{\Omega_\epsilon} ext{div} \overrightarrow{\mathbf{F}} dV = \iint_{\partial \Omega_\epsilon} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA$$

$$ullet 0 = \iiint_{\Omega_\epsilon} ext{div} \overrightarrow{\mathbf{F}} dV = \iint_{\partial \Omega_\epsilon} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA$$

$$ullet$$
 $\partial\Omega_{\epsilon}=S-\mathbb{S}_{\epsilon}$

$$oldsymbol{igg|} oldsymbol{igg|} oldsymbol{igg|}_S oldsymbol{oldsymbol{igg|}} oldsymbol{igchi} oldsymbol{oldsymbol{igg|}}_S oldsymbol{oldsymbol{igg|}} oldsymbol{igchi} oldsymbol{oldsymbol{igg|}} oldsymbol{igchi}_S oldsymbol{igchi} oldsymbol{igchi} oldsymbol{igchi}_S oldsymbol{igchi} oldsymbol{igchi} oldsymbol{igchi} oldsymbol{igchi}_S oldsymbol{igchi} oldsymbol{igchi} oldsymbol{igchi}_S oldsymbol{igchi} oldsymbol{igchi}_S oldsymbol{igchi} oldsymbol{igchi} oldsymbol{igchi}_S oldsymbol{igchi} oldsymbol{igchi}_S oldsymbol{igchi} oldsymbol{igchi}_S oldsymbol{igchi} oldsymbol{igchi}_S oldsymbol{igchi} oldsymbol{igchi}_S oldsymbol{igchi} oldsymbol{igchi}_S oldsymbol{igchi} oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{igchi}}_S oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{igchi}}_S oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{oldsymbol{igchi}} oldsymbol{oldsymbol{oldsymbol{oldsymbol{oldsymbol{oldsymbol{igchi}}} oldsymbol{$$

$$ullet$$
 $\mathbf{N}(p)=rac{p}{\epsilon}$

• Then

$$egin{aligned} \iint_{\mathbb{S}_{\epsilon}} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA &= \iint_{\mathbb{S}_{\epsilon}} rac{p}{\epsilon^3} \cdot rac{p}{\epsilon} dA \ &= \iint_{\mathbb{S}_{\epsilon}} rac{1}{\epsilon^2} dA \ &= 4\pi \end{aligned}$$

ELECTRIC FIELD

ELECTRIC FIELD

THEOREM

The flux of the electric field through a surface S is proportional to the enclosed charge.

• point charge:
$$\overrightarrow{\mathbf{E}} = \frac{1}{4\pi\epsilon_0} \frac{p}{r^3}$$

 multiple point charges: superposition (linearity)

MAXWELL'S EQUATIONS

MAXWELL'S EQUATIONS

THEOREM

$$egin{cases} \operatorname{div} \overrightarrow{\mathbf{E}} &= rac{
ho}{\epsilon_0} \ \operatorname{curl} \overrightarrow{\mathbf{E}} &= -\partial_t \overrightarrow{\mathbf{B}} \ \operatorname{div} \overrightarrow{\mathbf{B}} &= 0 \ \operatorname{curl} \overrightarrow{\mathbf{B}} &= \mu_0 \left(\overrightarrow{\mathbf{J}} + \epsilon_0 \partial_t \overrightarrow{\mathbf{E}}
ight) \end{cases}$$

FARADAY CAGE

THEOREM

A perfectly conducting, closed surface S shields any external electrostatic field.

$$egin{array}{c} \overrightarrow{\mathbf{E}} =
abla arphi \ arphi |_S \equiv ext{constant} \ \Delta arphi := ext{div} \,
abla arphi = 0 \ \end{aligned}$$