DIVERGENCE AND CURL

- Divergence
- Curl
- Green's Theorem

DIVERGENCE

DIVERGENCE

DEFINITION

The **divergence** of a vector field $\overrightarrow{\mathbf{F}}$ is the *scalar* field

$$\begin{aligned} \operatorname{div} \overrightarrow{\mathbf{F}} &= \nabla \cdot \overrightarrow{\mathbf{F}} \\ &= \partial_{x_1} F_1 + \cdots + \partial_{x_n} F_n \end{aligned}$$

EXAMPLES

EXAMPLE

$$\operatorname{div}\left(x\overrightarrow{\mathbf{e}_1} + y\overrightarrow{\mathbf{e}_2}\right) = 2$$

$$\operatorname{div}\left(x\overrightarrow{\mathbf{e}_1} - y\overrightarrow{\mathbf{e}_2}\right) = 0$$

$$\operatorname{div}\left(\overrightarrow{\mathbf{e}_1} + \overrightarrow{\mathbf{e}_2}\right) = 0$$

CURL

CURL

DEFINITION

The **curl** of $F = (P, Q, R)$ is the *vector* field

$$\begin{aligned}\operatorname{curl} F &= \nabla \times F \\ &= (\partial_y R - \partial_z Q)\overrightarrow{\mathbf{e}_1} \\ &\quad - (\partial_x R - \partial_z P)\overrightarrow{\mathbf{e}_2} \\ &\quad + (\partial_x Q - \partial_y P)\overrightarrow{\mathbf{e}_3}\end{aligned}$$

CURL

EXAMPLE

$$F = (x^2 z, e^y + xz, xyz)$$

$$\operatorname{curl} F = (xz - x)\overrightarrow{\mathbf{e}_1} + (x^2 - yx)\overrightarrow{\mathbf{e}_2} + z\overrightarrow{\mathbf{e}_3}$$

GREEN'S THEOREM

2D CURL

DEFINITION

For $\overrightarrow{\mathbf{F}} = (P, Q)$, the 2-d curl is the *scalar* field

$$\operatorname{curl}_2 \overrightarrow{\mathbf{F}} = \partial_x Q - \partial_y P$$

- $\operatorname{curl}_2(P, Q) = \operatorname{curl}(P, Q, 0) \cdot \overrightarrow{\mathbf{e}_3}$

GREEN'S THEOREM: CURL FORM

THEOREM

$$\int_C \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{T}} ds = \iint_D \operatorname{curl}_2 F dA$$

GREEN'S THEOREM: DIV FORM

THEOREM

$$\int_C \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} ds = \iint_D \operatorname{div} \overrightarrow{\mathbf{F}} dA$$

- C is oriented counter-clockwise and $\overrightarrow{\mathbf{N}}$ is the unit outer normal.

EXAMPLE

EXAMPLE

$$\overrightarrow{\mathbf{F}} = (x^3, y^3), \quad C = (\cos t, \sin t) \quad 0 \leq t \leq 2\pi$$