LINE INTEGRALS

- Scalar Line Integrals
- Vector Line Integrals

SCALAR LINE INTEGRALS

SCALAR LINE INTEGRALS

DEFINITION

The integral of f along C is

$$\int_C f ds = \int_a^b f(\mathbf{c}(t)) \left|\mathbf{c}'(t)\right| dt$$

SCALAR LINE EXAMPLES

EXAMPLE

$$\mathbf{c}(t) = (\cos t, \sin t, t)$$

$$f(x, y, z) = x^2 + y^2 + z^2$$

VECTOR LINE INTEGRALS

WORK

$$W = F \cdot V$$

WORK ALONG A CURVE

$$W = \int_C \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{T}} ds = \int_a^b \overrightarrow{\mathbf{F}}(\mathbf{c}(t)) \cdot \mathbf{c}'(t) dt$$

$$ds = |\mathbf{c}'| dt \quad \overrightarrow{\mathbf{T}} = \frac{\mathbf{c}'}{|\mathbf{c}'|}$$

EXAMPLE

EXAMPLE

Calculate $\int_C \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{T}} ds$ where

$$\overrightarrow{\mathbf{F}}(x, y) = (xy, x + y^2) \quad \mathbf{c}(t) = (t, t^2), 0 \leq t \leq 1$$

2D NOTATION

DEFINITION

For $\overrightarrow{\mathbf{F}} = (P, Q)$ and $c(t) = (x(t), y(t))$

$$\int_C \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{T}} ds = \int_a^b P\frac{dx}{dt}dt + Q\frac{dy}{dt}dt$$

$$= \int_C P dx + Q dy.$$

3D NOTATION

DEFINITION

For $\overrightarrow{\mathbf{F}} = (P, Q, R)$ and $c = (x(t), y(t), z(t))$

$$\int_C \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{T}} ds = \int_a^b P\frac{dx}{dt}dt + Q\frac{dy}{dt}dt + R\frac{dz}{dt}dt$$

$$= \int_C Pdx + Qdy + Rdz.$$

EXAMPLE

EXAMPLE

$$\int_C x\,dx + y\,dy + z\,dz$$

along $\mathbf{c}(t) = (\sin t, \cos t, t), 0 \leq t \leq 2\pi.$