FLUX INTEGRALS

- Normal Vector
- Flux Integrals

NORMAL VECTOR

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DEFINITION

The **positively oriented unit normal** in a parametrisation is

$$\overrightarrow{\mathbf{N}} = \frac{\overrightarrow{\mathbf{e}}_u \times \overrightarrow{\mathbf{e}}_v}{\left|\overrightarrow{\mathbf{e}}_u \times \overrightarrow{\mathbf{e}}_v\right|}$$

NORMAL VECTOR

$\overrightarrow{\mathbf{N}}$ is perpendicular to S

$$\overrightarrow{\mathbf{N}} \cdot \overrightarrow{\mathbf{e}}_u = \overrightarrow{\mathbf{N}} \cdot \overrightarrow{\mathbf{e}}_v = 0$$

ORIENTATION

DEFINITION

Positive orientation is the right hand rule. Negative orientation is the left hand rule.

ORIENTATION EXAMPLES

- Positive: $(\overrightarrow{\mathbf{e}_u}, \overrightarrow{\mathbf{e}_v}, \overrightarrow{\mathbf{N}}), (\overrightarrow{\mathbf{e}_v}, \overrightarrow{\mathbf{e}_u}, -\overrightarrow{\mathbf{N}})$
- Negative: $(\overrightarrow{\mathbf{e}_u}, \overrightarrow{\mathbf{e}_v}, -\overrightarrow{\mathbf{N}}), (\overrightarrow{\mathbf{e}_v}, \overrightarrow{\mathbf{e}_u}, \overrightarrow{\mathbf{N}})$

EXAMPLE

- Sphere: $\overrightarrow{\mathbf{N}}(p) = p$
- xy-plane: $\overrightarrow{\mathbf{N}} = \overrightarrow{\mathbf{e}_3}$
- Cylinder: $\overrightarrow{\mathbf{N}}(x, y, z) = (x, y, 0)$
- Graph $z = f(x, y)$: $\overrightarrow{\mathbf{N}} = \frac{\overrightarrow{\mathbf{e}_3} - \nabla f}{\sqrt{1 + |\nabla f|^2}}$

FLUX INTEGRALS

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DEFINITION

The **flux** of a vector field $\overrightarrow{\mathbf{F}}$ across a surface S is

$$\iint_S \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA = \iint_U \overrightarrow{\mathbf{F}} \cdot \frac{\overrightarrow{\mathbf{e}}_u \times \overrightarrow{\mathbf{e}}_v}{\left|\overrightarrow{\mathbf{e}}_u \times \overrightarrow{\mathbf{e}}_v\right|} \left|\overrightarrow{\mathbf{e}}_u \times \overrightarrow{\mathbf{e}}_v\right| dudv$$

$$= \iint_U \overrightarrow{\mathbf{F}} \cdot \left(\overrightarrow{\mathbf{e}}_u \times \overrightarrow{\mathbf{e}}_v\right) dudv$$

FLUX INTEGRALS

- Flux integrals are independent of parametrisation up to orientation
- Change of orientation $\vec{\mathbf{N}} \mapsto -\vec{\mathbf{N}}$ changes the **sign** of the flux integral

EXAMPLE

$$S = \{-1 < x < 1, -1 < y < 1, z = 0\}$$

$$\overrightarrow{\mathbf{F}} = (xz^2, e^y, xe^z)$$

EXAMPLE

$$S = \{x^2 + y^2 + z^2 = 1\}$$

$$\overrightarrow{\mathbf{F_1}} = (x, y, z), \overrightarrow{\mathbf{F_2}} = (-y, x, 0)$$

EXAMPLE

$$S = \{x^2 + y^2 = 1\}$$

$$\overrightarrow{\mathbf{F_1}} = (x, y, z), \overrightarrow{\mathbf{F_2}} = (-y, x, 0)$$