ORDINARY DIFFERENTIAL EQUATIONS

ODE

DEFINITION

An **O**rdinary **D**ifferential **E**equation (**ODE**) is an equation involving one or more derivatives of a function

ODE's are **everywhere**: biology, engineering, physics, economics, chemistry, geometry, robotics, computer science, etc.

FREE FALL UNDER GRAVITY

EXAMPLE

Newton's second law $F = ma$

$$z'' = -g$$

EXPONENTIAL GROWTH/DECAY

EXAMPLE

Rate of change proportional to amount (carbon dating, continuous compounding,...)

$$y' = ry$$

LOGISTIC GROWTH

EXAMPLE

In population modelling

$$p' = rp(K - p)$$

SOLUTIONS OF ODE'S

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DEFINITION

A *solution* of an ode is a function $f(t)$ that satisfies the ODE for every t.

FREE FALL UNDER GRAVITY

EXAMPLE

$$z = -\frac{gt^2}{2}$$

is a solution of

$$z'' = -g$$

EXPONENTIAL GROWTH/DECAY

EXAMPLE

$$y = e^{3t}$$

is a solution of

$$y' = 3y$$

LOGISTIC GROWTH

EXAMPLE

$$p = \frac{3}{1 + e^{-15t}}$$

is a solution of

$$p' = 5p(3 - p)$$

INITIAL CONDITION

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DEFINITION

ODE's describe how a function changes. To determine solutions we need somewhere to start. The starting values are called **initial conditions.**

FREE FALL

EXAMPLE

$$\begin{cases} z'' & = -g \\ z(0) & = 1 \\ z'(0) & = 0 \end{cases}$$

$$z(t) = \frac{-gt^2}{2} + 1$$

EXPONENTIAL GROWTH/DECAY

EXAMPLE

$$\begin{cases} y' &= 3y \\ y(0) &= 4 \end{cases}$$

$$y = 4e^{3t}$$

LOGISTIC GROWTH

EXAMPLE

$$\begin{cases} p' &= 5p(3-p) \\ p(0) &= 1 \end{cases}$$

$$p = \frac{3}{1+2e^{-15t}}$$