CONTINUITY

CONTINUITY AT A POINT

DEFINITION

A function $f : (a, b) \to \mathbb{R}$ is continuous at the point $x_0 \in (a, b)$ if

$$\lim_{x \to x_0} f(x) = f(x_0).$$

CONTINUITY AT ENDPOINTS

DEFINITION

A function $f : [a, b] \to \mathbb{R}$ is continuous at the left end-point point a if

$$\lim_{x \to a^+} f(x) = f(a).$$

It is continuous at the right end-point b if

$$\lim_{x \to b^-} f(x) = f(b).$$

CONTINUOUS FUNCTIONS

DEFINITION

A function defined on an interval is continuous if it is continuous at every point of the interval.

EXAMPLE

EXAMPLE

The function

$$f(x) = x^2 + 2 \quad \text{for} \quad x \in [3, 7]$$

is continuous.

EXAMPLE

EXAMPLE

Show the function,

$$f(x) = \frac{x-2}{x+1}$$

is continuous for every $x_0 \neq -1$.

CONTINUITY LAWS

CONTINUITY LAWS

THEOREM

Sums, products and quotients (at points where the denominator is non-zero) of continuous functions are continuous.

EXAMPLE

EXAMPLE

Show that the function

$$f(x) = 2x + x^3 \frac{x-2}{x+1}$$

is continuous for every $x \neq -1$.

COMPOSITION

THEOREM

If f and g are continuous functions, then the composition $f \circ g$ is continuous wherever it is defined.

EXAMPLE

EXAMPLE

$h(x) = \frac{1}{x^2+2}$ is continuous.

CONTINUITY OF STANDARD FUNCTIONS

STANDARD FUNCTIONS

THEOREM

The following functions are continuous

Polynomials: $c_n x^n + c_{n-1}x^{n-1} + \cdots + c_1 x + c_0$
Trig functions: $\sin, \cos, \tan, \sec$, etc.
Exponential and log: $e^x, \ln x, b^x, \log_b x$
Inverse trig: $\sin^{-1}, \cos^{-1}, \tan^{-1}$, etc.

EXAMPLE

EXAMPLE

Any polynomial $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ is continuous.

EXAMPLE

EXAMPLE

The function $\sin(x)$ is continuous at every $x_0 \in \mathbb{R}$.