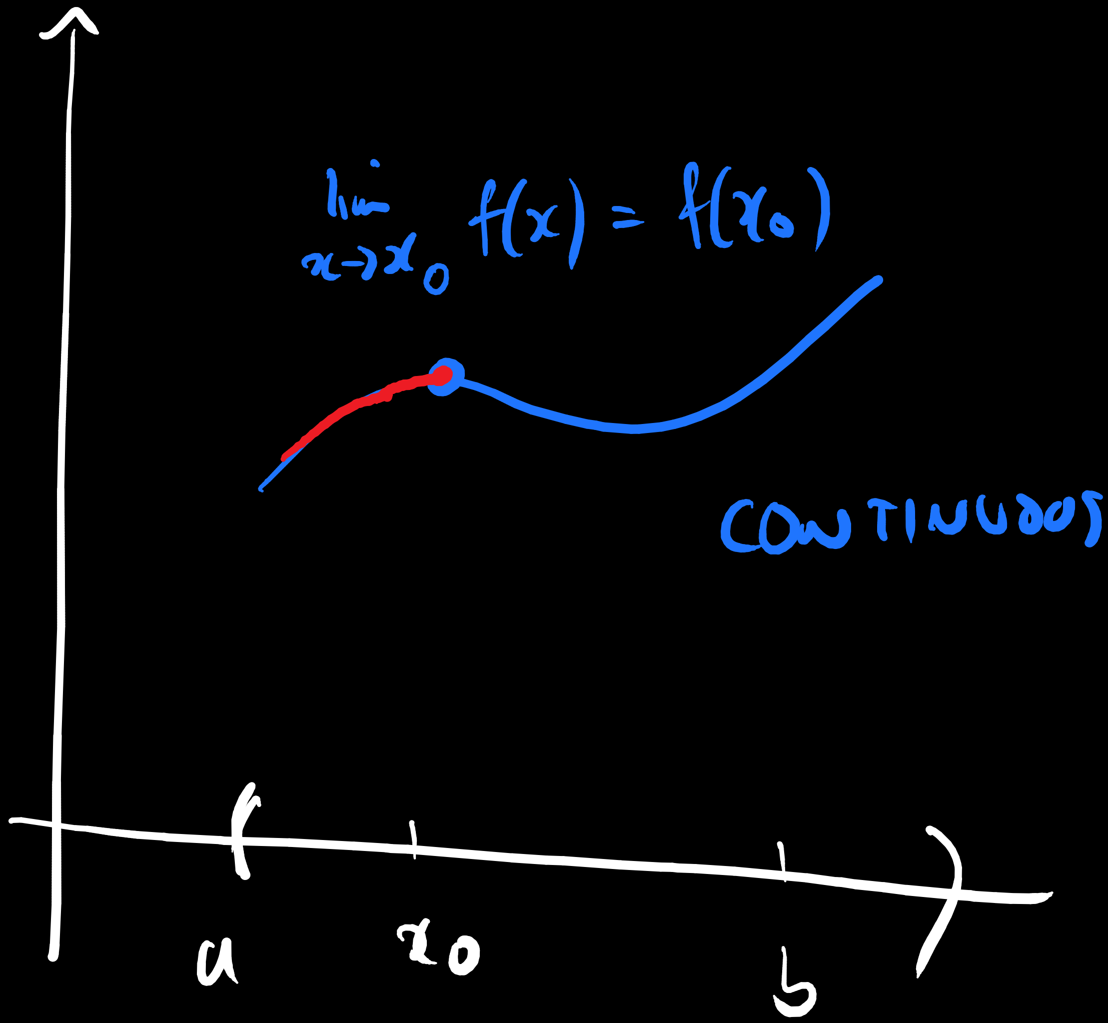
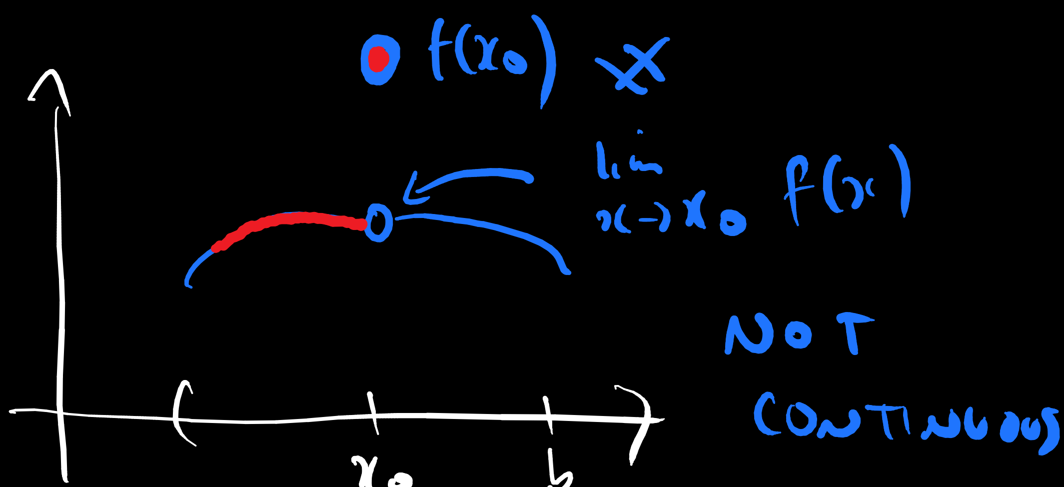
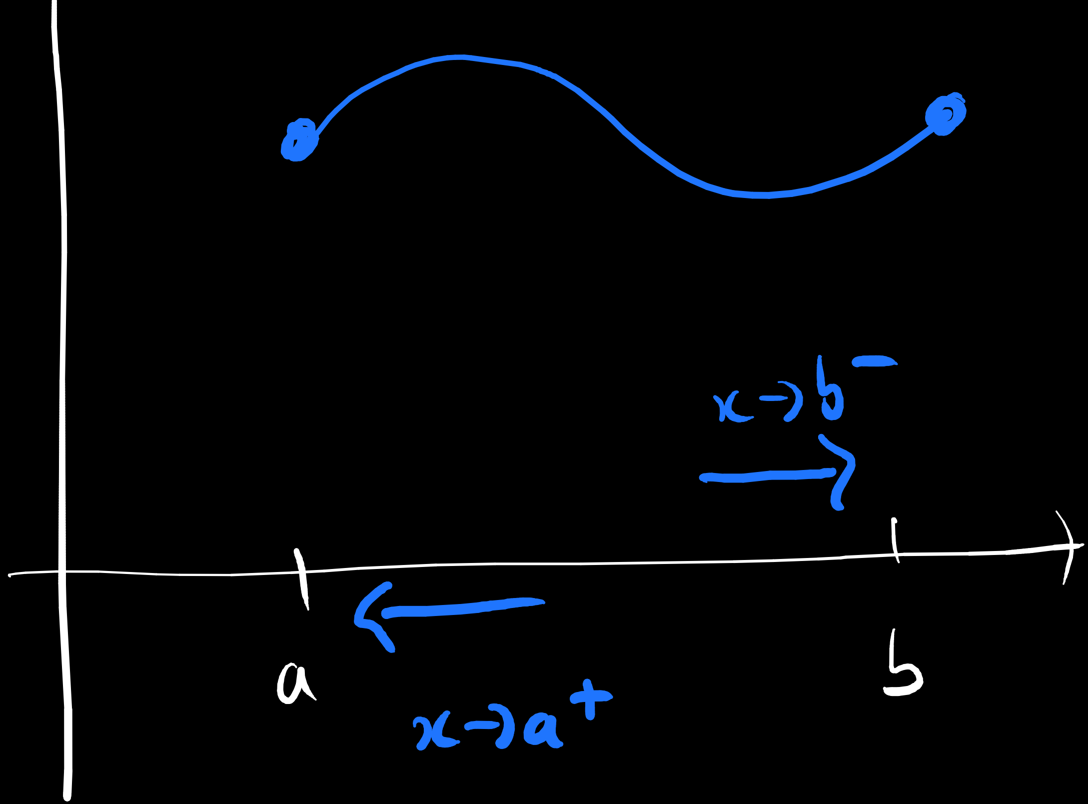

$\lim_{x \to x_0} f(x) = f(x_0)$
CONTINUOUS
a
x_0
b


$f(x_0)$ ✗
$\lim_{x \to x_0} f(x)$
NOT CONTINUOUS
x_0
b

$$f(a) = \lim_{x \to a^+} f(x)$$



$$f(x) = x^2 + 2 \quad , x \in [3, 7]$$

For $x_0 \in (3, 7)$

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} x^2 + 2$$

(LIMIT LAWS) $$= \left(\lim_{x \to x_0} x \right)^2 + 2$$

$$= x_0^2 + 2$$

$$= f(x_0)$$

For $x_0 = 3$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x^2 + 2)$$

(LIMIT EXISTS) $$= \lim_{x \to 3} (x^2 + 2)$$

$$= 3^2 + 2 = 11 = f(3)$$

similarly $\lim_{x \to 7^-} f(x) = f(7)$

$\therefore f$ is continuous.

$$f(x) = \frac{x-2}{x+1} \qquad x \neq -1$$

note f is defined

for $x \in (-\infty,-1) \cup (-1,\infty)$

i.e. all $x \neq -1$

For $x_0 \neq -1$

$$\lim_{x \to x_0} \frac{x-2}{x+1} = \frac{\lim_{x \to x_0} x-2}{\lim_{x \to x_0} x+1}$$

$$= \frac{x_0 - 2}{x_0 + 1}$$

OKAY since

$x_0 + 1 \neq 0$

$\therefore$ f is continuous

$$f(x) = \boxed{2x} + \boxed{x^3}\frac{\boxed{x-2}}{\boxed{x+1}} \quad x \neq 1$$

$$= g(x) + h(x) \cdot \frac{k(x)}{j(x)}$$

$$\lim_{x \to x_0} g(x) = \lim_{x \to x_0} 2x = 2x_0$$

$$= g(x)$$

$\therefore$ g is continuous.

$h = x^3$ is cts (continuous)

$k = x-2$ is cts

$j = x+1$ is cts, $x \neq -1$

$\therefore$ By continuity laws

f is continuous.

Let $h(x) = \dfrac{1}{x^2+2}$

Note for any x, $x^2 \geq 0$

$\therefore \quad x^2+2 \geq 0+2 = 2$

$\therefore \quad x^2+2 \neq 0$

Let $f(y) = \dfrac{1}{y}$ is continuous for $y \neq 0$

Let $g(x) = x^2+2$

$$f \circ g(x) = f(g(x))$$
$$= f(\underbrace{x^2+2}_{y = x^2+2})$$
$$= \frac{1}{x^2+2} = h(x)$$

$\therefore$ h is continuous for every x.

Let $f(x) = \underbrace{2x^3}_{f_1} + \underbrace{7x}_{f_2} - \underbrace{3}_{f_3}$

$f_1 = 2x^3$ is continuous

$f_2 = 7x$ is continuous

$f_3 = -3$ is continuous

$\therefore$ f is continuous.

e.g. $\lim_{x \to x_0} 2x^3$

$= 2\left(\lim_{x \to x_0} x\right)^3$

$= 2x_0^3 = f_1(x_0)$

$\therefore$ f_1 is cts.

$f(x) = \sin x$ is cts.

In the notes:

$$\lim_{x \to 0} \sin x = 0$$
$$= \sin(0)$$

$\therefore$ sin is continuous at $x = 0$

Recall angle sum formula

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

Pick any x_0 & write

$$x = \underbrace{x - x_0}_{a} + \underbrace{x_0}_{b}$$

$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$

$$a = x - x_0 , \qquad b = x_0$$

$$\begin{aligned}
\sin(x) &= \sin(x - x_0 + x_0) \\
&= \sin(a+b) \\
&= \sin(a)\cos(b) + \sin(b)\cos(a) \\
&= \sin(x - x_0)\cos(x_0) \\
&\quad + \sin(x_0)\cos(x - x_0)
\end{aligned}$$

$$\begin{aligned}
\lim_{x \to x_0} \sin(x) &= \lim_{x \to x_0} \left(\sin(x - x_0)\overbrace{\cos(x_0)}^{\text{constant}} + \underbrace{\sin(x_0)}_{\text{constant}}\cos(x - x_0) \right) \\
&= \cos(x_0) \lim_{x \to x_0} \sin(x - x_0) \\
&\quad + \sin(x_0) \lim_{x \to x_0} \cos(x - x_0)
\end{aligned}$$

$$\lim_{x \to x_0} \sin x = \left[\cos(x_0) \lim_{x \to x_0} \sin(x - x_0) + \sin(x_0) \lim_{x \to x_0} \cos(x - x_0) \right]$$

$\sin$ is cts at $y = 0$

$x - x_0$ ______ $x \to x_0$

$x = x_0 \Rightarrow y = x - x_0 = 0$

$\therefore \underbrace{\sin(x - x_0)}_{\text{composition of } \sin y \text{ \& } x - x_0}$ is cts at x_0

$$\therefore \lim_{x \to x_0} \sin(x - x_0) = \sin(x_0 - x_0) = \sin(0) = 0$$

Likewise $\cos(x - x_0)$ is cts.

$$\therefore \lim_{x \to x_0} \cos(x - x_0) = \cos(x_0 - x_0) = \cos(0) = 1$$

$$\lim_{x \to x_0} \sin x = \left[\cos(x_0) \lim_{x \to x_0} \sin(x - x_0) + \sin(x_0) \lim_{x \to x_0} \cos(x - x_0) \right]$$

$$= \cos(x_0) \sin(x_0 - x_0) + \sin(x_0) \cos(x_0 - x_0)$$

$$= \cos(x_0) \cdot 0 + \sin(x_0) \cdot 1$$

$$= \sin x_0$$

In summary

$$\lim_{x \to x_0} \sin x = \sin x_0$$

$\therefore$ sin is <u>cts.</u>