GAUSS' LAW

- Gauss' Law
- Electric Field
- Maxwell's Equations

GAUSS' LAW

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THEOREM

Let $\overrightarrow{\mathbf{F}}(p) = \frac{p}{r^3}$

Then for any closed surface S enclosing the region Ω

$$\iint_S \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA = \begin{cases} 4\pi & 0 \in \Omega \\ 0 & 0 \notin \overline{\Omega} \end{cases}$$

PROOF

- If $0 \notin \overline{\Omega}$ then $\overrightarrow{\mathbf{F}}$ is defined on Ω
- $\operatorname{div} \overrightarrow{\mathbf{F}} = 0$ on Ω
- Divergence theorem

$$\iint_S \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA = \iiint_\Omega \operatorname{div} \overrightarrow{\mathbf{F}} dV = 0$$

PROOF

- $0 \in \overline{\Omega}$: $\overrightarrow{\mathbf{F}}$ defined on $\Omega_\epsilon = \Omega \backslash \mathbb{B}_\epsilon(0)$
- $\operatorname{div} \overrightarrow{\mathbf{F}} = 0$ on Ω_ϵ
- Divergence theorem

$$0 = \iiint_{\Omega_\epsilon} \operatorname{div} \overrightarrow{\mathbf{F}} dV = \iint_{\partial\Omega_\epsilon} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA$$

PROOF

- $0 = \iiint_{\Omega_\epsilon} \operatorname{div} \overrightarrow{\mathbf{F}} dV = \iint_{\partial\Omega_\epsilon} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA$
- $\partial\Omega_\epsilon = S - \mathbb{S}_\epsilon$
- $\iint_S \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA = \iint_{\mathbb{S}_\epsilon} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA$

PROOF

- $\overrightarrow{\mathbf{N}}(p) = \frac{p}{\epsilon}$
- Then

$$\begin{aligned}\iint_{\mathbb{S}_\epsilon} \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{N}} dA &= \iint_{\mathbb{S}_\epsilon} \frac{p}{\epsilon^3} \cdot \frac{p}{\epsilon} dA \\ &= \iint_{\mathbb{S}_\epsilon} \frac{1}{\epsilon^2} dA \\ &= 4\pi\end{aligned}$$

ELECTRIC FIELD

ELECTRIC FIELD

THEOREM

The flux of the electric field through a surface S is proportional to the enclosed charge.

- point charge: $\overrightarrow{\mathbf{E}} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}$
- multiple point charges: superposition (linearity)

MAXWELL'S EQUATIONS

MAXWELL'S EQUATIONS

THEOREM

$$\begin{cases} \operatorname{div} \overrightarrow{\mathbf{E}} & = \dfrac{\rho}{\epsilon_0} \\ \operatorname{curl} \overrightarrow{\mathbf{E}} & = -\partial_t \overrightarrow{\mathbf{B}} \\ \operatorname{div} \overrightarrow{\mathbf{B}} & = 0 \\ \operatorname{curl} \overrightarrow{\mathbf{B}} & = \mu_0 \left(\overrightarrow{\mathbf{J}} + \epsilon_0 \partial_t \overrightarrow{\mathbf{E}} \right) \end{cases}$$

FARADAY CAGE

THEOREM

A perfectly conducting, closed surface S shields any external electrostatic field.

$$\overrightarrow{\mathbf{E}} = \nabla\varphi$$

$$\varphi|_S \equiv \text{ constant}$$

$$\Delta\varphi := \operatorname{div}\nabla\varphi = 0$$