L1: Kalman vs. Luenberger

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***Abstract*—Three predictor filters are studied and compared for a total of four datasets, three of which are synthetic and the last extracted from a real system. The filters under study are a Luenberg Observer, a Time Variant Kalman Filter and a Time Invariant one. The potential of these filters to make one-step ahead predictions is noted, and its tuning capabilities explored. Learned how to implement these three predictors and experimented with a Monte Carlo approach that appeared successful at tuning Kalman filters.**

I. Results

A. Filters on synthetic data

For the three synthetic datasets, innovations show no significant evolution over time, and resemble a normal distribution with zero average. Figures 1, 2 and 3 show the innovation distribution and evolution.

Mean squared errors (MSE) are the smallest for Dataset 1. They are also highest for dataset 2, which system has a different process noise. Finally, the filters applied on Dataset 3 have performed just slightly worse than those of Dataset 1. Figure 4 holds the MSE for each considered case.

It is relevant to note that these indicators are not suitable to compare the performance of each filter, since they were poorly tuned at the time of evaluation. Luenberg's filter has proved to be very precise if its poles are placed very close to zero. Also, while not as easy to optimize, Kalman filters still can and will be improved in a further section.

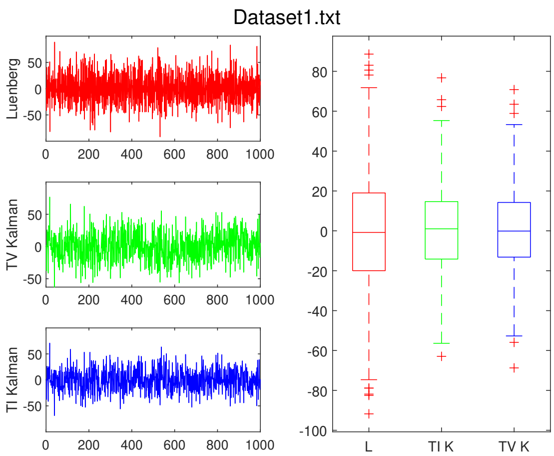

Fig. 1: Prediction error (innovation) for the three filters on Dataset 1. Time-series on the left and distribution on the right.

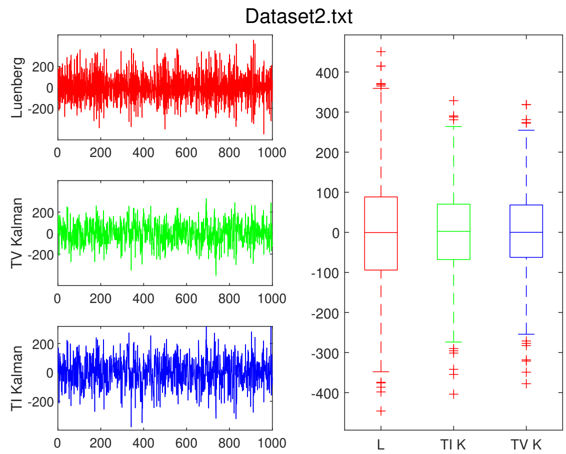

Fig. 2: Prediction error (innovation) for the three filters on Dataset 2. Time-series on the left and distribution on the right.

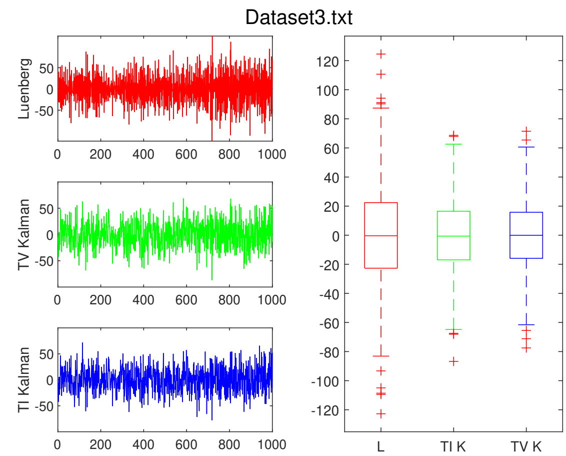

Fig. 3: Prediction error (innovation) for the three filters on Dataset 3. Time-series on the left and distribution on the right.

	Dataset 1	Dataset 2	Dataset 3
Luenberg	816.76	19596	1230
Time Variant Kalman	446.23	11303	584.57
Time Invariant Kalman	408.65	10466	543.79

Fig. 4: Mean square errors for the three filters on the three synthetic datasets.

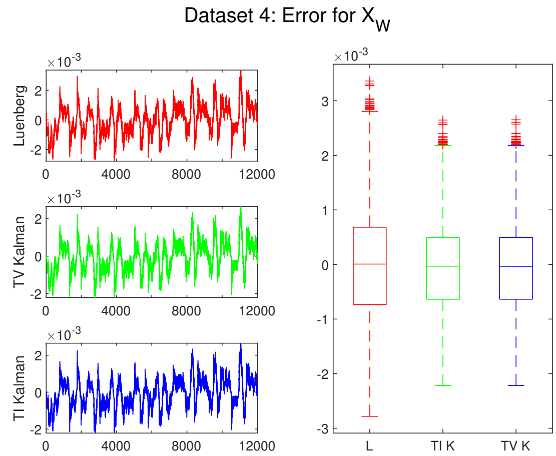

Fig. 5: Innovation of the X_w output channel from the real system Dataset 4 was extracted from, for the three filters.

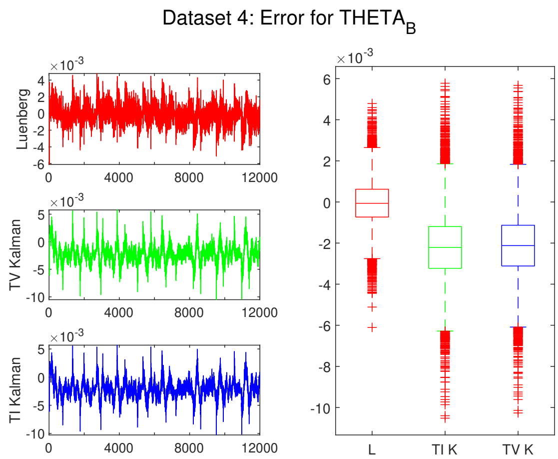

Fig. 6: Innovation of the $Theta_B$ output channel from the real system Dataset 4 was extracted from, for the three filters.

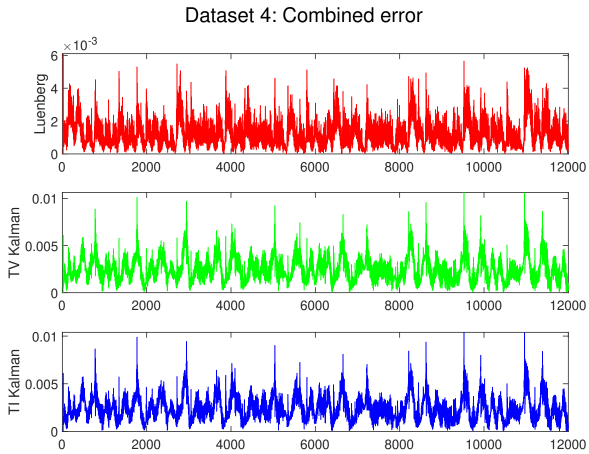

Fig. 7: Combined innovation of the real system's output over time, computed as the 2-norm of the two output channels.

	Luenberg	Kalman TV	Kalman TI
MSE Y1	9.1192e-07	5.8089e-07	5.8171e-07
MSE Y2	1.2339e-06	7.6343e-06	7.1106e-06
||MSE Y1, MSE Y2||_2	1.5343e-06	7.6563e-06	7.1343e-06

Fig. 8: MSEs for the three different filters on both output channels plus their combined MSE.

B. *Filters on real data*

The real data belongs to a single-input-multiple-output (SIMO) system. From the given template code, the space-state model matrix C is modified to reach full observability and controlability. The outputs of the system are the parameters X_W and $Theta_B$. The metrics to evaluate the filters are the MSE of each individual output and their 2-norm, which is referred to as combined MSE in this paper.

It is observed that, again, the tendency is the same. The performance of the three non-tuned filters keeps is kind of the same for X_W, the untuned Luenberg behaving slightly worse as in the previous section. However, the two Kalman filters seem to have optimized X_W better and show a slight bias when predicting $Theta_B$. This can be seen in figures 5 and 6. While its distribution is not representative, the evolution of the combined error can be seen in figure 7.

Overall, the MSE in one of the ouput channels always seem to be bigger than the other. Figure 8 displays the MSE of the different channels for each filter, plus their combined MSE.

C. *Kalman filter optimization*

The two kalman filters, time variant (TV) and invariant (TI) were optimized for the real dataset by Monte Carlo methods. Optimization parameters were set to be the matrices Q and R. Matrix Q was initialized as an identity matrix, while the R matrix was obtained from the data. The algorithm consisted on running each of the filters 20,000 times trying to minimize the combined MSE of the 100 first samples of output data. Each run, each value in the matrices Q and R was multiplied by a independent and random parameter extracted from a $N(1, 0.1)$ distribution.

Optimization was first attempted by hand-tuning but quicky got stuck at a suboptimal improvement: dialing 6 parameters at once is not an intuitive task at all. After running the iterative algorithm, it was seen that the some parameters required numbers as close to zero as possible while some others required values well above the initial unity.

The innovation of the tuned filters is shown in figures 9 and 10. It can be seen that the tuning has corrected the bias in the second output channel, for both filters.

Figure 11 holds the MSE for each of filter and channel, before and after tuning. While the unbiased output hasn't improved that much, the other one has improved its MSE significantly, leading to a reduction of the combined MSE of one order of magnitude.

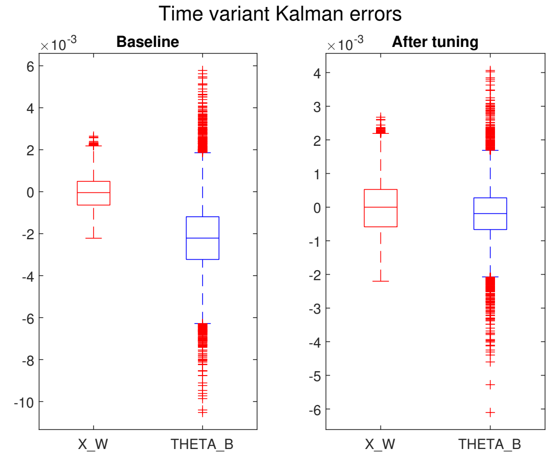

Fig. 9: Distribution of the innovations on both output channels of the real system for a Time Varian Kalman filter. Baseline stands for the untuned Kalman filter used in the previous sections. After tuning shows the new results after tuning the filter under a Monte Carlo approach.

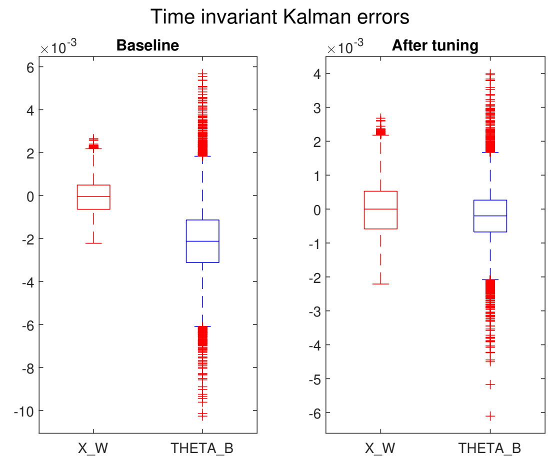

Fig. 10: Distribution of the innovations on both output channels of the real system for a Time Invariant Kalman filter. Baseline stands for the untuned Kalman filter used in the previous sections. After tuning shows the new results after tuning the filter under a Monte Carlo approach.

	TV Kalman		TI Kalman	
	Baseline	After tuning	Baseline	After tuning
MSE X_W	5.8089e-07	5.723e-07	5.8171e-07	5.7273e-07
MSE THETA_B	7.6343e-06	6.7374e-07	7.1106e-06	6.7235e-07
Combined MSE	7.6563e-06	8.84e-07	7.1343e-06	8.8322e-07

Fig. 11: MSEs of the different output channels for TV and TI Kalman filters, both before and after tuning. Baseline stands for the untuned Kalman filter described in the previous sections.

II. Conclusions

In this work, Lueneberg observers and Kalman filters were explored for various datasets. In general, their performance appears outstanding considering how simple their implemetation can be, provided that the responsible person has some experience on the topic. The performance of these filters can be further boosted by tuning. The main advantage of the Kalman filter is that it allows a one-step prediction based only on statistical parameters of the data, however correctly finding them can be a challenging task.

Acknowledgments: Thanks to Martin.