Lab Assignment-5

PHY617/473-Computational Physics

5th Feb, 2025

Explain the algorithm you are using for each question.

Question 1 [**10 marks**]
Write a Python function that represents the mathematical function

$$f(x) = 2.0 * exp(-x^2)cos(2\pi x)$$

on a mesh consisting of q+1 equally spaced points on [-1,1] and return 1) the interpolated function value at x = -0.45 and 2) the error in the interpolated value. Call the function and write out the error for q = 2,4,8,16. Use linear interpolation technique.

Question 2.
Imagine we have n+1 measurements of some quantity y that depends on $x : (x_0, y_0), (x_1, y_1), ..., (x_n, y_n)$. We may think of y as a function of x and ask what y is at some arbitrary point x not coinciding with any of the points $x_0, ..., x_n$. It is not clear how y varies between the measurement points, but we can make assumptions or models for this behavior using interpolation. One way to solve the interpolation problem is to fit a continuous function that goes through all the $n + 1$ points and then evaluate this function for any desired x. A candidate for such a function is the polynomial of degree n that goes through all the points. It turns out that this polynomial can be written as

$$p_L(x) = \sum_{k=0}^{n} y_k L_k(x), \tag{1}$$

where

$$L_k(x) = \prod_{i=0, i\neq k}^{n} \frac{x - x_i}{x_k - x_i}. \tag{2}$$

The polynomial $p_L(x)$ is known as Lagrange's interpolation formula, and the points $(x_0, y_0).....(x_n, y_n)$ are called interpolation points.

a) Make functions $p_L(x, xp, yp)$ and $L_k(x, k, xp, yp)$ that evaluate $p_L(x)$ and $L_k(x)$ by Eq. 1 and Eq. 2, respectively, at the point x. The arrays xp and yp contain the x and y coordinates of the $n+1$ interpolation points, respectively. That is, xp holds $x_0, ..., x_n$, and yp holds $y_0, ..., y_n$. [**5 marks**]

b) To verify the program, we observe that $L_k(x_k) = 1$ and that $L_k(x_i) = 0$ for $i \neq k$, implying that $p_L(x_k) = y_k$. That is, the polynomial p_L goes through all the points $(x_0, y_0)....., (x_n, y_n)$. Write a function test_p_L(xp, yp) that computes $|p_L(x_k) - y_k|$ at all the interpolation points (x_k, y_k) and checks that the value is approximately zero. Call test_p_L with xp and yp corresponding to 5 equally spaced points along the curve $y = sin(x)$ for $x \in [0, \pi]$. Thereafter, evaluate $p_L(x)$ for an x in the middle of two interpolation points and compare the value of $p_L(x)$ with the exact one. [**5 marks**]

Question 3
(a)Use Neville's method to compute y at $x = \pi/4$ from the data points [**5 marks**]

$x = [0, 0.5, 1, 1.5, 2]$
$y = [-1.00, 1.75, 4.00, 5.75, 7.00]$
(b) Interpolate the above datapoints to the new x values between 0 to 2 with an interval of 0.2 using Lagrange's polynomical method. [**5 marks**]

(c) Do the same using Neville's method. Plot the original data, interpolated data using Lagrange method and Nevlille's method in a same graph. [**5 marks**]