CS4341 - Project 2.

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PART A: SIMPLIFY, VALID..., TRUTH.

① $((p \to q) \land q) \lor p$

A) $\boxed{q \lor p}$

B) SATISFIABLE

C)

q	p	$q \lor p$	$p \to q$	$((p \to q) \land q) \lor p$
0	0	0	1	0
0	1	1	0	1
1	0	1	1	1
1	1	1	1	1

3 MODELS

② $(p \to r) \land (q \to \sim r)$

A) $\boxed{(\sim p \lor r) \land (\sim q \lor \sim r)}$

B) SATISFIABLE

C)

p	r	q	$p \to r$	$q \to \sim r$	$(p \to r) \land (q \to \sim r)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	0	1	0
1	0	1	0	1	0
1	1	0	1	1	1
1	1	1	1	0	0

4 MODELS

③ $(p \to (q \to r)) \land (p \to (r \to q))$

A) $p \to ((q \to r) \land (r \to q))$

$\sim p \lor ((\sim q \lor r) \land (\sim r \lor q))$

$\boxed{\sim p \lor (\sim q \land \sim r) \lor (q \land r)}$

B) SATISFIABLE

C)

p	q	r	q->r	r->q	p->(q->r)	p->(r->q)	FULL
0	0	0	1	1	1	1	1
0	0	1	1	0	1	1	1
0	1	0	0	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1
1	0	1	1	0	1	0	0
1	1	0	0	1	0	1	0
1	1	1	1	1	1	1	1

$\boxed{\text{6 MODELS}}$

④ $(\sim p \lor \sim q) \to (\sim p \land \sim q)$

A) $\sim(\sim p \lor \sim q) \lor (\sim p \land \sim q)$

$\cancel{\sim p \land \sim q}$ $\boxed{p \leftrightarrow q}$

B) SATISFIABLE

C)

p	q	~p∨~q	~p∧~q	FULL
0	0	1	1	1
0	1	1	0	0
1	0	1	0	0
1	1	0	0	1

$\boxed{\text{2 MODEL}}$

⑤ $p \land q \land (p \to \sim q)$

A) $p \land q \land (\sim p \lor \sim q)$

$p \land q \land \sim(p \land q) = (p \land q) \land \sim(p \land q)$

B) UNSATISFIABLE

C)

p	q	p->~q	p∧q	FULL
0	0	1	0	0
0	1	1	0	0
1	0	1	0	0
1	1	0	1	0

PART B: CONJUNCTIVE NORMAL FORM

① $(p \lor q) \to r$

$\sim(p \lor q) \lor r$

$(\sim p \land \sim q) \lor r$

$\boxed{(\sim p \lor r) \land (\sim q \lor r)}$

p	q	r	p∨q	FULL	CNF
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	0	0
1	1	1	1	1	1

✓

② $\sim(p \to (q \leftrightarrow r))$

$\sim(\sim p \lor (\sim q \land \sim r) \lor (q \land r))$

$\boxed{p \land (q \lor r) \land (\sim q \lor \sim r)}$

p	q	r	q<->r	FULL	CNF
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	0	0

✓

③ $\sim((p \wedge q) \vee (q \wedge r))$

DEMORGAN'S LAW

$\sim(p \wedge q) \wedge \sim(q \wedge r)$

$$\boxed{(\sim p \vee \sim q) \wedge (\sim q \vee \sim r)}$$

p	q	r	$p \wedge q$	$q \wedge r$	FULL	CNF
0	0	0	0	0	1	1
0	0	1	0	0	1	1
0	1	0	0	0	1	1
0	1	1	0	1	0	0
1	0	0	0	0	1	1
1	0	1	0	0	1	1
1	1	0	1	0	0	1
1	1	1	1	1	0	1

④ $((p \vee q) \wedge (p \vee r)) \rightarrow s$

$$\boxed{(\sim q \vee \sim r \vee s) \wedge (\sim p \vee s)}$$

$\sim((p \vee q) \wedge (p \vee r)) \vee s$

p	q	r	s	A $p \vee q$	B $p \vee r$	$A \wedge B$	FULL	CNF
0	0	0	0	0	0	0	1	1
0	0	0	1	0	0	0	1	1
0	0	1	0	0	1	0	1	1
0	0	1	1	0	1	0	1	1
0	1	0	0	1	0	0	1	1
0	1	0	1	1	0	0	1	1
0	1	1	0	1	1	1	0	0
0	1	1	1	1	1	1	1	1
1	0	0	0	1	1	1	0	0
1	0	0	1	1	1	1	1	1
1	0	1	0	1	1	1	0	0
1	0	1	1	1	1	1	1	1
1	1	0	0	1	1	1	0	0
1	1	0	1	1	1	1	1	1
1	1	1	0	1	1	1	0	0
1	1	1	1	1	1	1	1	1

PART C : THE PROOF

(1) $a \vee b$ (4) $\sim d$

(2) $b \rightarrow c$ (5) $\sim(a \leftrightarrow c)$ TO PROOF...

(3) $(a \wedge c) \rightarrow d$

WRITE ALL AS CNF.

(1) $a \vee b$ (3) $\sim a \vee \sim c \vee d$ (5) $(a \vee c) \wedge (\sim a \vee \sim c)$

(2) $\sim b \vee c$ (4) $\sim d$

(1) - (4) ARE OUR PREMISES.

NEGATE (5) : $(a \leftrightarrow c) = (\sim a \vee c) \wedge (a \vee \sim c)$

FROM RESOLUTION RULE

$$\frac{\alpha \vee \beta, \sim\beta \vee \gamma}{\alpha \vee \gamma}$$

(a) FROM (1) AND (2) : $a \vee c$ ~~YIELD AN EMPTY CLAUSE WHICH IS~~

(b) FROM (3) AND (4) : $\sim a \vee \sim c$

(c) FROM (1) AND (3) : $b \vee \sim c \vee d$

(d) FROM (2) AND (3) : $\sim b \vee \sim a \vee d$

(e) FROM (a) AND (5) : $\circ$ RESULTS IN AN EMPTY CLAUSE!

A CONTRADICTION.

IT HAS THEN BEEN PROVEN THAT

$\boxed{\sim(a \text{ IFF } c)}$