Macroeconomic Fundamentals in Range-Based Volatility Models

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Introduction

- Uncertainty dominates financial markets as one cannot be 100% certain of future direction of companies, governments and individual participants in the market.
- The measurement of volatility in these markets has been foundational in finance in terms of investment portfolio management, risk management, and asset pricing [6, 16, 20, 25].
- Estimating volatility requires thorough understanding of statistical properties of financial time series, so called stylized facts [20, 25].
- The stylized facts are generally concerned with nonconstant variance in time and nonnormality of returns.

Introduction

- As variance is nonconstant, volatility measurement should be conditional on existing information at current time that can explain or cause volatility
- A family of models that target dynamic variation of returns are the family of conditional heteroscedasticity models [11, 7, 15]
- With the growth of big financial databases able to extract intra-daily prices of financial assets [13, 17], measurement and modeling of realized volatility has flourished [3, 21].
- Two problems:
  1. much of the growth in this realm is based on nonstructural approaches which does not account for possible exogenous contributors to volatility dynamics [24]
  2. hurdles in acquiring intra-daily data for emerging markets such as the Philippines [19].

Introduction

- A possible framework:
  1. Engle, et al. [12] introduced the GARCH-mixed frequency data sampling [GARCH-MiDaS] volatility model which combines the GARCH specification for short-run volatility while the long-run component has a MiDaS structure [14] which considers covariates collected at lower frequency than the financial time series.
  2. Mapa [19] introduced the GARCH-PARK-R model for forecasting daily volatility through the Parkinson range [22], a substitute for realized volatility but easily available from free databases.
- The two approaches are combined, thus devising the GARCH-MiDaS-PARK-R, of which to model daily volatility with Parkinson Range and accounting for long-run and short-run volatility dynamics.

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Basic Concepts in Financial Time Series

- Transformation of the price series of a financial asset to its returns [25]

$$r_t = [\log P_t - \log P_{t-1}] \times 100\% \quad (1)$$

- Generally, stylized facts of financial time series can be described by [20, 25]:
  - Nonnormality of financial returns
    1. thicker tails than the normal distribution, which means higher or positively infinite kurtosis
    2. negative skewness or skewed in which tails are longer in the side of negative values, called leverage effects
  - Volatility clustering, modeled by the autoregressive conditional heteroscedasticity (ARCH) specifications by Engle [11] and are extended by Bollerslev [7] through the generalized ARCH (GARCH) models

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Volatility Models

Autoregressive Conditonal Heteroscedasticity [ARCH] model [11]: Let r_t be the log-return of a financial asset at time t as defined in equation 1. $r_t \sim ARCH(p)$ iff:

$$r_t = \epsilon_t, \epsilon_t \sim (0, h_t) \tag{2}$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 \tag{3}$$

Standardized errors should be serially uncorrelated with zero mean and variance h_t. The types of distributions often used are the normal or the Student's t distribution [20, 25]

Volatility Models

Autoregressive Conditonal Heteroscedasticity [ARCH] model [11]:

Equation 3 outlines the volatility dynamics characteristic of the ARCH process. The order p describes the number of past squared errors ϵ_t of which the conditional volatility h_t has dependence, of which each dependence is weighted by the parameters α_i.

If large errors within the p immediate past periods are observed, this implies an increase in uncertainty reflected by the increase in variance at time t.

By introducing nonconstant variation, thick tails can manifest on the return series to some limited extent by having a higher kurtosis than the normal distribution, even if the standardized error in equation 2 is modeled from the normal.

Volatility Models

Generalized ARCH [GARCH] model [7]:

$r_t \sim GARCH(p, q)$ if and only if equation 3 is respecified as:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{4}$$

The β_j parameters account for persistence of high volatility whilst the α_i parameters carried over from the ARCH process account for immediate response of volatility to large errors. When $q = 0$, the model specializes back to the ARCH specification. By this modification, low p and q are often required in financial time series modeling. It also leads to larger kurtosis.

Volatility Models

GJRGARCH model [15]:

$r_t \sim GJRGARCH(p, q)$ iff equation 3 is specified as:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \left[\alpha_i + \gamma_i I_{\{\epsilon_{t-i}<0\}}\right] \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j} \tag{5}$$

The γ_i parameters describe the increase in volatility when past errors $\epsilon_{t-i} < 0$, as indicated by the variable I_A which is equal to 1 when A is true and 0 otherwise. If $\gamma_i = 0$ for all past errors in the ARCH term, then it specializes to the GARCH specification. The GJRGARCH structure facilitates negative skewness as negative return values would tend to have thicker tails than positive values, addressing another stylized fact in a limited extent.

Volatility Models

GARCH-MiDaS model [12]:

The model is defined in the manner below where $r_{l,t}$ is the return series:

$$r_{l,t} = \sqrt{\tau_t}\epsilon_{l,t}, \tag{6}$$

$$\epsilon_{l,t} \sim (0, h_{l,t}) \tag{7}$$

$$h_{l,t} = (1 - \alpha_1 - \beta_1) + \alpha_1 \frac{\epsilon^2_{l-1,t}}{\tau_t} + \beta_1 h_{l-1,t} \tag{8}$$

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K} \phi_k(\omega_2) X_{t-k} \tag{9}$$

Volatility Models

GARCH-MiDaS model [12]:

- conditional variance $\sigma^2_{i,t} = \tau_t h_{I,t}$
- short-run volatility component $h_{I,t}$ is expressed in equation 8, which is an adjusted form of the GARCH(1,1) specification.
- long-run volatility component is expressed in equation 9 and is denoted by τ_t of which its logarithm is subjected to the MiDaS regression structure [14] with an exogenous regressor variable X_t.
- m is the intercept of the regression,
- θ is the parameter that describes whether the regressor X_t is relevant in the modeling of the underlying long-run volatility.

Volatility Models

GARCH-MiDaS model [12]:

The function $\phi_k(\omega_2)$ describes a distributed lag structure of weights which lays out how the lagged values of X_t contribute to the volatility dynamics. The weighting used by Amendola, et al [2]:

$$\text{Beta Function Weights: } \phi_k(\omega_2) = \frac{(1 - k/K)^{\omega_2 - 1}}{\sum_{s=1}^{K} (1 - s/K)^{\omega_2 - 1}} \tag{10}$$

The weights sum up to 1 and are monotone decreasing with respect to k. With respect to the choice of lags K, it is often the seasonal frequency of the regressor variable, e.g., $K = 4$ for quaterly regressors, or $K = 12$ for monthly regressors.

Volatility Models

GJRGARCH-MiDaS model [10]:

GJRGARCH-MiDaS simply adjusts the short-term volatility to a GJRGARCH structure:

$$h_{l,t} = (1 - \alpha_1 - \gamma_1/2 - \beta_1) + \left[\alpha_1 + \gamma_1 I_{\{\epsilon_{l-i,t}\}}\right] \frac{\epsilon^2_{l-1,t}}{\tau_t} + \beta_1 h_{l-1,t} \tag{11}$$

The GARCH-MiDaS and GJRGARCH-MiDaS models allow for more than 1 regressor in the MiDaS structure, though recent software on the method still has limited capabilities and thus the paper will devise using only 1 regressor as capabilities on such MiDaS structure is unconstrained.

Quasi-maximum likelihood estimation [QMLE] [26, 8] is utilized. Conrad and Kleen [10] outlines their QMLE approach in more detail.

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Realized Volatility

Introduced by Andersen and Bollerslev [3] in response to critique of the forecasting capability of GARCH models.

Setup: Let $r_{n,t} = log\left(P_{n,t}\right) - log\left(P_{n-1,t}\right)\ \left(0, \sigma_t^2\right)$ be the intra-daily return at time-point n within day t, in which:

- $P_{n,t}$ is the price of a financial asset at time point n within day t, with $P_{0,t}$ as the open price of the asset,
- $n = 1, 2, ..., N$, where N is the number of time points within a day
- $t = 1, 2, ..., T$, where T is the number of days
- σ_t^2 is the daily population variance

Realized Volatility

An unbiased and consistent estimator for σ_t^2 is given by [4]:

$$s_t^2 = \sum_{n=1}^{N} r_{n,t}^2 \tag{12}$$

The quantity s_t^2 is called realized variance at day t. This quantity is estimated when intra-daily data exists, which is typically available from databases with high subscription fees (e.g., Refinitiv, formerly Thomson Reuters Tick History) and are never published in business broadsheets.

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GARCH-PARK-R Model

Parkinson [22] defines the Parkinson range as:

$$R_{Park,t} = \frac{log\left(P_{H,t}\right) - log\left(P_{L,t}\right)}{\sqrt{4log(2)}} \times 100\% \tag{13}$$

where $P_{H,t}$ is the high price in day t while $P_{L,t}$ is the low price, for $t = 1, 2, ..., T$.

Such data for popular assets may be extracted from free databases or collated from business broadsheets.

Based on Parkinson's work, $E\left[R^2_{Park,t}\right] = \sigma^2_t$

GARCH-PARK-R Model

Mapa [19] devises the GARCH(p,q)-PARK-R:

$$R_{Park,t} = \mu_t \epsilon_t, \tag{14}$$

$$\epsilon_t | I_{t-1} \sim iid(1, \phi_t) \tag{15}$$

$$\mu_t = \omega + \sum_{j=1}^{p} \alpha_j R_{Park,t-j} + \sum_{j=1}^{q} \beta_j \mu_{t-j} \tag{16}$$

GARCH-PARK-R Model

To estimate the GARCH-PARK-R model, the quasi-maximum-likelihood estimation (QMLE) method [26, 8] is used with the following structure:

$$\sqrt{R_{Park,t}} = \sqrt{\mu_t}\nu_t, \tag{17}$$

$$\nu_t | I_{t-1} \sim iid(0,1) \tag{18}$$

$$\mu_t = \omega + \sum_{j=1}^{p} \alpha_j R_{Park,t-j} + \sum_{j=1}^{q} \beta_j \mu_{t-j} \tag{19}$$

It works because:

- $E\left[\sqrt{R_{Park,t}} | I_{t-1}\right] = 0$
- $Var\left[\sqrt{R_{Park,t}} | I_{t-1}\right] = \mu_t$
- the QML estimators are asymptotically normal and consistent even if the density of ν_t is misspecified [5]

Thus, estimating the GARCH-PARK-R model is similar to estimating a GARCH model on the square-root of the Parkinson range with zero error mean.

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GARCH-MiDaS-PARK-R Model

Following from Mapa [19], the GARCH(1,1)-MiDaS-PARK-R model is:

$$R_{Park,l,t} = \mu_{l,t}\epsilon_{l,t}, \tag{20}$$

$$\mu_{l,t} = \tau_t, h_{l,t}, \tag{21}$$

$$\epsilon_{l,t}|I_{l-1,t} \sim iid(1, \phi_{l,t}) \tag{22}$$

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K} \phi_k(\omega_2) X_{t-k} \tag{23}$$

$$h_{l,t} = (1 - \alpha_1 - \beta_1) + \alpha_1 \frac{R_{Park,l-1,t}}{\tau_t} + \beta_1 \frac{\mu_{l-1,t}}{\tau_t} \tag{24}$$

where:

- $l = 1, 2, \ldots, L_t$ and L_t is the number of trading days in the low-frequency period t
- $t = 1, 2, \ldots, T$ and T is the number of low-frequency periods (weeks, months, quarters, years)

GARCH-MiDaS-PARK-R Model

The QML specification for estimating the GARCH(p,q)-MiDaS-PARK-R model is:

$$\sqrt{R_{Park,l,t}} = \sqrt{\mu_{l,t}}\nu_{l,t}, \tag{25}$$

$$\mu_{l,t} = \tau_t, h_{l,t}, \tag{26}$$

$$\nu_{l,t}|I_{l-1,t} \sim iid(0,1) \tag{27}$$

$$\log(\tau_t) = m + \theta \sum_{k=1}^{K} \phi_k(\omega_2) X_{t-k} \tag{28}$$

$$h_{l,t} = \omega + \sum_{j=1}^{p} \alpha_j \frac{R_{Park,l-j,t}}{\tau_t} + \sum_{j=1}^{q} \beta_j \frac{\mu_{l-1,t}}{\tau_t} \tag{29}$$

which means it can be done by software that already runs GARCH-MiDaS models with QMLE, of which the program was designed by Candila [9]

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Application to Real Data

- Real Data: the Philippine stock exchange index [PSEI] adjusted closing, high, and low prices, from January 2008 to December 2019
- For the regressors of the MiDaS structure, three monthly variables from January 2007 to December 2019 were generated:
    - the year-on-year percentage change of the volume of production index (VoPI), and
    - the monthly realized volatility [RV] of the log returns, defined as $RV_t = \frac{1}{L_t}\sum_{i=1}^{L_t}(r_{i,t} - \bar{r}_t)^2$.
- $K = 12$ for the distributed MiDaS lag structure.
- the last 250 observations will be the designated test data set for out-of-sample evaluation while the rest will be used for model estimation

Application to Real Data

List of 15 Models; All GARCH models with $p = 1$ and $q = 1$ and GARCH-MiDaS models use weighing scheme based on beta function specification by Amendola [2].

Name	GJR Term	MiDaS term	Distribution	Parkinson Range
GARN	No	No	Normal	No
GART	No	No	Student's t	No
GJRN	Yes	No	Normal	No
GJRT	Yes	No	Student's t	No
GMNV	No	VoPI	Normal	No
GMNR	No	RV	Normal	No
GMTV	No	VoPI	Student's t	No
GMTR	No	RV	Student's t	No
JMNV	Yes	VoPI	Normal	No
JMNR	Yes	RV	Normal	No
JMTV	Yes	VoPI	Student's t	No
JMTR	Yes	RV	Student's t	No
PGAR	No	No	Normal (QMLE)	Yes
PGMV	No	VoPI	Normal (QMLE)	Yes
PGMR	No	RV	Normal (QMLE)	Yes

Application to Real Data

- For goodness-of-fit, the Akaike information criterion [1] was used.
- For within-sample and out-of-sample performance, we used the mean square error (MSE) with respect to squared daily returns, and squared Parkinson range given a vector of predictions $\hat{\sigma}$ [19, 23]:

$$MSE\left(\underline{\hat{\sigma}}^2; r_{l,t}^2\right) = \frac{1}{\#obs} \sum_{l,t} \left(r_{l,t}^2 - \hat{\sigma}_{l,t}^2\right)^2 \tag{30}$$

$$MSE\left(\underline{\hat{\sigma}}; R_{Park,l,t}\right) = \frac{1}{\#obs} \sum_{l,t} \left(R_{Park,l,t} - \hat{\sigma}_{l,t}\right)^2 \tag{31}$$

$$MSE\left(\underline{\hat{\sigma}}^2; R_{Park,l,t}^2\right) = \frac{1}{\#obs} \sum_{l,t} \left(R_{Park,l,t}^2 - \hat{\sigma}_{l,t}^2\right)^2 \tag{32}$$

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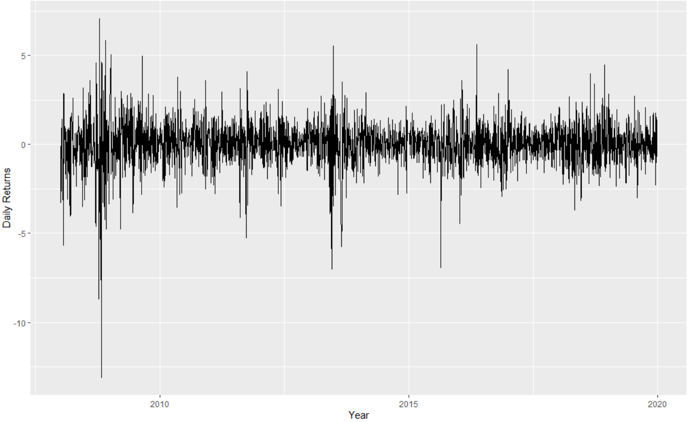
Descriptive Analysis

Figure: Philippine Stock Exchange Index, 2008 - 2019

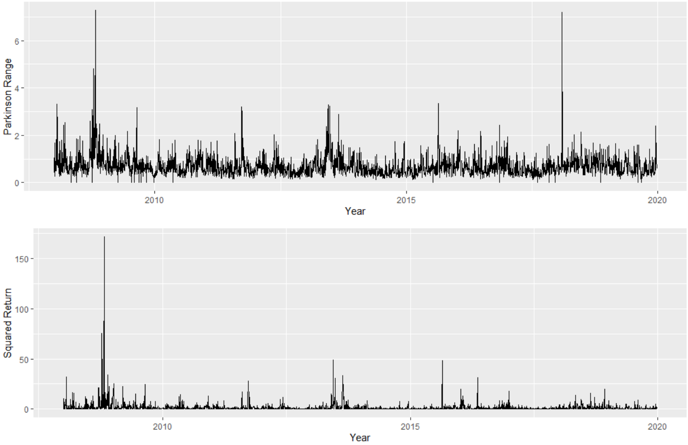
Descriptive Analysis

Figure: Daily Returns of the Philippine Stock Exchange Index, 2008 - 2019



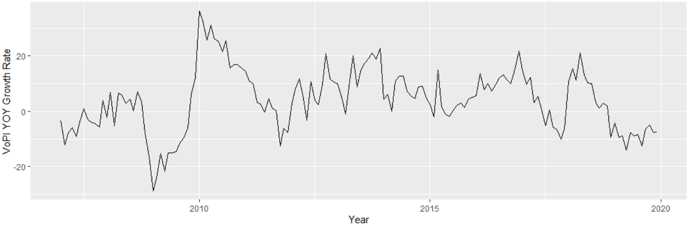
Descriptive Analysis

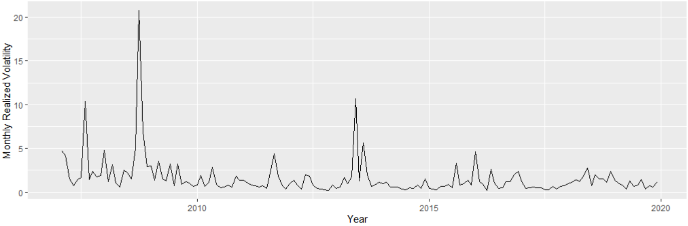
Figure: Parkinson Range and Squared Returns of the Philippine Stock Exchange Index, 2008 - 2019



Descriptive Analysis

Figure: Monthly Variables, 2007 - 2019





Descriptive Analysis

Table: Summary Statistics of the Time Series Data

Summary Statistics	PSEI Returns	VoPI Growth Rate
Minimum	-13.089	-28.700
First Quartile	-0.582	-4.290
Median	0.049	4.400
Mean	0.027	4.075
Third Quartile	0.680	11.107
Maximum	7.056	36.200
Variance	1.489	128.031
Skewness	-0.770	0.039
Kurtosis (=3 means Normal)	11.536	3.145
KPSS [18] test stat (Ho: Stationary)	0.151	0.235
KPSS 10% Critical Value	0.347	0.347
KPSS 5% Critical Value	0.463	0.463

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Forecasting Performance

Table: In-Sample Performance Metrics

Model	AIC	MSE (Sq Ret)	MSE (Park R)	MSE (Sq Park R)
GARN	2.921	7.590	0.333	2.546
GART	2.870	7.611	0.332	2.542
GJRN	2.895	7.484	0.341	2.873
GJRT	2.859	7.469	0.331	2.730
GMNV	6933.384	7.598	0.347	2.639
GMNR	6924.232	7.623	0.340	2.627
GMTV	4101.440	7.610	0.340	2.590
GMTR	4096.382	7.632	0.338	2.616
JMNV	6867.086	7.498	0.356	3.002
JMNR	6851.571	7.443	0.343	2.867
JMTV	4070.364	7.449	0.339	2.776
JMTR	4059.322	7.424	0.333	2.740
PGAR	2.412	8.358	0.139	1.777
PGMV	-533.959	9.597	0.361	2.188
PGMR	-528.670	9.598	0.361	2.188

Forecasting Performance

Table: Out-of-Sample Performance

Model	MSE (Sq Ret)	Rank	MSE (Park R)	Rank	MSE (Sq Park R)	Rank
GARN	2.480	1	0.214	3	0.609	4
GART	2.568	2	0.211	2	0.600	3
GJRN	2.788	3	0.206	1	0.569	2
GJRT	3.300	6	0.237	7	0.679	9
GMNV	3.376	12	0.281	13	0.853	15
GMNR	3.322	9	0.244	9	0.741	11
GMTV	3.340	11	0.267	10	0.799	13
GMTR	3.309	7	0.238	8	0.716	10
JMNV	3.328	10	0.275	12	0.802	14
JMNR	3.261	5	0.226	6	0.650	8
JMTV	3.310	8	0.270	11	0.771	12
JMTR	3.251	4	0.221	4	0.628	7
PGAR	3.477	13	0.221	4	0.348	1
PGMV	3.974	14	0.288	14	0.612	5
PGMR	3.980	15	0.295	15	0.615	6

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Conclusion

The GARCH-MiDaS-PARK-R model seeks to address the scenario of forecasting volatility with a reliable proxy and when strong evidence of dependence of volatility to macroeconomic low frequency data is present. This research was devised to assess the performance of the GARCH-MiDaS-PARK-R approach when face with problems in real application and in competition with other models.

From the application of the model to the PSEI data with VoPI and RV as potential regressors for GARCH-MiDaS fitting and benchmarking with various heteroscedasticity models, the results have showed that in-sample fit has been promising and much room for improvement and steps forward for out-of-sample forecasting.

The work sets forth an idea for more creative volatility model design and contributes to the idea of expanding the potential sources of dependence of volatility to other macroeconomic or low-frequency time series data.

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Thank you very much and have a g'day!

Maraming salamat sa inyong lahat!