# Interdisciplinary Analysis of Energy Consumption in Various States

# 1. Introduction

Global warming, acidification, and sea level rising have become serious environmental issues of concern due to the improper and unsustainable usage of energy. During the industrial and post-industrial revolutions, much fossil fuel energy was used to promote factory yield at a high rate. However, the drawbacks of fossil fuel overuse now prompt humans to find more efficient supplemental energies for the future.

In light of the environmental issues caused by unoptimized energy usage, this paper mainly gives profiles of energy usage for Arizona (AZ), California (CA), New Mexico (NM), and Texas (TX) from 1960 to 2009. Models are also included in order to evaluate these profiles (the trend of energy use), particularly as a way to interpret whether the states are using their energy effectively.

# 2. Notation

t: Time;
$\mathrm{Y(t)}$: Total Energy Consumption;
$\mathrm{L(t)}$: Renewable Energy Consumption;
$\mathrm{K(t)}$: Non-Renewable Energy Consumption;
$\alpha_{ij}$: Competition Index;
$\varepsilon(\mathrm{t})$: Dominant Index of Non-Renewable Energy;
$\varepsilon_i(t)$: Refined Dominant Index of Non-Renewable Energy;
$\bar{\varepsilon}(t)$: Average Dominant Index;
$\mathrm{A(t)}$: Characteristic function for each state;
$\varphi(t)$: Average Price of Energy;
$\tau(t)$: Average Price of Renewable energy;
$\bar{\tau}(t)$: Average Price of Renewable Energy;
$N_i(t)$: Consumption of the i th Category of Energy;
REII: Renewable Energy Integration Index;

# 3. Energy Profile of Four States

In order to show the relationship between time and energy as well as indicate the fraction of each energy category in the total consumption, an area chart was used to represent the energy profiles.

Because it is not practical to display all 48 categories of energy indicated in the dataset on the graph, the categories of energy were refined to 10 more general types. According to the standard made by the U.S. Energy Information Administration (EIA), energy categories can be simplified to: Biomass, Coal, Hydroelectricity, Jet Fuel, LPG, Motor Gasoline (Excluding Fuel Ethanol), Natural Gas (Including Supplemental Gaseous Fuels), Other Petroleum Products, Renewable Energy, and Residential Fuel Oil.

Based on the MSN codes and their definitions provided in the existing data set, we developed a library system with Python to search and reorder the overwhelming amount of data. After filtration, 2,000 of the 105,745 total pieces of data (500 pieces of data per state) were chosen to build each state energy profile.

The area charts made by RStudio are shown below:

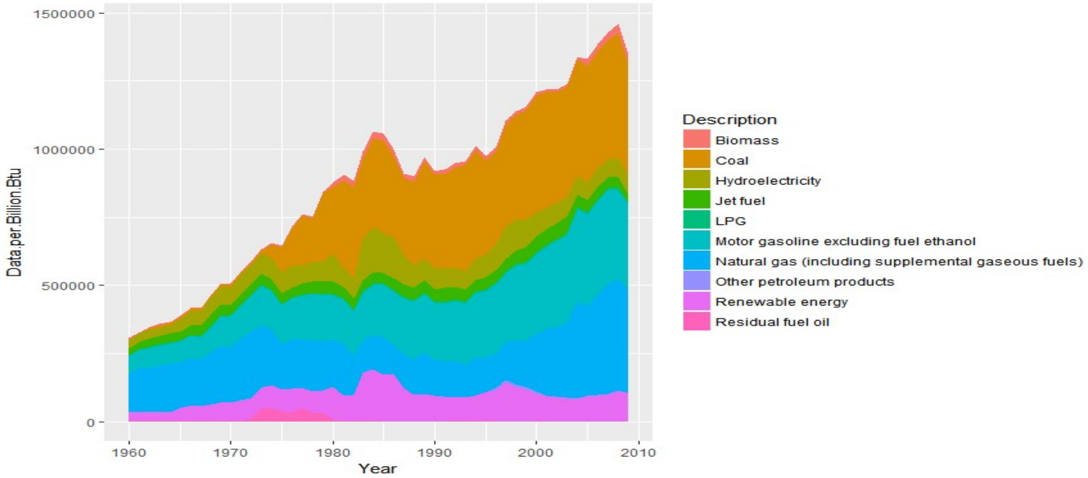



Figure (1): Energy Profile of Arizona

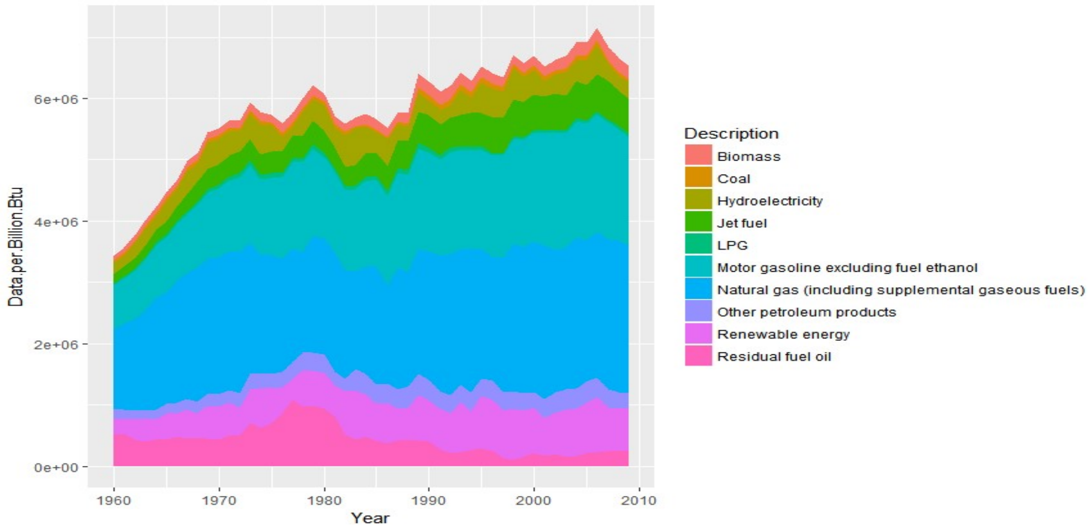



Figure (2): Energy Profile of California

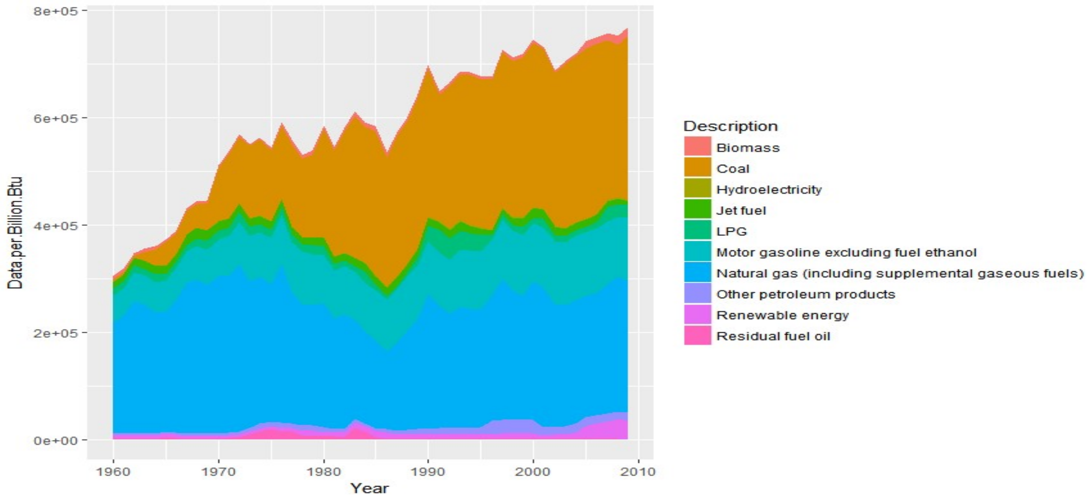



Figure (3): Energy Profile of New Mexico

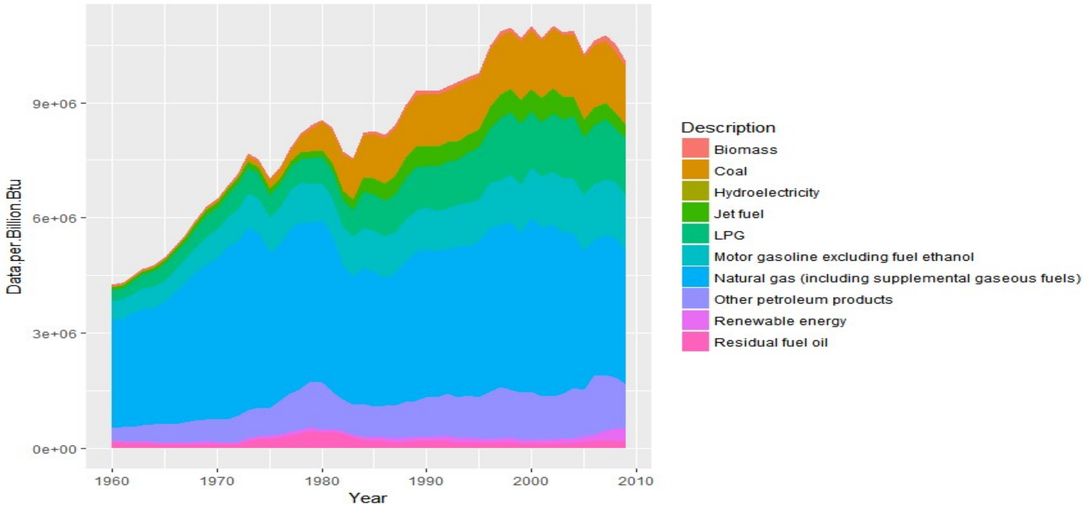



Figure (4): Energy Profile of Texas

From the energy profiles, it is shown that in all states except for Arizona, natural gas occupies a large portion of energy use (California, New Mexico, and Texas). Coal is another major source of energy for in Arizona and New Mexico. Comparatively, California is more diverse than other states in its energy usage, with biomass, jet fuel, and residential oil contributing a higher share to the total. However, renewable energy in general still only stands for a small portion of energy usage in all states.

# 4. Energy Evaluation Model

### 3.1 Introduction of Energy Evaluation Model

The energy profile is composed by two parts: the general trend of energy consumption and each energy's individual consumption. Based on this feature, the Energy Evaluation Model (EEM) should also be composed of two parts: The Energy Usage Prediction (EUP) function used to evaluate the general trend and the Energy Rival (ER) Model used to evaluate the change of each energy category. The two parts form a recursive system.

### 3.2 The generalization and development of "Cobb-Douglas Production Function"

The total energy usage is mainly influenced by renewable energy and non-renewable energy. It is convenient to separate into these two categories because renewable energy and non-renewable energy are complements of each other, so the model of profile can be related to the Cobb-Douglas Production Function:

$$\mathrm{Y} = \mathrm{A}K^{\alpha}L^{b}$$

However, since it is not certain whether using more energy is beneficial, $\alpha + \beta = 1$ (Constant Elasticity of Substitution) is set to keep the conclusion general.

Additionally, because the total energy usage $\mathrm{Y}$ as well as usage of renewable energy and non-renewable energy varies by years, the original model can be developed into the following form:

$$\mathrm{Y(t) = A(t-1)K(t-1)^{\varepsilon}L(t-1)^{1-\varepsilon}}$$

We have translated this production function Y(t) into the Energy Usage Prediction (EUP) function, in which $\mathrm{Y(t)}$ represents the energy consumption (in billion Btu) in year $\mathrm{t}$; $\mathrm{L(t-1)}$ represents the renewable energy consumption (in billion Btu) in the year previous to $t$; $K(t-1)$ represents the non-renewable energy consumption (in billion Btu) in the year previous to $t$; $\varepsilon(t)$ is denoted as the "Dominant Index" of non-renewable energy; $A(t-1)$ is a characteristic function for each state.

The EUP function of each state can use the data of renewable and non-renewable energy usage to predict the total energy usage of the following year.

### 3.3 Calculation of the Energy Usage Prediction Function

In order to derive the EUP function, it is important to know about the characteristic function $A(t)$.

Because the data of renewable energy from 1970 to 2009 is accessible, $A(t)$ can be calculated using its data and induction methods. In total, because of the 1-year-lag between $Y(t)$ and $L(t), K(t)$, $A(t)$ of 39 years (from 1970 to 2008) can be calculated. If those 39 results of $A(t)$ are scattered in a plot and form a strong relationship (no matter the form of the relationship), then this indicates that the EUP function is useful in this situation. If no relationship is formed, then EUP function fails.

Based on the categories in Chapter 2, Biomass, Hydroelectricity, and other renewable energy are categorized in $L(t)$, while the other 7 types of energy are categorized in $K(t)$.

Developed from the "Cobb-Douglas Production Function", the dominant index $\varepsilon$ is determined by total energy consumption $Y(t)$, total renewable energy consumption $L(t)$, average price of energy $\sigma$, and average price of renewable energy $\tau$:

$$\varepsilon(t) = \frac{\varphi(t)L(t)}{\tau(t)Y(t)}$$

$$\text{Then } A(t) = \frac{Y(t+1)}{K(t)^{\varepsilon(t)} \cdot L(t)^{1-\varepsilon(t)}}$$

Using the data of $F(t), L(t), K(t), \varphi(t)$ and $\tau(t)$, the $A(t)$ vs. $t$ graphs of four states are printed as following:

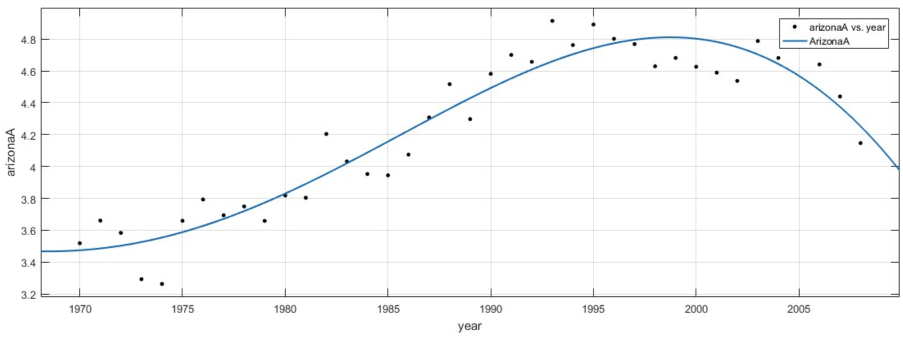
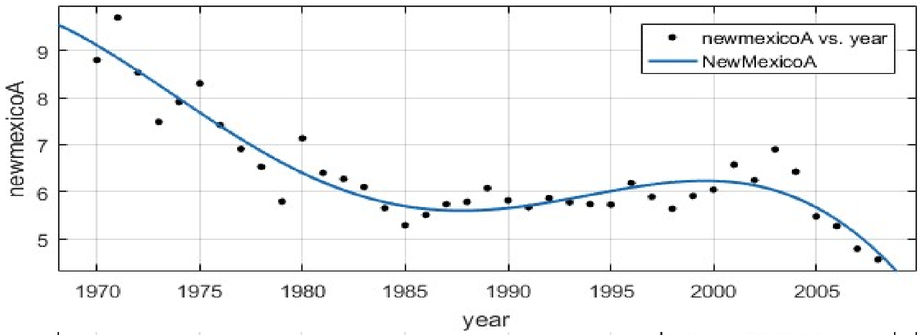


Figure (5): A(t) graph of Arizona

$$A_a(t) = 85.58\sin(0.05192x + 62.53) + 82.07\sin(0.05366x + 225.6)$$

Figure (6): A(t) graph of California

$$A_c(t) = 3.769\sin(0.0006886x + 163.6) + 0.08925\sin(0.2558x - 178.5)$$



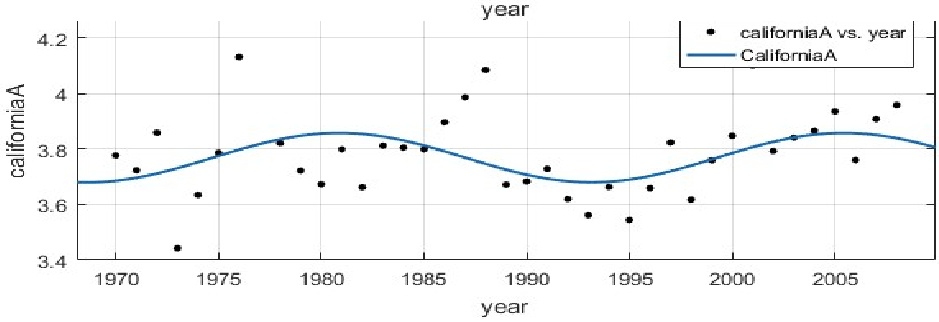




Figure (7): A(t) graph of New Mexico

$$A_N(t) = 62.72\sin(0.07689x + 12.56) + 56.69\sin(0.0808x + 165.3)$$

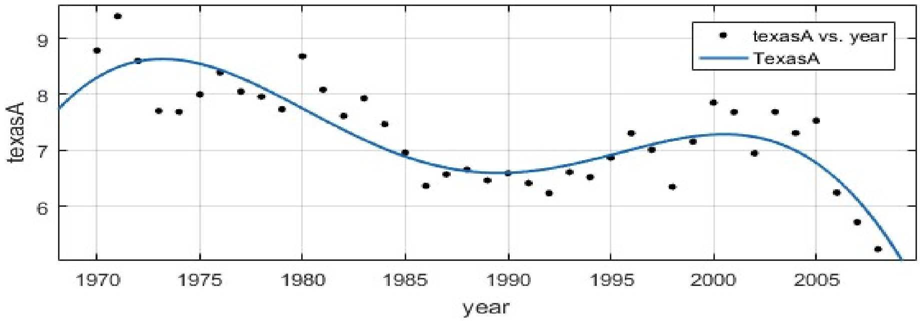



Figure (8): A(t) graph of Texas

$$A_T(t) = 77.49\sin(0.09199x - 17.88) + 70.81\sin(0.09832x + 136.1)$$

The goodness of fit of the four functions are all relatively high:

| State | R-square | Adjusted R-square |
|---|---|---|
| Arizona | 0.912 | 0.8987 |
| California | The graph is close to y=3.8 due to steady total consumption of energy | |
| New Mexico | 0.8722 | 0.8528 |
| Texas | 0.722 | 0.6799 |

Chart (1): Goodness of fit of four functions

This indicates that points in the graph have strong relationships, which further indicates our EUPF can be applied to predictions.

**3.4 "Competition" between energy and the Energy Rival Model**

After analyzing the evolution of the total energy consumption, the evolution of each energy category must be further explored.

Most of the energies have a relationship of competition: that is, energy is the substitute of each other. For example, electricity and petroleum can both be used to start a car, but the user will only choose one of them. Therefore, a rivalling relationship between energies can be concluded.

In order to show the consumption of different energies more explicitly, the energy consumption situation can become an analogy of an environmental circle. Recall that in a single year, the total energy consumption is fixed (because the data is discrete), which corresponds to the limited resources in the environment. The categories of energy are like species in the environment, which retain a relationship of rivalry: under a fixed need, more consumption of energy A will cause a consumption decrease of energy B.

The competition between several species can be explained by Gause-Lotka-Volterra Model (GLV), which has the formula:

$$\frac{dN_i(t)}{dt} = \gamma_i N_i(t)\left[1 - \sum_{j=1}^{n} \alpha_{ij} N_j(t)\right], j \neq i$$

By modifying the GLV, an Energy Rival (ER) Model can be built. The model contains an equation set including 11 differential equations due to 10 main categories of energy mentioned in Chapter 2. The equation set is now complete, shown below:

$$\begin{cases} \sum_{i=1}^{n=10} N_i(t) = Y(t) \\ \dfrac{dN_i(t)}{dt} = \widetilde{\varepsilon_i}(t) N_i(t)\left[1 - \sum_{j=1}^{n} k_i N_j(t)\right], j \neq i \end{cases}$$

Note: $\begin{cases} \widetilde{\varepsilon_i}(t) = \varepsilon_i(t) \text{ (when } N_i(t) \text{ is non-renewable)} \\ \widetilde{\varepsilon_i}(t) = 1 - \varepsilon_i(t) \text{ (when } N_i(t) \text{ is renewable)} \end{cases}$

- $1 \leq j \leq 10$ and $k_i$ is a function depends on each state i
- $Y(t)$ represents the total energy consumption in year t
- $N_i(t)$ represents the consumption of the I th category of the energy
- $\varepsilon_i(t)$ is the Dominant Index; $\widetilde{\varepsilon_i}(t)$ is denoted as Refined Dominant Index

**3.5 Computation of Energy Rival Model**

To calculate the differential equation set of different states and energy, it is essential to determine the function $k_i$ for each state.

If the total energy consumption is low last year, it will restrain the total energy consumption this year. Therefore, $k_i$ is related to $F(t-1)$. Also, because the part $[1 - \sum_{j=1}^{n} k_i N_j(t)]$ has no unit, $k_i$ can be defined as $1/F(t-1)$. Then the general solution to the differential equation is. (Implicit function)

$$N_i(T) = Ce^{\widetilde{\varepsilon_i}(t)[1 - \sum_{j=1, i \neq j}^{10} N_j(t)]}$$

The general solution to the equation is not as useful as in theory, simply because the solution of the function is recursive, as seen by $N_i(T)$ calling $N_j(t)$. Thus, approximation methods must be utilized to retrieve the values. As long as the $A(t)$ from 3.3 is known, we can use these methods to verify the equation. In this case, we use Runge Kutta to calculate the values of $N_i(T)$ from $\frac{dN_i(t)}{dt}$.

**3.6 Explanation of Energy Evaluation Model**

The EEM is easy to understand when it is related to production and environmental competition.

Recall the general model:

$$\mathrm{A(t)} = \frac{Y(t+1)}{K(t)^{\varepsilon(t)} \cdot L(t)^{1-\varepsilon(t)}} \quad (1)$$

$$\sum_{i=1}^{n=10} N_i(t) = Y(t) \quad (2)$$

$$\frac{dN_i(t)}{dt} = \tilde{\varepsilon}_i(t) N_i(t)\left[1 - \sum_{j=1}^{n} k_i N_j(t)\right] \quad (3)$$

In equation (1), the energy consumptions of renewable and non-renewable energy are like the capital input of an investment. The predicted profit will remain unchanged even if the investment increases, and both two types of capital have the same effect, so the factor that determines future profit (future energy consumption) is the "dominant index", which is determined by consumption of renewable energy and price. Furthermore, the $\mathrm{A(t)}$ acts as a factor that determines the future profit overall, which is like the policy influence on the future consumption energy.

In equation (2) and (3), the energy consumptions of each categories are comparable species (energies, for the equations) competing for limited resources (demand for energy). The existence of other species will threaten the growth of one species (in equation (3)), and the impact depends on the current importance of those species (dominant index), the numbers of the species (energy consumption), as well as the former capacity of the environment (inverse of $\mathrm{F(t-1)}$).

Comparing the four states, we see that New Mexico and Texas have relatively high $\mathrm{A(t)}$—around 6 to 9—while Arizona and California remain an $\mathrm{A(t)}$ value around 4. This means that the policies or industries of Arizona and California are not as energy-intensive as those of New Mexico and Texas, partially because the latter two are states with more heavy industries.

Also, the dominant indices of Arizona and California are higher than those of New Mexico and Texas, which means the renewable energies in former two states have more important status. The renewable energies are expensive in Texas, so it only occupies a small portion of energy in total, while in New Mexico, the growth rate is too low keep the dominance of renewable energy in the market. (Further exploration will be in chapter 4) The reason of the difference is likely because Arizona and California are on-shore or close-to-on-shore states with an easier access/ways to transfer different renewable energy than New Mexico and Texas, as well as different population culture and state policies regarding energy usage.

Overall, the factors that may affect the EEM include location, policy, industry, economic development and so on. The location directly or indirectly decides the accessibility to renewable energy due to the effect of transportation and climate. Policy decides whether the governors support renewable energy consumption and further decides the price as well as dominant index. The impact of heavy industries on the state economy and the population decides the demand of energy in total. Combined with the effect of policy, changing the partition of those demands can make a great influence on the energy profile of the state.

Eventually, EUP and ER models form a recursive loop. Since $\mathrm{A(t)}$ is a function that is known to each state based on the database, $Y(t+1)$ can be derived from $\mathrm{L(t), K(t)}$, and $\varepsilon(t)$ by the EUP model. As $Y(t+1)$ is known, $\mathrm{L(t+1), K(t+1)}$ can be derived by the ER model using Runge Kutta: $\mathrm{L(t+1)} = \sum N_i(t)$, $(N_i(t) \in \{renewable\})$ $\mathrm{K(t+1)} = \sum N_i(t)$, $(N_i(t) \in \{nonrenewable\})$, and $\varphi(t), \tau(t)$ can be derived by the relationship between price and demand, which finds $\varepsilon(t+1)$. $Y(t+2)$ can be indicated based on $\mathrm{L(t+1), K(t+1)}, \varepsilon(t+1)\ldots$ The EUP model and ER Model alternate with each other to derive future sets of data.

Overall, an equilibrium recursion model can be built. If the renewable function is dominating the market, then the demand of it next year will increase, which causes the dominant index $\varepsilon$ to increase. The increase will cause the demand of non-renewable energy to decrease next year, which keep the renewable energy and non-renewable energy consumptions in a natural, dynamic balance. Furthermore, if there are no policies posed by the government ideally, the balance will be kept until the day when non-renewable energy is depleted, and the system will finally crash abruptly. Therefore, the formation of this recursive system proves that governors should get involved in energy consumption to prevent the crash of system.

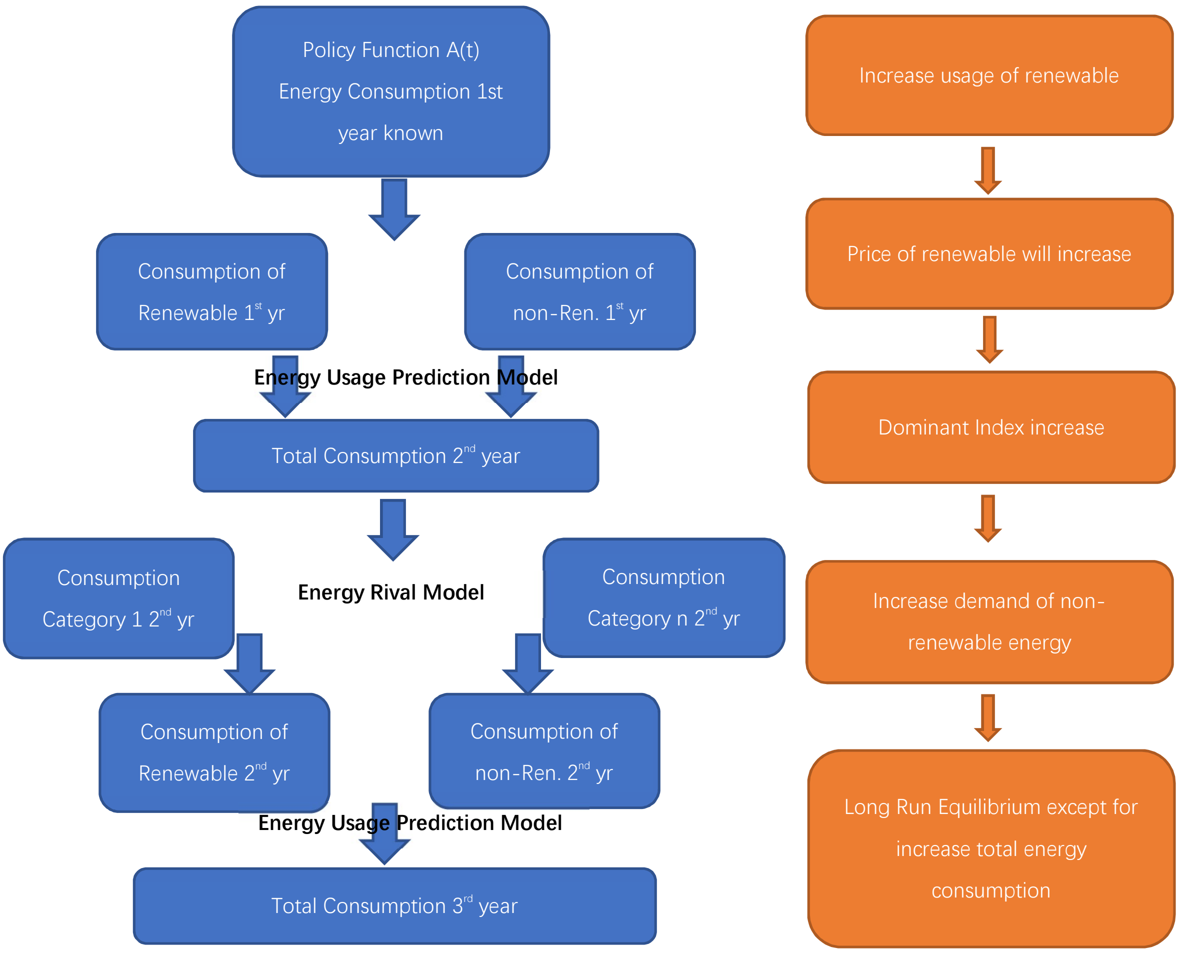



Figure (9) The mechanism of Energy Evaluate Model & the long-run equilibrium

# 5. Renewable Energy CDF Model

## 4.1 Introduction of Renewable Energy CDF Model

The fact that renewable energy is a cleaner alternative to its counterpart has caused many people think renewable energy should be used as much as possible for its sustainability. However, this theory is not true all the time. Though renewable energy has its advantage, overusing it can cause many problems such as inflation of renewable energy, monopoly or oligopoly, crash of traditional industry, possibly becoming non-renewable, etc. (Recall that wood is a renewable energy, but its regrowth rate is outpaced by the rate of deforestation, making it unrenewable in certain areas)

Therefore, the criteria of whether a state makes proper use of renewable energy should contain two parts: the state neither refuses using renewable energy nor overusing renewable energy. That is, either extreme will harm renewable energy consumption.

Based on the fact that renewable energy should be moderately used, the Renewable Energy CDF Model is built to evaluate the usage.

Because of the symmetry of normal distribution, the Renewable Energy CDF Model is mainly built on the bell-shape graph with the mean $\mu = 0$ and standard deviation $\sigma = 1$. Then a vertical line $x = k$ named "Status Line" is drawn to show the current renewable energy consumption status. The x-coordinate of the Status Line depends on factors such as growth rate of renewable energy $dN_i/dt$, Dominant Index $\varepsilon$, and price $\tau(t)$. Whether the state is overusing or underusing renewable energy can be judged based on the graph in the following:

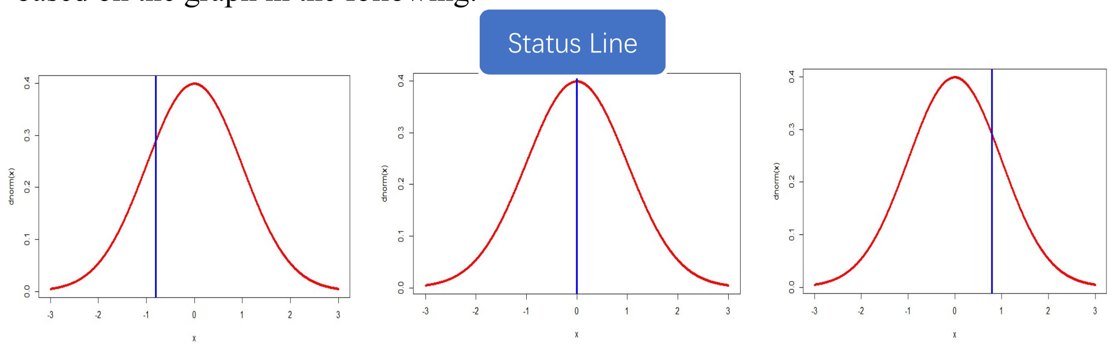



Figure (10): Graphs of Under Using, Perfect Using (Idealistic), and Over Using

## 5.2 Computation of Renewable Energy CDF Model

According to the fact that the optimal solution should be at $x = 0$ and the total area under the normal distribution curve is 1, the Renewable Energy Integration Index is defined as the following:

$$\text{REII} = 4\int_{-\infty}^{x} Norm(x)dx \int_{x}^{\infty} Norm(x)dx \qquad \text{REII} \in (0,1)$$

The graph of normal distribution decides that the optimal solution should be $x = 0$. Because $\int_{-\infty}^{x} Norm(x)dx + \int_{x}^{\infty} Norm(x)dx = \int_{-\infty}^{\infty} Norm(x)dx = 1$, when the Status Line moves away from $x = 0$, the product of two areas will diminish.

However, recall that the formula of normal distribution is like the following:

$$\text{Norm}(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{(-\frac{x-\mu}{2\sigma})^2} = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

The derivative of the function is:

$$\mathrm{Norm}'(x) = -\frac{x}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

Because there is an e under the power of $x^2$, the derivative changes rapidly when x increases from 1 to $\infty$, which makes the REII diminish faster than expected. Therefore, a t-distribution with a low degree of freedom n ($n < 30$)should be used to make the curve smoother and flatter.

The general formula of t-distribution curve is the following:

$$\mathrm{t(x)} = \frac{Gam\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}Gam\left(\frac{n}{2}\right)}(1+\frac{x^2}{n})^{-\frac{n+1}{2}}$$

$$\text{in which } \mathrm{Gam(x)} = \int_0^\infty t^{x-1}e^{-t}dt$$

When n is fixed, the derivative of t-distribution curve is

$$t'(\mathrm{x}) = -\frac{(n+1)Gam\left(\frac{n+1}{2}\right)}{n\sqrt{n\pi}Gam\left(\frac{n}{2}\right)}(1+\frac{x^2}{n})^{-\frac{n-1}{2}}$$

Compared with the derivative of normal distribution, because the power is fixed to $-\frac{n-1}{2}$, the distribution does not change too rapidly when x value sway from 0, and when the positive n (degree of freedom) gets smaller, the curve becomes flatter.

In 10 categories listed in Chapter 2, 3 of them are renewable energy. Therefore, assume $n = 3$ is right and check the result:

| REII | x = −3 | x = −2 | x = −1 | x = 0 | x = 1 | x = 2 | x = 3 |
|---|---|---|---|---|---|---|---|
| n = 1 | 0.3677 | 0.5032 | 0.7500 | 1.0000 | 0.7500 | 0.5032 | 0.3677 |
| n = 3 | 0.1120 | 0.2592 | 0.6291 | 1.0000 | 0.6291 | 0.2592 | 0.1120 |
| n = 10 | 0.0265 | 0.1414 | 0.5655 | 1.0000 | 0.5655 | 0.1414 | 0.0265 |
| n = 30 | 0.0108 | 0.1063 | 0.5448 | 1.0000 | 0.5448 | 0.1063 | 0.0108 |
| Normal | 0.0053 | 0.0889 | 0.5339 | 1.0000 | 0.5339 | 0.0889 | 0.0053 |

Chart (2): Relationship between REII, Status Line, and degree of freedom

In the common range [-3,3] using for graphing distribution curves, $n = 3$ seems like a proper value to adjust the curve. Furthermore, as 3 is the number of renewable energy type, this indicates that when the renewable energy becomes more various, which means renewable energy is going to dominate the market and become more important, any changes in status line make a larger difference. Therefore, t-distribution with degree of freedom df=3 should be a better choice.

In 4.1, it is noted that the factors that decide the Status Line include growth rate, dominant index, and price. The equation of x should be:

$$\mathrm{x} = a_1 f_1(average(\frac{dN_i(t)}{dt})) + a_2 f_2(\varepsilon(t)) + a_3 f_3(\tau(t)) + R$$

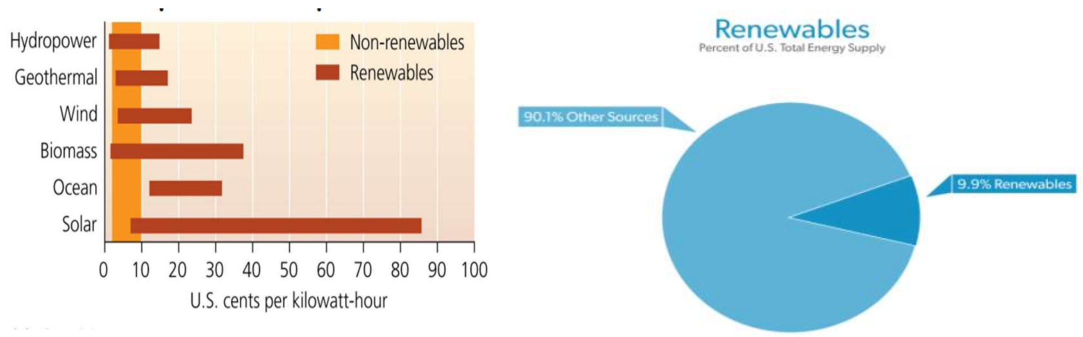
($a_1, a_2, a_3$ are coefficients, and R is a randomized function)

When x is positive, it indicates the renewable energy is being overused and vice versa. It is easy to decide that $a_1 > 0, a_2 > 0, a_3 < 0$, because usually a higher growth rate and dominant place in the market indicate overusing, while higher price indicates underusing.

The function $f_1, f_2, f_3$ are used to control the value, for $\varepsilon(t), \tau(t)$ which are always greater than 0. For $f_1$, it is used to adjust the order of magnitude, and $f_2, f_3$ are used to compare the value of a state to the value of the national average. This is to show the comparative dominant index and price as well as control the x within the range of [-3,3] to avoid extreme output.

From the data of Institute of Energy Research (IER) and internet, shown below:

Figure (11): Price of Renewable Energy and Percentage of Consumption




Then the average dominant index of the US will be:

$$\bar{\varepsilon} = \frac{Avg\ Price \times Percentage\ (Re)}{Avg\ Price\ (all) \times 100\%}$$

Take the average price of renewable energy as approximately 70 cents per kWh (207921.7 USD/Billion Btu) and the non-renewable energy as 3 cents per kWh (1 billion Btu = 293071 kWh). The percentage of renewable energy is 9.9%. After weighting the prices, the average energy cost found to be approximately 28612.99 USD/Billion Btu. Then $\bar{\varepsilon} \approx \frac{207921.7 \times 9.9\%}{28612.99} = 0.7194$ $\quad \bar{\tau} = 207.9217 \frac{\text{USD}}{\text{Million}} Btu$

In order to restrict the data to the range of [-3,3], a log function can be used to shrink the magnitude of degree. (Note for $f_1$, assume 10% should be a normal growth rate)

$$\begin{cases} f_1 = \log(x/1000) - 2 \\ f_2 = \frac{1}{\bar{\varepsilon}}(x - \bar{\varepsilon}) \\ f_3 = -\frac{1}{\bar{\tau}}(x - \bar{\tau}) \end{cases}$$

The formula for the model should be:

$$\text{REII}(x) = 4 \int_{-\infty}^{x} \frac{\int_0^\infty te^{-t}dt}{\sqrt{n\pi}\int_0^\infty t^{-\frac{1}{2}}e^{-t}dt}\left(1 + \frac{y^2}{3}\right)^{-2} dy \cdot \int_{x}^{\infty} \frac{\int_0^\infty te^{-t}dt}{\sqrt{n\pi}\int_0^\infty t^{-\frac{1}{2}}e^{-t}dt}\left(1 + \frac{y^2}{3}\right)^{-2} dy$$

Where $x = f_1 + f_2 + f_3$

### 5.3 Judge the Best Usage of Renewable Energy using REII

Find the $average(\frac{dN_i(t)}{dt})$, $\varepsilon(t)$, and $\tau(t)$ of the four states in 2009, and plug them in the Renewable Energy CDF Model:

| | $f_1$ | $f_2$ | $f_3$ | Sum (x) |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| AZ | -0.5677 | 0.1275 | -0.0780 | -0.5182 |
| CA | 0.1250 | 0.0502 | 0.2488 | 0.424 |
| NM | -1.1250 | 0.0035 | -0.1567 | -1.2782 |
| TX | -0.1540 | 0.0360 | -0.7462 | -0.8642 |

Chart (3): Status Line values of each state

From the result of sum, which represents the vertical line, it can be discovered that except for California, the other three states (AZ, NM, and TX) are underusing their renewable energy, while California is overusing slightly.

Find the REII for the four states:

$$REII_{AZ} \approx 0.8705 \quad REII_{CA} \approx 0.9101 \quad REII_{NM} \approx 0.4975 \quad REII_{TX} \approx 0.6986$$

It can be concluded that California has the best profile of renewable energy, while New Mexico has the worst. For each category of judgement, New Mexico's growth rate is too low, and Texas's price of renewable energy is too high.

The graphs of REII are shown in the following:

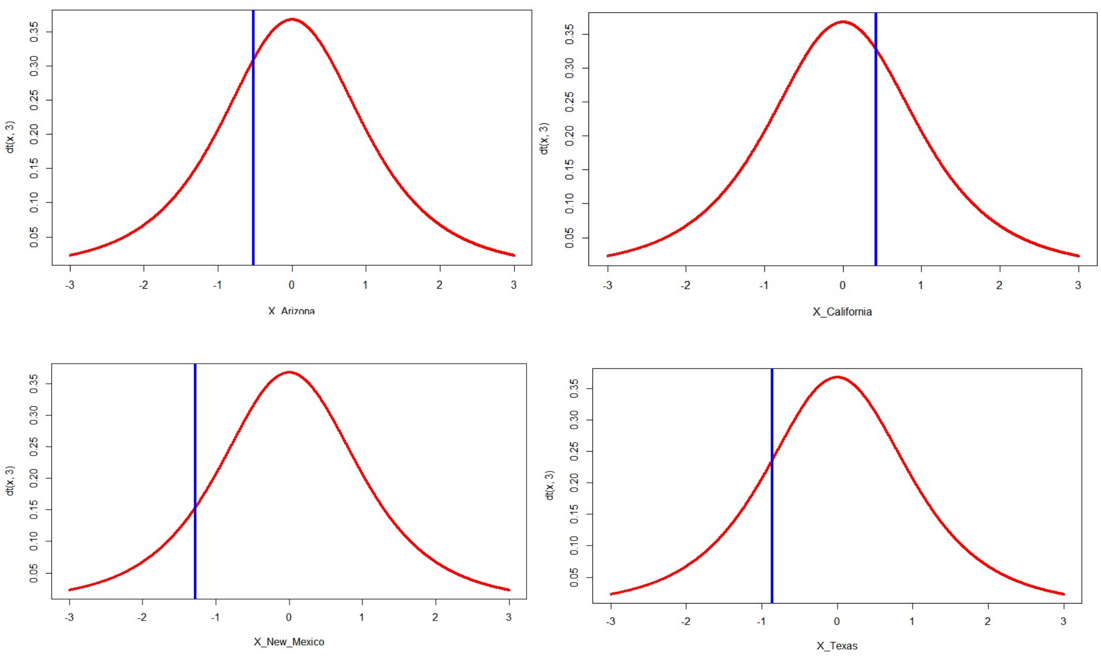



Figure (12): REII graphs of Arizona, California, New Mexico, and Texas

## 6. Estimation of Energy Profile from 2025 to 2050

According to the Energy Evaluation Model, the energy profile estimation of four states are shown in the following by the same area graph as in Chapter 2.

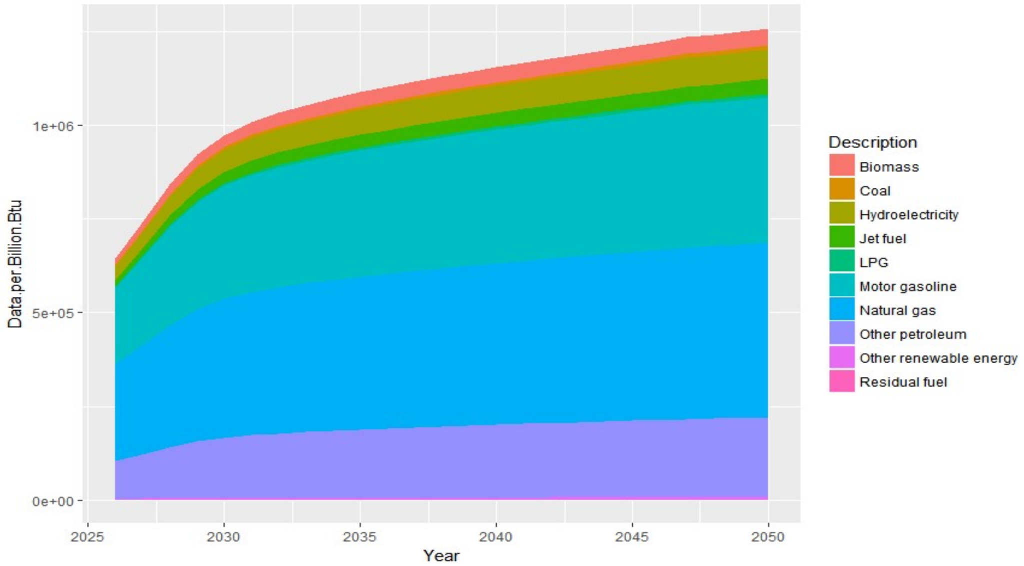



Figure (13): Energy Profile of Arizona (2025~2050) Estimation

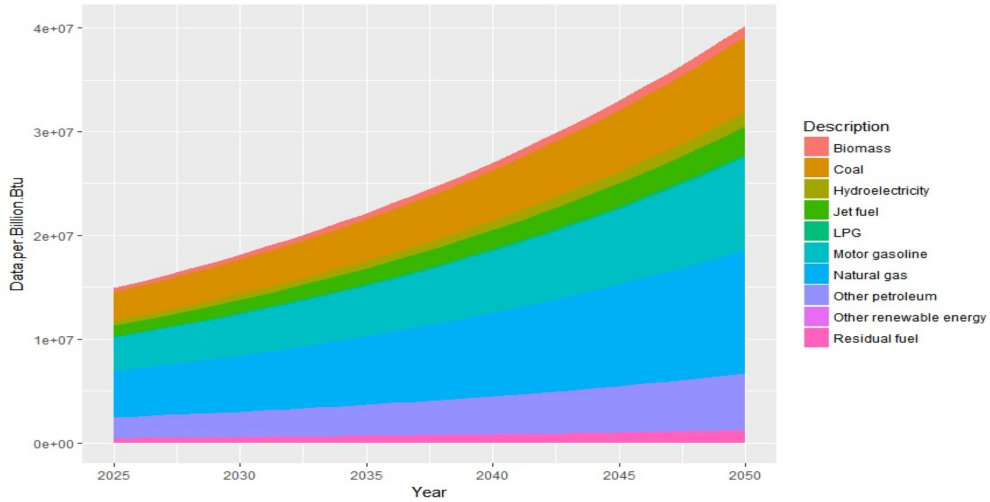



Figure (14): Energy Profile of California (2025~2050) Estimation

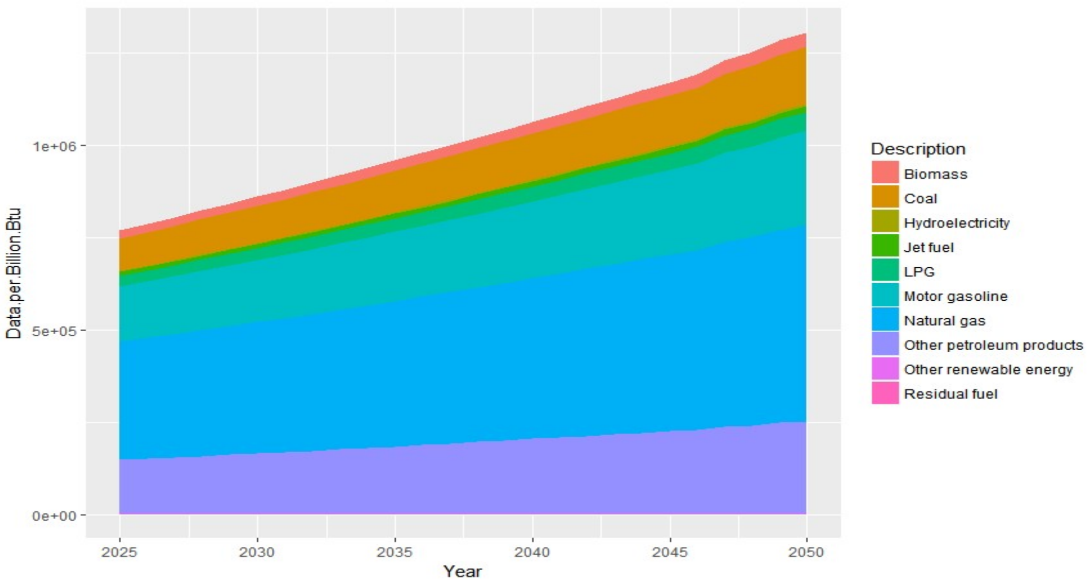


Figure (15): Energy Profile of New Mexico (2025~2050) Estimation

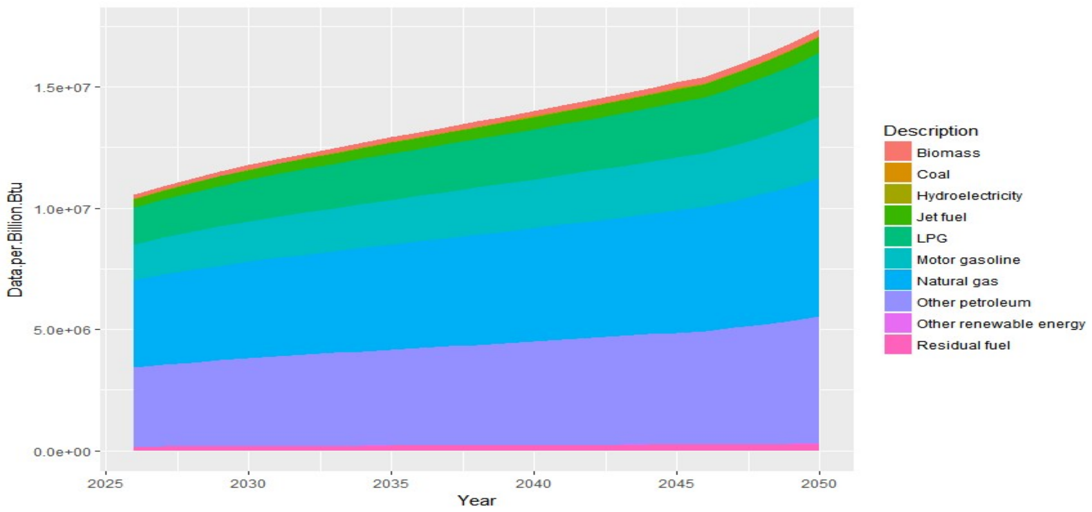



Figure (16): Energy Profile of Texas (2025~2050) Estimation

From the graph, it is shown that if there is no policy involving in managing energy consumption, the growth of energy consumption of each state will form a smooth curve, either concave up or concave down. Also, the renewable energy and non-renewable energy consumption will both increase in proportion. That is, the difference between consumption of non-renewable energy and renewable energy will increase, and the dominant energy (natural gas and petroleum) will continue to be dominant.

# 7. Renewable Energy Target of 2025 & 2050

Based on the different renewable energy consumption status of each state, the target of 2025 & 2050 should be varied. Because the REII index can be calculated, the future target of renewable energy can be quantified: set the final target of REII to be greater than 0.95, which means that renewable energy is being properly used. (Neither underuse so that non-renewable energy will be depleted, nor overuse so that the using speed is faster than technology growth which creates inefficiency.)

States can be categorized into three groups due to their REII value:

| Group | REII (2009) | REII Target (2025) | REII Target (2050) |
|---|---|---|---|
| I (CA) | $\mathrm{REII} \geq 0.9$ | $\mathrm{REII} \geq 0.95$ | $\mathrm{REII} \geq 0.95$ |
| II (AZ/TX) | $0.6 < \mathrm{REII} < 0.9$ | $\mathrm{REII} \geq 0.9$ | $\mathrm{REII} \geq 0.95$ |
| III (NM) | $\mathrm{REII} \leq 0.6$ | $\mathrm{REII} \geq 0.5 + \mathrm{REII}(2009)/2$ | $\mathrm{REII} \geq 0.95$ |

Chart (4): Renewable Energy Target

Then use REII value to solve for x (Status Line): (In Chapter 4)

$$x \in [\mathrm{invT}\left(\frac{1-\sqrt{1-REI}}{2}\right), \mathrm{invT}\left(\frac{1+\sqrt{1-REII}}{2}\right)]$$

(invT means inverse function of t distribution)

When $\mathrm{REII} = 0.95$, the range of x (Status Line) should be $[-0.3106, 0.3106]$. This means in order to reach the final target in 2050, the value of Status Line should be restricted into the interval above.

Recall that $x = f_1 + f_2 + f_3$. As $f_1, f_2, f_3$ are linearly independent, the feasible region can be presented in a Cartesian Coordinate System.

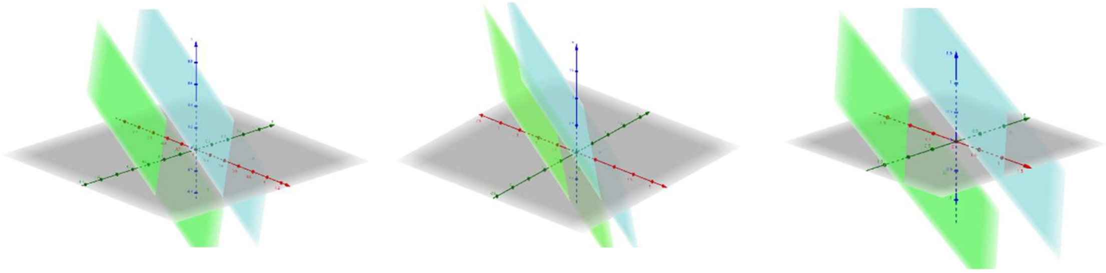


Figure: The feasible region of x. (Area between two planes)
(x-axis: red; y-axis: green; z-axis: blue)

Because $f_1, f_2, f_3$ can all be positive or negative, in order to simplify the problem, restrict the $f_1, f_2, f_3$ to the first octant. That is,

$$|f_1| + |f_2| + |f_3| \leq \text{invT}\left(\frac{1+\sqrt{1-REII}}{2}\right) \ldots\ldots (a)$$

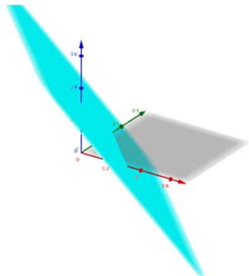


Figure (17): First Octant Graph

Therefore, the situation when positive and negative $f_1, f_2, f_3$ counteract each other and finally results in a good REII is prevented. In order not to allow an extreme $f$ to exist, the values of $f_1, f_2, f_3$ are further restricted to $|f| \leq 0.3106/3 \approx 0.1035$. This is the smallest cube that can contain in the tetrahedron (a).

According to the estimated price of renewable energy $\varepsilon(t)$, estimated price of energy in total $\tau(t)$, estimated renewable and total energy consumption, we can find out the $\bar{\varepsilon}, \bar{\tau}$ value of year 2025 and 2050: (take REII $= 0.95$ as an example)

$$\overline{\varepsilon_{2025}} \approx 0.556 \quad \overline{\tau_{2025}} \approx 318.8 \frac{USD}{\text{Million}} Btu$$

$$\overline{\varepsilon_{2050}} \approx 0.453 \quad \overline{\tau_{2050}} \approx 392.6 \frac{USD}{Million} Btu$$

Solve for $|f_1| \leq 0.1035$:

$$78795\ Billion\ Btu/yr \leq dN/dt \leq 126911\ Billion\ Btu/yr$$

Solve for $|f_2| \leq 0.1035$: (for 2025)

$$0.4985 = 0.8965\overline{\varepsilon_{2025}} \leq \varepsilon \leq 1.1035\overline{\varepsilon_{2025}} = 0.6135$$

Furthermore, it can be implied that $\text{renewable\%} = \frac{Price\ total}{Price\ renewable} \times \varepsilon$

$$4.44\% \leq \text{renewable\%} \leq 5.46\%$$

Solve for $|f_2| \leq 0.1035$: (for 2025)

$$285.8042 \frac{USD}{Million} Btu = 0.8965\overline{\tau_{2025}} \leq \tau \leq 1.1035\overline{\tau_{2025}} \approx 351.7958 \frac{USD}{Million} Btu$$

Therefore, for the energy target listed in chart (4), the REII values can be interpreted into specific goals like the following:

| Group | Target for 2025 | Target for 2050 |
|---|---|---|
| I | Control the renewable energy growth rate within $[78795, 126911]$Billion Btu/yr. | Control the renewable energy growth rate within $[78795, 126911]$Billion Btu/yr. |

| | | |
|---|---|---|
| | Control the renewable energy consumption ratio to total energy within $[4.44\%, 5.46\%]$. Control the renewable energy price within [285.8042,351.7958] USD/Million Btu. | Control the renewable energy consumption ratio to total energy within $[4.44\%, 5.46\%]$. Control the renewable energy price within [351.9659,433.2341] USD/Million Btu. |
| II | Control the renewable growth rate within [70843,141156] Billion Btu/yr. Control the renewable energy consumption ratio to total energy within [4.24%,5.73%]. Control the renewable energy price within [271.0756,366.5243] USD/Million Btu. | Control the renewable energy growth rate within $[78795, 126911]$Billion Btu/yr. Control the renewable energy consumption ratio to total energy within $[4.44\%, 5.46\%]$. Control the renewable energy price within [351.9659,433.2341] USD/Million Btu. |
| III | Control the renewable energy growth rate within $[10^{4+t_1}, 10^{4+t_2}]$ Control the renewable energy consumption ratio to total energy within $[4.95t_1, 4.95t_2]$%. Control the renewable energy price within $[318.8t_1, 318.8t_2]$ | Control the renewable energy growth rate within $[78795, 126911]$Billion Btu/yr. Control the renewable energy consumption ratio to total energy within $[4.44\%, 5.46\%]$. Control the renewable energy price within [351.9659,433.2341] USD/Million Btu. |

(Note: $t_1 = \left(1 + \frac{invT\left(1-\frac{\sqrt{1-REI}}{2}\right)}{3}\right)$,$t_2 = \left(1 + \frac{invT\left(1+\frac{\sqrt{1-REI}}{2}\right)}{3}\right)$.)

Chart (5): Interpretation of renewable energy target.

Because of the different situation of each state, it is hard to unify the energy that should be developed by each state. Instead, as far as the combination of all renewable energy follows the chart above, it will be an optimal target to the state.

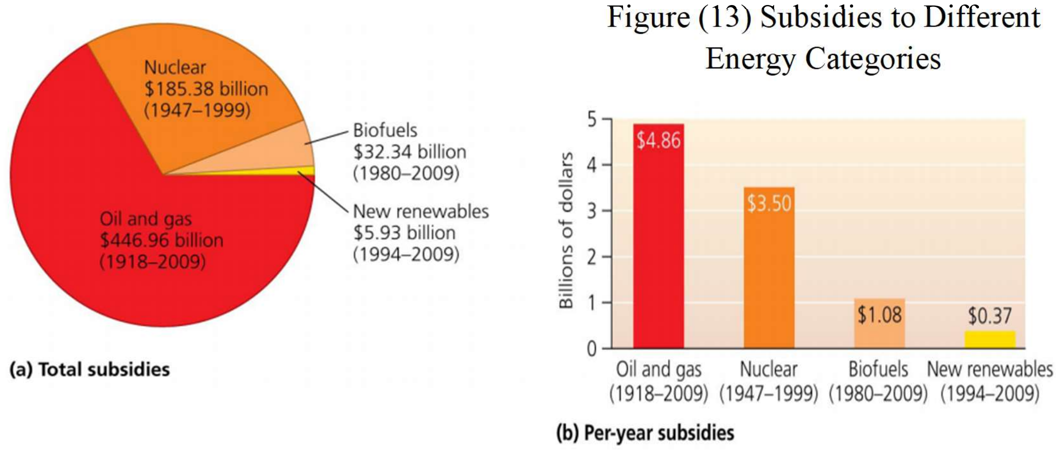
# 8. Policy used to reach the goal

Based on chapter 4, it can be concluded that most states are underusing their renewable energy. In order to promote consumption of renewable energy properly as well as restrict consumption of non-renewable energy, actions such as subsides, price control, and restricting the total energy consumption.

**7.1 Subsidies**

A production subsidy policy can be applied to reduce the market price of renewable energy, such that the competitiveness of the renewable energy can be highly improved.

Figure (13) Subsidies to Different Energy Categories




Suppose in the most idealistic situation, all the subsidies convert to decrease in prices. Suppose $s(t)$ represents subsidies to renewable energy per year, then

$$\Delta\varphi(t) = s(t)/L(t)$$

However, in actual cases, subsidies are not usually 100% effective. That is, $\Delta\varphi(t) \leq s(t)/L(t)$ due to companies do not always want to give out all the subsidies to the society, so it is reasonable to use randomized function to evaluate the reduction of price:

$$\Delta\varphi(t) = Ran(0.5,1] \times s(t)/L(t)$$

**8.2 Price Control**

Although promoting the use of renewable energy is a consensus, it is not technologically mature to support the whole energy structure. Also, an economy based on non-renewable energy is detrimental to the future development. Therefore, both overuse or underuse of renewable energy is not desirable to the economy.

In order to regulate the proportion of renewable energy production, a price ceiling (pc) and a price floor (pf) of the average price for renewable energy can be applied.

The price ceiling provides a buffer, which prevents the renewable energy market from overheating, which deteriorates the performance of energy structure. On the other hand, the price floor guarantees the minimum profits for renewable energy companies to maintain their production, such that their market share can be well protected.

**7.3 Total Energy Consumption Restriction**

The total energy consumption restriction includes many methods such as increase research budget of machinery efficiency, exploitation restriction, and additional tax on non-renewable energies. Suppose that each company has approximately the same investment on using energy, then an additional tax rate of $\gamma$ can reduce the consumption of non-renewable energy to $1/(1+\gamma)^{elas}$ the original amount, for energy is an inelastic good.

# 9. Strengths and Weaknesses

**Strengths:**

- The models are interdisciplinary and understandable. In our EEM, the relationship of energy consumption is developed from economic (Cobb-Douglas Function) and environment (Gause-Lotka-Volterra Model). Instead of plainly copying those models, we develop index such as “Policy Function” $A(t)$ and “Dominant Index” $\varepsilon$ to make EEM account for real environment. As those simulations are close to lives, the math model becomes easier to interpret.
- The models are delicate with mathematical skills. For example, the RECDFM make use of statistic knowledge: t-distribution, and the usage of Cartesian Coordination to quantify the renewable energy consumption target makes the result more accurate.
- Instead of making a single model, our research forms an energy evaluation system. The EEM model (with the two models inside) and RECDFM are interrelated, using many variables in common, which makes the research become more cohesive and applicable.
- The models are creative. People will usually think of a specific function to evaluation the consumption of energy. Instead of using a function for each energy categories, we built a recursion model to show the interrelationship between the variables more clearly and close to reality.

**Weakness:**

- The model can be further developed. Though the system of evaluation is built, because of the time limitation, we only take 10 categories into account. The standard of dividing variables can be refined in order to make the model more comprehensive and can indicate the energy consumption of sub-categories.
- The usage of the model can be broadened. In the paper, we use the model to evaluate consumption of each energy category. In actuality, we can also use the model to evaluate energy input in different sectors. It should work theoretically, but we didn’t include it into the paper.
- The quality of the graph and code can be improved. Though we use RStudio, MATLAB, Python, and Geogebra to assist us to form the model, we can make the graph and code become more readable.

From: Team#80749
To: The Western Interstate Energy Compact
Subject: Analysis of the Energy Market in 2009.
Date: February, 12, 2018

Summary of Energy Profiles;

Since the 1960s, the total energy consumption of the four states has increased dramatically. However, the growth rate of non-renewable energy in the energy market is clearly greater than that of renewable energy. Therefore, in 2009, the market share of renewable energy only occupies approximately 6.67%, 14.28%, 4.90%, and 3.33%, in Arizona, California, New Mexico, and Texas, respectively.

In terms of the use of non-renewable energy, the profiles of the four states show that while all of the four highly rely on the use of natural gases, Arizona and New Mexico also heavily depend on the use of coal. The use of petroleum products occupies a large energy market share in the four states, ranging from approximately 20% to 40%. In general, non-renewable energy still dominates the energy market.

Predictions:

Because of the differences in geography, population, and politics, the total consumption of energy in the four states will follow an increasing, smooth curve, either concave up or concave down. The estimation shows that Arizona will follow a concave down pattern of growth, while California will follow a concave up pattern of growth. The pattern of total consumption growth for New Mexico and Texas will first follow a concave down curve and then a concave up curve.

Also, without any policy interventions, non-renewable energy will continuously dominate the energy market and keep shrinking the market share of renewable energy in the future, until the non-renewable energy is depleted and people cannot find substitutes. Therefore, new policies are in need to protect the environment.

Recommended goals:

We provide an index REII to measure the performance of energy market. The higher the REII value, the better the energy market performs:

1. California: Since California has the most mature energy market and the highest market share of renewable energy, its final target should be to achieve 0.95 REII value before 2025.
2. Arizona and Texas: They have a moderate energy market and a moderate share of renewable energy, so they should achieve 0.9 REII value before 2025, and then achieve 0.95 REII value in 2050;
3. New Mexico: It has the less mature energy market and the smallest market share of renewable energy. Accordingly, it should achieve of an REII value of (0.5+0.5*REII(2009)) before 2025, and achieve the final target of 0.95 in 2050.

# Works Cited

Renewable Energy. Washington, DC: The Institute of Energy Research; [accessed 2018 Feb 11]. https://instituteforenergyresearch.org/topics/encyclopedia/renewable-energy/

California State Profile and Energy Estimates. October 19, 2017. Washington, DC: U.S. Energy Information Administration;
[accessed 2018 Feb 10]. https://www.eia.gov/state/?sid=CA

# Appendix

Language: Python

Libraries: openpyxl – read and write Excel data

```
from openpyxl import load_workbook
from openpyxl import Workbook
```

Function implementations:

```
def calcSigma(rConsump, rPrice , tPrice, tConsump):
    return (rConsump*rPrice)/(tConsump*tPrice)

#Parameters in t except tConsump in t+1, solve for proportion in t
def calcKnownProportions(nConsump, rConsump, tConsump, sigma):
    return tConsump/((nConsump**(sigma)*rConsump**(1-sigma)))

#Given parameters in t, calculate consumption for t+1
def calcConsumption(proportion, nConsump, rConsump, sigma):
    return proportion*nConsump**(sigma)*rConsump**(1-sigma)

#calculate next year's total energy consumption
def calcF(bigA, k, bigQ, sigma): #calc renewable & nonRenewable from NList
    return bigA*k**(sigma)*bigQ**(1-sigma)

#Combine N_i(t) from NList into total renewable and nonrenewable energy
def combine(yr, state, NList, currentF, sigma):
    k = 0
    bigQ = 0
    smallq = calcRenewableAvgPrice(yr, state)
    bigA = calcProportion(yr, state)
    #print("bigA", str(bigA))
    for key in NList:
        if key in ourKeys and ourKeys[key]:
            bigQ += NList[key].y
        elif key in ourKeys and not ourKeys[key]:
            k += NList[key].y
        else:
            bigQ += NList[key].y
    #print("bigQ", str(bigQ))
    p = calcPTotal(yr, state, bigQ, currentF)
    #print("sigma1", str(sigma))
    #k = currentF - bigQ
    #print("k", str(k))
    if yr == 2025 or yr == 2050:
        print("yr", yr)
```

```
        print("smallq", str(smallq))
        print("p", str(p))
    sigma = calcSigma(bigQ, smallq, p, currentF)
    #print("sigma2", str(sigma))
    nextF = calcF(bigA, k, bigQ, sigma)
    #print("nextF", nextF)
    coordList = {key: Coord(yr, NList[key].y) for key in NList.keys()}
    return [nextF, coordList]

#call runge kutta and split F(t) into N_i(t)
def splitToNList(yr, endYr, prevF, currentF, coordList, state, prevNList,
sigma):
    global r
    #print("F", yr, currentF)
    NList = {key: approx(0, 1, coordList[key], 1, yr, prevF, key, prevNList,
sigma) for key in prevNList.keys()} #set up N(t = start)
    #print("NList")
    for key in NList:
        #print(key, NList[key].y)
        if key != "ORETCB":
            sheet.cell(row=r, column=5).value =
msnTable[key][0].replace("total consumption", "")
        else:
            sheet.cell(row=r, column=5).value = "Other renewable energy
consumption"
        r+=1
    if yr == endYr:
        return NList
    combineArr = combine(yr, state, NList, currentF, sigma)
    yr+=1
    return splitToNList(yr, endYr, currentF, combineArr[0], combineArr[1],
state, prevNList, sigma)

#call recursive model
def predict(yr, NList, currentTConsump, endYr, state):
    k = 0
    bigQ = 0
    smallq = calcRenewableAvgPrice(yr, state)
    bigA = calcProportion(yr, state)
    for key in NList:
        if key in ourKeys and ourKeys[key]:
            bigQ += NList[key]
        else:
            k += NList[key]
```

```
    sumOtherRenewables = 0
    for i in otherRenewableKeys:
        sumOtherRenewables += table[i][stateNum][yr]
    bigQ += sumOtherRenewables
    print(bigQ/currentTConsump)
    p = calcPTotal(yr, state, bigQ, currentTConsump)
    sigma = calcSigma(bigQ, smallq, p, currentTConsump)
    nextF = calcF(bigA, k, bigQ, sigma)
    coordList = {key: Coord(yr, NList[key]) for key in NList.keys()}
    yr+=1
    return splitToNList(yr, endYr, currentTConsump, nextF, coordList, state,
NList, sigma)

#Runge Kutta: dn(), k1(), k2(), k3(), k4(), approx()
class Coord:
    def __init__(self, x, y):
        self.x = x
        self.y = y

    def __str__(self):
        return str(self.x) + " " + str(self.y)

def dn(c, yr, carryingCapacity, keyI, nList, sigma):
    interSpecies = 0 #interSpecies competition
    for key in nList.keys():
        if key != keyI:
            interSpecies += float(1/carryingCapacity)*nList[key]
    return sigma*c.y*(1-interSpecies)

def k1(h, c, yr, carryingCapacity, keyI, nList, sigma):
    return h*dn(c, yr, carryingCapacity, keyI, nList, sigma)

def k2(h, c, yr, carryingCapacity, keyI, nList, sigma):
    return h*dn(Coord(c.x+h/2, c.y+k1(h, c, yr, carryingCapacity, keyI,
nList, sigma)/2), yr, carryingCapacity, keyI, nList, sigma)

def k3(h, c, yr, carryingCapacity, keyI, nList, sigma):
    return h*dn(Coord(c.x+h/2, c.y+k2(h, c, yr, carryingCapacity, keyI,
nList, sigma)/2), yr, carryingCapacity, keyI, nList, sigma)

def k4(h, c, yr, carryingCapacity, keyI, nList, sigma):
    return h*dn(Coord(c.x+h/2, c.y+k3(h, c, yr, carryingCapacity, keyI,
nList, sigma)), yr, carryingCapacity, keyI, nList, sigma)
```

```
def approx(index, maxIndex, c, h, yr, carryingCapacity, keyI, nList, sigma):
#keyI is the key that matches so that dn will not overlap keys
    if index == maxIndex:
        return c
    next = Coord(c.x+h, c.y+float(1/6)*(k1(h, c, yr, carryingCapacity, keyI,
nList, sigma)+2*k2(h, c, yr, carryingCapacity, keyI, nList, sigma)+2*k3(h,
c, yr, carryingCapacity, keyI, nList, sigma)+k4(h, c, yr, carryingCapacity,
keyI, nList, sigma)))
    return approx(index+1, maxIndex, next, h, yr, carryingCapacity, keyI,
nList, sigma) #returns a coord
```