

Team notebook

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1 DP

1.1 ConvexHull

```
// Description: Container where you can add lines of the form kx+m, and query
// maximum values at points x.
#pragma once
struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};
struct LineContainer : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};
```

1.2 SOS

```
//iterative version
for(int mask = 0; mask < (1<<N); ++mask){
    dp[mask][1] = A[mask]; //handle base case separately (leaf states)
    for(int i = 0; i < N; ++i){
        if(mask & (1<<i))
            dp[mask][i] = dp[mask][i-1] + dp[mask^(1<<i)][i-1];
        else
            dp[mask][i] = dp[mask][i-1];
    }
    F[mask] = dp[mask][N-1];
}
//memory optimized, super easy to code.
```

```
for(int i = 0; i < (1<<N); ++i)
    F[i] = A[i];
for(int i = 0; i < N; ++i) for(int mask = 0; mask < (1<<N); ++mask){
    if(mask & (1<<i))
        F[mask] += F[mask^(1<<i)];
}
```

1.3 divide-and-conquer

```
/*
DP[i][j] = min( DP[i-1][k] + C[k][j] )
K[i][j] <= K[i][j+1]
*/
ll lastDP[tam], DP[tam];
int C[tam][tam]; // Cambiar a una funcion de costo si pre-procesar ocupa mucha
// memoria
void DC(int b, int e, int KL, int KR) {
    int mid = (b + e) / 2;
    pair<ll, int> best = mp(-1, KL);
    for (int k = KL; k < min(mid, KR+1); k++)
        best = max( best, mp(lastDP[k] + C[k+1][mid], k) );
    DP[mid] = best.first;
    int K = best.second;
    if (b <= mid-1)
        DC(b, mid-1, KL, K);
    if (mid+1 <= e)
        DC(mid+1, e, K, KR);
}
```

1.4 knuth

```
fore(sz, 0, n) {
    for (int i = 0; i + sz < n; i++) {
        int j = i+sz;
        // CASOS BASE
        if (sz <= 1) { // Barra inexistente o con cero cortes en medio
            DP[i][j] = 0;
            continue;
        }
        if (sz == 2) { // Barra con un solo corte posible en medio
            K[i][j] = i+1;
            DP[i][j] = C[i][j];
            continue;
        }
        int KL = K[i][j-1];
        int KR = K[i+1][j];
        DP[i][j] = INF;
        for (int k = KL; k <= KR; k++) {
            int newVal = DP[i][k] + DP[k][j] + C[i][j];
            if (newVal < DP[i][j]) {
```

```

    K[i][j] = k;
    DP[i][j] = newVal;
  }
}
}
}

```

2 DataStructures

2.1 link-cut

```

namespace linkcut {
//no queries on path
#include <bits/stdc++.h>
#define fore(x,a,b) for(int x=a,qwe=b; x<qwe; x++)
using namespace std;
//Most changes required by problems of linkcut with values on vertex and
//query on path are usually only on the 4 lines below
const int N_DELTA = 0, N_VALUE = 0;
inline int modifyOp(int x, int y){return x+y;}
inline int queryOp(int lval, int rval){return lval + rval;}
inline int dOnSeg(int d, int len){return d==N_DELTA ? N_DELTA : d*len;}
//all code below is mostly generic
//join delta with value or another delta
inline int joinVD(int v, int d){ return d==N_DELTA ? v : modifyOp(v, d);}
inline int joinDeltas(int d1, int d2){
  if(d1==N_DELTA)return d2; if(d2==N_DELTA)return d1; return modifyOp(d1, d2);
}
//node structure
struct Node_t{
  int sz, nVal, stVal, d;
  bool rev;
  Node_t *ch[2], *p;
  Node_t(int v) : sz(1), nVal(v), stVal(v), d(N_DELTA), rev(0), p(0){
    ch[0]=ch[1]=0;
  }
  bool isRoot(){return !p || (p->ch[0] != this && p->ch[1] != this);}
  void push(){
    if(rev){
      rev=0; swap(ch[0], ch[1]);
      fore(x,0,2)if(ch[x])ch[x]->rev^=1;
    }
    nVal=joinVD(nVal, d); stVal=joinVD(stVal, dOnSeg(d, sz));
    fore(x,0,2)if(ch[x])ch[x]->d=joinDeltas(ch[x]->d, d);
    d=N_DELTA;
  }
  void upd();
};
typedef Node_t* Node;
int getSize(Node r){return r ? r->sz : 0;}
int getstVal(Node r){
  return r ? joinVD(r->stVal, dOnSeg(r->d,r->sz)) : N_VALUE;}

```

```

void Node_t::upd(){
  stVal = queryOp(queryOp(getstVal(ch[0]), joinVD(nVal, d)), getstVal(ch[1]));
  sz = 1 + getSize(ch[0]) + getSize(ch[1]);
}
//splay related functions
void connect(Node ch, Node p, int isl){if(ch)ch->p=p; if(isl>=0)p->ch[1-isl]=ch;}
void rotate(Node x){
  Node p = x->p, g = p->p;
  bool gCh=p->isRoot(), isl = x==p->ch[0];
  connect(x->ch[isl],p,isl);connect(p,x,!isl);connect(x,g,gCh?-1:(p==g->ch[0]));
  p->upd();
}
void splay(Node x){
  while(!x->isRoot()){
    Node p = x->p, g = p->p;
    if(!p->isRoot())g->push();
    p->push(); x->push();
    if(!p->isRoot())rotate((x==p->ch[0])==(p==g->ch[0])? p : x);
    rotate(x);
  }
  x->push(); x->upd();
}
Node expose(Node x){
  Node last=0;
  for(Node y=x; y; y=y->p)splay(y),y->ch[0]=last,y->upd(),last=y;
  splay(x);
  return last;
}
//only new Node_t(v) and the functions below should be used
Node findRoot(Node x){expose(x); while(x->ch[1])x=x->ch[1]; splay(x); return x;}
Node lca(Node x, Node y){expose(x); return expose(y);}
void makeRoot(Node x){expose(x); x->rev^=1;}
bool connected(Node x, Node y){if(x==y)return 1; expose(x);expose(y); return
  x->p;}
void link(Node x, Node y){makeRoot(x); x->p=y;}
void cut(Node x, Node y){makeRoot(x); expose(y); y->ch[1]->p = 0; y->ch[1]=0;}
int query(Node x, Node y){makeRoot(x); expose(y); return getstVal(y);}
void modify(Node x, Node y, int d){makeRoot(x); expose(y); y->d =
  joinDeltas(y->d, d);}
}

```

2.2 segment-tree

```

struct mylazy { ll lazy1 = 0,lazy2 = INF;
  bool operator==(const mylazy& otr) {
    return lazy1 == otr.lazy1 && lazy2 == otr.lazy2; }
  mylazy& operator=(const mylazy& otr) { lazy1 = otr.lazy1; lazy2 = otr.lazy2;
    return *this;}
};
struct node{
  ll sumpw2, sum;
  size_t csz = 0;

```

```

mylazy lazy;
using value = ll;
using lzy = mylazy;
void upd(value val0) { //only if node is leaf
    sum = val0;
    sumpw2 = val0 * val0;
    csz = 1;
}
};
void calclazy(node& act) { //take care
    if (act.lazy.lazy2 != INF) {
        act.sumpw2 = (act.lazy.lazy2 * act.lazy.lazy2) * act.csz;
        act.sum = act.lazy.lazy2 * act.csz;
    }
    if (act.lazy.lazy1 != 0) {
        act.sumpw2 += (act.lazy.lazy1 * act.lazy.lazy1) * act.csz + 2 * act.sum *
            act.lazy.lazy1;
        act.sum += act.lazy.lazy1 * act.csz;
    }
} //lazy still exist

node join(const node izq, const node der) {
    static node dum1; dum1 = izq; calclazy(dum1);
    static node dum2; dum2 = der; calclazy(dum2);
    node res;
    res.sumpw2 = dum1.sumpw2 + dum2.sumpw2;
    res.sum = dum1.sum + dum2.sum;
    res.csz = dum1.csz + dum2.csz;
    return res;
}

void pushlazy(node &act, const int& l, const int& r, node* t) {
    if (act.lazy == mylazy{0, INF}) return;
    calclazy(act);
    if (l == r) { act.lazy.lazy1 = 0; act.lazy.lazy2 = INF; return; }
    if (act.lazy.lazy2 != INF) { t[l].lazy = t[r].lazy = {0, act.lazy.lazy2}; }
    t[l].lazy.lazy1 += act.lazy.lazy1, t[r].lazy.lazy1 += act.lazy.lazy1;
    act.lazy = {0, INF};
}

template <typename nodo> // sgtree is index 1
class Segtree { //nodo needs val
#define idxtree int mid = (b + e) / 2, l = node * 2 + 1, r = l + 1;
using T = typename nodo::value;
using L = typename nodo::lzy; // lazy
public:
    T ar[tam];
    void init(int b, int e, int node) {
        if (b == e) {
            t[node].upd(ar[b]);
            return;
        }
        int mid = (b + e) / 2, l = node * 2 + 1, r = l + 1;
        init(b, mid, l); init(mid + 1, e, r);
        t[node] = join(t[l], t[r]);
    }
    nodo qry(int b, int e, int node, int i, int j) { //check if b>e & i>j
        idxtree;

```

```

        pushlazy(t[node], l, r, t);
        if (b >= i && e <= j)
            return t[node];
        if (mid < i)
            return qry(mid + 1, e, r, i, j);
        if (mid >= j)
            return qry(b, mid, l, i, j);
        return join(qry(b, mid, l, i, j), qry(mid + 1, e, r, i, j));
    }
    void upd0(int b, int e, int node, int pos, const T val) {
        idxtree;
        pushlazy(t[node], l, r, t);
        if (b == e) { t[node].upd(val); return; }
        if (mid < pos)
            upd(mid + 1, e, r, pos, val);
        else
            upd(b, mid, l, pos, val);
        t[node] = join(t[l], t[r]);
    }
    void upd1(int b, int e, int node, int i, int j, const L& val) {
        if (b > e || b > j || e < i) return;
        idxtree;
        pushlazy(t[node], l, r, t);
        if (b >= i && e <= j) {
            t[node].lazy = val;
            pushlazy(t[node], l, r, t);
            return;
        }
        upd1(b, mid, l, i, j, val);
        upd1(mid + 1, e, r, i, j, val);
        t[node] = join(t[l], t[r]);
    }
    nodo t[4 * tam];
};

```

2.3 sparse-table

```

template<typename it, typename bin_op>
struct sparse_table {
    using T = typename remove_reference<decltype(*declval<it>())>::type;
    vector<vector<T>> t; bin_op f;
    sparse_table(it first, it last, bin_op op) : t(1), f(op) {
        int n = distance(first, last);
        t.assign(32 - __builtin_clz(n), vector<T>(n));
        t[0].assign(first, last);
        for (int i = 1; i < t.size(); i++)
            for (int j = 0; j < n - (1 << i) + 1; j++)
                t[i][j] = f(t[i-1][j], t[i-1][j + (1 << (i-1))]);
    }
    // returns f(a[l..r]) in O(1) time
    T query(int l, int r) {
        int h = floor(log2(r-l+1));

```

```

    return f(t[h][l], t[h][r-(1<<h)+1]);
}
};
sparse_table g(all(vec), [(ll x, ll y){
    return __gcd(x, y);
}]);

```

2.4 treap

```

struct item {
    int key, pri, siz;
    item *l, *r;
    item() {}
    item(int key) : key(key), siz(1), pri(rand()), l(0), r(0) {}
};
typedef item* pitem;
int sz(pitem t) {
    return (t?t->siz:0);
}
void up_sz(pitem t) {
    if(t) t->siz = sz(t->l) + 1 + sz(t->r);
}
void split(pitem t, pitem &l, pitem &r, int val) {
    if(!t) r = l = NULL;
    else if(t->key < val) split(t->r, t->r, r, val), l = t;
    else split(t->l, l, t->l, val), r = t;
    up_sz(t);
}
void merge(pitem &t, pitem l, pitem r) {
    if(!l || !r) t=(l?l:r);
    else if(l->pri >= r->pri) merge(l->r, l->r, r), t = l;
    else merge(r->l, l, r->l), t=r;
    up_sz(t);
}

```

3 Flows

3.1 Kunhs

```

bool try_kuhn(int v) { // 0(nm) tested
    if (used[v])
        return false;
    used[v] = true;
    for (int to : g[v]) {
        if (mt[to] == -1 || try_kuhn(mt[to])) {
            mt[to] = v;
            return true;
        }
    }
}

```

```

    return false;
}
void doit(){ mt.assign(k, -1);
    fore(v,0,n){
        used.assign(n, false);
        try_kuhn(v);
    }
    //fore(i,0,k) if (mt[i] != -1)cout<<mt[i]+1<<' '<<i+1<<'\n';
}

```

3.2 dinic

```

namespace dinic { // can someone please test_me >_<
    int s, t;
    int gp[110][110], usados[110], lev[110];
    vi g[110];
    bool bfs(int maxscal) {
        int node; queue<int> que;
        memset(lev, -1, sizeof(lev));
        que.push(s); lev[s]=0;
        while(!que.empty()) {
            node = que.front(); que.pop();
            for(int i=0; i<g[node].size(); i++) {
                if(lev[g[node][i]]==-1 && gp[node][g[node][i]] >= maxscal) {
                    que.push(g[node][i]); lev[g[node][i]] = lev[node] + 1;
                }
            }
        }
        return lev[t]!=-1;
    }
    int dfs(int node, int f) {
        if(node==t) return f;
        for(; usados[node]<g[node].size(); usados[node]++) {
            int u = g[node][usados[node]];
            if(lev[u]!=lev[node]+1 || gp[node][u] < f) continue;
            int flow = dfs(u, min(gp[node][u], f));
            if(flow>0) {
                gp[node][u]-=flow; gp[u][node]+=flow; return flow;
            }
        }
        return 0;
    }
    int maxFlow() {
        int flujo=0, ax;
        for(int scaling=30; scaling>=0; scaling--) { //pragma? 0(VElogF)
            while(bfs(1<<scaling)) {
                memset(usados, 0, sizeof(usados));
                while(true) {
                    ax=dfs(s, 1<<scaling);
                    if (ax == 0) break;
                    flujo+=ax;
                }
            }
        }
    }
}

```

```

} //with scaling
/* without scaling
* bool bfs()
* if(lev[g[node][i]]==-1 && gp[node][g[node][i]]>0)
* if(lev[u]!=lev[node]+1 || gp[node][u] <= 0 ) continue;
* while(bfs())
* ax=dfs(s,INT_MAX);
* */

```

3.3 hungarian

```

/* Description: Given a weighted bipartite graph, matches every node on the
left with a node on the right such that no nodes are in two matchings and
the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] =
cost for L[i] to be matched with R[j] and returns (min cost, match), where
L[i] is matched with R[match[i]]. Negate costs for max cost. Time: O (N^2 M)*/
pair<int, vi> hungarian(const vector<vi> &a) { // IS A DENSE GRAPH matching is N,M
    if (a.empty()) return {0, {}};
    int n = sz(a) + 1, m = sz(a[0]) + 1;
    vi u(n), v(m), p(m), ans(n - 1);
    fore(i,1,n) {
        p[0] = i;
        int j0 = 0; // add dummy worker 0
        vi dist(m, INT_MAX), pre(m, -1);
        vector<bool> done(m + 1);
        do { // d i j k s t r a
            done[j0] = true;
            int i0 = p[j0], j1, delta = INT_MAX;
            fore(j,1,m) if (!done[j]) {
                auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
                if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
                if (dist[j] < delta) delta = dist[j], j1 = j;
            }
            fore(j,0,m) {
                if (done[j]) u[p[j]] += delta, v[j] -= delta;
                else dist[j] -= delta;
            }
            j0 = j1;
        } while (p[j0]);
        while (j0) { // update alternating path
            int j1 = pre[j0];
            p[j0] = p[j1], j0 = j1;
        }
        fore(j,1,m) if (p[j]) ans[p[j] - 1] = j - 1;
        return {-v[0], ans/}; // min cost
    }
}

```

3.4 min-cost-max-flow

```

typedef ll tf; //copy paste vasito :0
typedef ll tc;
const tf INFFLOW=1e9;
const tc INFCOST=1e9;
struct MCF{
    int n;
    vector<tc> prio, pot; vector<tf> curflow; vector<int> prevedge,prevnode;
    priority_queue<pair<tc, int>, vector<pair<tc, int>>, greater<pair<tc, int>>> q;
    struct edge{int to, rev; tf f, cap; tc cost;};
    vector<vector<edge>> g;
    MCF(int n):n(n),prio(n),curflow(n),prevedge(n),prevnode(n),pot(n),g(n){}
    void add_edge(int s, int t, tf cap, tc cost) {
        g[s].pb((edge){t,SZ(g[t]),0,cap,cost});
        g[t].pb((edge){s,SZ(g[s])-1,0,0,-cost});
    }
    pair<tf,tc> get_flow(int s, int t) {
        tf flow=0; tc flowcost=0;
        while(1){
            q.push({0, s});
            fill(ALL(prio),INFCOST);
            prio[s]=0; curflow[s]=INFFLOW;
            while(!q.empty()) {
                auto cur=q.top();
                tc d=cur.fst;
                int u=cur.snd;
                q.pop();
                if(d!=prio[u]) continue;
                for(int i=0; i<SZ(g[u]); ++i) {
                    edge &e=g[u][i];
                    int v=e.to;
                    if(e.cap<=e.f) continue;
                    tc nprio=prio[u]+e.cost+pot[u]-pot[v];
                    if(prio[v]>nprio) {
                        prio[v]=nprio;
                        q.push({nprio, v});
                        prevnode[v]=u; prevedge[v]=i;
                        curflow[v]=min(curflow[u], e.cap-e.f);
                    }
                }
            }
            if(prio[t]==INFCOST) break;
            fore(i,0,n) pot[i]+=prio[i];
            tf df=min(curflow[t], INFFLOW-flow);
            flow+=df;
            for(int v=t; v!=s; v=prevnode[v]) {
                edge &e=g[prevnode[v]][prevedge[v]];
                e.f+=df; g[v][e.rev].f-=df;
                flowcost+=df*e.cost;
            }
        }
        return {flow,flowcost};
    }
};

```

4 Geometry

4.1 centroid

```
// calcula el centro de masa de un poligono antihorario
point cen(vector<point> p) {
    double x = 0, y = 0, area = 0, ax;
    int n = p.size()-1;
    for(i, 0, n) {
        ax = (p[i] ^ p[i+1]) / 2;
        area += ax;
        x += ax * (p[i].x + p[i+1].x) / 3;
        y += ax * (p[i].y + p[i+1].y) / 3;
    }
    return point(x / area, y / area);
}
```

4.2 hull

```
// devuelve horario
vector<point> hull(vector<point> p) {
    int n = p.size();
    vector<point> h;
    sort(all(p));
    for(i, 0, n) {
        while(h.size() >= 2 && p[i].left(h[sz(h) - 2], h.back())) h.pop_back();
        h.push_back(p[i]);
    }
    h.pop_back();
    int k = h.size();
    for(int i = n-1; i > -1; i--) {
        while(h.size() >= k + 2 && p[i].left(h[sz(h) - 2], h.back())) h.pop_back();
        h.pb(p[i]);
    }
    h.pop_back();
    return h;
}
```

4.3 circle2ptsrad

```
bool circle2PtsRad(point a, point b, double r, point &c) { //dados 2 puntos y un
    radio
    double det = (a.x-b.x)*(a.x-b.x) + (a.y-b.y)*(a.y-b.y);
    det = r * r / det - 0.25;
    if(det < 0.0) return false;
    det = sqrt(det);
    c.x = (a.x + b.x) * 0.5 + (b.y-a.y) * det;
    c.y = (a.y + b.y) * 0.5 + (a.x-b.x) * det;
    return true;
}
```

```
}
```

4.4 halfplane

```
const double DINF=1e100;
struct pt { // for 3D add z coordinate
    double x,y;
    pt(double x, double y):x(x),y(y){}
    pt(){}
    double norm2(){return *this**this;}
    double norm(){return sqrt(norm2());}
    bool operator==(pt p){return abs(x-p.x)<=EPS&&abs(y-p.y)<=EPS;}
    pt operator+(pt p){return pt(x+p.x,y+p.y);}
    pt operator-(pt p){return pt(x-p.x,y-p.y);}
    pt operator*(double t){return pt(x*t,y*t);}
    pt operator/(double t){return pt(x/t,y/t);}
    double operator*(pt p){return x*p.x+y*p.y;}
    // pt operator^(pt p){ // only for 3D
    //     return pt(y*p.z-z*p.y,z*p.x-x*p.z,x*p.y-y*p.x);}
    double angle(pt p){ // redefine acos for values out of range
        return acos(*this*p/(norm()*p.norm()));}
    pt unit(){return *this/norm();}
    double operator%(pt p){return x*p.y-y*p.x;}
    // 2D from now on
    bool operator<(pt p)const{ // for convex hull
        return x<p.x-EPS||(abs(x-p.x)<=EPS&&y<p.y-EPS);}
    bool left(pt p, pt q){ // is it to the left of directed line pq?
        return (q-p)%(*this-p)>EPS;}
    pt rot(pt r){return pt(*this*r,*this*r);}
    pt rot(double a){return rot(pt(sin(a),cos(a)));}
};
pt ccw90(1,0);
pt cw90(-1,0);
int sgn2(double x){return x<0?-1:1;}
struct ln {
    pt p,pq;
    ln(pt p, pt q):p(p),pq(q){}
    ln(){}
    bool has(pt r){return dist(r)<=EPS;}
    bool seghas(pt r){return has(r)&&(r-p)*(r-(p+pq))<=EPS;}
    // bool operator /(ln l){return (pq.unit()^l.pq.unit()).norm()<=EPS;} // 3D
    bool operator/(ln l){return abs(pq.unit()%l.pq.unit())<=EPS;} // 2D
    bool operator==(ln l){return *this/l&&has(l.p);}
    pt operator^(ln l){ // intersection
        if(*this/l)return pt(DINF,DINF);
        pt r=l.p+l.pq*((p-l.p)%pq/(l.pq%pq));
        // if(!has(r)){return pt(NAN,NAN,NAN);} // check only for 3D
        return r;
    }
    double angle(ln l){return pq.angle(l.pq);}
    int side(pt r){return has(r)?0:sgn2(pq%(r-p));} // 2D
    pt proj(pt r){return p+pq*((r-p)*pq/pq.norm2());}
}
```

```

    pt ref(pt r){return proj(r)*2-r;}
    double dist(pt r){return (r-proj(r)).norm();}
    // double dist(ln l){ // only 3D
    //     if(*this/l)return dist(l.p);
    //     return abs((l.p-p)*(pq^l.pq))/(pq^l.pq).norm();
    // }
    ln rot(auto a){return ln(p,p+pq.rot(a));} // 2D
};
ln bisector(ln l, ln m){ // angle bisector
    pt p=l^m;
    return ln(p,p+l.pq.unit()+m.pq.unit());
}
ln bisector(pt p, pt q){ // segment bisector (2D)
    return ln((p+q)*.5,p).rot(ccw90);
}
// polygon intersecting left side of halfplanes
struct halfplane:public ln{
    double angle;
    halfplane(){}
    halfplane(pt a,pt b){p=a; pq=b-a; angle=atan2(pq.y,pq.x);}
    bool operator<(halfplane b)const{return angle<b.angle;}
    bool out(pt q){return pq%(q-p)<=-EPS;}
};
vector<pt> intersect(vector<halfplane> b){
    vector<pt>bx={{DINF,DINF},{-DINF,DINF},{-DINF,-DINF},{DINF,-DINF}};
    fore(i,0,4) b.pb(halfplane(bx[i],bx[(i+1)%4]));
    sort(all(b));
    int n=sz(b),q=1,h=0;
    vector<halfplane> c(sz(b)+10);
    fore(i,0,n){
        while(q<h&&b[i].out(c[h]^c[h-1])) h--;
        while(q<h&&b[i].out(c[q]^c[q+1])) q++;
        c[++h]=b[i];
        if(q<h&&abs(c[h].pq*c[h-1].pq)<EPS){
            if(c[h].pq*c[h-1].pq<=0) return {};
            h--;
            if(b[i].out(c[h].p)) c[h]=b[i];
        }
    }
    while(q<h-1&&c[q].out(c[h]^c[h-1]))h--;
    while(q<h-1&&c[h].out(c[q]^c[q+1]))q++;
    if(h-q<=1)return {};
    c[h+1]=c[q];
    vector<pt> s;
    fore(i,q,h+1) s.pb(c[i]^c[i+1]);
    return s;
}
struct pol {
    int n;vector<pt> p;
    pol(){}
    pol(vector<pt> _p){p=_p;n=p.size();}
    double area(){
        double r=0.;
        fore(i,0,n)r+=p[i]%p[(i+1)%n];
        return abs(r)/2; // negative if CW, positive if CCW
    }
};

```

```

    }
};

```

4.5 line

```

struct line {
    double a, b, c;
    line(point p, point q) {
        a = p.y - q.y;
        b = q.x - p.x;
        c = -a * p.x - b * p.y;
    };
    void setOrigin(point p) { c += a * p.x + b * p.y; } //trasladar linea como si p
    fuera el origen
};
double det(double a, double b, double c, double d) {
    return a * d - b * c;
}
point intersec(line a, line b) { //primero estar seguro si no son paralelas
    double d = -det(a.a, a.b, b.a, b.b);
    return point(det(a.c, a.b, b.c, b.b) / d, det(a.a, a.c, b.a, b.c) / d);
}

```

4.6 minkowski

```

typedef vector<point> poly;
void norm(poly &pol) {
    int pos = 0;
    fore(i, 0, pol.size()) {
        if(pol[i] < pol[pos])
            pos = i;
    }
    rotate(pol.begin(), pol.begin() + pos, pol.end());
}
poly minkos(poly &a, poly &b) {
    norm(a);
    norm(b);
    int posa = 0, posb = 0, ta = a.size(), tb = b.size();
    poly res;
    ll cro;
    while(posa < ta || posb < tb) {
        res.pb(a[(posa) % ta] + b[(posb) % tb]);
        cro = (a[(posa + 1) % ta] - a[posa % ta]) ^ (b[(posb + 1) % tb] - b[posb %
            tb]);
        if(cro == 0)
            posa++, posb++;
        else if(cro < 0)
            posb++;
        else posa++;
    }
}

```



```

    return res;
}

```

4.7 point-in-poly

```

// logaritmico counterclockwise
bool inpol(poly &pol, point p) {
    int n = pol.size();
    if(((pol[1] - pol[0]) ^ (p - pol[0])) < 0 || ((pol[n - 1] - pol[0]) ^ (p -
        pol[0])) > 0)
        return 0;
    int lo = 1, hi = n - 2, mid, res;
    while(lo <= hi) {
        mid = (lo + hi) / 2;
        if(((pol[mid] - pol[0]) ^ (p - pol[0])) >= 0)
            res = mid, lo = mid + 1;
        else
            hi = mid - 1;
    }
    return ((pol[res + 1] - pol[res]) ^ (p - pol[res])) >= 0;
}

```

5 Graph

5.1 2sat

```

namespace sat2{
    set<int> G[tam], Ginv[tam];
    int N, mark[tam], mark_comp[tam], valor[tam];
    int neg(const int& x) { return (x>=N)? x - N : x + N;}
    void add_(const int& x,const int& y) {G[x].insert(y);Ginv[y].insert(x);}
    void addor(const int x,const int y) {add_(neg(x),y);add_(neg(y),x);}
    void dfs0(int u, vector<int>& orden) { mark[u] = 1;
        for(auto& v: G[u]) {
            if (!mark[v]) dfs0(v,orden);
        } orden.push_back(u);
    }
    void dfs1(int u, const int& cmp) { mark_comp[u] = cmp;
        for(auto& v: Ginv[u]) {
            if (!mark_comp[v]) dfs1(v,cmp);
        }
    }
    bool check() { bool impos = false;
        for(int i = 0; i < N; i++) {
            impos |= (mark_comp[i] == mark_comp[neg(i)]);
            valor[i] = (mark_comp[i] > mark_comp[neg(i)]) ;}
        return !impos;
    }
}

```

5.2 articulation-bridges-biconnected

```

namespace art_bic {
    vector<int> g[tam];int n;
    struct edge {int u,v,comp;bool bridge;};
    vector<edge> e;
    void add_edge(int u, int v){
        g[u].pb(e.size());g[v].pb(e.size());
        e.pb((edge){u,v,-1,false});
    }
    int D[tam],B[tam],T;
    int nbc; // number of biconnected components
    int art[tam]; // articulation point iff !=0
    stack<int> st; // only for biconnected
    void dfs(int u,int pe){
        B[u]=D[u]=T++;
        for(int ne:g[u])if(ne!=pe){
            int v=e[ne].u^e[ne].v^u;
            if(D[v] < 0){
                st.push(ne);dfs(v,ne);
                if(B[v]>D[u])e[ne].bridge = true; // bridge
                if(B[v]>=D[u]){
                    art[u]++; // articulation
                    int last; // start biconnected
                    do {
                        last=st.top();st.pop();
                        e[last].comp=nbc;
                    } while(last!=ne);
                    nbc++; // end biconnected
                }
                B[u]=min(B[u],B[v]);
            }
            else if(D[v]<D[u])st.push(ne),B[u]=min(B[u],D[v]);
        }
    }
    void doit(){
        memset(D,-1,sizeof(D));memset(art,0,sizeof(art));
        nbc=T=0;
        fore(i,0,n)if(D[i]<0)dfs(i,-1),art[i]--;
    }
}

```

5.3 bellman-ford

```

int n; vector<ii> g[tam]; // u->[(v,cost)]
ll dist[tam];
void bford(int src){ // 0(nm)
    fill(dist,dist+n,INF);dist[src]=0;
    fore(_,0,n)fore(x,0,n)if(dist[x]!=INF)for(auto t:g[x]){
        dist[t.fst]=min(dist[t.fst],dist[x]+t.snd);
    }
    fore(x,0,n)if(dist[x]!=INF)for(auto t:g[x]){

```

```

if(dist[t.fst]>dist[x]+t.snd){
    // neg cycle: all nodes reachable from t.fst have -INF distance
    // to reconstruct neg cycle: save "prev" of each node, go up from t.fst
    // until repeating a node. this node and all nodes between the two
    // occurrences form a neg cycle
} } }

```

5.4 centroid

```

namespace cent_{
    vector<int> g[tam]; int n;
    bool tk[tam];
    int fat[tam]; // father in centroid decomposition
    int sz[tam]; // size of subtree
    int calcsz(int x, int f){
        sz[x]=1;
        for(auto y:g[x]) if(y!=f&&!tk[y]) sz[x]+=calcsz(y,x);
        return sz[x];
    }
    void cdfs(int x=0, int f=-1, int sz=-1){ // O(nlogn)
        if(sz<0) sz=calcsz(x,-1);
        for(auto y:g[x]) if(!tk[y]&&sz[y]*2>=sz){
            sz[x]=0; cdfs(y,f,sz); return;
        }
        tk[x]=true; fat[x]=f; // next is ops
        for(auto y:g[x]) if(!tk[y]) cdfs(y,x);
    }
    void centroid(){memset(tk,false,sizeof(tk)); cdfs();}
}

```

5.5 dynamic-connectivity

```

namespace dyn_con {
    struct UnionFind {
        int n, comp;
        vector<int> uf, si, c;
        UnionFind(int n=0): n(n), comp(n), uf(n), si(n,1){
            for(i,0,n) uf[i]=i;
        }
        int find(int x){return x==uf[x]?x:find(uf[x]);}
        bool join(int x, int y){
            if((x=find(x))==(y=find(y))) return false;
            if(si[x]<si[y]) swap(x,y);
            si[x]+=si[y]; uf[y]=x; comp--; c.pb(y);
            return true;
        }
        int snap(){return c.size();}
        void rollback(int snap){
            while(c.size()>snap){
                int x=c.back(); c.pop_back();
                si[uf[x]]-=si[x]; uf[x]=x; comp++;
            }
        }
    };
}

```

```

    }
} };
enum {ADD,DEL,QUERY};
struct Query {int type,x,y;};
struct DynCon {
    vector<Query> q;
    UnionFind dsu;
    vector<int> mt;
    map<pair<int,int>,int> last;
    DynCon(int n):dsu(n){}
    void add(int x, int y){
        if(x>y) swap(x,y);
        q.pb((Query){ADD,x,y}); mt.pb(-1); last[{x,y}]=q.size()-1;
    }
    void remove(int x, int y){
        if(x>y) swap(x,y);
        q.pb((Query){DEL,x,y});
        int pr=last[{x,y}]; mt[pr]=q.size()-1; mt.pb(pr);
    }
    void query(){q.pb((Query){QUERY,-1,-1}); mt.pb(-1);}
    void process(){ // answers all queries in order
        if(!q.size()) return;
        for(i,0,q.size()) if(q[i].type==ADD&&mt[i]<0) mt[i]=q.size();
        go(0,q.size());
    }
    void go(int s, int e){
        if(s+1==e){
            if(q[s].type==QUERY) // answer query using DSU
                printf("%d\n",dsu.comp); // can ask current state UnionFind
            return;
        }
        int k=dsu.snap(), m=(s+e)/2;
        for(int i=e-1; i>=m; --i) if(mt[i]>=0&&mt[i]<s) dsu.join(q[i].x,q[i].y);
        go(s,m); dsu.rollback(k);
        for(int i=m-1; i>=s; --i) if(mt[i]>=e) dsu.join(q[i].x,q[i].y);
        go(m,e); dsu.rollback(k);
    }
} };
}

```

5.6 edmonds-blossom

```

namespace ed_bls{ // undirected G
    vector<int> g[tam];
    int n,m,mt[tam],qh,qt,q[tam],ft[tam],bs[tam];
    bool inq[tam],inb[tam],inp[tam];
    int lca(int root, int x, int y){
        memset(inp,0,sizeof(inp));
        while(1){
            inp[x=bs[x]]=true;
            if(x==root) break;
            x=ft[mt[x]];
        }
    }
}

```

```

    while(1){
        if(inp[y==bs[y]])return y;
        else y=ft[mt[y]];
    }
}

void mark(int z, int x){
    while(bs[x]!=z){
        int y=mt[x];
        inb[bs[x]]=inb[bs[y]]=true;
        x=ft[y];
        if(bs[x]!=z)ft[x]=y;
    }
}

void contr(int s, int x, int y){
    int z=lca(s,x,y);
    memset(inb,0,sizeof(inb));
    mark(z,x);mark(z,y);
    if(bs[x]!=z)ft[x]=y;
    if(bs[y]!=z)ft[y]=x;
    fore(x,0,n)if(inb[bs[x]]){
        bs[x]=z;
        if(!inq[x])inq[q[++qt]=x]=true;
    }
}

int findp(int s){
    memset(inq,0,sizeof(inq));
    memset(ft,-1,sizeof(ft));
    fore(i,0,n)bs[i]=i;
    inq[q[qh=qt=0]=s]=true;
    while(qh<=qt){
        int x=q[qh++];
        for(int y:g[x])if(bs[x]!=bs[y]&&mt[x]!=y){
            if(y==s||mt[y]>=0&&ft[mt[y]]>=0)contr(s,x,y);
            else if(ft[y]<0){
                ft[y]=x;
                if(mt[y]<0)return y;
                else if(!inq[mt[y]])inq[q[++qt]=mt[y]]=true;
            }
        }
    }
    return -1;
}

int aug(int s, int t){
    int x=t,y,z;
    while(x>=0){
        y=ft[x];
        z=mt[y];
        mt[y]=x;mt[x]=y;
        x=z;
    }
    return t>=0;
}

int edmonds(){ // 0(n^2 m)
    int r=0;
    memset(mt,-1,sizeof(mt));

```

```

        fore(x,0,n)if(mt[x]<0)r+=aug(x,findp(x));
        return r;
    }
}

```

5.7 eulerian-path

```

// Directed version (uncomment commented code for undirected)
struct edge {
    int y;
    // list<edge>::iterator rev;
    edge(int y):y(y){ }
};
list<edge> g[MAXN];
void add_edge(int a, int b){
    g[a].push_front(edge(b));//auto ia=g[a].begin();
    // g[b].push_front(edge(a));auto ib=g[b].begin();
    // ia->rev=ib;ib->rev=ia;
}
vector<int> p;
void go(int x){
    while(g[x].size()) {
        int y=g[x].front().y;
        //g[y].erase(g[y].front().rev);
        g[x].pop_front();
        go(y); }
    p.push_back(x);}
vector<int> get_path(int x){ // get a path that begins in x
// check that a path exists from x before calling to get_path!
    p.clear();go(x);reverse(p.begin(),p.end());
    return p;
}

```

5.8 hld

```

vector<int> parent, depth, heavy, head, pos;
int cur_pos;
int dfs(int v, vector<vector<int>> const& adj) {
    int size = 1;
    int max_c_size = 0;
    for (int c : adj[v]) {
        if (c != parent[v]) {
            parent[c] = v, depth[c] = depth[v] + 1;
            int c_size = dfs(c, adj);
            size += c_size;
            if (c_size > max_c_size)
                max_c_size = c_size, heavy[v] = c;
        }
    }
    return size;
}

```

```

void decompose(int v, int h, vector<vector<int>> const& adj) {
    head[v] = h, pos[v] = cur_pos++;
    if (heavy[v] != -1)
        decompose(heavy[v], h, adj);
    for (int c : adj[v]) {
        if (c != parent[v] && c != heavy[v])
            decompose(c, c, adj);
    }
}

void init(vector<vector<int>> const& adj) {
    int n = adj.size();
    parent = vector<int>(n);
    depth = vector<int>(n);
    heavy = vector<int>(n, -1);
    head = vector<int>(n);
    pos = vector<int>(n);
    cur_pos = 0;

    dfs(0, adj);
    decompose(0, 0, adj);
    // init segtree with base[pos[i]]=val[i]
}

int query(int a, int b) { // for max
    int res = 0;
    for (; head[a] != head[b]; b = parent[head[b]]) {
        if (depth[head[a]] > depth[head[b]])
            swap(a, b);
        int cur_heavy_path_max = segQuery(pos[head[b]], pos[b]);
        res = max(res, cur_heavy_path_max);
    }
    if (depth[a] > depth[b])
        swap(a, b);
    int last_heavy_path_max = segQuery(pos[a], pos[b]);
    res = max(res, last_heavy_path_max);
    return res;
}

```

6 Math

6.1 LinearRecurrence

```

/*
Description: Generates the kth term of an n-order linear recurrence
S[i] = S[i - j] * tr[j], given S[0 . . . n - 1] and tr[0 . . . n - 1].
Faster
than matrix multiplication. Useful together with BerlekampMassey.
Usage: linearRec({0, 1}, {1, 1}, k) // kth Fibonacci number
Time: O(n^2 log k)*/
typedef vector<ll> Poly;
#define sz(x) (int)(x).size()
ll linearRec(Poly S, Poly tr, ll k) {
    int n = sz(tr);

```

```

auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    fore(i, 0, n+1) fore(j, 0, n+1)
        res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) fore(j, 0, n)
        res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
    res.resize(n + 1);
    return res;
};
Poly pol(n + 1), e(pol);
pol[0] = e[1] = 1;
for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
}
ll res = 0;
fore(i, 0, n) res = (res + pol[i + 1] * S[i]) % mod;
return res;
}

```

6.2 berlekamp

```

/*BerlekampMassey.h
Description: Recovers any n-order linear recurrence relation from the first
2n terms of the recurrence. Useful for guessing linear recurrences after brute-
forcing the first terms. Should work on any field, but numerical stability for
floats is not guaranteed. Output will have size n.
Usage: berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}
Time: O(N^2)*/
vector<ll> berlekampMassey(vector<ll> s) {
    int n = sz(s), L = 0, m = 0;
    vector<ll> C(n), B(n), T;
    C[0] = B[0] = 1;
    ll b = 1;
    fore(i, 0, n) { ++m;
        ll d = s[i] % mod;
        fore(j, 1, L+1) d = (d + C[j] * s[i - j]) % mod;
        if (!d) continue;
        T = C; ll coef = d * modpow(b, mod-2) % mod;
        fore(j, m, n) C[j] = (C[j] - coef * B[j - m]) % mod;
        if (2 * L > i) continue;
        L = i + 1 - L; B = T; b = d; m = 0;
    }
    C.resize(L + 1); C.erase(C.begin());
    for (ll& x : C) x = (mod - x) % mod;
    return C;
}

```

6.3 catalan

```
// Catalan, parentesis balanceados, arboles binarios, triangulacion poligono
// convexo de n + 2 lados, caminos en grilla sin atravesar diagonal
// Cat[n] = C(2n, n) / (n + 1)
// C(n, k) es el coeficiente binomial
Cat[0] = 1;
Cat[n+1] = Cat[n] * 2 * (2 * n + 1) / (n + 2);
Cat[n] = Cat[n-1] * 2 * (2 * n - 1) / (n + 1);
```

6.4 chinese-remainder

```
constexpr long long safe_mod(long long x, long long m) {
    x %= m;
    if (x < 0) x += m;
    return x;
}

constexpr std::pair<long long, long long> inv_gcd(long long a, long long b) {
    a = safe_mod(a, b);
    if (a == 0) return {b, 0};
    long long s = b, t = a;
    long long m0 = 0, m1 = 1;
    while (t) {
        long long u = s / t;
        s -= t * u;
        m0 -= m1 * u;
        auto tmp = s;
        s = t;
        t = tmp;
        tmp = m0;
        m0 = m1;
        m1 = tmp;
    }
    if (m0 < 0) m0 += b / s;
    return {s, m0};
}

std::pair<long long, long long> crt(const std::vector<long long>& r,
    const std::vector<long long>& m) {
    assert(r.size() == m.size());
    int n = int(r.size());
    long long r0 = 0, m0 = 1;
    for (int i = 0; i < n; i++) {
        assert(1 <= m[i]);
        long long r1 = safe_mod(r[i], m[i]), m1 = m[i];
        if (m0 < m1) {
            std::swap(r0, r1);
            std::swap(m0, m1);
        }
        if (m0 % m1 == 0) {
            if (r0 % m1 != r1) return {0, 0};
            continue;
        }
        long long g, im;
        std::tie(g, im) = inv_gcd(m0, m1);
```

```
        long long u1 = (m1 / g);
        if ((r1 - r0) % g) return {0, 0};
        long long x = (r1 - r0) / g % u1 * im % u1;
        r0 += x * m0;
        m0 *= u1;
        if (r0 < 0) r0 += m0;
    }
    return {r0, m0};
}

cin >> a >> b >> c >> d;
extendedEuclid(b, d);
mul = b / g * d;
b /= g;
d /= g;
cout << (mulmod(x, mulmod(b, c, mul), mul) + mulmod(y, mulmod(d, a, mul), mul)) %
    mul << ' ' << mul << '\n';
```

6.5 exteded-euclid

```
int x, y, d;
void extendedEuclid(int a, int b) //ecuacion diofantica ax + by = d
{
    if (b == 0) {x = 1; y = 0; d = a; return;}
    extendedEuclid(b, a % b);
    int x1 = y;
    y = x - (a / b) * y;
    x = x1;
}
```

6.6 fast-gcd

```
int gcd(int a, int b) {
    if (!a || !b)
        return a | b;
    unsigned shift = __builtin_ctz(a | b);
    a >>= __builtin_ctz(a);
    do {
        b >>= __builtin_ctz(b);
        if (a > b)
            swap(a, b);
        b -= a;
    } while (b);
    return a << shift;
}
```

6.7 fft-operations

```

// MAXN must be power of 2 !!
// MOD-1 needs to be a multiple of MAXN !!
// big mod and primitive root for NTT:
typedef int tf;
typedef vector<tf> poly;
const tf MOD=998244353,RT=3,MAXN=1<<16;
tf addmod(tf a, tf b){tf r=a+b;if(r>=MOD)r-=MOD;return r;}
tf submod(tf a, tf b){tf r=a-b;if(r<0)r+=MOD;return r;}
tf mulmod(ll a, ll b){return a*b%MOD;}
tf pm(ll a, ll b){
    ll r=1;
    while(b){
        if(b&1) r=mulmod(r,a); b>>=1;
        a=mulmod(a,a);
    }
    return r;
}
tf inv(tf a){return pm(a,MOD-2);}
// FFT
/*struct CD {
    double r,i;
    CD(double r=0, double i=0):r(r),i(i){}
    double real()const{return r;}
    void operator/=(const int c){r/=c, i/=c;}
};
CD operator*(const CD& a, const CD& b){
    return CD(a.r*b.r-a.i*b.i,a.r*b.i+a.i*b.r);}
CD operator+(const CD& a, const CD& b){return CD(a.r+b.r,a.i+b.i);}
CD operator-(const CD& a, const CD& b){return CD(a.r-b.r,a.i-b.i);}
const double pi=acos(-1.0);*/
// NTT
struct CD {
    tf x;
    CD(tf x):x(x){}
    CD(){}
};
CD operator*(const CD& a, const CD& b){return CD(mulmod(a.x,b.x));}
CD operator+(const CD& a, const CD& b){return CD(addmod(a.x,b.x));}
CD operator-(const CD& a, const CD& b){return CD(submod(a.x,b.x));}
vector<tf> rts(MAXN+9,-1);
CD root(int n, bool inv){
    tf r=rts[n]<0?rts[n]=pm(RT,(MOD-1)/n):rts[n];
    return CD(inv?pm(r,MOD-2):r);
}
CD cp1[MAXN+9],cp2[MAXN+9];
int R[MAXN+9];
void dft(CD* a, int n, bool inv){
    fore(i,0,n)if(R[i]<i)swap(a[R[i]],a[i]);
    for(int m=2;m<=n;m*=2){
        //double z=2*pi/m*(inv?-1:1); // FFT
        //CD wi=CD(cos(z),sin(z)); // FFT
        CD wi=root(m,inv); // NTT
        for(int j=0;j<n;j+=m){
            CD w(1);

```

```

            for(int k=j,k2=j+m/2;k2<j+m;k++,k2++){
                CD u=a[k];CD v=a[k2]*w;a[k]=u+v;a[k2]=u-v;w=w*wi;
            }
        }
    }
    //if(inv)fore(i,0,n)a[i]/=n; // FFT
    if(inv){ // NTT
        CD z(pm(n,MOD-2)); // pm: modular exponentiation
        fore(i,0,n)a[i]=a[i]*z;
    }
}
poly multiply(poly& p1, poly& p2){
    int n=p1.size()+p2.size()+1;
    int m=1,cnt=0;
    while(m<=n)m*=m,cnt++;
    fore(i,0,m){R[i]=0;fore(j,0,cnt)R[i]=(R[i]<<1|((i>>j)&1));}
    fore(i,0,m)cp1[i]=0,cp2[i]=0;
    fore(i,0,p1.size())cp1[i]=p1[i];
    fore(i,0,p2.size())cp2[i]=p2[i];
    dft(cp1,m,false);dft(cp2,m,false);
    fore(i,0,m)cp1[i]=cp1[i]*cp2[i];
    dft(cp1,m,true);
    poly res;
    n-=2;
    //fore(i,0,n)res.pb((tf)floor(cp1[i].real()+0.5)); // FFT
    fore(i,0,n)res.pb(cp1[i].x); // NTT
    return res;
}
//Polynomial division: O(n*log(n))
//Multi-point polynomial evaluation: O(n*log^2(n))
//Polynomial interpolation: O(n*log^2(n))
//Works with NTT. For FFT, just replace addmod,submod,mulmod,inv
poly add(poly &a, poly &b){
    int n=SZ(a),m=SZ(b);
    poly ans(max(n,m));
    fore(i,0,max(n,m)){
        if(i<n) ans[i]=addmod(ans[i],a[i]);
        if(i<m) ans[i]=addmod(ans[i],b[i]);
    }
    while(SZ(ans)>1&&!ans.back())ans.pop_back();
    return ans;
}
poly invert(poly &b, int d){
    poly c = {inv(b[0])};
    while(SZ(c)<=d){
        int j=2*SZ(c);
        auto bb=b; bb.resize(j);
        poly cb=multiply(c,bb);
        fore(i,0,SZ(cb)) cb[i]=submod(0,cb[i]);
        cb[0]=addmod(cb[0],2);
        c=multiply(c,cb);
        c.resize(j);
    }
    c.resize(d+1);
    return c;
}

```

```

}
pair<poly,poly> divslow(poly &a, poly &b){
    poly q,r=a;
    while(SZ(r)>=SZ(b)){
        q.pb(mulmod(r.back(),inv(b.back())));
        if(q.back()) fore(i,0,SZ(b)){
            r[SZ(r)-i-1]=submod(r[SZ(r)-i-1],mulmod(q.back(),b[SZ(b)-i-1]));
        }
        r.pop_back();
    }
    reverse(ALL(q));
    return {q,r};
}
pair<poly,poly> divide(poly &a, poly &b){ //returns {quotient,remainder}
    int m=SZ(a),n=SZ(b),MAGIC=750;
    if(m<n) return {{0},a};
    if(min(m-n,n)<MAGIC)return divslow(a,b);
    poly ap=a; reverse(ALL(ap));
    poly bp=b; reverse(ALL(bp));
    bp=invert(bp,m-n);
    poly q=multiply(ap,bp);
    q.resize(SZ(q)+m-n-SZ(q)+1,0);
    reverse(ALL(q));
    poly bq=multiply(b,q);
    fore(i,0,SZ(bq)) bq[i]=submod(0,bq[i]);
    poly r=add(a,bq);
    return {q,r};
}
vector<poly> tree;
void filltree(vector<tf> &x){
    int k=SZ(x);
    tree.resize(2*k);
    fore(i,k,2*k) tree[i]={submod(0,x[i-k]),1};
    for(int i=k-1;i--){
        tree[i]=multiply(tree[2*i],tree[2*i+1]);
    }
}
vector<tf> evaluate(poly &a, vector<tf> &x){
    filltree(x);
    int k=SZ(x);
    vector<poly> ans(2*k);
    ans[1]=divide(a,tree[1]).snd;
    fore(i,2,2*k) ans[i]=divide(ans[i>>1],tree[i]).snd;
    vector<tf> r; fore(i,0,k) r.pb(ans[i+k][0]);
    return r;
}
poly derivate(poly &p){
    poly ans(SZ(p)-1);
    fore(i,1,SZ(p)) ans[i-1]=mulmod(p[i],i);
    return ans;
}
poly interpolate(vector<tf> &x, vector<tf> &y){
    filltree(x);
    poly p=derivate(tree[1]);
    int k=SZ(y);
    vector<tf> d=evaluate(p,x);
    vector<poly> intree(2*k);

```

```

    fore(i,k,2*k) intree[i]={mulmod(y[i-k],inv(d[i-k]))};
    for(int i=k-1;i--){
        poly p1=multiply(tree[2*i],intree[2*i+1]);
        poly p2=multiply(tree[2*i+1],intree[2*i]);
        intree[i]=add(p1,p2);
    }
    return intree[1];
}
int main(){FIN;
    int m,k; cin>>m>>k;
    int top=max(k,m)+2;
    vector<int> x,y;
    int ac=0;
    fore(i,0,top){
        ac=addmod(ac,pm(i,k));
        x.pb(i); y.pb(ac);
    }
    poly p=interpolate(x,y);
    vector<int> xs;
    fore(i,0,m){
        ll x; cin>>x; x%=MOD;
        xs.pb(x);
    }
    while(SZ(xs)!=top) xs.pb(0);
    vector<int> ans=evaluate(p,xs);
    fore(i,0,m)cout<<ans[i]<<" ";cout<<"\n";
}

```

6.8 fht

```

ll c1[tam+9],c2[tam+9]; // tam must be power of 2 !!
void fht(ll* p, int n, bool inv){
    for(int l = 1; 2 * l <= n; l *= 2)
        for(int i = 0; i < n; i += 2 * l)
            fore(j, 0, l)
            {
                ll u = p[i + j], v = p[i + l + j];
                if(!inv) p[i + j] = u + v, p[i + l + j] = u - v; // XOR
                else p[i + j] = (u + v) / 2, p[i + l + j] = (u - v) / 2;
                //if(!inv) p[i + j] = v, p[i + l + j] = u + v; // AND
                //else p[i + j] = -u + v, p[i + l + j] = u;
                //if(!inv) p[i + j] = u + v, p[i + l + j] = u; // OR
                //else p[i + j] = v, p[i + l + j] = u - v;
            }
    }
// like polynomial multiplication, but XORing exponents
// instead of adding them (also ANDing, ORing)
vector<ll> multiply(vector<ll>& p1, vector<ll>& p2){
    int n = 1<<(32-__builtin_clz(max(sz(p1), sz(p2)) - 1));
    fore(i, 0, n) c1[i] = 0, c2[i] = 0;
    fore(i, 0, sz(p1)) c1[i] = p1[i];
    fore(i, 0, sz(p2)) c2[i] = p2[i];

```

```

    fht(c1, n, false); fht(c2, n, false);
    fore(i, 0, n) c1[i] *= c2[i];
    fht(c1, n, true);
    return vector<ll>(c1, c1 + n);
}
void fht(vector<ll>& p, bool inv) {
    fore(i, 0, sz(p)) c1[i] = p[i];
    fht(c1, sz(p), inv);
    fore(i, 0, sz(p)) p[i] = c1[i];
}

```

6.9 formulas

//Stirling number of the second kind is the number of ways to partition a set of n objects into k non-empty subsets.

$S(n, k) = ks(n-1, k) + S(n-1, k-1)$, where $S(0, 0) = 1, s(n, 0) = s(0, n) = 0$

$S(n, 2) = 2^{n-1} - 1$

$S(n, k)$ k != number of ways to color n nodes using colors from 1 to k such that each color is used at least once.

An r -associated Stirling number of the second kind is the number of ways to partition a set of n objects into k subsets, with each subset containing at least r elements. It is denoted by $Sr(n, k)$ and obeys the recurrence relation.

$Sr(n+1, k) = kSr(n, k) + C(n, r-1)Sr(n-r+1, k-1)$

//The Stirling numbers of the first kind count permutations according to their number of cycles (counting fixed points as cycles of length one).

$S(n, k)$ counts the number of permutations of n elements with k disjoint cycles.

$S(n, k) = (n-1)S(n-1, k) + S(n-1, k-1)$, where, $s(0, 0) = 1, S(n, 0) = s(0, n) = 0$

$\text{Sum}(k, 0, n) \quad S(n, k) = n!$

The unsigned Stirling numbers may also be defined algebraically, as the coefficient of the rising factorial: $x^{(\sim n)} = x(x+1)(x+2)\dots = \text{Sum}(k, 0, n)s(n, k)x^k$

//Bell number count the number of partition of a set

$B_{n+1} = \text{Sum}(k, 0, n)\{C(n, k)B_k\}$

$B_n = S \text{ Sum}(k, 0, n)Sr(n, k)$, where Sr is Stirling number of 2kind

//Formally, for a sequence of numbers $\{a_i\}$, we define the ordinary generating function (OGF) of a to be $A(x) = \text{Sum}(i, 0, \text{inf}) a_i x^i$.

$1/(1-x) = 1 + x + x^2 + \dots = \text{Sum}(n, 0, \text{inf}) x^n$

$\ln(1-x) = x + x^2/2 + x^3/3 + \dots = \text{Sum}(n, 0, \text{inf}) x^n/n$

$e^x = 1 + x + x^2/2! + x^3/3! + \dots = \text{Sum}(n, 0, \text{inf}) x^n/n!$

$(1-x)^{-k} = C(k-1, 0)x^0 + C(k-1, 1)x^1 + C(k-1, 2)x^2 + \dots = \text{Sum}(n, 0, \text{inf}) C(n+k-1, n)x^n$

For OGF, $C(x) = A(x)^k$ generates the sequence

$cn = \text{Sum}(i_1 \dots i_k, i_1 + i_2 + \dots + i_k = n) (a_{i_1} a_{i_2} \dots a_{i_k})$

For EGF, $C(x) = A(x)^k$ generates the sequence

$cn = \text{Sum}(i_1 \dots i_k, i_1 + i_2 + \dots + i_k = n) (a_{i_1} a_{i_2} \dots a_{i_k}) n! / (i_1! \dots i_k!)$

Suppose want to generate the sequence $cn = a_0 + a_1 + \dots + a_n$. Then, we can take

$C(x) = 1/(1-x) * A(x)$.

6.10 gauss

```

// resuelve Ax = b, dada la matriz a de n * (m + 1), n ecuaciones y m variables,
// siendo la ultima columna el vector b
// The function returns the number of solutions of the system (0,1,or ). if
// there's at least a solution, it's in ans
const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity or a big number
int gauss (vector < vector<double> > a, vector<double> & ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;
    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;
        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }
    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }
    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}

```

6.11 interpol-o(n)

```

// evaluar un "polinomio interpolado" en o(nlogMOD)
// debe cumplir xi+1 - xi = xj+1 - xj for all i, j < n
// recibe vector de ys tal que f(i) = y[i]
ll eval(vll ys, ll x) {
    int n = ys.size();

```



```

if(x < n) return ys[x];
ll res = 0, up = 1, dow = 1;
for(i, 1, n)
    dow = dow * (MOD - i) % MOD;
    up = up * (x - i) % MOD;
for(i, 1, n) {
    up = up * (x - i + 1) % MOD * pot(x - i, MOD - 2) % MOD;
    dow = dow * i % MOD * pot(MOD - (n - i), MOD - 2) % MOD;
    res = (res + ys[i] * up % MOD * pot(dow, MOD - 2) % MOD) % MOD;
}
return res;
}

```

6.12 karatsuba

```

typedef ll tp;
// #define add(n,s,d,k) for(i,0,n)(d)[i]+=(s)[i]*k
#define add(n,s,d,k) for(i,0,n)(d)[i]+=(s)[i]*k%MOD, (d)[i] = ((d)[i] % MOD + MOD) % MOD;
tp* ini(int n){tp *r=new tp[n];fill(r,r+n,0);return r;}
void karatsura(int n, tp* p, tp* q, tp* r){
    if(n<=0)return;
    // if(n<35)for(i,0,n)for(j,0,n)r[i+j]+=p[i]*q[j];
    if(n<35)for(i,0,n)for(j,0,n)r[i+j]+=p[i]*q[j] % MOD, r[i + j] %= MOD;
    else {
        int nac=n/2,nbd=n-n/2;
        tp *a=p,*b=p+nac,*c=q,*d=q+nac;
        tp *ab=ini(nbd+1),*cd=ini(nbd+1),*ac=ini(nac*2),*bd=ini(nbd*2);
        add(nac,a,ab,1);add(nbd,b,ab,1);
        add(nac,c,cd,1);add(nbd,d,cd,1);
        karatsura(nac,a,c,ac);karatsura(nbd,b,d,bd);
        add(nac*2,ac,r+nac,-1);add(nbd*2,bd,r+nac,-1);
        add(nac*2,ac,r,1);add(nbd*2,bd,r+nac*2,1);
        karatsura(nbd+1,ab,cd,r+nac);
        free(ab);free(cd);free(ac);free(bd);
    }
}
vector<tp> multiply(vector<tp> p0, vector<tp> p1){
    int n=max(p0.size(),p1.size());
    tp *p=ini(n),*q=ini(n),*r=ini(2*n);
    for(i,0,p0.size())p[i]=p0[i];
    for(i,0,p1.size())q[i]=p1[i];
    karatsura(n,p,q,r);
    vector<tp> rr(r,r+p0.size()+p1.size()-1);
    free(p);free(q);free(r);
    return rr;
}

```

6.13 matrix-determinant

```

const double EPS=1e-4;
double reduce(vector<vector<double> >& x){ // returns determinant
    int n=x.size(),m=x[0].size();
    int i=0,j=0;double r=1.;
    while(i<n&&j<m){
        int l=i;
        for(k,i+1,n)if(abs(x[k][j])>abs(x[l][j]))l=k;
        if(abs(x[l][j])<EPS){j++;r=0.;continue;}
        if(l!=i){r=-r;swap(x[i],x[l]);}
        r*=x[l][j];
        for(int k=m-1;k>=j;k--)x[i][k]/=x[i][j];
        for(k,0,n){
            if(k==i)continue;
            for(int l=m-1;l>=j;l--)x[k][l]-=x[k][j]*x[i][l];
        }
        i++;j++;
    }
    return r;
}

```

6.14 matrix-fast-pow

```

typedef vector<vector<ll> > Matrix;
Matrix ones(int n) {
    Matrix r(n,vector<ll>(n));
    for(i,0,n)r[i][i]=1;
    return r;
}
Matrix operator*(Matrix &a, Matrix &b) {
    int n=SZ(a),m=SZ(b[0]),z=SZ(a[0]);
    Matrix r(n,vector<ll>(m));
    for(i,0,n)for(j,0,m)for(k,0,z)
        r[i][j]+=a[i][k]*b[k][j],r[i][j]%=mod;
    return r;
}
Matrix be(Matrix b, ll e) {
    Matrix r=ones(SZ(b));
    while(e){if(e&1LL)r=r*b;b=b*b;e/=2;}
    return r;
}

```

6.15 moebius

```

//f(n)=sum(d|n,g(d))=>g(n)=sum(d|n,f(d)*mu(n/d))
//f(n)=sum(i->inf,f(i*n)*mu(i));f(n)=#f(a)->n;g(n)=#f(a)->xn
int mu[tam], is_prime [tam];
for(i, 0, tam) mu[i]=is_prime[i]=1;
for(i, 2, tam) if(is_prime[i]) {
    for(j, i, tam, i) {

```

```

    if(j > i) is_prime[j] = 0;
    if(j / i % i == 0) mu[j]=0;
    mu[j] = -mu[j];
}
}

```

6.16 pollard-rho-miller-rabil

```

ll gcd(ll a, ll b){return a?gcd(b%a,a):b;}
ll mulmod(ll a, ll b, ll m) {
    ll r=a*b-(ll)((long double)a*b/m+.5)*m;
    return r<0?r+m:r;
}
ll expmod(ll b, ll e, ll m){
    if(!e)return 1;
    ll q=expmod(b,e/2,m);q=mulmod(q,q,m);
    return e&1?mulmod(b,q,m):q;
}
bool is_prime_prob(ll n, int a){
    if(n==a)return true;
    ll s=0,d=n-1;
    while(d%2==0)s++,d/=2;
    ll x=expmod(a,d,n);
    if((x==1)|| (x+1==n))return true;
    for(;;s-1){
        x=mulmod(x,x,n);
        if(x==1)return false;
        if(x+1==n)return true;
    }
    return false;
}
bool rabin(ll n){ // true iff n is prime
    if(n==1)return false;
    int ar[]={2,3,5,7,11,13,17,19,23};
    for(i,0,9)if(!is_prime_prob(n,ar[i]))return false;
    return true;
}
// optimized version: replace rho and fact with the following:
const int MAXP=1e6+1; // sieve size
int sv[MAXP]; // sieve
ll add(ll a, ll b, ll m){return (a+b)<m?a+b-m;}
ll rho(ll n){
    static ll s[MAXP];
    while(1){
        ll x=rand()%n,y=x,c=rand()%n;
        ll *px=s,*py=s,v=0,p=1;
        while(1){
            *py+=y=add(mulmod(y,y,n),c,n);
            *py+=y=add(mulmod(y,y,n),c,n);
            if((x=*px++)==y)break;
            ll t=p;
            p=mulmod(p,abs(y-x),n);

```

```

        if(!p)return gcd(t,n);
        if(++v==26){
            if((p=gcd(p,n))>1&&p<n)return p;
            v=0;
        }
        if(v&&(p=gcd(p,n))>1&&p<n)return p;
    }
}
void init_sv(){
    for(i,2,MAXP)if(!sv[i])for(ll j=i;j<MAXP;j+=i)sv[j]=i;
}
void fact(ll n, map<ll,int>& f){ // call init_sv first!!!
    for(auto&& p:f){
        while(n%p.f==0){
            p.s++; n/=p.f;
        }
    }
    if(n<MAXP)while(n>1)f[sv[n]]++,n/=sv[n];
    else if(rabin(n))f[n]++;
    else {ll q=rho(n);fact(q,f);fact(n/q,f);}
}

```

6.17 simplex

```

vector<int> X,Y;
vector<vector<double>> > A;
vector<double> b,c;
double z;
int n,m;
void pivot(int x,int y){
    swap(X[y],Y[x]);
    b[x]/=A[x][y];
    for(i,0,m)if(i!=y)A[x][i]/=A[x][y];
    A[x][y]=1/A[x][y];
    for(i,0,n)if(i!=x&&abs(A[i][y])>EPS){
        b[i]-=A[i][y]*b[x];
        for(j,0,m)if(j!=y)A[i][j]-=A[i][y]*A[x][j];
        A[i][y]=-A[i][y]*A[x][y];
    }
    z+=c[y]*b[x];
    for(i,0,m)if(i!=y)c[i]-=c[y]*A[x][i];
    c[y]=-c[y]*A[x][y];
}
pair<double,vector<double>> simplex( // maximize c^T x s.t. Ax<=b, x>=0
    vector<vector<double>> > _A, vector<double> _b, vector<double> _c){
    // returns pair (maximum value, solution vector)
    A=_A;b=_b;c=_c;
    n=b.size();m=c.size();z=0.;
    X=vector<int>(m);Y=vector<int>(n);
    for(i,0,m)X[i]=i;
    for(i,0,n)Y[i]=i+m;

```

```

while(1){
    int x=-1,y=-1;
    double mn=-EPS;
    fore(i,0,n)if(b[i]<mn)mn=b[i],x=i;
    if(x<0)break;
    fore(i,0,m)if(A[x][i]<=-EPS){y=i;break;}
    assert(y>=0); // no solution to Ax<=b
    pivot(x,y);
}
while(1){
    double mx=EPS;
    int x=-1,y=-1;
    fore(i,0,m)if(c[i]>mx)mx=c[i],y=i;
    if(y<0)break;
    double mn=1e200;
    fore(i,0,n)if(A[i][y]>EPS&&b[i]/A[i][y]<mn)mn=b[i]/A[i][y],x=i;
    assert(x>=0); // c^T x is unbounded
    pivot(x,y);
}
vector<double> r(m);
fore(i,0,n)if(Y[i]<m)r[Y[i]]=b[i];
return mp(z,r);
}

```

6.18 stirling y bell

```

// stir[n][k] cantidad de formas de paritcionar un conjunto de n elementos en k
// conjuntos
// bell[n] cantidad de formas de particionar un conjunto
ll stir[tam][tam];
ll bell[tam];
void stirBell() {
    fore(i, 1, tam) {
        stir[i][1] = 1;
        fore(j, 2, 1010)
            stir[i][j] = (j * stir[i - 1][j] % MOD + stir[i - 1][j - 1]) % MOD;
    }
    fore(i, 1, tam)
        fore(j, 1, i + 1)
            bell[i] = (bell[i] + stir[i][j]) % MOD;
}

```

7 Shortcuts

7.1 dsu

```

int cnt[maxn];
void dfs(int v, int p, bool keep){
    int mx = -1, bigChild = -1;

```

```

for(auto u : g[v])
    if(u != p && sz[u] > mx)
        mx = sz[u], bigChild = u;
for(auto u : g[v])
    if(u != p && u != bigChild)
        dfs(u, v, 0); // run a dfs on small childs and clear them from cnt
if(bigChild != -1)
    dfs(bigChild, v, 1); // bigChild marked as big and not cleared from cnt
for(auto u : g[v])
    if(u != p && u != bigChild)
        for(int p = st[u]; p < ft[u]; p++)
            cnt[ col[ ver[p] ] ]++;
cnt[ col[v] ]++;
//now cnt[c] is the number of vertices in subtree of vertex v that has color c.
// You can answer the queries easily.
if(keep == 0)
    for(int p = st[v]; p < ft[v]; p++)
        cnt[ col[ ver[p] ] ]--;
}

```

7.2 mo

```

void remove(idx);
void add(idx);
int get_answer();
int block_size;
struct Query {
    int l, r, idx;
    bool operator<(Query other) const
    {
        if (l / block_size != other.l / block_size)
            return mp(l, r) < mp(other.l, other.r);
        return ((l / block_size) & 1) ? (r < other.r) : (r > other.r);
    }
};
vector<int> mo_s_algorithm(vector<Query> queries) {
    vector<int> answers(queries.size());
    sort(queries.begin(), queries.end());
    int cur_l = 0;
    int cur_r = -1;
    for (Query q : queries) {
        while (cur_l > q.l) {
            cur_l--;
            add(cur_l);
        }
        while (cur_r < q.r) {
            cur_r++;
            add(cur_r);
        }
        while (cur_l < q.l) {
            remove(cur_l);
            cur_l++;
        }
    }
}

```

```

    }
    while (cur_r > q.r) {
        remove(cur_r);
        cur_r--;
    }
    answers[q.idx] = get_answer();
}
return answers;
}

```

7.3 shortcuts

```

// Better random mt19937_64 para 64 bits
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
cout << rng() << endl;
shuffle(permutation.begin(), permutation.end(), rng);
// while TLE
double t = clock(), TLE = 3;
while((clock() - t) / CLOCKS_PER_SEC < TLE);
// ordered_set
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef
    tree<int,null_type,less<int>,rb_tree_tag,tree_order_statistics_node_update>
    ordered_set;
// find_by_order kth largest 0 indexed, order_of_key finds how many are less than
// Faster map gp_hash_table<int,int,my_hash> m;
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
struct my_hash {
    const uint64_t RANDOM = chrono::steady_clock::now().time_since_epoch().count();
    static uint64_t splitmix64(uint64_t x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }
    size_t operator()(uint64_t x) const {
        return splitmix64(x + RANDOM);
    }
};

```

8 Strings

8.1 aho-corasick

```

struct vertex {
    int go[26], pch, par, link = -1, super = -1, leaf = 0;

```

```

    vertex(): link(0), super(0) { mem(go, -1); }
    vertex(int ch, int from): pch(ch), par(from) { mem(go, -1); }
};
vector<vertex> t(1);
void add(string &s, int pos) {
    int node = 0;
    for(char ch : s) {
        ch -= 'a';
        if(t[node].go[ch] == -1)
            t[node].go[ch] = t.size(); t.emplace_back(ch, node);
        node = t[node].go[ch];
    }
    t[node].leaf = 1;
}
int go(int node, char c);
int suff(int node) {
    if(t[node].link == -1)
        t[node].link = t[node].par == 0 ? 0 : go(suff(t[node].par), t[node].pch);
    return t[node].link;
}
int go(int node, char ch) {
    if(t[node].go[ch] == -1)
        t[node].go[ch] = node == 0 ? 0 : go(suff(node), ch);
    return t[node].go[ch];
}
int super(int v) {
    if(t[v].super == -1)
        t[v].super = t[suff(v)].leaf ? suff(v) : super(suff(v));
    return t[v].super;
}

```

8.2 hsh-128

```

#define bint __int128
struct Hash {
    bint MOD=212345678987654321LL,P=1777771,PI=106955741089659571LL;
    vector<bint> h,pi;
    Hash(string& s){
        assert((P*PI)%MOD==1);
        h.resize(s.size()+1);pi.resize(s.size()+1);
        h[0]=0;pi[0]=1;
        bint p=1;
        for(i,1,s.size()+1){
            h[i]=(h[i-1]+p*s[i-1])%MOD;
            pi[i]=(pi[i-1]*PI)%MOD;
            p=(p*P)%MOD;
        }
    }
    ll get(int s, int e){
        return (((h[e]-h[s]+MOD)%MOD)*pi[s])%MOD;
    }
};

```

8.3 manacher

```
int d1[MAXN]; //d1[i] = max odd palindrome centered on i
int d2[MAXN]; //d2[i] = max even palindrome centered on i
//s aabbaacaabbaa
//d1 1111117111111
//d2 0103010010301
void manacher(string& s){
    int l=0,r=-1,n=s.size();
    for(i,0,n){
        int k=i>r?1:min(d1[l+r-i],r-i);
        while(i+k<n&&i-k>=0&&s[i+k]==s[i-k])k++;
        d1[i]=k--;
        if(i+k>r)l=i-k,r=i+k;
    }
    l=0;r=-1;
    for(i,0,n){
        int k=i>r?0:min(d2[l+r-i+1],r-i+1);k++;
        while(i+k<n&&i-k>=0&&s[i+k-1]==s[i-k])k++;
        d2[i]=--k;
        if(i+k-1>r)l=i-k,r=i+k-1;
    }
}
```

8.4 prefix-function

```
vector<int> prefix_function(string &s) {
    int n = (int)s.length();
    vector<int> pi(n);
    for (int i = 1; i < n; i++) {
        int j = pi[i-1];
        while (j > 0 && s[i] != s[j])
            j = pi[j-1];
        if (s[i] == s[j])
            j++;
        pi[i] = j;
    }
    return pi;
}

void compute_automaton(string &s, vector<vector<int>>& aut) {
    s += '#'; int n = s.size();
    vector<int> pi = prefix_function(s);
    aut.assign(n, vector<int>(26));
    for (int i = 0; i < n; i++) {
        for (int c = 0; c < 26; c++) {
            if (i > 0 && 'a' + c != s[i])
                aut[i][c] = aut[pi[i-1]][c];
            else
                aut[i][c] = i + ('a' + c == s[i]);
        }
    }
}
```

8.5 suffix-array

```
vector<vector<int>> table;
vector<int> suffixa(string &s){
    int n = s.size(), cc, ax;
    vector<int> sa(n), sa1(n), col(n), col1(n), head(n);
    for(i, 0, n) sa[i] = i;
    auto cmp = [&](int a, int b){ return s[a] < s[b]; };
    stable_sort(sa.begin(), sa.end(), cmp);
    head[0] = col[sa[0]] = cc = 0;
    for(i, 1, n){
        if(s[sa[i]] != s[sa[i-1]])
            cc++, head[cc] = i;
        col[sa[i]] = cc;
    }
    table.pb(col);
    for(int k = 1; k < n; k *= 2){
        for(i, 0, n){
            ax = (sa[i] - k + n) % n;
            sa1[head[col[ax]]++] = ax;
        }
        swap(sa, sa1);
        col1[sa[0]] = head[0] = cc = 0;
        for(i, 1, n){
            if(col[sa[i]] != col[sa[i-1]] || col[(sa[i] + k) % n] != col[(sa[i-1] + k) % n])
                cc++, head[cc] = i;
            col1[sa[i]] = cc;
        }
        swap(col, col1); table.pb(col);
        if(col[sa[n-1]] == n-1) break;
    }
    return sa;
}

pair<int, int> query(int b, int e){
    int lev = 31 - __builtin_clz(e - b + 1);
    return mp(table[lev][b], table[lev][e - (1 << lev) + 1]);
}

bool comp(int b1, int e1, int b2, int e2){
    int siz = min(e1 - b1, e2 - b2);
    ii le = query(b1, b1 + siz), ri = query(b2, b2 + siz);
    if(le == ri)
        return e1 - b1 < e2 - b2;
    return le < ri;
}

vector<int> lcp(string &s, vector<int> &sa){
    int n = s.size(), k, z = 0;
    vector<int> sa1(n), lcp(n);
    for(i, 0, n) sa1[sa[i]] = i;
    for(i, 0, n){
        k = sa1[i];
```

```

    if(k < n - 1)
        while(s[i + z] == s[sa[k+1] + z])
            z++;
    lcp[k] = z; z = max(z-1, 0);
}
return lcp;
}

```

8.6 suffix-automata

```

struct state {int len,link;map<char,int> next;}; //clear next!!
state st[100005];
int sz,last;
void sa_init(){
    last=st[0].len=0;sz=1;
    st[0].link=-1;
}
void sa_extend(char c){
    int k=sz++,p;
    st[k].len=st[last].len+1;
    for(p=last;p!=-1&&!st[p].next.count(c);p=st[p].link)st[p].next[c]=k;
    if(p==-1)st[k].link=0;
    else {
        int q=st[p].next[c];

```

```

        if(st[p].len+1==st[q].len)st[k].link=q;
    else {
        int w=sz++;
        st[w].len=st[p].len+1;
        st[w].next=st[q].next;st[w].link=st[q].link;
        for(;p!=-1&&st[p].next[c]==q;p=st[p].link)st[p].next[c]=w;
        st[q].link=st[k].link=w;
    }
}
last=k;
}
// input: abcbcbc
// i,link,len,next
// 0 -1 0 (a,1) (b,5) (c,7)
// 1 0 1 (b,2)
// 2 5 2 (c,3)
// 3 7 3 (b,4)
// 4 9 4 (c,6)
// 5 0 1 (c,7)
// 6 11 5 (b,8)
// 7 0 2 (b,9)
// 8 9 6 (c,10)
// 9 5 3 (c,11)
// 10 11 7
// 11 7 4 (b,8)

```
