Theorem 1: Inverse Function Theorem

Let f be a continuously differentiable function from an open set $U \subset \mathbb{R}^n$ to $\mathbb{R}^n$, and let $a \in U$. If the Jacobian determinant $J_f(a)$ is nonzero, then there exists an open set V containing a and an open set W containing $f(a)$ such that f is one-to-one on V and such that the inverse function f^{-1} is continuously differentiable on W.

Theorem 2: Implicit Function Theorem

Let f be a continuously differentiable function from an open set $U \subset \mathbb{R}^{n+m}$ to $\mathbb{R}^m$, and let $(a, b) \in U$. If $f(a, b) = 0$ and the Jacobian matrix $J_f(a, b)$ has rank m, then there exist open sets V and W in $\mathbb{R}^n$ and $\mathbb{R}^m$, respectively, with $(a, b) \in V \times W \subset U$, and a continuously differentiable function $g : V \to W$ such that $f(x, g(x)) = 0$ for all $x \in V$.

Theorem 3: Lagrange Multiplier Theorem

Let f and $g_1, \ldots, g_m$ be continuously differentiable functions from an open set $U \subset \mathbb{R}^n$ to $\mathbb{R}$. If $a \in U$ is a local extremum of f subject to the constraints $g_1(x) = \cdots = g_m(x) = 0$, and if the gradients $\nabla g_1(a), \ldots, \nabla g_m(a)$ are linearly independent, then there exist constants $\lambda_1, \ldots, \lambda_m$ such that

$$\nabla f(a) = \lambda_1 \nabla g_1(a) + \cdots + \lambda_m \nabla g_m(a).$$