Using R in Production

Lessons from The Observatory of Economic Complexity

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Data

Sources

All of the product data shown on the OEC site is classified using either

- SITC (Standard International Trade Classification)
- *HS* (Harmonized System).

For historical SITC classification data (1962 - 2000), the OEC is using data from [Feenstra et al., 2005]. For more recent data (2001 - 2016), the OEC is using data provided by UN COMTRADE.

Availability

Classification	Availability
HS rev 1992	1992 – 2016
HS rev 1996	1996 – 2016
HS rev 2002	2002 - 2016
HS rev 2007	2007 - 2016
HS rev 2012	2012 - 2016
SITC rev 2	2000 - 2016

Tidy Data

- I followed Tidy Data principles to obtain an output that in our opinion can be useful for others.
- Tidy Data principles are closely tied to those of relational databases and Codd's relational algebra.
- I did not innovate at this point and I only limited to follow the principles exposed in [Wickham, 2014b] and [Wickham and Grolemund, 2016] above all matters related to performing code and coding style.

Tidy Data

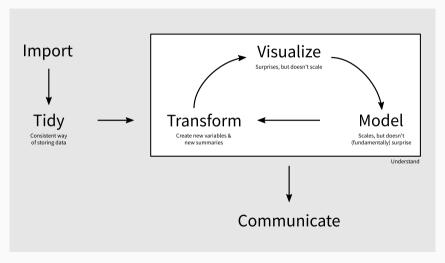


Figure 1: Data pipeline

Filling some gaps in our data

- For each NA or 0 import/export value I tried to fill the gap.
- If country A reported NA or 0 exports (imports) of product B to (from) country C, then I searched what country C reported of imports (exports) of product B from (to) country A.

Filling some gaps in our data

I provide column marker indicating those replacements under this labels:

meaning
imports with replacements
exports with replacements
imports and exports with replacements
no replacements needed

Filling some gaps in our data



Figure 2: Example of gap in data

Countries not included in rankings and indicators

- The curated data includes all the countries available from UN Comtrade data.
- RCA based calculations such as ECI, PCI, Proximity consider 128 countries that account for
 - 99% of world trade
 - 97% of the world's total GDP
 - and 95% of the world's population according to [Hidalgo et al., 2014].

Countries not included in rankings and indicators

I considered simultaneously:

- Countries with population greater or equal to 1.2 million
- Countries whose traded value is greater or equal than 1 billion

Countries not included in rankings and indicators

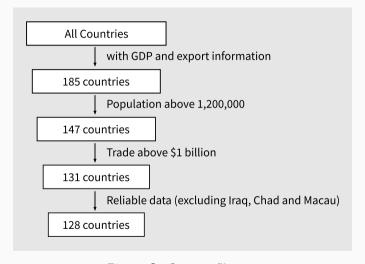


Figure 3: Country filtering

Hardware & Software

Hardware

- Intel[®] Xeon 2.27GHz processor (eight cores)
- 32 GB (four DDR3 cards of eight gigabytes each)

Software

- Ubuntu Server 16.04
- **R** 3.4.3
- RStudio Server Pro 1.1
- I built R from binaries and linked to Intel MKL 2017 so I benefit from multi-threaded BLAS/LAPACK libraries

Packages

- packrat
- pacman
- data.table
- dplyr
- tidyr
- doParallel
- Matrix
- RcppArmadillo
- feather
- RPostgreSQL

Limitations

- The project is divided in three big tasks:
 - download raw data
 - clean data following Tidy Data principles
 - write data to PostgreSQL DB

Limitations

- Functions such as download (i.e run wget on eight cores) do not suppose a problem
- Functions that involve matrix computation (i.e compute economic complexity rankings) were run on four cores because I detected a *large* overhead due to data communication with cores when using more cores.

Slowness has several explanations

- Running many processes
- Hardware bottlenecks (i.e faster RAM versus more RAM, same for HDD)
- Numerical libraries
- Coding

Numerical libraries can make a difference

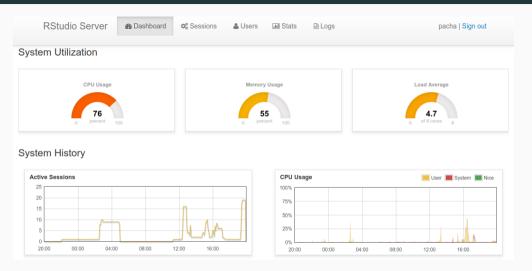


Figure 4: RStudio admin panel while running scripts to clean data and compute economic complexity rankings

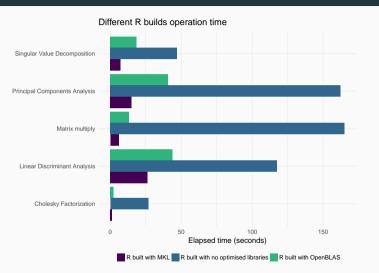


Figure 5: Performance exhibited running unmodified Microsoft benchmark script (edited R builds location)

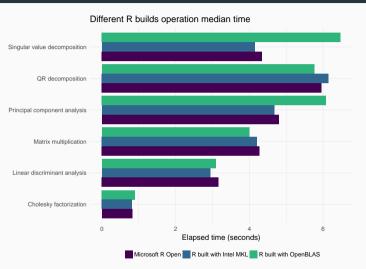


Figure 6: Performance exhibited running modified ATT script (modified to run fresh 100 times and then store the median result)

Coding

Syntax

- I used Tidyverse Style Guide
- As cornerstone references for performant code I followed [Wickham, 2014a] and [Peng et al., 2017]
- Readability is really important (specially after one ytd and you don't remember what you were doing)
- Over this project I moved from doing most of tasks with data.table to tidyverse because of readability

Performant code

- Some matrix operations are written in Rcpp to take advantage of C++ speed
- To take full advantage of hardware and numerical libraries I are using sparse matrices as it is explained in [Ni et al., 2018]
- As a general reccomendation using sparse matrices is a good practise with respect to both speed and memory usage

Reproducibility

- The full project is organized in different GitHub repositories
- Uploading to GitHub is not enought
- For full reproducibility I thought about
 - Using packrat to have an isolated repository of packages
 - Making packages bundle available
 - Considering literate programming to write scripts
 - Documenting everything

Reproducibility

- I have reproducility flaws
 - UN COMTRADE access
 - Parallelization is OS-specific
- Potential solutions
 - Collaborate with UN COMTRADE
 - Move to a parallelization type different than FORK

Economic Complexity

What is Economic Complexity?

- We owe to Adam Smith the idea that the division (specialization) of labor is the secret of the wealth of nations.
- The division of labor into markets and organizations is what allows the knowledge held by few to reach many, making us collectively wiser.

What is Economic Complexity?

- The complexity of an economy is related to the multiplicity of useful knowledge embedded in it.
- Because individuals are limited in what they know, the only way societies can expand their knowledge base is by facilitating the interaction of individuals in increasingly complex networks in order to make products.
- We can measure economic complexity by the mix of these products that countries are able to make.

What is Economic Complexity?

- Some products, like medical imaging devices or jet engines, embed large amounts of knowledge and are the results of very large networks of people and organizations.
- These products cannot be made in simpler economies that are missing parts of this network's capability set.
- Economic complexity, therefore, is expressed in the composition of a country's productive output and reflects the structures that emerge to hold and combine knowledge.

Let $x_{c,p}$ represent the exports of country c in product p, we can express the Revealed Comparative Advantage that country c has in product p as:

$$RCA_{c,p} = \frac{x_{c,p}}{\sum_{c} x_{c,p}} / \frac{\sum_{p} x_{c,p}}{\sum_{c} \sum_{p} x_{c,p}}$$
(1)

RCA is the basic indicator to measure economic complexity

Exercise

- Open RStudio and load fantasy_world_long.rdata to your workspace
- Use dplyr to compute RCA
- Remember RCA definition

$$RCA_{c,p} = \frac{x_{c,p}}{\sum_{c} x_{c,p}} / \frac{\sum_{p} x_{c,p}}{\sum_{c} \sum_{p} x_{c,p}}$$

Exploring the dataset

```
library(dplyr)
load("fantasy world long.rdata")
fantasy_world_long %>% print(n = 3)
## # A tibble: 108 x 3
    country product export val
##
##
    <chr>
                  <chr>
                              <int>
## 1 patolandia alpha
## 2 mordor
           alpha
## 3 neverneverland alpha
## # ... with 105 more rows
```

```
rca long <- fantasy world long %>%
  rename(c = country,
         p = product,
         xcp = export val) %>%
  group_by(c) %>%
  mutate(sum_c_xcp = sum(xcp)) %>%
  group_by(p) %>%
  mutate(sum p xcp = sum(xcp)) %>%
  ungroup() %>%
  mutate(sum c p xcp = sum(xcp)) %>%
  mutate(rca = (xcp / sum c xcp) /
           (sum_p_xcp / sum_c_p_xcp))
```

```
## # A tibble: 108 \times 3
##
                         rca
    <chr>
               <chr> <dbl>
##
## 1 patolandia alpha
## 2 mordor
               alpha 0.
## 3 neverneverland alpha 0.
## 4 thematrix
                  alpha 12.1
## 5 lilliput alpha 0.
## # ... with 103 more rows
```

Exercise

• Now create a matrix *M* with entries

$$m_{c,p} = \begin{cases} 1 & \text{if } RCA_{c,p} \ge 1 \\ 0 & \text{otherwise} \end{cases}$$
 (2)

Try to create it as if the dataset was actually large

```
rca_matrix <- rca_long %>%
    select(c, p, rca) %>%
    mutate(rca = ifelse(rca > 1, 1, 0)) %>%
    spread(p, rca)
```

Smooth Revealed Comparative Advantage (SRCA)

- In the OEC real data we use a modified RCA to reduce trade fluctuations
- We smooth changes in export volumes induced by the price fluctuation of commodities by using a modification of (1) in which $x_{c,p}$ is averaged over the previous three years by using weights:

$$SRCA_{c,p}^{(t)} = \frac{\hat{x}_{c,p}^{(t)}}{\sum_{c} \hat{x}_{c,p}^{(t)}} / \frac{\sum_{p} \hat{x}_{c,p}^{(t)}}{\sum_{c} \sum_{p} \hat{x}_{c,p}^{(t)}}$$

Where

$$\hat{x}_{c,p}^{(t)} = \frac{2x_{c,p}^{(t)} + x_{c,p}^{(t-1)} + x_{c,p}^{(t-2)}}{4}$$

- With *M* defined as in the previous sections, we can measure Diversity and Ubiquity simply by summing over the rows or columns of that matrix.
- Diversity:

$$k_c^{(0)} = \sum_p m_{c,p}$$

Ubiquity:

$$k_p^{(0)} = \sum_c m_{c,p}$$

Exercise

- Compute diversity and ubiquity
- ullet Store the result in two matrices D and U

```
diversity <- rca_long %>% select(c) %>% distinct()
ubiquity <- tibble(p = colnames(rca_matrix)) %>%
  filter(row_number() > 1)

rca_matrix <- rca_matrix %>%
  select(-c) %>%
  as.matrix()
```

```
# convert to sparse class
library(Matrix)
rca matrix <- Matrix(rca matrix, sparse = T)</pre>
diversity <- diversity %>%
  mutate(val = rowSums(rca_matrix)) %>%
  filter(val > 0)
ubiquity <- ubiquity %>%
  mutate(val = colSums(rca matrix)) %>%
  filter(val > 0)
```

```
rownames(rca_matrix) <- diversity$c

D <- as.matrix(diversity$val, ncol = 1)
U <- as.matrix(ubiquity$val, ncol = 1)</pre>
```

- The information that diversity and ubiquity carry can be used each one to correct the other.
- For countries, this is to calculate the average ubiquity of the products that it exports.
- For products, this is to calculate the average diversity of the countries that make them.

The last slide can be expressed by the recursion:

$$k_c^{(n)} = \frac{1}{k_c^{(0)}} \sum_p m_{c,p} k_p^{(n-1)}$$
(3)

$$k_{\rho}^{(n)} = \frac{1}{k_{\rho}^{(0)}} \sum_{c} m_{c,\rho} k_{c}^{(n-1)} \tag{4}$$

We then insert (4) into (3) to obtain:

$$k_c^{(n)} = \sum_c \left[\frac{1}{k_c^{(0)}} \sum_p m_{c,p} \frac{1}{k_p^{(0)}} m_{c,p} \right] k_c^{(n-2)}$$
 (5)

Exercise

- lacktriangledown Remove null rows and columns from rca_matrix using the names in D and U
- Store the result as Mcp and remove rca_matrix
- Save D and U using numeric class as kc0 and kp0 respectively

```
# remove null rows and cols
Mcp <- rca matrix[</pre>
  which(rownames(rca matrix) %in% unlist(diversity$c)) .
  which(colnames(rca matrix) %in% unlist(ubiquity$p))]
rm(rca_matrix)
# diversity and ubiquity following the Atlas notation
kc0 <- as.numeric(D)
kp0 <- as.numeric(U)</pre>
```

Exercise

- Using kc0 and kp0 create two matrices kc and kp with 20 columns each
- Compute $kc j^{th}$ column from of $kp j 1^{th}$ column
- ullet Compute $kp\ j^{th}$ column from $kc\ j-1^{th}$ column

Remember

$$k_c^{(n)} = \frac{1}{k_c^{(0)}} \sum_p m_{c,p} k_p^{(n-1)} \qquad k_p^{(n)} = \frac{1}{k_p^{(0)}} \sum_c m_{c,p} k_c^{(n-1)}$$

```
kcinv \leftarrow 1 / kc0
kpinv \leftarrow 1 / kp0
# create empty matrices
kc <- Matrix(0, nrow = length(kc0), ncol = 20, sparse = T)
kp <- Matrix(0, nrow = length(kp0), ncol = 20, sparse = T)</pre>
# fill the first column with kcO and kpO to start iterating
kc[.1] \leftarrow kc0
kp[,1] \leftarrow kp0
```

```
# compute cols 2 to 20 by iterating from col 1
for (c in 2:ncol(kc)) {
   kc[ ,c] <- kcinv * (Mcp %*% kp[ ,(c - 1)])
   kp[ ,c] <- kpinv * (t(Mcp) %*% kc[ ,(c - 1)])
}</pre>
```

- The interpretation of the scores changes when considering odd or even iteration order n
- High-order iterations are difficult to interpret, and the process asymptotically converges to a trivial fixed point
- ullet To compute rankings I used kc 19^{th} column and kp 20^{th} column

Economic Complexity Index (ECI)

From the Reflections Method, the Economic Complexity Index (ECI) is defined as:

$$ECI_c = \frac{v_c - \mu_v}{\sigma_v} \tag{6}$$

Where

- \vec{v} is defined as v < kc[.19].
- $\mu_v = \sum_c v_c/C$ (mean of \vec{v}) $\sigma_v = \sqrt{\sum_c (v_c \mu_v)^2/(C-1)}$ (standard deviation of \vec{v})

Product Complexity Index (PCI)

Similar to the Economic Complexity Index (ECI), the Product Complexity Index (PCI) is defined as:

$$PCI_{p} = \frac{w_{p} - \mu_{w}}{\sigma_{w}} \tag{7}$$

Where

- \vec{w} is defined as w <- kp[,20]
- $\mu_w = \sum_p w_p/P$ (mean of \vec{w})
- $\sigma_w = \sqrt{\sum_p (w_p \mu_w)^2/(P-1)}$ (standard deviation of \vec{w})

Exercise

- Compute ECI and PCI
- Arrange the results in decreasing order
- Show the results and conclude

```
eci_reflections <- as_tibble(
        (kc[ ,19] - mean(kc[ ,19])) / sd(kc[ ,19])
) %>%
      mutate(country = diversity$c) %>%
      select(country, value) %>%
      arrange(desc(value))
```

Table 3: ECI for Fantasy World (top 5)

country	value
narnia	0.903
patolandia	0.823
pandora	0.604
neverneverland	0.417
xanadu	0.326

Table 4: PCI for Fantasy World (top 5)

product	value
theta	0.782
epsilon	0.606
eta	0.538
mu	0.530
alpha	0.513

- To make products you need chunks of embedded knowledge which we call capabilities.
- The capabilities needed to produce one good may or may not be useful in the production of other goods.
- Capabilities are not observed directly,
- Proximity is a measure that infers the similarity between the capabilities required by a pair of goods by looking at the probability that they are coexported.

Example

- In the year 2008, 17 countries exported wine, 24 exported grapes and 11 exported both, all with *RCA* > 1.
- Then, the product proximity between wines and grapes is 11/24=0.46.
- Note that I divide by 24 instead of 17 to minimize false positives

For a pair of goods p and p' Product Proximity $\Phi \in \mathbb{R}^{P \times P}$ is defined as:

$$\Phi = (M^t M) \odot U$$

Where \odot denotes element-wise multiplication and

$$u_{p,p'} = 1/\max(k_p^{(0)}, k_{p'}^{(0)})$$

In other terms, each entry of $\boldsymbol{\Phi}$ corresponds to:

$$\phi_{p,p'} = \frac{\sum_{c} m_{c,p} m_{c,p'}}{\max(k_p^{(0)}, k_{p'}^{(0)})}$$

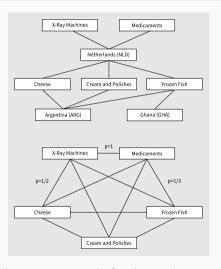


Figure 7: An illustrative example for the product proximity measure

Country Proximity $\Lambda \in \mathbb{R}^{C \times C}$ is similarly defined:

$$\Lambda = (MM^t) \odot D$$

Where

$$d_{c,c'} = 1/\max(k_c^{(0)}, k_{c'}^{(0)})$$

In other terms, each entry of $\boldsymbol{\Lambda}$ corresponds to:

$$\lambda_{c,c'} = \frac{\sum_{p} m_{c,p} m_{c,p'}}{\max(k_c^{(0)}, k_{c'}^{(0)})}$$

Exercise

- Write an Rcpp function titled proximity_products_denominator
- Use that function to make the next code work:

```
Phi pp <- (t(Mcp) %*% Mcp) /
  proximity products denominator(Mcp, U, cores = n cores)
Phi pp 1 <- Phi pp
Phi pp l[upper.tri(Phi pp l, diag = T)] <- NA
Phi_pp_long <- as_tibble(as.matrix(Phi pp 1)) %>%
  mutate(id = rownames(Phi pp)) %>%
  gather(id2, value, -id) %>%
  filter(!is.na(value))
```

```
#include <omp.h>
#include <RcppArmadillo.h>
// [[Rcpp::depends(RcppArmadillo)]]
// [[Rcpp::plugins(openmp)]]
using namespace Rcpp;
// [[Rcpp::export]]
arma::mat proximity_products_denominator(
    arma::sp_mat Mcp, arma::mat U, int cores = 1) {
  // Constants
  int N = (int) Mcp.n cols;
```

```
// Output
  arma::mat Phi_down(N,N);
  // Filling with ones
  Phi_down.ones();
  // Number of cores
  omp set num threads(cores);
```

```
#pragma omp parallel for shared(Mcp, U, N, Phi down) default(none)
 for (int i=0; i<N; i++)</pre>
   for (int j=0; j<=i; j++) {
      // Fill the lower part
      Phi down.at(i,j) = std::max(U(i,0), U(j,0));
      // Fill the upper part
      Phi_down.at(j,i) = Phi_down.at(i,j);
   return Phi down;
```

```
Rcpp::sourceCpp("proximity products denominator.cpp")
n cores <- 4
Phi pp <- (t(Mcp) %*% Mcp) /
  proximity_products_denominator(Mcp, U, cores = n_cores)
Phi pp 1 <- Phi pp
Phi pp l[upper.tri(Phi pp l, diag = T)] <- NA
Phi pp long <- as_tibble(as.matrix(Phi pp 1)) %>%
 mutate(id = rownames(Phi_pp)) %>%
  gather(id2, value, -id) %>%
  filter(!is.na(value))
```

Table 5: Exploring proximity results

id	id2	value
beta	alpha	0.00
delta	alpha	1.00
epsilon	alpha	0.25
eta	alpha	0.00
gamma	alpha	0.00

Exercise

- Write an Rcpp function titled proximity_countries_denominator
- Use that function to make the next code work:

```
Phi cc <- (Mcp ** t(Mcp)) /
  proximity countries denominator(Mcp, D, cores = n cores)
Phi cc l <- Phi cc
Phi cc l[upper.tri(Phi_cc_l, diag = T)] <- NA
Phi cc long <- as tibble(as.matrix(Phi cc 1)) %>%
  mutate(id = rownames(Phi cc)) %>%
  gather(id2, value, -id) %>%
  filter(!is.na(value))
```

```
#include <omp.h>
#include <RcppArmadillo.h>
// [[Rcpp::depends(RcppArmadillo)]]
// [[Rcpp::plugins(openmp)]]
using namespace Rcpp;
// [[Rcpp::export]]
arma::mat proximity_countries_denominator(
    arma::sp mat Mcp, arma::mat D, int cores = 1) {
  // Constants
  int M = (int) Mcp.n rows;
```

```
// Output
  arma::mat Phi_down(M,M);
  // Filling with ones
  Phi_down.ones();
  // Number of cores
  omp set num threads(cores);
```

```
#pragma omp parallel for shared(Mcp, D, M, Phi down) default(none)
 for (int i=0; i<M; i++)</pre>
   for (int j=0; j<=i; j++) {
      // Fill the lower part
      Phi down.at(i,j) = std::max(D(i,0), D(j,0));
      // Fill the upper part
      Phi_down.at(j,i) = Phi_down.at(i,j);
   return Phi down;
```

```
Rcpp::sourceCpp("proximity_countries_denominator.cpp")
n cores <- 4
Phi cc <- (Mcp ** t(Mcp)) /
  proximity_countries_denominator(Mcp, D, cores = n_cores)
Phi cc l <- Phi cc
Phi cc l[upper.tri(Phi cc l, diag = T)] <- NA
Phi cc long <- as tibble(as.matrix(Phi cc 1)) %>%
  mutate(id = rownames(Phi cc)) %>%
  gather(id2, value, -id) %>%
  filter(!is.na(value))
```

Table 6: Exploring proximity results

id	id2	value
mordor	patolandia	0.0
neverneverland	patolandia	0.4
thematrix	patolandia	0.0
lilliput	patolandia	0.0
narnia	patolandia	0.5

- To visualize the product space we use some simple design criteria.
- First, we want the visualization of the product space to be a connected network.
- The second criteria is that we want the network visualization to be relatively sparse.
- Trying to visualize too many links can create unnecessary visual complexity where the most relevant connections will be occluded.

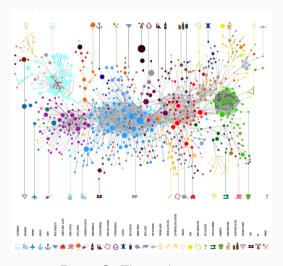


Figure 8: The product space

Exercise

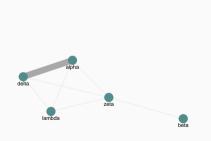
- Create a tibble that includes all pairs with a product proximity higher or equal than 0.4
- Use ggraph package to sketch Fantasy World's Product Space
- Ignore any disconnected products for now

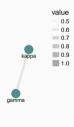
```
if (!require("pacman")) install.packages("pacman")
p_load(igraph, ggraph)
```

```
set.seed(1717)
Phi pp long %>%
  filter(value >= 0.4) %>%
  graph_from_data_frame() %>%
 ggraph(lavout = "fr") +
  geom_edge_link(aes(edge alpha = value, edge width = value),
                 edge colour = "#a8a8a8") +
  geom node point(color = "darkslategrav4", size = 8) +
  geom node text(aes(label = name), viust = 2.2) +
  ggtitle("Sketch of Fantasy World's Product Space") +
  theme void(base size = 15)
```

Sketch of Fantasy World's Product Space







Exercise

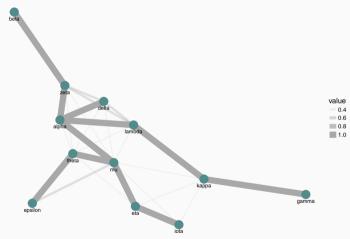
- Obtain a Maximum Spanning Tree by using ape package
- Append missing pairs of products with a proximity higher than 0.3 to your MST
- Sketch the connected Product Space

```
p_load(ape)
Phi pp 2 <- (t(Mcp) %*% Mcp) /
  proximity products denominator(Mcp, U, cores = n cores)
Phi_pp_3 <- -1 * Phi_pp_2
mst pp <- ape::mst(Phi pp 3)</pre>
mst_pp[upper.tri(mst_pp, diag = T)] <- NA</pre>
class(mst_pp) <- "matrix"</pre>
```

```
Phi_pp_2[upper.tri(Phi_pp_2, diag = T)] <- NA
additions_pp_long <- as_tibble(as.matrix(Phi pp 2)) %>%
  mutate(id = rownames(mst pp)) %>%
  gather(id2, value, -id) %>%
  filter(!is.na(value),
         value >= 0.3) \% %
  anti_join(mst_pp_long, by = c("id", "id2"))
graph long <- mst pp long %>%
  bind rows(additions pp long)
```

```
set.seed(1717)
graph long %>%
  graph_from_data_frame() %>%
 ggraph(lavout = "fr") +
  geom edge link(aes(edge alpha = value, edge width = value),
                 edge colour = "#a8a8a8") +
  geom node point(color = "darkslategrav4", size = 8) +
  geom node text(aes(label = name), viust = 2.2) +
 ggtitle("Sketch of Fantasy World's Product Space") +
  theme void(base size = 15)
```

Sketch of Fantasy World's Product Space



Country Space

Exercise

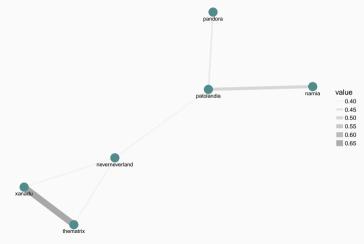
- Create a tibble that includes all pairs with a country proximity higher or equal than 0.4
- Use ggraph package to sketch Fantasy World's Country Space
- Ignore any disconnected countries for now

Country Space

```
set.seed(1717)
Phi cc long %>%
  filter(value >= 0.4) %>%
  graph from data frame() %>%
 ggraph(lavout = "fr") +
  geom_edge_link(aes(edge alpha = value, edge width = value),
                 edge colour = "#a8a8a8") +
  geom node point(color = "darkslategrav4", size = 8) +
  geom node text(aes(label = name), viust = 2.2) +
  ggtitle("Sketch of Fantasy World's Product Space") +
  theme void()
```

Country Space

Sketch of Fantasy World's Country Space



Questions?

Contact

Twitter: @pachamaltese

Blog: pacha.hk

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