2. Discrete time signals and systems.

1. A discrete time signal $x[n]$ is defined as

$$x[n] = \begin{cases} 1+\frac{n}{3} & -3 \le n \le -1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere.} \end{cases}$$

a) determine its values and sketch the signal $x[n]$.

for $n=$

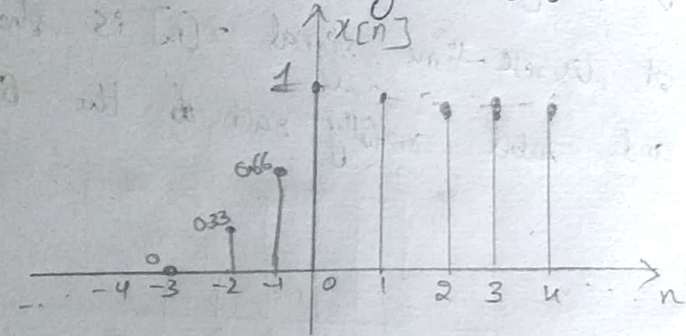
$-3 \Rightarrow 1+\frac{-3}{3} = 1-1=0$

$-2 \Rightarrow 1+\frac{-2}{3} = 0.33$

$-1 \Rightarrow 1+\frac{-1}{3} = 0.66$

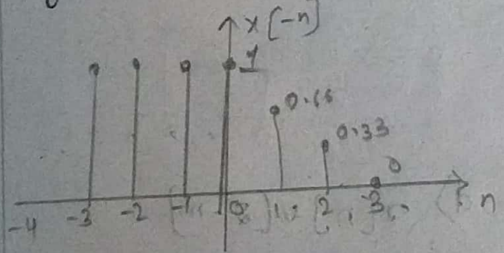
$0 \Rightarrow 1$

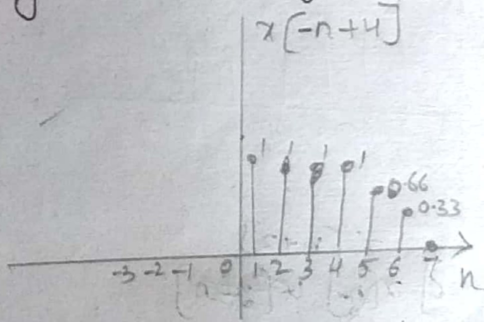
$1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 1$



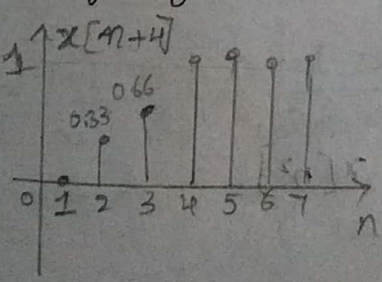
b) Sketch the signals that result if we .

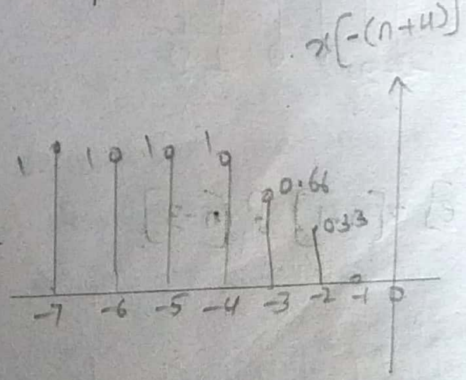
1) first fold $x(n)$ and then delay the resulting signal by four samples .



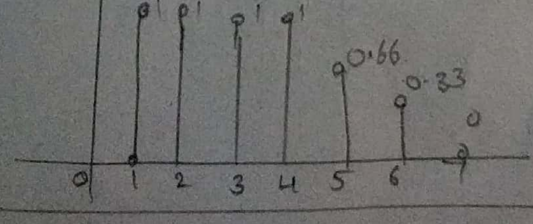


2) first delay $x[n]$ by four samples and then fold the resulting signal .





c) Sketch the signal $x[-n+4]$.

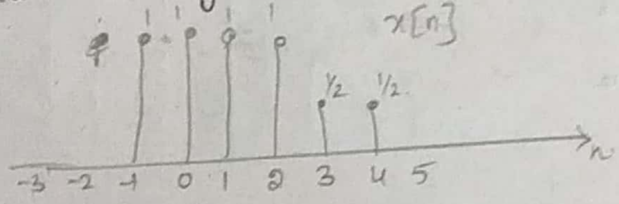


d) Compare the result in parts (b) and (c) and derive a rule for obtaining the signal $x[-n+k]$ from $x[n]$.

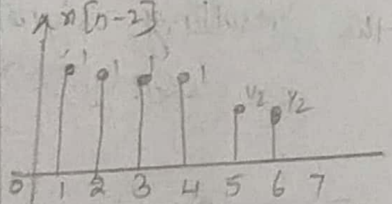
→ For any signal we may have only three operations delay ~~shift~~ advance and folding.

⇒ for geting that Expression first we have to fold the given signal and then perform scaling operations.

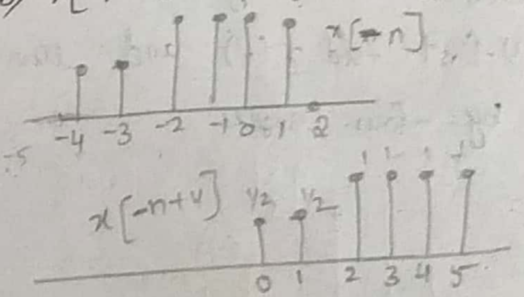
2) A discrete-time signal $x[n]$ is shown in fig. Sketch and label carefully each of the following signals.



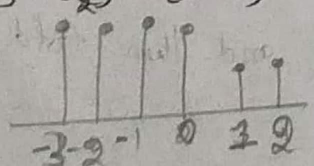
a) $x[n-2]$.



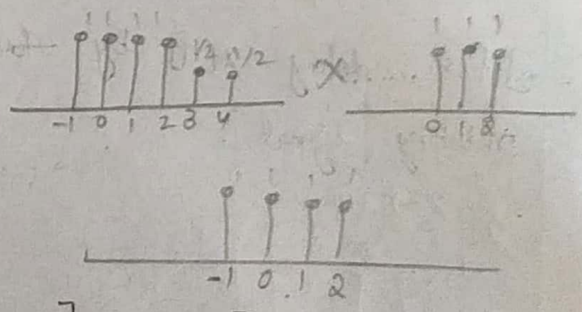
b) $x[4-n] = x[-(n-4)]$



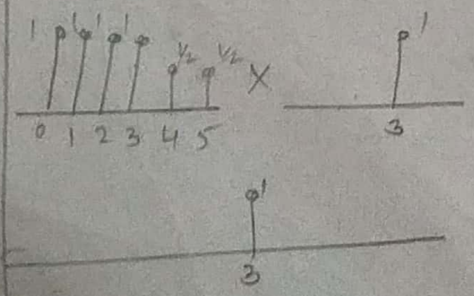
c) $x[n+2]$



d) $x[n]\,u[2-n]$

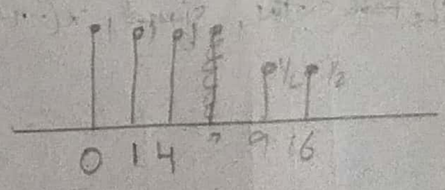


e) $x[n-1]\,\delta[n-3]$



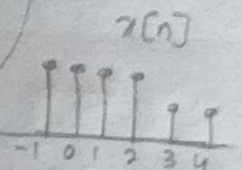
f) $x[n^2]$

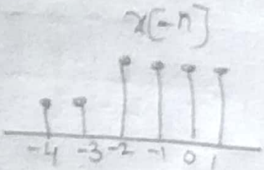
$-1 \Rightarrow 1 \quad 0 \Rightarrow 0 \quad 1 \Rightarrow 1 \quad 2 \Rightarrow 4 \quad 3 \Rightarrow 9 \quad 4 \Rightarrow 16$



g) Even part of $x[n]$.

$$x_e[n] = \frac{x[n]+x[-n]}{2}.$$



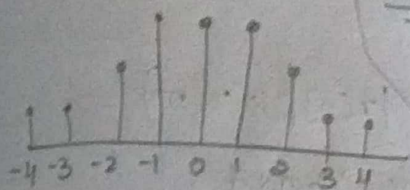


$4 \,\&\, -4 \Rightarrow \frac{0+1/2}{2} = 1/4$

$3 \,\&\, -3 \Rightarrow \frac{0+1/2}{2} = 1/4$

$2 \,\&\, -2 = \frac{0+1}{2} = 1/2$

$-1 \,\&\, 1 \,\&\, 0 \,\& \Rightarrow \frac{1+1}{2} = 1$



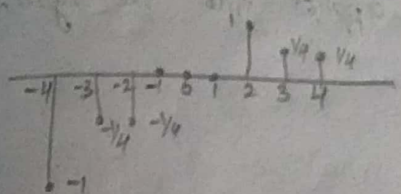
h] odd parts of $x[n]$.

$$x_o[n] = \frac{x[n]-x[-n]}{2}$$

$-3 \,\&\, -4 \Rightarrow \frac{0-1/2}{2} = -1/4$ $\quad 3 \,\&\, 4 \Rightarrow \frac{1/2-0}{2} = 1/4$

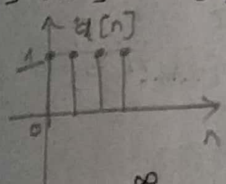
$-2 \Rightarrow \frac{0-1}{2} = -1/2$ $\quad 2 \Rightarrow \frac{1-0}{2} = 1$

$1, 0, -1 \Rightarrow \frac{1-1}{2} = 0$

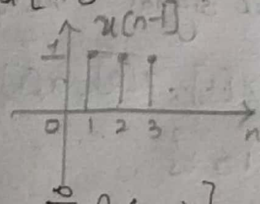


3] Show that:

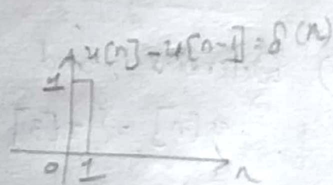
a] $\delta[n] = u[n] - u[n-1]$



$-$

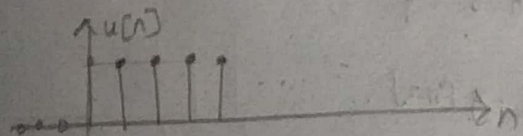


$=$

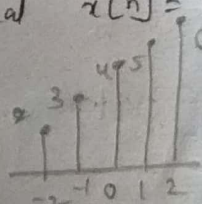


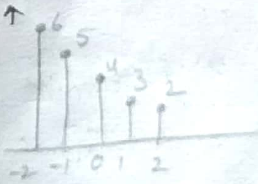
b] $u[n] = \sum_{n=-\infty}^{\infty} \delta(k) = \sum_{n=0}^{\infty} \delta(n-k)$.

$x(n) = \delta[n] + \delta[n-1] + \delta(n-2) + \delta[n-3] + \dots + \delta[n-k]$.



4] Show that any signal can be decomposed into an even and and an odd component. Is the decomposition unique? illustrate your arguments using the signal $x[n] = [2, 3, \underset{\uparrow}{4}, 5, 6]$





$$x_e[n] = \frac{x[n]+x[-n]}{2}$$

$x[-2] = \frac{2+6}{2} = 4$

$x[-1] = \frac{3+5}{2} = 4$

$x[0] = \frac{4+4}{2} = 4$

$x[1] = \frac{5+3}{2} = 4$

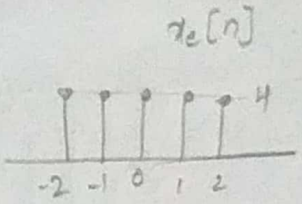
$x[2] = \frac{6+2}{2} = 4$

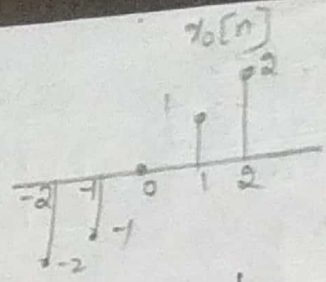
$$x_o[n] = \frac{x[n]+x[-n]}{2}$$

$x[-2] = \frac{2-6}{2} = -2$ $\quad x[1] = \frac{5-3}{2} = 1$

$x[-1] = \frac{3-5}{2} = -1$ $\quad x[2] = \frac{6-2}{2} = 2.$

$x[0] = \frac{4-4}{2} = 0$





6] Consider the system. $y[n] = T[x[n]] = x(n^2)$

a) Determine if the system is time invariant.

The system is time variant.

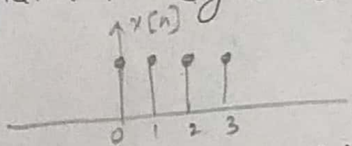
$x[n] \to y[n] = x[n^2]$

$x[n-k] \to y[n^2 - 2nk + k^2] \neq x[n^2]$.

b) To clarify the result in part (a) assume that the signal

$x[n] = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$ is applied into the system.

1) Sketch the signal $x[n]$.



2) Determine and sketch the signal $x[n] = T[x[n]]$.

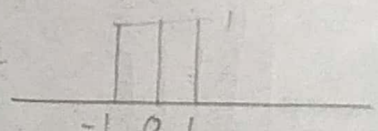
$y[n] = \{x[3], x[2], x[1], x[0], x[1], x[4], x[9]\}$

$= \{0, 0, 1, \underset{\uparrow}{1}, 1, 0, 0\}$.

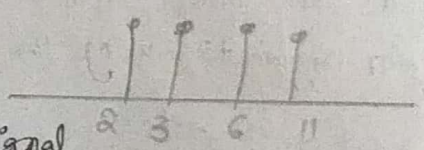
$y[n] = x[n^2] = \{x[0], x[1], x[4], x[9]\}$.

$= \{\underset{\uparrow}{1}, 1, 0, 0, \ldots\}$

3] Sketch the signal $y_2'[n] = y[n-2]$.

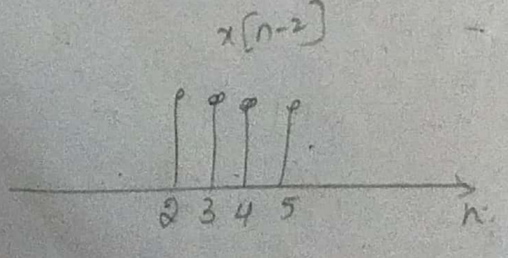


$y_2'[n] = y[n-2] = \{x[2], x[illegible], x[illegible],$



4) Determine and sketch the signal

$x_2[n] = x[n-2]$.



5) Determine & sketch the signal $y_2(n) = T[x_2(n)]$.

sol: $y_2[n] = x_2[n^2]$ $\{..0,1,0,0,0,1,0\}$

$n=0 \Rightarrow x[0]$

$n=-1$ & $1 \Rightarrow x[1]$

$n=-2$ & $2 \Rightarrow x[4]$.

6) Compare the signals $y_2(n)$ and $y[n-2]$ what is your conclusion.

$y_2[n] \neq y[n-2]$

As the system is time Varient.

Q.7 A discrete-time system can be.

1] static & dynamic:

2) Linear & nonlinear

3) Time invarient & time varying

4) Causal & non causal.

5) Stable & unstable.

Examine the following systems w.r.t to the properties above.

a) $y[n] = \cos[x(n)]$.

1] It is static system. because it depends only on present values of i/p.

2) It is Non-linear..

If $x=0$ $y=0$

$\cos(x+y) \neq \cos x + \cos y = 1+1=2.$

$x(0+0) = 1$

3) It is time invarient.

$y(n) = \cos[x(n)]$

$y[n-n_0] = \cos[x(n-n_0)]$

4] It is causal system. As the o/p. depends only on present i/p terms.

5] It is stable system. If we give any bounded i/p we get bounded o/p.

b) $y(n) = \sum_{-\infty}^{n+1} x(k)$

1) It is dynamic system because o/p it depends on previous input values.

2) Linear.

Ex: $x(k_0)+x(k_1)+x(k_2)\ldots = x(n)$

$x(k_0+k_1+k_2+\cdots) = x(n_0)$.

$x(n) = x(n_0)$

3) Time invarient system. If we pass deplay at the i/p side & in system we get same o/p.

4) Non-causal system. It depends on its past values

5) Un stable. Because we may not get bounded o/p for bounded i/p.

c) $y[n] = x[n] \cos[\omega_0 n]$.

1) static system
2) Linear system
3] Time varient.
4) causal system.
5] stable system.

d) $y[n] = x[2-n]$.

1) Dynamic system.
2) Linear system.
3) Time varient system.
4) Causal system.
5) Stable system.

e) $y[n] = Trun|x(n)|$, where $Trun|x(n)|$ dendes the integer part of $x[n]$ obtained by truncation.

1) Static system.
2) Non-linear system.
3) Time invarient, 4) Causal 5) Stable.

f) $y[n]$ = Round $[x[n]]$, where Round $[x[n]]$ denotes the integer part of $x[n]$ obtained by rounding

1) static 2) Non linear 3) time In variant
4) Causal 5) stable.

g) $y[n] = |x[n]|$.

1) Static 2) Non-linear 3) Time invariant 4) Causal.
4) Causal 5) stable.

h) $y[n] = x[n]\, u[n]$
1) Static 2) Linear 3) time in Varient 4) Causal 5) stable.

i) $y[n] = x[n] + n\, x[n+1]$
1) Dynamic 2) linear 3) time varient 5) Non-causal 5) unstable.

j) $y[n] = x[2n]$
1) Dynamic 2) Linear 4) time varient 4) non-causal 5) stable.

k) $y[n] = \begin{cases} x[n], & \text{if } x[n] \geq 0 \\ 0, & \text{if } x[n] < 0 \end{cases}$

1) Static 2) Non-linear 3) time in varient 4) Causal 5) Stable.

l) $y[n] = x[-n]$
1) Dynamic 2) Non-linear 3) Time invarent 4) Causal. 5) stable

m) $y[n] = \text{sign}[x[n]]$
1) Static 2) linear 3) Time invarent. 4) Causal 5) stable.

n) The ideal sampling system with input $x_0(t)$ and output $x[n] = x_0(nT)$; $-\infty < n < \infty$.
1) Static 2) Linear 3) Time invarent 4) Causal 5) stable

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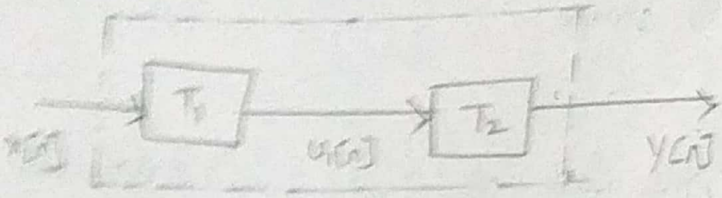
2.8 Two discrete-time systems T_1 and T_2 are connected in cascade to form a new system T as shown in fig. prove & disprove the following statements



$T = T_1 T_2$

a) If T_1 and T_2 are linear, then T is linear {i.e, the cascade connection of two linear systems is linear}.

$v_1[n] = T_1[x_1[n]]$ and $v_2[n] = T_1[x_2[n]]$ then

$$\alpha_1 x_1[n] + \alpha_2 x_2[n] \longrightarrow \alpha_1 v_1[n] + \alpha_2 v_2[n]$$

By the linearity property of T_1, similarly, if

$y_1[n] = T_2[v_1[n]]$ and $y_2[n] = T_2[v_2[n]]$

~~then~~

$$\beta_1 v_1[n] + \beta_2 v_2[n] \rightarrow y[n] = \beta_1 y_1[n] + \beta_2 y_2[n]$$

by the linearity property of T_2 since.

$v_1[n] = T_1[x_1[n]]$ and $v_2[n] = T_2[x_2[n]]$

it follows that

$$A_1 x_1(n) + A_2 x_2[n] \longrightarrow A_1 T[x_1[n]] + A_2 T[x_2[n]].$$

where $T = T_1 T_2$ Hence T is linear.

b) If T_1 and T_2 are time invariant then T is time invariant.

If for T_1

$x[n] \longrightarrow v[n]$

& $x[n-k] \longrightarrow v[n-k]$.

If for T_2

$v[n] \longrightarrow y[n]$

$v[n-k] \longrightarrow y[n-k]$

for T_1, T_2; if $x[n] \longrightarrow y[n]$

$x[n-k] \longrightarrow y[n-k]$.

c) If T_1 & T_2 are causal then T is causal.

T_1 is causal $\Rightarrow v[n]$ depends only on $x(k)$ for $k \leq n$.

T_2 is causal $\Rightarrow y[n]$ depends only on $v[k]$ for $k \leq n$.

$\therefore y[n]$ depends only on $x(k)$ for $k \leq n$. Hence, T is causal.

d) If T_1 and T_2 are linear and time invariant the same holds for T.

combine ⓐ & ⓑ then we will get to understand that T_1 and T_2 are linear and time invaried that holds for T.

e) If T_1 and T_2 are linear and time invariant. Then interchanging their order doesn't change s/m. T.

This follows from $h_1[n] * h_2[n] = h_2[n] * h_1[n]$.

f) As in part (e) except that T_1, T_2 are now time varying

T_1; $y[n] = n\,x[n]$

T_2; $y[n] = n\,x[n+1]$

$T_2[T_1[\delta[n]]] = T_2[0] = 0$

$T_1[T_2[\delta[n]]] = T_1[\delta[n+1]] = 0$.

No, they are time invarient.

g) If T_1 and T_2 are non-linear, then T is nonlinear.

false:- Ex:- T_1; $y[n] = x[n] + b$

T_2; $y[n] = x[n] - b$, where $b \neq 0$.

$T[x[n]] = T_2[T_1[x[n]]] = T_2[x[n]+b] = x[n]$

Hence T is linear.

h] If T_1 and T_2 are stable, then T is stable.

T_1 is stable $\Rightarrow$ $\omega[n]$ is bounded if $x[n]$ is bounded

T_2 is stable $\Rightarrow$ $y[n]$ is bounded if $\omega[n]$ is bounded

Then $y[n]$ is bounded if $x[n]$ is bounded.

Then the system is bounded.

i) Show by an example that the inverse of parts (c) and (h) do not hold in general.

Inverse of (c). T_1 and for T_2 are non causal $\Rightarrow$ T is non causal

Ex:- T_1; $y[n] = x[n+1]$

T_2; $y[n] = x[n-2]$

T; $y[n] = x[n-1]$

which is causal. Hence, the inverse of (c) is false. Inverse of (h); T_1 and (or) T_2 is unstable, implies T is unstable

Ex:- $y[n] = e^{x[n]}$, stable and

T_2; $y[n] = \ln[x[n]]$ which is unstable.

But T; $y[n] = x[n]$, which is stable. Hence the inverse of (h) is false.

2.9) Let T be an LTI relaxed and BIBO stable system with input $x[n]$ and output $y[n]$ Show that:

a) If $x[n]$ is periodic with period N. {i.e, $x[n] = x[n+N]$ for all $n \geq 0$]. the output $y[n]$ tends to a periodic signal with same period.

b) If $x[n]$ is bounded and tends to a constant. the output will also tend to a constant.

c) If $x[n]$ is an Energy signal. the output $y[n]$ will also be an Energy signal.

Sol:- $x[n] = x[n+N]$ $\forall$ $n \geq 0$

$$y[n] = \sum_{k=-\infty}^{n} h(k)\, x(n-k)$$

$$y[n+N] = \sum_{k=n+N}^{n+N} h(k)\, x(n-k) + \sum_{k=-\infty}^{n} h(k)\, x(n-k)$$

$$y[n+N] = y(n) + \sum_{k=n+1}^{n+N} h(k)\, x(n-k)$$

for BIBO system $\lim_{n\to\infty} |h(n)| = 0$

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)\, x(n-k) = 0$$

$$\lim_{n\to\infty} y(n+N] = y[n]$$

$$\therefore y[N] = y[n+N]$$

b) $x[n] = x_0[n] + \alpha u[n]$

where $x_0[n] \rightarrow$ bounded with $\lim_{n\to\infty} x_0[n] = 0$

$$\sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_0^2[n] < \infty$$

hence $\lim_{n\to\infty} |y_0[n]| = 0$

$$\alpha \sum_{k=0}^{n} h(k) = \text{constant}$$

c) $y[n] = \sum_k h(k)\, x(n+k)$

$$\sum_{-\infty}^{\infty} y^2[n] = \sum_{-\infty}^{\infty} \left[\sum_k h(k)\, x(n-k)\right]^2$$

$$= \sum_k \sum_y h(k)\, h(l) \sum_n x(n-k)\, x(n-l)$$

but $\sum_n x(n-k)\, x(n-l) \le \sum_n x^2(n)\, |h(l)|$

for BIBO stable system $\sum_k |h(k)| < M$

Hence $E_y \le M^2 E_x$, So that $E_y < 0$ if $E_x < 0$

2.10 The following input-output pairs have been observed during the operation of a time-invarient system.

$x_1[n] = [\underset{\uparrow}{1},0,2] \xleftrightarrow{T} y_1[n] = [\underset{\uparrow}{0},1,2]$

$x_2[n] = [0,0,3] \xleftrightarrow{T} y_2[n] = [0,\underset{\uparrow}{1},0,2]$

$x_3[n] = [\underset{\uparrow}{0},0,0,1] \xleftrightarrow{T} y_3[n] = [1,\underset{\uparrow}{2},1]$

Can you draw any conclusions regarding the linearity of the system. What is the impulse response of the system.

sol. As this is a time-invarient system
$y_2[n]$ should have only 3 elements and
$y_3[n]$ should have 4 elements wrt to inputs
so, it is non-linear.

11 The following input-output pairs have been observed

$x_3[n] = \{0,\underset{\uparrow}{1}\} \xrightarrow{T} y_3[n] = \{1,2,1\}$

Can you draw any conclusions about the time invarient of this system?

$x_1[n] = \{-1,\underset{\uparrow}{2},1\} \xleftrightarrow{T} y[n] = \{1,\underset{\uparrow}{2},-1,0,1\}$

$x_2[n] = \{1,\underset{\uparrow}{-1},-1\} \xleftrightarrow{T} y_3[n] = \{\underset{\uparrow}{1},2,1\}$

$x_1[n]+x_2[n] = \delta[n]$ and system is linear the impulse response of the system?

$y_1[n] + y_2[n] = \{0,3,-1,2,1\}$

If s/m are time invarient the response of $x_3[n]$ would be $\{3,2,3,1\}$.

12 The only available information about a system consists of N input-output pairs of signals

$y_i[n] = T[x_i[n]] \quad , i = 1,2,\cdots,N$.

a] What is the class of input signals for which we can determine the output, using the information above. if the system is known to be linear?

sol. (13)

b] The same as above, if the system is known to be time invarient.?

a] Linear combination of signal in the form of

$x_i[n]$; $i = 1, 2, 3, \ldots\ldots$

because, if we take $i = 1, 3, \ldots$

$y_1[n] = x_1[n]$

$y_3[n] = x_3[n]$

$y[n] = y_1[n] + y_3[n] = x_1[n] + x_3[n]$

b] Same repeat, for the s/m is invarient Any $x_i[n-k]$ where k is any integer, $i = 1, 2, \ldots N$

1st replace $n = n - n_0 \Rightarrow x_i[n - n_0 = k]$

$x[n]$ by $x[n-n_0] \Rightarrow x_i[n-k-n_0]$

Time invarient.

13] Show that the necessary and suffiont condition for a relaxed LTI system to be BIBO stable is.

$\sum_{n=-\infty}^{\infty} |h(n)| \leq M_h < \infty$ for some constant M_h.

A system is to be BIBO stable only when bounded output should produce bounded input

$$y[n] = \sum_k h(k) \cdot x(n-k)$$

$$|y(n)| = \sum_k |h(k)| \cdot |x(n-k)|$$

$$= \sum_k |x(n-k)| \leq M_n \{\text{some constant}\}$$

So $\sum_{n=-\infty}^{\infty} |y(n)|$

A system to be BIBO stable only when bounded input produce bounded output.

$$|y(n)| = \sum_{k=-\infty}^{\infty} |h(k)| \, |x(n-k)|$$

as $\sum_{k=-\infty}^{\infty} |x(n-k)| \leq M_n$ for some constant

$|y(n)|$ if and only if $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$.

so $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$.

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14) Show that

a) A relaxed linear system is causal if & only if for any input $x[n]$ such that $x(n)=0$ for $n<n_0 \Rightarrow y(n)$ for $n<n_0$.

If a system is causal output depends only on the present and past inputs as $x(n)=0$ for $n<0$

$y(n)$ also becomes 0 for $n<0$.

b) A relaxed LTI system is causal if and only if

$h(n)=0$ for $n<0$.

$$y(n) = \sum_{k=0}^{\infty} h(k)\,x[n-k]$$

$h(n)=0$, $n<0$ and $n\geq m$.

So $y(n)$ reduces to $y(n) = \sum_{k=0}^{m-1} h(k)\cdot x(n-k]$

if it is infinite impulse response

Then $y(n) = \sum_{k=0}^{\infty} h(k)\,x[n-k]$.

15

a) Show that for any real or complex constant a, and any finite integer numbers M and N, we have.

$$\sum_{n=M}^{N} a^n = \begin{cases} \dfrac{a^M - a^{N+1}}{1-a} & \text{if } a \neq 1 \\ N-M+1 & \text{if } a=1 \end{cases}$$

for $a=1$; $\sum_{n=M}^{N} a^n = N-M+1$

for $a\neq 1$; $\sum_{n=M}^{N} a^n = \dfrac{a^M - a^{N+1}}{1-a}$

$$(1-a)\sum_{n=M}^{N} a^n = a^M + a^{M+1} + a^{M+2} + \cdots \; a^{N} - a^{N} - a^{1}$$

$$= a^M - a^{N+1}.$$

b) Show that if $|a|<1$, then $\sum_{n=0}^{\infty} a^n = \dfrac{1}{1-a}$

for $M=0$, $|a|<1$ and $N\to\infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad , |a|<1$$

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a) If $y[n] = x[n] * h[n]$, show that $\Sigma_y = \Sigma_x \Sigma_h$, where $\Sigma_x = \sum_{n=-\infty}^{\infty} x[n]$.

$y[n] = \sum_k h(k)\, x(n-k)$

$\sum_n y[n] = \sum_n \sum_k h(k) \cdot x(n-k)$

$\sum_n y[n] = \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k)$

$\sum_n y[n] = \left(\sum_k h(k)\right) \cdot \left(\sum_n x[n]\right)$

b] Compute the convolution $y[n] = x[n] * h[n]$ of the following signals and check the corrections of the results by using the test in (a).

i) $x[n] = \{1, 2, 4\}$, $h[n] = \{1, 1, 1, 1, 1\}$

$y[n] = \{1, 3, 7, 7, 7, 6, 4\}$

$\sum_n y[n] = \sum_n x[n] \sum_n h[n]$

$35 = 7 \times 5$

$35 = 35$

h(n) \ x(n)	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

ii) $x[n] = \{1, 2, -1\}$, $h[n] = x[n]$.

$x[n] = \{1, 2, -1\}$, $h[n] = \{1, 2, -1\}$

$y[n] = x[n] * h[n]$

$y[n] = \{1, 4, 2, -4, 1\}$

$\sum_n y[n] = 4$; $\sum_n x[n] = 2$.

$\sum_n y(n) = \sum_n x(n)\, h(n)$

$4 = 2 \times 2 = 4$

h(n) \ x(n)	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

3] $x[n] = \{0, 1, -2, 3, -4\}$ $h[n] = \{1/2, 1/2, 1, 1/2\}$

$y[n] = \{0, 1/2, -1/2, 3/2, -2, 0, -5/2, 2\}$

$\sum_n y[n] = \sum_n x(n)\, h(n)$

$-5 = -2 \times 5/2$

$-5 = -5$

	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

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iv) $x[n] = \{1,2,3,4,5\}$, $h[n] = \{1\}$

$y[n] = \{1,2,3,4,5\}$

$\sum_n y[n] = \sum_n x[n] \cdot \sum_n h[n]$

$15 = 15 \times 1$

$15 = 15$

v) $x[n] = \{1,-2,3\}$ $h[n] = \{0,0,1,1,1,1\}$

h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

$y[n] = \{0,0,1,-1,-2,2,1,3\}$

$\sum_n y[n] = \sum_n x(n) \cdot h[n]$

$8 = 2 \times 4$

$8 = 8$.

vi) $x[n] = \{0,0,1,1,1,1\}$, $h[n] = \{1,-2,3\}$

h(n) \ x(n)	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$y[n] = \{0,0,1,-1,2,2,1,3\}$.

$\sum y(n) = 8$; $\sum_n x(n) = 2$ $\sum_n h[n] = 4$

$\sum y[n] = \sum_n x(n)\, h[n]$

$8 = 8$.

vii) $x[n] = \{0,1,4,-3\}$, $h[n] = \{1,0,-1,-1\}$

h(n) \ x(n)	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

$y[n] = \{0,1,4,-4,-5,-1,3\}$

$\sum_n y[n] = \sum_n x[n]\, h[n]$

$-2 = -2 \times 1$

$-2 = -2$

viii) $x[n] = \{1,1,2\}$ $h[n] = u[n]$

h(n) \ x(n)	1	1	2
1	1	1	2
1	1	1	2
1	1	1	2

$y[n] = \{\underset{\uparrow}{1},2,4,3,2\}$

$\sum_n y[n] = \sum_n x[n]\, h[n]$

$12 = 4 \times 3$.

$12 = 12$.

1x) $x[n] = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$ $h[n] = \{1, -2, -3, 4\}$

$y[n] = \{1, -1, -5, 2, \underset{\uparrow}{3}, -5, 1, 4\}$

h(n) \ x(n)	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
4	4	4	0	4	4

$\sum_n y[n] = \sum_n x[n] \, h[n]$

$0 = 4 \times 0$

$0 = 0$.

x) $x[n] = \{1, 2, \underset{\uparrow}{0}, 2, 1\}$ $h[n] = x[n]$

$y[n] = \{1, 4, 4, 4, \underset{\uparrow}{10}, 4, 4, 4, 1\}$.

h(n) \ x(n)	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

$\sum_n y[n] = \sum_n x[n] \, h[n]$

$36 = 6 \times 6$

x1) $x[n] = \left[\frac{1}{2}\right]^n \cdot u[n]$, $h[n] = \left(\frac{1}{4}\right)^n \cdot u[n]$.

$y[n] = \left[2 (1/2)^n - \left(\frac{1}{4}\right)^n\right] u[n]$

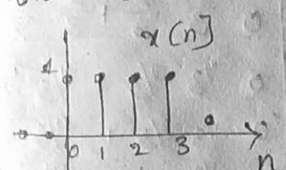
$\sum_n y[n] = \sum_n h[n] \, x[n]$

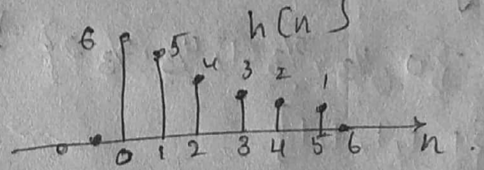
$\frac{8}{3} = \frac{4}{3} \times 2$.

$\frac{8}{3} = \frac{8}{3}$.

17. Compute and plot the convolutions $x[n] * h[n]$ and $h[n] * x[n]$ for the pairs of signals shown in figures.

(a)



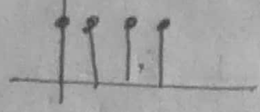


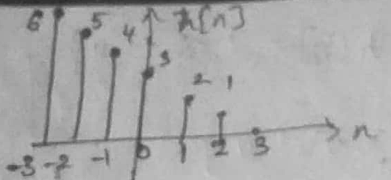
$x[n] = \{1, 1, 1, 1\}$ $h[n] = \{6, 5, 4, 3, 2, 1\}$

$y[n] = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

h(n) \ x(n)	1	1	1	1
6	6	6	6	6
5	5	5	5	5
4	4	4	4	4
3	3	3	3	3
2	2	2	2	2
1	1	1	1	1

b]

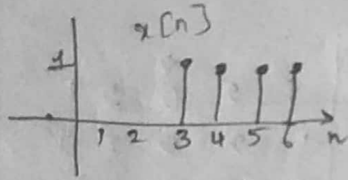


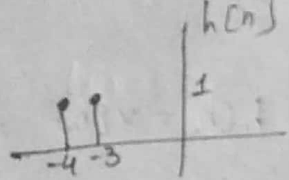


$x[n] = \{1,1,1,1\}$ $h[n] = \{6,5,4,3,2,1\}$

$y[n] = \{6,11,15,18,14,10,6,3,1\}$.

c)



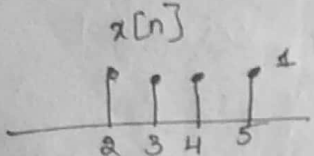


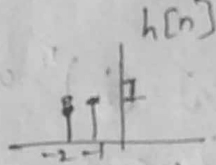
h(n) \ x(n)	1	1	1	1
1	1	1	1	1
1	1	1	1	1

$x(n) = \{1,1,1,1\}$ $h[n] = \{1,1\}$.

$y[n] = \{1,2,2,2,1\}$.

d)





	0	1	1	1	1
1	0	1	1	1	1
1	0	1	1	1	1
0	0	0	0	0	0

$x[n] = \{0,1,1,1,1\}$ $h[n] = \{1,1,0\}$

$y[n] = \{0,1,2,2,2,1,0\}$.

18) Determine and sketch the convolution $y[n]$ of the signals

$$x[n] = \begin{cases} \frac{1}{3}n & 0 \le n \le 6 \\ 0 & \text{elsewhere} \end{cases} \qquad h[n] = \begin{cases} 1 & -2 \le n \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

a] Graphically.

h(n) \ x(n)	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2
1	0	1/3	2/3	1	4/3	5/3	2

$x[n] = \{0, 1/3, 2/3, 1, 4/3, 5/3, 2\}$

$h[n] = \{1,1,1,1,1\}$

$y[n] = \{0, 1/3, 1, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

b] Analytically.

$x[n] = \frac{1}{3}n\{u[n] - u[n-1])$

$h[n] = u[n+2] - u[n-3]]$

$y[n] = \frac{1}{3}n\left[u[n] - u[n-3] - x((n+2) - u[n-3])\right]]$

20

[illegible]

[illegible]

Consider the following [illegible] equation

Ex: Compute the convolution $y[n]$ of the signals

$$x[n] = \begin{cases} 5^n & -3 \le n \le 5 \\ 0 & \text{otherwise} \end{cases} \qquad h[n] = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$$

Sol:

$$y[n] = \sum_{k=0}^{4} h[k]\, x(n-k)$$

$$x[n] = \{5^{-3}, 5^{-2}, 5^{-1}, 1, 5, \ldots 5^{5}\}$$

$$h[n] = \{1, 1, 1, 1, 1\}$$

$$y[n] = \begin{cases} \sum_{k=0}^{4} x(n-k), & -3 \le n \le 9 \\ 0 & \text{otherwise} \end{cases}$$

$y(-3) = 5^{-3}$

$y(-2) = x(-3) + x(-2) = 5^{-3} + 5^{-2}$

$y(-1) = 5^{-3} + 5^{-2} + 5^{-1}$

$y(0) = 5^{-3} + 5^{-2} + 5^{-1} + 1$

$y(1) = 5^{-3} + 5^{-2} + 5^{-1} + 1 + 5$

$y(2) = 5^{-2} + 5^{-1} + 1 + 5 + 5^{2}$

$y(3) = 5^{-1} + 1 + 5 + 5^{2} + 5^{3}$

$y(4) = 5^{4} + 5^{3} + 5^{2} + 5 + 1$

$y(5) = 5 + 5^{2} + 5^{3} + 5^{4} + 5^{5}$

$y(6) = 5^{2} + 5^{3} + 5^{4} + 5^{5}$

$y(7) = 5^{3} + 5^{4} + 5^{5}$

$y(8) = 5^{4} + 5^{5}$

$y(9) = 5^{5}$

20 Consider the following three operations.

a) Multiply the integer numbers : 131 and 122.

$131 \times 122 = 15982$.

b) Compute the convolution of signals $(1,3,1) * (1,2,2)$

$y[n] = \{1, 5, 9, 8, 2\}$.

	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

c) Multiply the polynomials $1+3z+z^2$ and $1+2z+2z^2$.

$= (z^2+3z+1) * (2z^2+2z+1)$

$= 2z^4 + 8z^3 + 9z^2 + 5z + 1$

d) Repeat part (a) for the numbers 1.31 and 12.2.

$1.31 * 12.2 = 15.98$

e) Comment on your result.

These are different ways to perform convolution.

21) Compute the convolution $y[n] = x[n] * h[n]$ of the following pairs of signals.

a) $x[n] = a^n u[n]$, $h[n] = b^n u[n]$ when $a \neq b$ and when $a = b$.

$y[n] = x[n] * h[n]$

$= a^n u[n] * b^n u[n]$.

$= [a^n * b^n] u[n]$

$$y[n] = \sum_{k=0}^{n} a^k u[n] . b^{n-k} u(n-k)$$

$$= b^n \sum_{k=0}^{n} a^k u(k) b^{-k}.$$

$$= b^n \sum_{k=0}^{n} (ab)^{-k}.$$

if $a \neq b$ then $y[n] = \dfrac{b^{n+1} - a^{n+1}}{b-a} u[n]$

if $a = b \Rightarrow b^n(n+1) . u[n]$.

b) $x[n] = \begin{cases} 1 & n = -2, 0, 1 \\ 2 & n = -1 \\ 0 & \text{elsewhere} \end{cases}$

$h[n] = \delta[n] - \delta[n-1] + \delta(n-4) + \delta(n-5)$

$x[n] = \{1, 2, 1, 1\}$

$h[n] = \{1, -1, 0, 0, 1, 1\}$

$y[n] = \{1, 1, -1, 0, 0, 3, 3, 2, 1\}$.

h(n) \ x(n)	1	2	1	1
1	1	2	1	1
-1	-1	-2	-1	-1
0	0	0	0	0
0	0	0	0	0
1	1	2	1	1
1	1	2	1	1

c) $x[n] = u(n+1) - u(n-4) - \delta(n-5)$

$h[n] = [u[n+2) - u(n-3)] \cdot (3 - |n|)$

$x(n) = \{1, 1, 1, 1, 1, 0, -1\}$ $\quad h[n] = \{1, 2, 3, 2, 1\}$

$y[n] = 1, 3, 6, 8, 5, 11, -2, -3, -1\}$

x(n) \ h(n)	1	1	1	1	1	0	-1
1	1	1	1	1	1	0	-1
2	2	2	2	2	2	0	-2
3	3	3	3	3	3	0	-3
2	2	2	2	2	2	0	-2
1	1	1	1	1	1	0	-1

d) $x(n) = u[n] - u[n-5]$

$h[n] = u(n-2] - u(n-8] + u(n-11)$
$\quad - u(n-17)$

$x(n) = \{1, 1, 1, 1, 1\}$

$h[n] = \{0, 0, 1, 1, 1, 1, 1, 1\}$

$h[n] = h^1[n] + h^1[n-9)$

$y[n] = y'(n) + y'(n-9)$

$y'(n) = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

2) Let $x[n]$ be the input signal to a ~~distr~~ discrete-time filter with impulse response $h_i(n)$ and let $y(n)$ be the corresponding output.

a) Compute and sketch $x[n]$ and $y[n]$ in the following cases using the same scale in all figures.

$x[n] = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$h_1[n] = [1, 1]$

$h_2[n] = [1, 2, 1]$

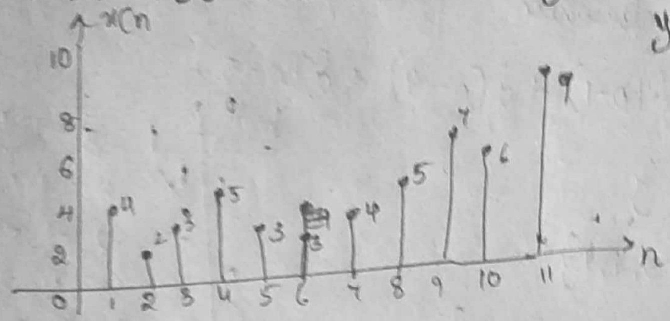
$h_3[n] = [\frac{1}{2}, \frac{1}{2}]$

$h_4[n] = [\frac{1}{4}, \frac{1}{2}, \frac{1}{4}]$

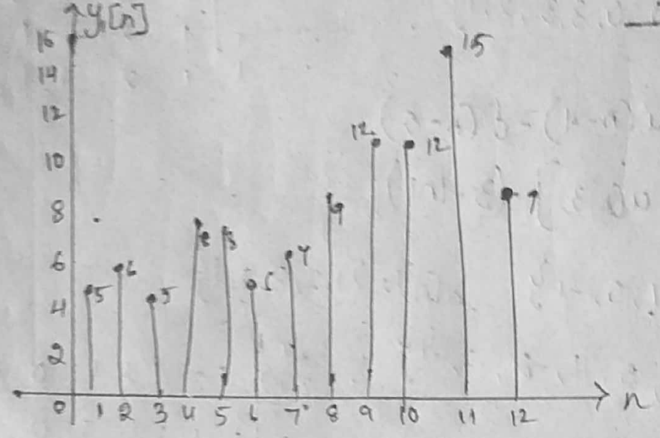
$h_5[n] = [\frac{1}{4}, -\frac{1}{2}, \frac{1}{4}]$

Sketch $x[n]$, $y[n]$, $y_2[n]$ on one graph and $x[n]$, $y_3[n]$, $y_4[n]$, $y_5[n]$ on another graph.

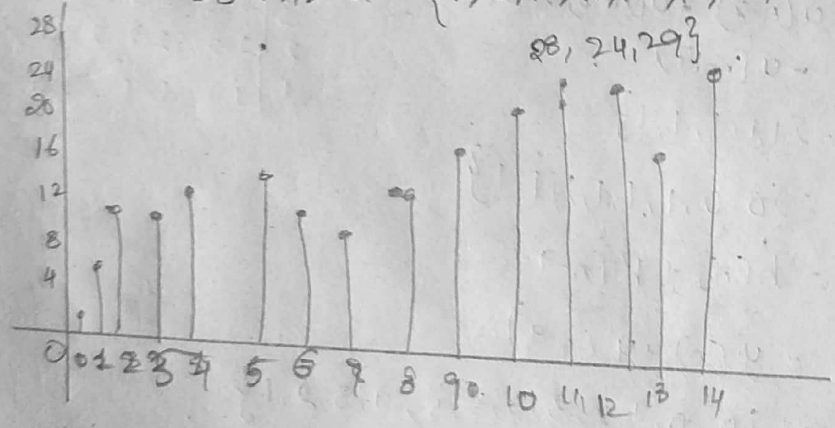
$y_1[n] = x[n] * h_1[n]$

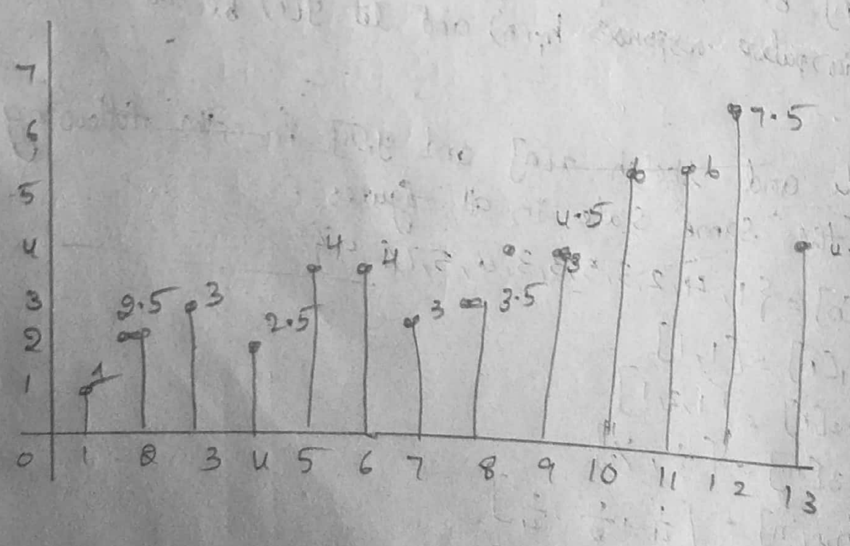


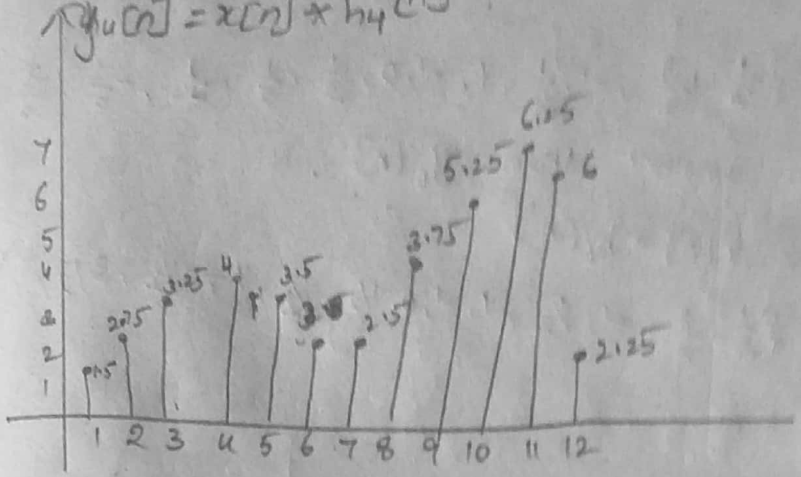
	1	4	2	3	5	3	3	4	5
1	1	4	2	3	5	3	3	4	5
1	1	4	2	3	5	3	3	4	5

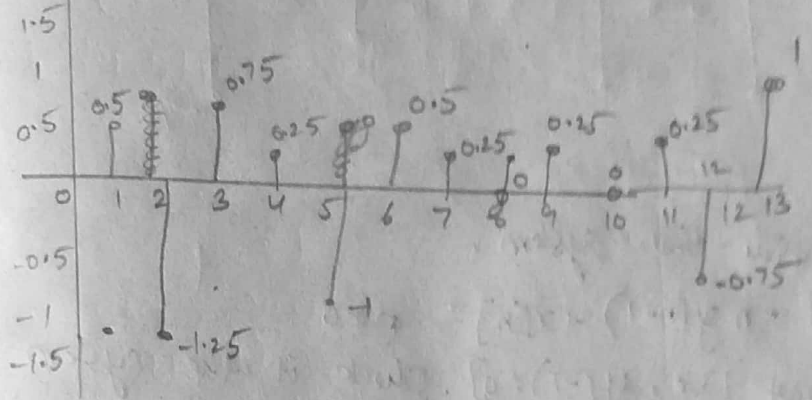


$y_2[n] = x[n] * h_2[n] = \{1, 6, 11, 11, 13, 16, 14, 13, 16, 24, 25, 28, 24, 29\}$









b) What is the difference between $y_1[n]$ and $y_2[n]$ and between $y_3[n]$ and $y_4[n]$.

$y_3[n] = \frac{1}{2} y_1[n]$, because

$h_3[n] = \frac{1}{2} h_1[n]$

$y_4[n] = \frac{1}{4} y_2[n]$, because $h_4[n] = \frac{1}{4} h_2[n]$.

c) comment on the smotness of $y_2[n]$ and $y_4[n]$ and between $y_3[n]$ and $y_4[n]$?

$y_2[n]$ and $y_4[n]$ are smother than $y_1[n]$, but $y_4[n]$ will appear even smother because of the smaller scalar factor.

d) Compare $y_4[n]$ with $y_5[n]$. What is the difference? Can you Explain it?

$y_4[n]$ System results in a smother output the negative value of $h_5[0]$ is responsible for the

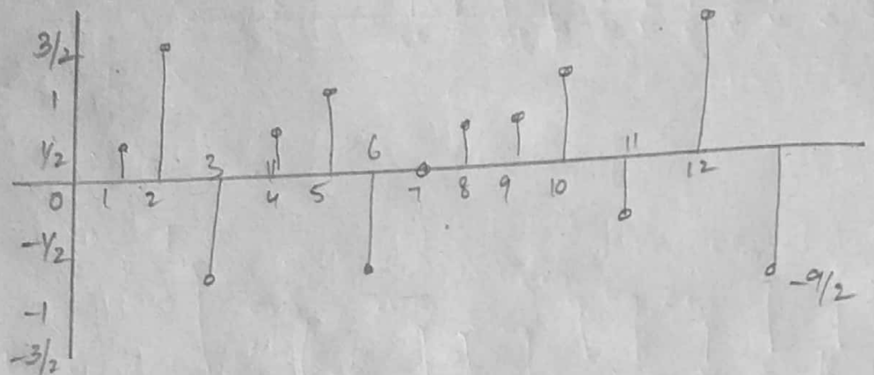
Non-Smother characteristics of $y_5[n]$.

e) $y_6[n] = \{\frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{3}{2}, \frac{-3}{2}\}$

$y_5[n]$ is smother than $y_6[n]$.

$y_6[n] = x[n] * h_6[n]$

$y_6[n] = \{\frac{1}{2}, \frac{3}{2}, -1, 1/2, 1, -1, 0, 1/2, 1/2, 1, -1/2, 3/2, -9/2\}$.



23 The discrete-time system.

$y[n] = n\,y(n-1) + x[n] \quad n \geq 0$

is at rest [i.e., $y(-1) = 0$]. Check if the system is linear time invarent and BIBO stable.

⇒ Express the output $y[n]$ of a linear time-invarent system with impulse response $h[n]$ in terms of its step response and the input $x[n]$.

$\delta[n] = h[n] * u[n]$ and the input $x[n]$.

We can express $\delta[n] = u[n] - u[n-1]$

$h[n] = h[n] * \delta[n]$

$= h[n] * [u[n] - u[n-1]]$

$= h[n] * u[n] - h[n] * u[n-1]$

$= \delta[n] - \delta(n-1)$

then $y[n] = h(n) * x[n]$

$= [\delta[n] - \delta[n-1]] * x[n]$

$= \delta[n] * x[n] - \delta(n-1) * x(n)$

24. Conside the signal $r[n] = a^n u[n]$, $0 < a < 1$

a) show that any sequence $x[n]$ can be decomposed as $x[n] = \sum_{n=-\infty}^{\infty} c_k\, r(n-k)$ and express c_k in terms of $x[n]$

$$\gamma[n] = \delta[n] - a\,\delta[n-1]$$

$$\gamma[n-k] = \delta[n-k] - a\,\delta(n-k-1)$$

$$x[n] = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\left[\delta(n-k) - a\,\delta(n-k-1)\right]$$

$$x[n] = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k-1)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) - a\sum_{k=-\infty}^{\infty} x(k-1)\,\delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left[x(k) - a\,x(k-1)\right]\delta(n-k)$$

thus $c_k = x[k] - a\,x(k-1)$.

b) Use the properties of linearity and time invariance to express the output $y[n] = T[x[n]]$ in terms of the input $x[n]$ and the signal $g[n] = T[\gamma[n]]$ where $T[\cdot]$ is an LTI system.

$$y[n] = T[x[n]]$$

$$= T\left[\sum_{k=-\infty}^{\infty} c_k\,\delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} c_k\,T[\gamma[n-k]]$$

$$= \sum_{k=-\infty}^{\infty} c_k\,\delta[n-k]$$

c) Express the impulse response $h[n] = T[\delta[n]]$ in terms of $g[n]$.

$$h[n] = T[\delta[n]]$$

$$h[n] = T[\delta[n] - a\,\delta[n-1]]$$

$$= g[n] - a\,g[n-1]$$

25] Determine the zero-input response of the system described by the second order difference equation $x[n] - 3y[n-1] - 4y[n-2] = 0$.

Sol:-

$x[n] - 3y[n-1] - 4y[n-2] = 0$.

with $x[n] = 0$

$-3y[n-1] - 4y[n-2] = 0 \quad \{\div(-3)\}$

$y[n-1] + \frac{4}{3} y[n-3] = 0$

at $n=0$

$y[-1] = \frac{-4}{3} y[-3]$

at $n=1$

$y[0] = \frac{-4}{3} y[-1] = \left(\frac{-4}{3}\right)^2 y[-2]$.

$y[1] = \left[\frac{-4}{3}\right]^3 y[-2]$

$y[k] = \left[\frac{-4}{3}\right]^{k+2} y[-2]$.

26] Determine the particular solution of the difference equation. $y[n] = \frac{5}{6} y[n-1] - \frac{1}{6} y[n-2] + x[n]$. When the forcing function is $x[n] = 2^n u[n]$.

Sol:-

$y[n] = \frac{5}{6} y[n-1] - \frac{1}{6} y[n-2] + x[n]$

$x[n] = y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2]$

Characteris Equation is

$\lambda^2 = \frac{5}{6} x + \frac{1}{6} = 0 \quad ; \lambda = 1/2, 1/3$

so $y_h[n] = C_1[1/2]^n + C_2[1/3]^n$

$x(n) = 2^n u[n]$

$y_p[n] = k[2^n] u[n]$

so, $k(2^n) \cdot u[n] - k(5/6)(2^{n-1}) u(n-1) + k(1/6) 2^n \cdot u[n-2]$

$= 2^n u[n]$.

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for $n=2$.

$$4K - \frac{5K}{3} + \frac{K}{6} = 4 \Rightarrow k = \frac{8}{5}$$

Total solution is $y_p[n] + y_h[n] = y[n]$.

$$y[n] = \frac{8}{5} 2^n \cdot u[n] + C_1 (1/2)^n \cdot u[n] + C_2 [1/3]^n \cdot u[n]$$

Assume $y[-2] = y[-1] = 0$,

so, $y[0] = 1$.

then $y(1) = \frac{5}{6} y(0) + 2 = \frac{17}{6}$

so $\frac{8}{5} + C_1 + C_2 = 1$

$$C_1 + C_2 = 3/5 \longrightarrow ①$$

$$\frac{16}{5} + 1/2 C_1 + \frac{1}{3} C_2 = \frac{17}{6}$$

$$3C_1 + 2C_2 = -\frac{11}{5} \rightarrow ②$$

By solving ① & ② we get $C_1 = -1$ and $C_2 = 2/5$.

so the total solution is

$$y[n] = \left[8/5 (2)^n - (1/2)^n + \frac{2}{5} (1/3)^n \right] \cdot u[n]$$

27. Determine the response $y[n]$, $n \geq 0$ of the system described by the second order difference equation $y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$ to the input $x[n] = 4^n u[n]$.

Sol:- $y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$

Charecteristic equation is $\lambda^2 - 3\lambda - 4 = 0$

$$\lambda = 4, -1$$

so, $y_h[n] = C_1 \times 4^n + C_2 (-1)^2$.

$x[n] = 4^n u[n]$.

$y_p[n] = K_n 4^n u[n]$

$K_n 4^n u[n] - 3k(n-1)\, 4^{n-1} u(n-1) - 4\, x(n-2)\, 4^{n-2} \cdot u[n-2] =$

$4^n u[n] - 1 \cdot 2(4)^{n-1} u[n-1]$

$n=2 \quad K(32-2) = 4^2+8 = 24$.

$K = 6/5$

The total solution is $y[n] = y_p[n] + y_h[n]$.

$= \{6/5\, n \cdot 4^{n-1} + C_1 4^n + C_2 (-1)^n\} u[n]$

to find C_1 and C_2, let $y[-2]$ & $y[-1] = 0$ then

$y(1) = 3y(0) + 4 + 2 = 9$.

$C_2 + C_1 = 1 \rightarrow ①$

$\frac{24}{5} + 4C_1 - C_2 = 9 \Rightarrow 4C_1 - C_2 = \frac{21}{5} \rightarrow ②$

from ① & ②

$C_1 = \frac{26}{25}$; $C_2 = \frac{-1}{25}$

so, $y[n] = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n\right] u[n]$

28) Determine the impulse response of the following causal system.

$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$

Charecteristic Equation $\lambda^2 - 3\lambda - 4 = 0$

$\lambda = -4, 1$

$y_h[n] = C_1 4^n + C_2 (-1)^n$

$x[n] = \delta[n]$

$y[0] = 1$ and $y[1] = 3 \cdot y[0] + 2$

$y[1] = 5$

so, $C_2 + C_1 = 1 \rightarrow ①$

$4C_1 - C_2 = 5 \rightarrow ②$

from ① & ② $\quad C_1 = 6/5$ and $C_2 = -1/5$

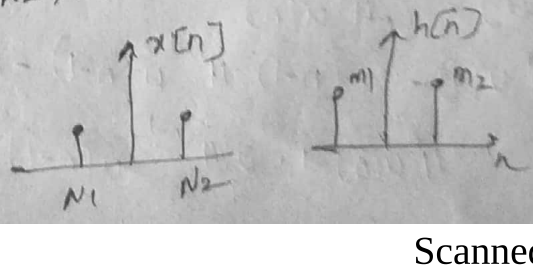
$h[n] = \left[\frac{6}{5} 4^n - \frac{1}{5} (-1)^n\right] u[n]$

29) Let $x[n]$, $N_1 \le n \le N_2$ and $h[n]$, $M_1 \le n \le M_2$ be two finite - discrete signals.

a] Determine the range $L_1 \le n \le L_2$ of their convolution in terms of N_1, N_2, M_1 and M_2.

$L_1 = N_1 + M_1$

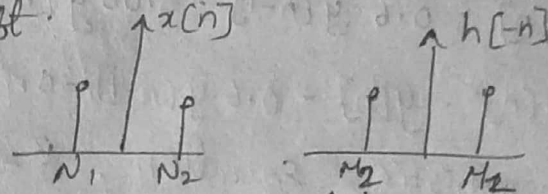
$L_2 = N_2 + M_2$



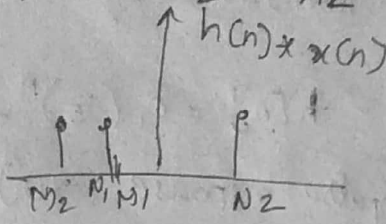
b) Determine the limits of the cases of partial overlap from the left, full overlap, and partial overlap from the right. For conveniance, assume that $h[n]$ has shorter duration than $x[n]$.

partial overlap from left.

$\rightarrow x[n] * h[n] \Rightarrow$



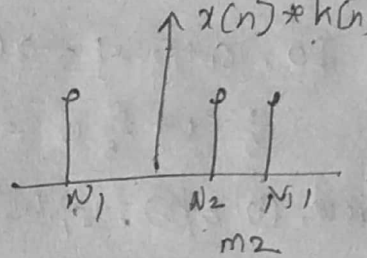
low N_1+M_1 & high M_2+N_1-1



If fully overlap then

N_1+m_2 (low) & high N_2+m_1

partial overlap from the right



low $\Rightarrow N_2+m_1+1$

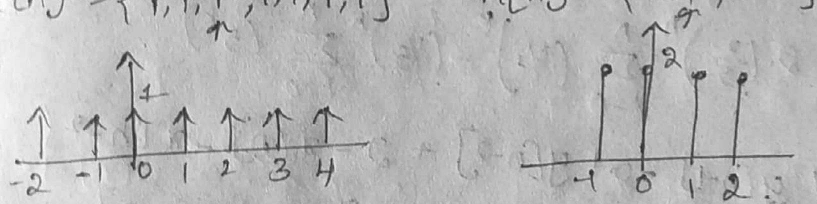
high $\Rightarrow N_2+m_2$.

if fully overlapped high N_2+m ; low $=N_1+m_2$

c) Illustrate the validity of your results by computing the convolution of the signals. $x(n) = \begin{cases} 1 & ; 2 \le n \le 4 \\ 0 & \text{otherwise} \end{cases}$

$h[n] = \begin{cases} 2 & -1 \le n \le 2 \\ 0 & \text{elsewhere} \end{cases}$

$x[n] = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$ $h[n] = \{2, \underset{\uparrow}{2}, 2, 2\}$

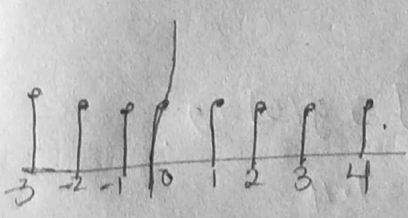


$N_1 = 2$; $N_2 = 4$ $M_1 = -1$; $M_2 = 2$

partial overlap from left.

Low $N_1+m_1 = -3$

high $M_2+N_1-1 = 2-2-1 = -1$



full overlap $n=0, n=3$.

partial right, $n=4, n=6$,

$L_2 = 6$

3) Determine the impulse response and the unit step response of the systems described by the difference Equation.

a] $y[n] = 0.6\, y[n-1] - 0.08\, y[n-2] + x[n]$.

$x[n] = y[n] - 0.6\, y[n-1] - 0.08\, y[n-2]$

Charactristic Equation. $\lambda^2 - 0.6\lambda + 0.08 = 0$

$\lambda = 1/2,\ 2/5$.

$h[n] = c_1 (1/5)^n + c_2 (2/5)^n$

Impulse response $x[n] = \delta[n]$, with $y[0] = 1$

$y[1] - 0.6\, y[0] = 0 \Rightarrow y[1] = 0.6$

So, $c_1 + c_2 = 1 \longrightarrow ①$

$\frac{1}{5} c_1 + \frac{2}{5} c_2 = 0.6 \longrightarrow ②$

from ① & ②. $c_1 = -1$ and $c_2 = 3$.

$h[n] = \left[-(1/5)^n + 2(2/5)^n \right] u[n]$

Step response $x[n] \cdot u[n]$.

$s[n] = \sum_{k=0}^{n} h[n-k]$, $n \geq 0$

$= \sum_{k=0}^{n} \left[2(2/5)^{n-k} - (1/5)^{n-k} \right]$.

$= 2 \cdot \left(\frac{2}{5}\right)^n \sum_{k=0}^{n} (5/2)^k - (1/5)^n \sum_{k=0}^{n} (5^k)$.

b] $y[n] = 0.7\, y[n-1] - 0.1\, y[n-2] + 2 - x[n] - x[n-2]$.

$2x[n] - x[n-2] = y[n] - 0.7\, y[n-1] + 0.1\, y[n-2]$

charactristic equation.

$\lambda^2 - 0.7\lambda + 0.1 = 0$

$\lambda = 1/2,\ 1/5$

$y \cdot h[n] = c_1 (1/2)^n + c_n (1/5)^n$

Impulse response $x[n] = \delta[n]$; $y[0] = 2$

$y[1] - 0.7\, y(0) = 0 \Rightarrow y(1) = 1.4$

$c_1 + c_2 = 0$.

$$\frac{1}{2}c_1 + \frac{1}{5}c_2 = \frac{7}{5} \rightarrow (1)$$

$$c_1 + \frac{2}{5}c_2 = \frac{14}{5} \rightarrow (2)$$

solving (1) & (2)

$c_1 = 14/3$, $c_2 = -4/3$.

so $h[n] = \left[\frac{10}{3}(1/2)^n - \frac{4}{3}(1/5)^n\right] u[n]$.

step response $s[n] = \sum_{k=0}^{n} h[n-k]$

$$= \frac{10}{3}\sum_{k=0}^{n}(1/2)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}(1/5)^{n-k}$$

$$= \frac{10}{3}(1/2)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}(1/5)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left[(1/2)^n \cdot (2^{n+1}-1)\cdot u[n]\right] - \frac{4}{3}\left[(1/5)^n\left(5^{n+1} - \frac{1}{4}\right)\right]$$

31. Consider a system with impulse respone

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n & 0 \leq n \leq 4 \\ 0 & \text{elsewhere.} \end{cases}$$

determine the ~~output~~ input $x[n]$ for $0 \leq n \leq \infty$ that will generate the output sequence.

$h[n] = \left\{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\right\}$

$y[n] = \{1, \underset{\uparrow}{2}, 2.5, 3, 3, 3, 2, 1, 0, \ldots\}$

$y[0] = x(0) \cdot h(0)$

$y(0) = x(0) \cdot 1 \Rightarrow x(0) = 1$

$y(1) = x(1) + h(1) \cdot x(0)$

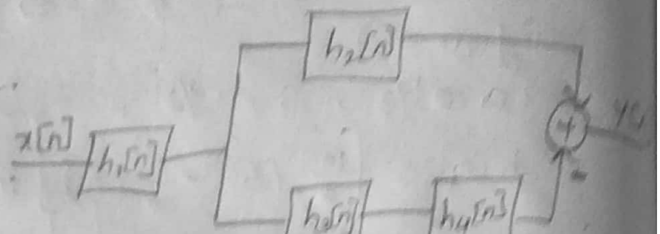
$2 = x(1) + \frac{1}{2}(1) \Rightarrow x(1) = 3/2$

$y(2) = x(2) + h(2) \cdot x(1) + h_1\, x(0)$

$2.5 = x(2) + \frac{1}{4}(3/2) + \frac{1}{2}(1)$

So $x(n) = \left\{1, \frac{3}{2}, 3/2, \frac{7}{4}, 3/2 \ldots\right\}$

32. Consider the interconnection of LTI system as shown in fig.
a) Express the overall impulse response in terms of $h_1[n]$, $h_2[n]$, $h_3[n]$ and $h_4[n]$.



b) Determine $h[n]$ when.

$h_1[n] = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{2}\}$

$h_2[n] = h_3[n] = (n+1)u[n]$

$h_4[n] = \delta[n-2]$.

c) Determine the response of the system in part (b) if

$x[n] = \delta[n+2] + 3\delta[n-1] - 4\delta[n-3]$.

Sol:- a) $h[n] = h_1[n] * [h_2[n] - \{h_3[n] * h_4[n]\}]$

b) $h_3[n] * h_4[n] = (n+1)\,u[n] * \delta[n-2]$

$= (n+1) * u[n-2]$

$h_2[n] - [h_3[n] * h_4[n]] = (n+1)\,u[n] - (n+1)\,u[n+1]$

$= 2 \cdot u[n] - \delta[n]$

$h_1[n] = \frac{1}{2}\delta[n] + \frac{1}{4}\delta[n-1] + \frac{1}{2}\delta[n-2]$

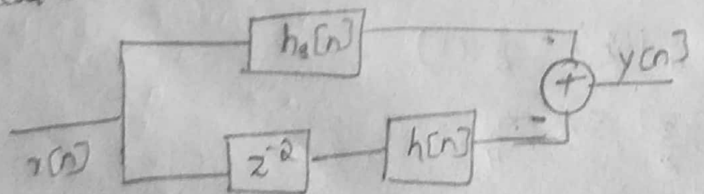
$h[n] = [\frac{1}{2}\delta[n] + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)] * [2u[n] - \delta[n]]$

$-\frac{1}{2}\delta[n] + \frac{5}{4}\delta[n-1) + 2\delta(n-2) + \frac{5}{2}u(n-3)$

c) $x[n] = \{1, 0, \underset{\uparrow}{0}, 0, 3, 0, -4\}$

$y[n] = \{\frac{1}{2}, 5/4, \underset{\uparrow}{2}, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0, \cdots\}$

83. Consider the system fig with $h[n] = a^n u[n]\ (-1 < a < 1)$. Determine the response $y[n]$ of the system to the excitation. $x[n] = u[n+5] - u[n-10]$.



$\delta[n] = u[n] * h[n]$

$\delta[n] = \sum_{k=-\infty}^{\infty} u[k] \cdot h[n-k]$.

$$= \sum_{k=0}^{\infty} h(n-k)$$

$$= \sum_{k=0}^{\infty} a^{n-k}.$$

$$= \frac{a^{n+1} - 1}{a-1} \;; n \geq 0 \quad \{\text{Sum of no } \S$$

for $x[n] = u[n+5] - u[n-10]$

then $\delta[n+5] \cdot 5(n-10) = \frac{a^{n+6}-1}{a-1} u[n+5] - \frac{a^{n-9}-1}{a+1} u[n]$

form given figure $g[n] = x[n] * h[n] - x[n] * h[n] - z^{-2}$

$$y[n] = \frac{a^{n+6}-1}{a-1} u[n+5] - \frac{a^{n-9}-1}{a-1} u(n-10) - \frac{a^{n+4}-1}{a-1} u[n+3]$$
$$+ \frac{a^{n-11}-1}{a-1} u[n-12].$$

$$y[n] = \frac{1}{M} \sum_{k=0}^{M-1} x[n-k]$$

$$h[n] = \frac{[u(n) - u(n-m)]}{M}$$

$$\delta[n] = \sum_{k=-\infty}^{\infty} u(k) h(n-k)$$

34) 23 The discrete time system $y[n] = n y[n-1] + x[n] \;; n \geq 0$ is at rest [i.e: $y(-1) = 0$] check if the system is linear time invarient and BIBO stable.

Sol: $y[n] = n \cdot y[n-1] + x[n] \;; n \geq 0$

$$\left.\begin{aligned} y_1[n] &= n y_1[n-1] + x_1[n] \\ y_2[n] &= n y_2[n-1] + x_2[n] \end{aligned}\right\} \Rightarrow y[n] = n y_1[n-1] + x_1[n] + n y_2[n-1] + x_2[n]$$

$$y[n] = n y[n-1] + x[n]$$

$$x(n) = a x_1[n] + b \cdot x_2[n]$$

$$y[n] = a \cdot y_1[n] + b y_2[n]$$

Hence the system is linear.

$\Rightarrow y[n-1] = (n-1) y(n-2) + x(n-1)$

delayed $\Rightarrow y[n-1] = n \cdot y[n-2] + x[n-1]$

So the system is time varient.

$\Rightarrow$ If $x[n] = u[n]$ then $|x[n]| \leq 1$ for this bounded input, output is $y(0) = 0$, $y(1) = 2$, $y(2) = 5$ ... unbounded

So the system is unstable.

35. Determine the range of values of the parameters a for which the linear time invariant system with impulse response

$$h[n] = \begin{cases} a^n & n \geq 0 \text{ ; } n \text{ Even} \\ 0 & \text{other wise} \end{cases}$$ is stable.

Sol:- $$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0, n=even.}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n} = \frac{1}{1-|a|^2}$$

it is stable if $|a| < 1$.

34] Compute and sketch the step response of the system

$$y[n] = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

Sol:- $$h[n] = \left[\frac{u[n] - u[n+1]}{m}\right]$$

$$\delta[n] = \sum_{n=-\infty}^{\infty} u(k) h(n-k)$$

$$= \sum_{k=0}^{m} h(n-k)$$

$$= \begin{cases} \frac{n+1}{M} & n < m \\ 1 & n > M. \end{cases}$$

36. Determine the response of the system with impulse response $h[n] = a^n u[n]$ to the input signal $x[n] = u[n] - u[n-10]$.

Sol:- $$h[n] = a^n u[n]$$

$$y_1[n] = \sum_{k=0}^{\infty} u[k] . h[n-k]$$

$$= \sum_{k=0}^{n} a^{n-k}$$

$$= a^n \sum_{k=0}^{n} a^{-k}$$

$$= \frac{1-a^{n+1}}{1-a} u[n]$$

$$y[n] = y_1[n] - y_1[n-10]$$
$$= \frac{1}{1-a}\left[(1-a^{n+1})u(n) - (1-a^{n-9})u[n-10]\right]$$

87. Determine the response of the (relaxed) system characterized by the impulse response $h[n] = (1/2)^n u[n]$ to the input signal

$$x[n] = \begin{cases} 1 & 0 \le n < 10 \\ 0 & \text{otherwise} \end{cases}$$

sol: $a = 1/2$ from above problem

$$y(n) = 2\{1-(1/2)^{n+1}\} u[n] - 2\{1-(1/2)^{n-9}\} u[n-?]$$

so $y(n) = 2\left[1-(1/2)^{n+1} - 2(1-(1/2)^{n-9}\right] u[n] \cdot u(0-0)$

$$y(n) = 2\left\{(1-(1/2)^{n+1}) - (1-(1/2)^{(n-9)})\right\} u(n)\, u(0)$$

88. Determine the response of the (relaxed) system characterized by the impulse response $h[n] = (1/2)^n u[n]$ to the input signal.

a) $x[n] = 2^n u[n]$.

sol $y(n) = \sum_{n=-\infty}^{\infty} h(k) \cdot x(n-k)$

$$= \sum_{k=0}^{\infty} (1/2)^k 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} (1/4)^k$$

$$= \frac{2}{3}\left[2^{n+1} - (1/2)^{n+1}\right] u[n].$$

b) $x[n] = u[-n]$

sol: $y[n] = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$

$$= \sum_{k=0}^{\infty} h(k)$$

$$= \sum_{k=0}^{\infty} (1/2)^k - \sum_{k=0}^{n-1} (1/2)^k$$

$$= 2 - \left(\frac{1-(1/2)^n}{1/2}\right)$$

$$= 2(1/2)^n \;;\; n \ge 0.$$

39 Three systems with impulse response $h_1[n] = \delta[n] - \delta[n-1]$, $h_2[n] = h[n]$ and $h_3[n] = u[n]$ are connected in cascade.

a) What is the impulse response, $h_i[n]$ of the overall system

sol

$$h_i[n] = h_1[n] * h_2[n] * h_3[n]$$

$$= \left(\delta[n] - \delta[n-1]\right) * u[n] * h[n]$$

$$= \left[u[n] - u[n-1]\right] * h[n]$$

$$= \delta[n] * h[n] \quad = h[n]$$

b) Does the order of inter connection effect the overall gain?

sol No.

40. @ prove and explain graphically the difference between the relation $x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$ and

$$x[n] * \delta[n-n_0] = x[n-n_0]$$

sol:- $x[n]\,\delta(n-n_0) = x[n_0]$ The only value of $x[n]$ at $n = n_0$ is intersect

$x[n] * \delta[n-n_0] = x[n-n_0]$ Thus, we obtained shifted version of ~~a~~ extendention of input

(b) Show that a discrete time system, which is described by a convolution summation is LTI and relaxd.

sol:-

$$y[n] = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$= h[n] * x[n]$$

Linearity :- $x_1[n] \to y[n] = h[n] * x_1[n]$

$$= x_2[n] \to y[n] = h[n] * x_2[n]$$

$$= \alpha h[n] * x_1[n] + \beta h(n) * x_2(n)$$

$$= \alpha y_1[n] + \beta y_2[n]$$

Time invarience :-

$$x[n] \to y_1[n] = h[n] * x[n]$$

$$x[n-n_0] \to y_1[n] = h[n] * x[n-n_0]$$

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$= \sum_k h(k) \cdot x(n - n_0 - k)$

$= y(n - n_0)$

) What is the response of the system described by
$y(n) = x(n - n_0)$.

$h(n) = \delta(n - n_0)$.

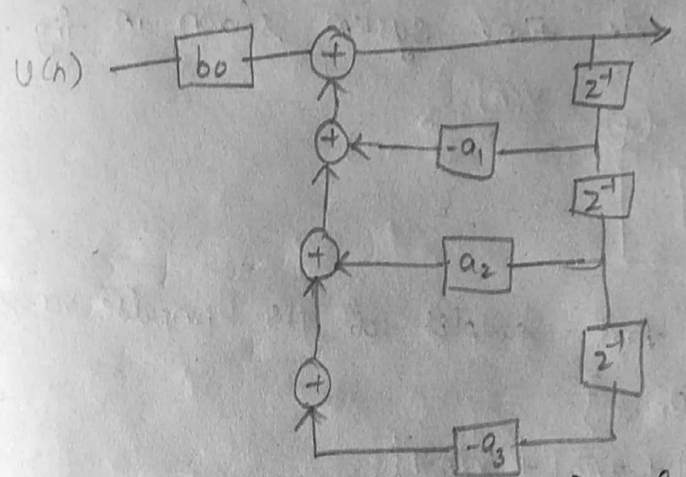
47. Two signals $s(n)$ and $u(n)$ are related through the following difference equations.

$s(n) + a_1 s(n-1) \ldots + a_N s(n-N) = b_0 u(n)$

Design the block diagram realized of.

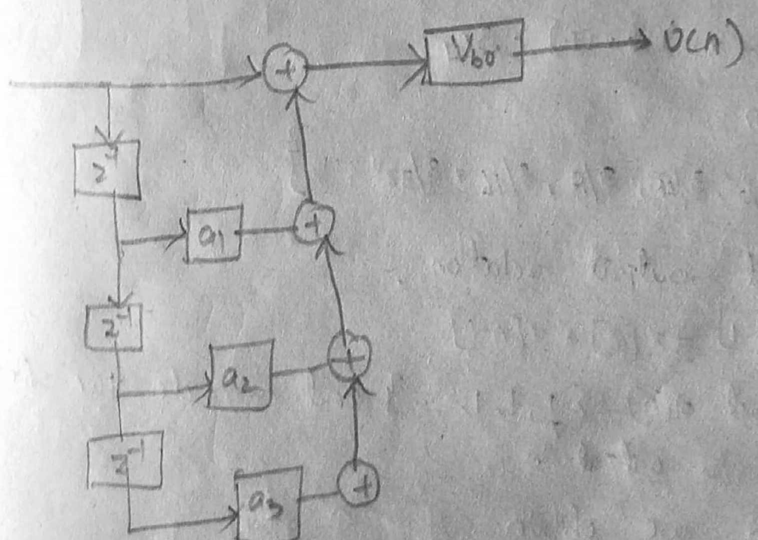
a) The system that generates $s(n)$ when excited by $u(n)$

$s(n) = -a_1\delta(n-1) - c_2\delta(n-1) - c_2\delta(n-2) \ldots + a_N\delta(n-N) \cdot b_0(v(n))$



b) The system that generates $u(n)$ when excited by $s(n)$

$u(n) = \frac{1}{b_0}\left[\delta(n) + a_1\delta(n-1) + a_2\delta(n-2) + \ldots + a_N\delta(n-N)\right]$



42. Compute the zero state response of the system described by the difference Equation.

$y(n) + \frac{1}{2} y(n-1) = x(n) + 2x(n-2)$ to the input

$x[n] = [1, 2, 3, 4, 2, 1\}$ by solving the difference Equation recursively.
$\uparrow$

sol:- $y[n] = \frac{-1}{2} y[n-1] + x[n] + 2 \cdot x[n-2]$

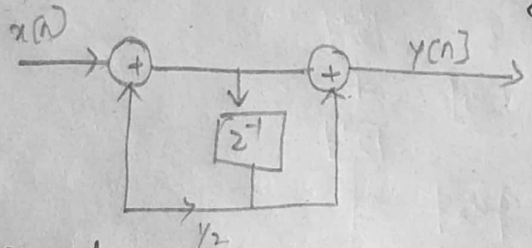
at $y[-2] = -\frac{1}{2} y[-3] + x(-2) + 2x(-4) = 1$

$y(-1) = \frac{-1}{2} y[-2] + x(-1] + 2x(-3) = 3/2$

$y(0) = \frac{-1}{2} y(-1] + 2x(-2) + x(0) = 1/4$

$y(1) = -1/2\, y(0) + x(1) + 2x(-1) = 4\, 1/8$.

Q4. Consider the discrete time system shown in fig.



a) Compute the 10 first samples of its impulse response.

sol:- $x[n] = \{ \frac{1}{1}, 0, 0, \ldots \}$

$y[n] = \frac{1}{2} y[n-1] + x[n] + x[n-1]$

$y[0] = x[0] = 1$

$y[5] = \frac{1}{2} y(0) + x(1) + x(0) = 3/2$.

$y(2] = 1/2\, y(1) + x(2) + x(1) = 3/4$

Thus we obtain

$y(n] = \{1, 3/2, 3/4, 3/8, 3/16, 3/32, \cdots\}$

b) find the input-output relation.

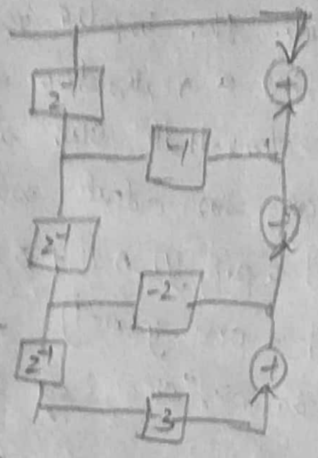
sol:- $y(n) = \frac{1}{2} y[n-1] + x[n] + x[n-1]$

c) Apply the input $x(n] = \{ \underset{\uparrow}{1}, 1, 1, \ldots \}$ and compute the first 10 samples of the output.

sol:- As in part (a) we obtain

$y[n] = \{1, 5/2, 13/4, 29/8, 61/16, \cdots\}$

d) Compute the first 10 samples of the output for the input given in part (c) by using convolution.



Sol: $y(n) = u(n) * h(n)$

$= \sum_k u(k) \cdot h(n-k)$

$= \sum_{k=0}^{n} h(n-k)$

$y(0) = h(0) = 1$

$y(1) = h(0) + h(1) = 5/2$

$y(2) = h(0) + h(1) + h(2) = 13/4$

⋮

e) Is the system causal? Is it stable.

Sol: From part (a), $h(n) = 0$ for $n < 0$ the system is causal

$\sum_{n=0}^{\infty} |h(n)| = 1 + 3/2(1 + 1/2 + 1/4 + \cdots) = 4$

system is stable.

45. Consider the system described by the difference equation.

$y[n] = ay[n-1] + bx[n]$.

a) Determine b in terms of a so that $\sum_{n=-\infty}^{\infty} h(n) = 1$

$y[n] = a\, y[n-1] + b\, x[n]$

$h[n] = b a^n \cdot u[n]$

$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$ then $b = 1-a$.

b) Compute the zero-state step response $s(n)$ of the system and choose b so that $s(\infty) = 1$

Sol: $s(n) = \sum_{k=0}^{n} h(n-k)$

$= b\left[\frac{1-a^{n+1}}{1-a}\right] u[n]$

$s(\infty) = \frac{b}{1-a} = 1$ then $b = 1-a$.

c) Compare the values of b obtained in parts (a) and (b). What do you notice?

Sol: $b = 1-a$ in both the cases.

2.46 A discrete-time system is realized by the structure in fig.

a) Determine the impulse response.

b) Determine a realization for its inverse sys, that sys which produces $x[n]$ as an output when $y[n]$ used as an input.

Sol:- a) $y[n] = 0.8\, y[n-1] + 2x[n] + 3x[n-1]$

$y[n] - 0.8\, y[n-1] = 2x[n] + 3x[n-1]$

The characteristic eq is $\lambda - 0.8 = 0$

$\lambda = 0.8$

$y_h[n] = c(0.8)^n$

Let us first consider the response of the sys $y[n] - 0.8\, y[n-1] = x[n]$ to $x[n] = \delta[n]$ since $y[0] = 1$, it flows that $c = 1$. Then, the impulse response of the original sys is

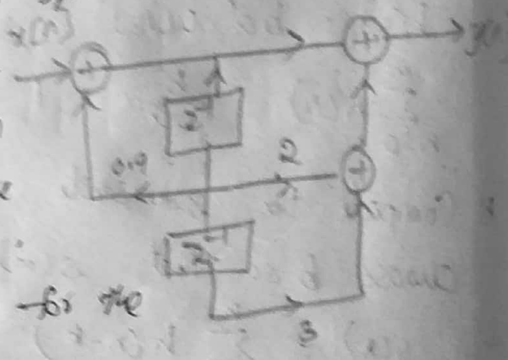
then $= 2(0.8)^n u[n] + 3(0.8)^{n-1} u(n-1)$

$= 2\,\delta[n] + 4.6\,(0.8)^{n-1} u[n-1]$

b) The inverse sys is characterized by the difference equation

$x[n] = -1.5\, x[n-1] + \frac{1}{2} y[n] - 0.4\, y[n-1]$.

2.47. Consider the discrete-time system shown in fig.



a) Compute the first six values of the impulse response of the system

b) Compute the first six values of the zero-state response of the system.

c) Determine an analytical expression for the impulse response of the system.

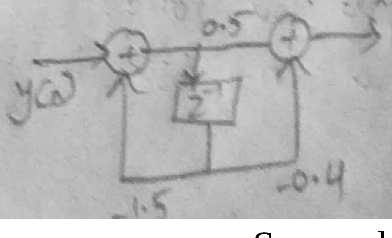
Sol:- $y[n] = 0.9\, y[n-1] + x[n] + 2x[n-1] + 3x[n-2]$

$y[n] - 0.9\, y[n-1] = x[n] + 2x[n-1] + 3x[n-2]$.

a) for $x[n] = \delta[n]$, we have

$y(0) = 1$, $y[1] = 2.9$, $y[2] = 5.61$, $y(3) = 5.049$

$y(4) = 4.544$, $y[5] = 4.090$.



b) $s(0) = y(0) = 1$

$s(1) = y(0) + y(1) = 3.91$

$s(2) = y(0) + y(1) + y(2) = 9.51$

$s(3) = y(0) + y(1) + y(2) + y(3) = 14.56$

$s(4) = \sum_{0}^{4} y(n) = 19.10$

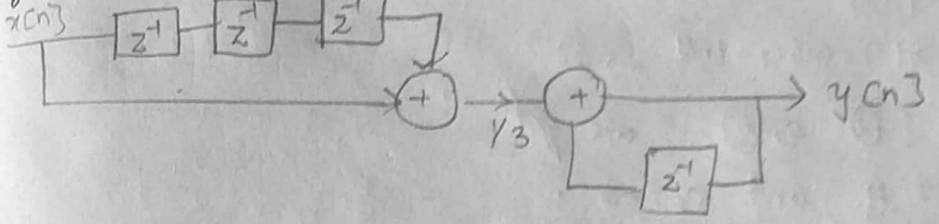
$s(5) = \sum_{0}^{5} y(n) = 23.17$

c] $h[n] = (0.9)^n u[n] + 2(0.9)^{n-1} u[n-1] + 3(0.9)^{n-2} u[n-2]$

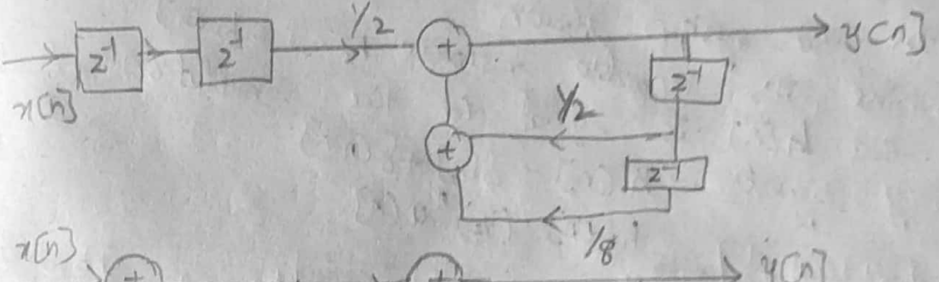
$= s(n) + 2.9\, s(n-1) + 5.6\, \delta (0.9)^{n-2} u(n-2)$

48. Determine and sketch the impulse response of the following systems for $n = 0, 1, \dots 9$.

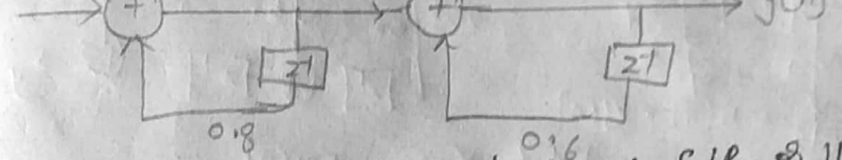
a)



b)



c)



d) classify the systems above as FIR & IIR

e) find an explicit expression for the impulse response of the system in part (c).

Sol:-

a) $y(n) = \frac{1}{3} x[n] + \frac{1}{3} x[n-3] + y[n-1]$

for $x(n) = \delta[n]$ we have

$h[n] = \{1/3, 1/3, 1/3, 2/3, 2/3, 2/3, 2/3, \dots\}$.

b) $y[n] = 1/2\, y[n-1] + 1/8\, y(n-2) + 1/2\, x(n-2)$

with $x[n] = \delta[n]$ and

$y(-1) = y(-2) = 0$, we obtain

$h[n] = \{0, 0, 1/2, 1/4, 3/16, 1/8, 11/128, 15/256, 10/\dots \dots\}$

c) $y[n] = 1.4\,y[n-1] - 0.48\,y[n-2] + x[n]$

As $x[n] = \delta[n]$ and:

$y(-1) = y(-2) = 0$ we obtain.

$h[n] = \{1, 1.4, 1.48, 1.4, 1.2496, 1.0774, 0.9086 \ldots\}$

d) All these systems are IIR.

e) $y[n] = 1.4\,y[n-1] - 0.48\,y[n-2] + x[n]$

The characteristic equation is

$\lambda^2 - 1.4\lambda + 0.48 = 0$ hence $\lambda = 0.8, 0.6$ and

$y_h[n] = c_1 (0.8)^n + c_2 (0.6)^n$ for $x[n] = \delta[n]$ we have

$c_1 + c_2 = 1 \rightarrow ①$ and,

$0.84\,c_1 + 0.6\,c_2 = 1.4 \rightarrow ②$

from ① & ② we get

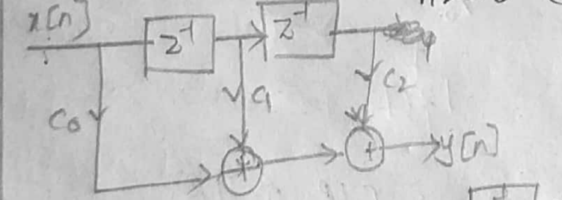
$c_1 = 4$ and $c_2 = -3$.

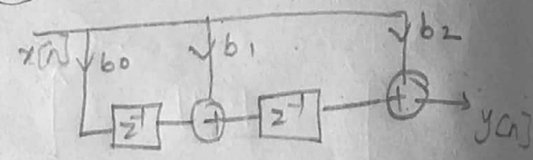
4.9 Consider the system shown

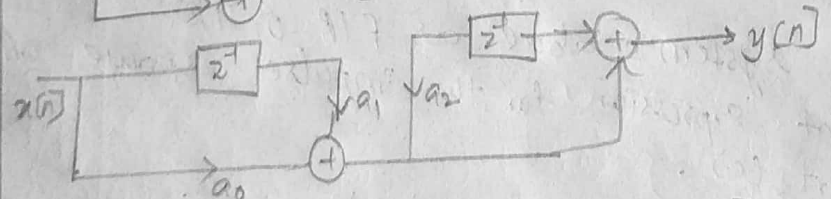
a) Determine its impulse response $h_1[n]$ & $h_2[n]$

b) Show that $h_3[n]$ is equal to the convolution of the following signals. $h_1[n] = \delta[n] + \delta[n-1]$

$h_2[n] = (1/2)^n u[n]$







Sol:

a) $h_1[n] = c_0\delta[n] + c_1\delta[n-1] + c_2\,\delta[n-2]$

$h_2[n] = b_2\delta[n] + b_1\,\delta[n-1] + b_0\delta[n-2]$

$h_3[n] = a_0\delta[n] + (a_1 + a_0 a_2)\,\delta[n-1] + a_1 a_2\,\delta[n-2]$

b) The only question is whether

$h_3[n] = h_2[n] = h_1[n]$ let $a_0 = c_0$

$a_1 + a_2 c_0 = c_1 \Rightarrow a_1 + a_2 c_0 - c_1 = 0$

$a_2 a_1 = c_2 \Rightarrow \frac{c_2}{a_2} = a_1$

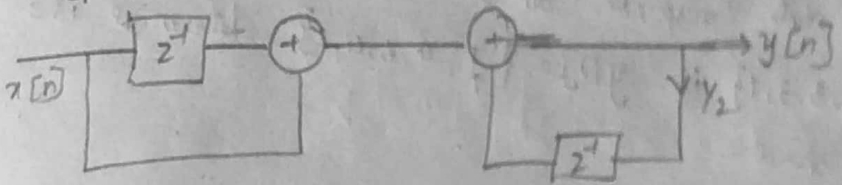
$\Rightarrow \frac{c_2}{a_2} + a_2 c_0 - c_1 = 0$

$\Rightarrow c_0 a_2^2 - c_1 a_2 + c_2 = 0$.

for $c_0 \neq 0$ the quadratic has a real solution if and only if

$c_1^2 - 4c_0c_2 \geq 0$

50]



a) determine its impulse response $h[n]$.

sol:- for $y[n] - \frac{1}{8}y[n-1] = x[n] + x[n-1]$

$$h[n] = (\tfrac{1}{2})^n u[n] + [\tfrac{1}{2}]^{n-1} u[n-1].$$

b) show that $h[n]$ is equal to the convolution of the following sig

$$h_1[n] = \delta[n] + \delta[n-1] \quad \text{and} \quad h_2[n] = (\tfrac{1}{2})^n u[n]$$

sol:- $h_1[n] * [\delta[n] + \delta[n-1]] = [\tfrac{1}{2}]^n u[n] + (\tfrac{1}{2})^{n-1} u[n-1]$.

51. Compute the sketch the convolution $y_1[n]$ and correlation $r_1[n]$ sequence for the following pair of signals and comment on the results obtained.

a] $x_1[n] = \{\underset{\uparrow}{1}, 2, 4\}$ $\quad h_1[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

Convolution ; $y_1[n] = \{1, 3, 7, 7, 7, 6, 4\}$

Corelation ; $r_1[n] = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

b] $x_2[n] = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$ $\quad h_2[n] = \{\frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}\}$

Convolution ; $y_2[n] = \{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, -\frac{5}{2}, -2\}$

Corelation ; $r_2[n] \because h_2[-n] = h_2[n]$.

c) $x_3[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $\quad h_3[n] = \{\underset{\uparrow}{4}, 3, 2, 1\}$.

Convolution ; $y_3[n] = \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$

Corelation ; $r_3[n] = \{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}$

d) $x_4[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $\quad h_4[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Convolution ; $y_4[n] = \{1, 4, 10, 20, 25, 24, 16\}$

Corelation ; $r_4[n] = \{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\}$

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Note that $h_3[-n] = h_4[n+3]$

hence $y_3[n] = y_4[n+3]$

$h_4[-n] = h_3[n+3]$

$y_4[n] = y_3[n+3]$.

52) The zero-state response of a causal LTI system to the input $x[n] = \{\underset{\uparrow}{1}, 3, 3, 1\}$ is $y[n] = \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$ determine its impulse response.

Sol:- $x[n] * y[n] = h[n]$

$h[n] = \{h_0, h_1\}$

$h_0 = 1$ and $3h_0 + h_1 = 4 \Rightarrow h_1 = 1$.

53) prove by direct substitution the Equation (2.5-9) and (2.5-10) which describe the discrete from II structure to the relation (2.5-6) which describes the direct from I structure.

Sol:- (2.5.6) $y[n] = -\sum_{k=1}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x(n-k)$

(2.5-9) $w[n] = -\sum_{k=1}^{N} a_k w[n-k] + x[n]$

(2.5-10) $y[n] = \sum_{k=0}^{M} b_k w[n-k]$.

from 2.5.9 we obtain $x[n] = w[n] + \sum_{k=1}^{N} a_k w[n-k] \rightarrow ①$

by substituting (2.5.10) for $y[n]$ and eq ① into (2.5-6) we obtain L.H.S = R.H.S.

54. Determine the response $y[n]$, $n \geq 0$ of the system described by the second-order difference equation when the input is

$y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$

$x[n] = (-1)^n u[n]$.

Sol:- In initial credits are $y[-1] = y[-2] = 0$

The characteristic equation is... $\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$

Hence $y_h[n] = c_1 2^n + c_2 n 2^n$.

The particular solution is $y_p[n] = k(-1)^n u[n]$.

substituting this solution into the difference equation we obtain $k(-1)^n u[n] - 4k(-1)^{n-1} u[n-1] + 4k(-1)^{n-2} u[n-2] =$
$(-1)^n u[n] - (-1)^{n-1} u[n-1]$

for n=2,

$k(1+4+4)=2 \Rightarrow k = 2/9$.

The total solution is:

$y[n] = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right]u[n]$

from the initial conditions; we obtain $y(0)=1$, $y(1)=2$, then

$c_1 + \frac{2}{9} = 1 \Rightarrow c_1 = 7/9$.

$2c_1 + 2c_2 - \frac{2}{9} = 2 \Rightarrow c_2 = 1/3$

55. determine the impulse response $h[n]$ for the s/m described by the 2nd order difference equation:

$y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$.

sol:- from problem <57>

$h[n] = [c_1 2^n + c_2 n 2^n]\,u[n]$

$y(0)=1$, $y(1)=3$, we have. $c_1 = 1$; $2c_1 + 2c_2 = 3 \Rightarrow c_2 = 1/2$

Thus $h[n] = \left[2^n + \frac{1}{2} n 2^n\right]u[n]$.

56) Show that any discrete-time signal $x[n]$ can be expressed as $x[n] = \sum_{k=-\infty}^{\infty}[x(k) - x(k-1)]\,u[n-k]$.

where $u[n-k]$ is a unit step delayed by k units in time.

that is $u[n-k] = \begin{cases} 1 & n \geq k \\ 0 & \text{otherwise} \end{cases}$

sol:-

$x[n] = x[n] * \delta[n]$

$= x[n] * [u[n] - u[n-1]]$

$= [x[n] - x[n-1]] * u[n]$

$x[n] = \sum_{k=-\infty}^{\infty}[x(k) - x(k-1)]\,u[n-k]$.

57. Show that the output of an LTI system can be expressed in terms of it's unit step response system as follows.

$y[n] = \sum_{k=-\infty}^{\infty}[s(k) - s(k-1)]\,x(n-k]$

$= \sum_{k=-\infty}^{\infty}[x(k) - x(k-1)]\,s(n-k]$.

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Sol:- Let $h[n]$ be the impulse response of the system

$$\delta(k) = \sum_{M=-\infty}^{\infty} h(M)$$

$$h(k) = \delta(k) - \delta(k-1)$$

$$y[n] = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left\{ [\delta(k) - \delta(k-1)]\, x[n-k] \right\}$$

58. Compute the correlation sequence $r_{xx}(l)$ and $r_{xy}(l)$ for the following signal sequences. $x[n] = \begin{cases} 1 & n_p - N \le n \le n_p + N \\ 0 & \text{otherwise} \end{cases}$

$$y[n] = \begin{cases} 1 & -N \le n \le N \\ 0 & \text{otherwise} \end{cases}$$

Sol:-

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

The range of non-zero values of $r_{xx}(l)$ is determined by

$n_0 - N \le n \le n_0 + N$.

$n_0 - N \le n - l \le n_0 + N$.

which implies $-2N \le l \le 2N$.

for a given shift l, the number of terms in summation for which the both $x[n]$ and non zero is $2N+1 \to (l)$ and the value of each term is

$$r_{xx}(l) = \begin{cases} 2N + l - |l| & ; -2N < l \le 2N \\ 0 & ; \text{otherwise} \end{cases}$$

for $r_{xy}(l)$ we have.

$$r_{xy}(l) = \begin{cases} 2N + l - |l - N| & n_0 - 2N \le n_0 + 2N \\ 0 & \text{otherwise} \end{cases}$$

59. Determine the autocorrelation sequence of the following signals a) $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$

b) $y(n) = \{\underset{\uparrow}{1}, 1, 2, 1\}$

What is your conclusion..?

Sol: (a) $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$

$r_{xx}(-3) = x(0)\,x(3) = 1$

$r_{xx}(-2) = x(0)\,x(1) + x(1)\,x(2) + x(2)\,x(3) = 5$

$r_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$

also $r_{xx}(-l) = r_{xx}(l)$

$\therefore r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$.

(b) $r_{yy}(l) = \sum_{n=-\infty}^{\infty} y[n]\, y[n-l]$

we obtain $r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

We obtain $y[n] = x[n+3]$ which is equivalent to reversing the sequence $x[n]$. This has not changed the auto correlation sequence ...

Q) What is the normalized autocorrelation sequence of the signal $x[n]$ given by $x[n] = \begin{cases} 1 & -N \le n \le N \\ 0 & \text{otherwise} \end{cases}$

Sol: $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x[n]\, x[n-l]$

$= \begin{cases} 2N+1-|l| & -2N \le l \le 2N \\ 0 & \text{otherwise.} \end{cases}$

$r_{xx}(0) = 2N+1$.

$\therefore$ The normalized autocorrelation is $\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|) \\ 0 \end{cases}$

6] An audio signal $s(t)$ generated by a loudspeaker is reflected at two different walls with reflection coefficient r_1 and r_2. The signal $x(t)$ recorded by a microphone close to the loudspeaker, after sampling is

$x[n] = s[n] + r_1\, s(n-k_1) + r_2\, s(n-k_2)$.

where k_1 and k_2 are the delays of the two echoes.

a] Determine the autocorrelation $r_{xx}(l)$ of the signal $x[n]$

b] Can we obtain r_1, r_2, k_1 and k_2 by observing $r_{xx}(l)$?

c) what happens if $r_2 = 0$?

Sol:- (a) $\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x[n]\, x[n-l]$

$= \sum_{n=-\infty}^{\infty} \left[\delta[n] + \gamma_1 \delta[n-k_1] + \gamma_2 \delta[n-k_2]\right] *$

$\left[\delta(n-l) + \gamma_1 \delta(n-l-k_1) + \gamma_2 \delta(n-l-k_2)\right]$

$= (1+\gamma_1^2+\gamma_2^2)\,\gamma_{ss}(l) + \gamma_1\left[\gamma_{ss}(l+k_1) + \gamma_{ss}(l-k_1)\right]$

$+ \gamma_2\left[\gamma_{ss}(l) + \gamma_{ss}(l-k_2)\right] + \gamma_1\gamma_2\left[\gamma_{ss}(l+k_1-k_2) +\right.$

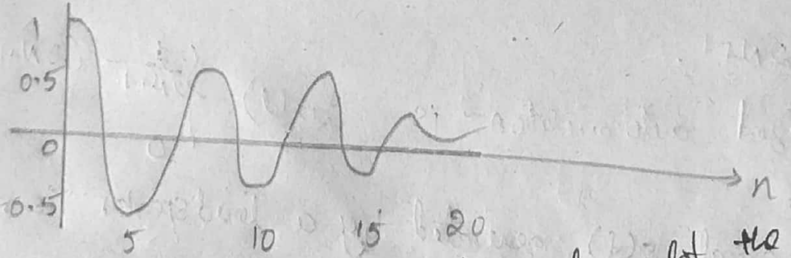
$\left.\gamma_{ss}(l+k_2-k_1)\right]$

(b) $\gamma_{xx}(l)$ has peaks at $l=0, \pm k, \pm k_2$ and $\pm(k_1+k_2)$. Suppose that $k_1<k_2$ then we can determine γ_1 and k_1. The problem is to determine γ_2 and k_2 from the other peaks

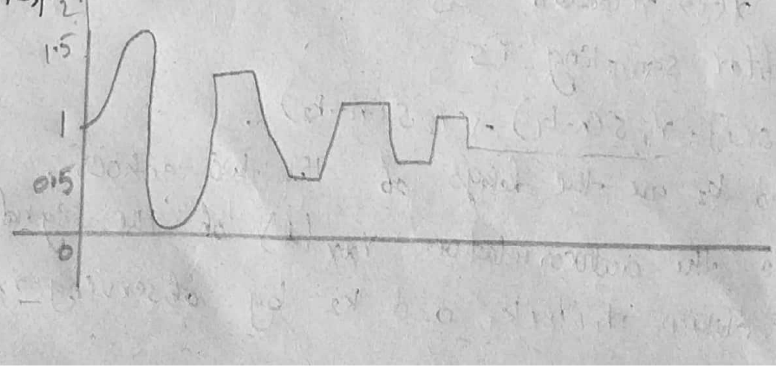
(c) If $\gamma_2=0$ the peaks occur at $l=0$ and $l=\pm k$, then it is easy to obtain γ_1 and k_1

62) Implementation of LTI systems. Consider the recursive discrete time system described by the difference equation.

a) write a program to computed and plot the impulse response $h[n]$ of the system for $0 \le n \le 47$.



b) write program to compute and plot the zero state step response $s[n]$ of the system for $0 \le n \le 100$.

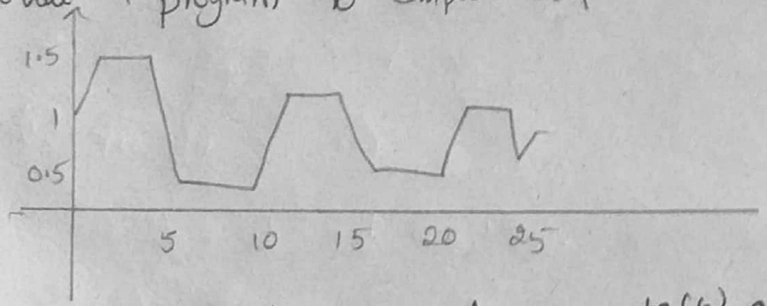


c) Define an FIR system with impulse response $h_{FIR}(n)$

given by $h_{FIR}(n) = \begin{cases} h[n] & 0 \le n \le 19 \\ 0 & \text{otherwise} \end{cases}$.

Where $h[n]$ is the impulse responses computed in part (a)

write a program to compute & plot its step response.



d) Compare the results obtained in parts (b) and (c) and Explain their similarities and differences.

sol:- c, b are similar except c have steady state after $n=20$ where b have nearly at $n=30$.

63) Write a computer program that computes the overall input response $h[n]$ of the system shown below for $0 \le n \le 99$. The systems T_1, T_2, T_3 and T_4 are specified by

$T_1 : h_1[n] = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}\}$

$T_2 : h_2[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$T_3 : y_3[n] = \frac{1}{4}x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2]$

$T_4 : y[n] = 0.9\,y[n-1] - 0.81\,y[n-2] + v[n] + v[n-1]$.

plot $h[n]$ for $0 \le n \le 99$.