CFDS® – Chartered Financial Data Scientist

Introduction to Python

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Statistical Methods in Data Science

- In this part, we study a number of statistical methods that have become very popular in Data Science applications:
- Statistical Learning methods are often classified into:
    - Regression versus classification
    - Supervised versus unsupervised learning (versus reinforcement learning)
- The methods studied here cover these different aspects:
    - Ridge regression and Lasso (regression, supervised)
    - Logistic regression (classification, supervised)
    - Principal components analysis (regression, unsupervised)

Ridge regression, Lasso and Elastic Net

- In linear regression, we assume a linear relationship between the response Y and the feature vector X:

  $$Y = a + b_1 X_1 + b_2 X_2 + \cdots + b_m X_m + \epsilon,$$

  where $a, b_1, \ldots, b_m$ are constants and ϵ is the error term.

- The ordinary least squares (OLS) estimates of a, b minimise the errors

  $$\sum_{i=1}^{n} \epsilon^2 = \sum_{i=1}^{n} (Y_i - a - b_1 X_{i1} - b_2 X_{i2} - \cdots - b_m X_{im})^2.$$

- In machine learning, especially when the number of features is high and when features are highly correlated, overfitting can occur.
- One way of dealing with this is known as **regularisation**.
- The most popular regularisation methods are:
    - Ridge regression
    - Lasso
    - Elastic net

Ridge regression

- In statistics, **ridge regression** is known as **Tikhonov regularisation** or L_2 **regularisation**.
- Building on OLS, a term is added to the objective function that places a **penalty** on the size of the coefficients $b_1, \ldots, b_m$, by minimising:

  $\sum_{i=1}^{n}(Y_i - a - b_1X_{i1} - b_2X_{i2} - \cdots - b_mX_{im})^2 + \lambda \sum_{j=1}^{m} b_j^2.$

- The constant λ is called **tuning parameter** or **hyperparameter** and controls the strength of the penalty factor.
- The term $\lambda \sum_{j=1}^{m} b_j^2$ is called the **shrinkage penalty**, as it will shrink the estimates of $b_1, \ldots, b_m$ towards zero.
- Selecting a good value of λ is critial and can be achieved, for example, by **cross-validation**.

Ridge regression

- The OLS estimates do not depend on the magnitude of the independent variables: multiplying X_j by a constant c leads to a scaling of the OLS-coefficient by $1/c$.
- This is different in ridge regression (and Lasso, see below): the estimated coefficients can change substantially when re-scaling independent variables.

- Therefore, it is custom, to [illegible] the features:

$$\tilde{x}_{ij} = \frac{x_{ij}}{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_{ij} - \overline{x}_j)^2}},$$

so that all variables are on the same scale, i.e., they all have a standard deviation of one.

Lasso

- **Lasso (Least absolute shrinkage and selection operator)**, also known as L_1 **regularisation** adds a different penalty:

  $$\sum_{i=1}^{n}(Y_i - a - b_1X_{i1} - b_2X_{i2} - \cdots - b_mX_{im})^2 + \lambda \sum_{j=1}^{m} |b_j|.$$

- This has the interesting effect that the less relevant features are completely eliminated.
- For this reason, Lasso is also often used as a feature selection or variable selection method.

Elastic net regression

- **Elastic net regression** is a mixture of ridge regression and Lasso:

  $\sum_{i=1}^{n}(Y_i - a - b_1X_{i1} - b_2X_{i2} - \cdots - b_mX_{im})^2 + \lambda_1 \sum_{j=1}^{m} b_j^2 + \lambda_2 \sum_{j=1}^{m} |b_j|$

  .

- Combining the effects of ridge regression and Lasso means that simultaneously

  - some coefficients are reduced to zero (Lasso),
  - some coefficients are reduced in size (ridge regression).

Example

- The following application predicts house prices based on different features of the property.
- The data set is from

  Hull: Machine Learning in Business. 3rd edition, independently published, 2021.

In [87]:
```
import pandas as pd # python's data handling package
import numpy as np # python's scientific computing package
import matplotlib.pyplot as plt # python's plotting package
import seaborn as sns

from sklearn.metrics import mean_squared_error as mse
from sklearn.model_selection import train_test_split
# The sklearn library has cross-validation built in!
# https://scikit-learn.org/stable/modules/cross_validation.html
from sklearn.model_selection import cross_val_score
```

In [88]:
```
# Both features and target have already been scaled: mean = 0; SD = 1
data = pd.read_csv('data/Houseprice_data_scaled.csv')
# data = pd.read_csv('https://raw.githubusercontent.com/packham/Python_
```

In [89]:
```
X = data.drop('Sale Price', axis=1)
y = data['Sale Price']
```

- `sklearn` can split training and testing data randomly.

In [90]:
```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2
```

Linear Regression

In [91]:
```
from sklearn.linear_model import LinearRegression
```

In [92]:
```
lr=LinearRegression()
lr.fit(X_train,y_train)
```

Out[92]:
```
▾ LinearRegression
LinearRegression()
```

In [93]:
```
pred = lr.predict(X_test)
mse(y_test, pred)
```

Out[93]:
```
0.13712723961133336
```

- Observe how the OLS coefficients are all non-zero.
- However, some coefficients are negative where a positive coefficient would be expected (e.g. `FullBath`).
- This is an indication that the model is struggling to fit the large number of features.

In [94]:
```
# Create dataFrame with corresponding feature and its respective coeffi
coeffs = pd.DataFrame([['intercept'] + list(X_train.columns),[lr.interc
coeffs
```

Out[94]:

0	1
intercept	682.894934
LotArea	0.114813
OverallQual	0.164816
OverallCond	0.080186
YearBuilt	0.133657
YearRemodAdd	0.079848
BsmtFinSF1	0.162612
BsmtUnfSF	0.033426
TotalBsmtSF	0.086656
1stFlrSF	0.412417
2ndFlrSF	0.475636
GrLivArea	-0.231936

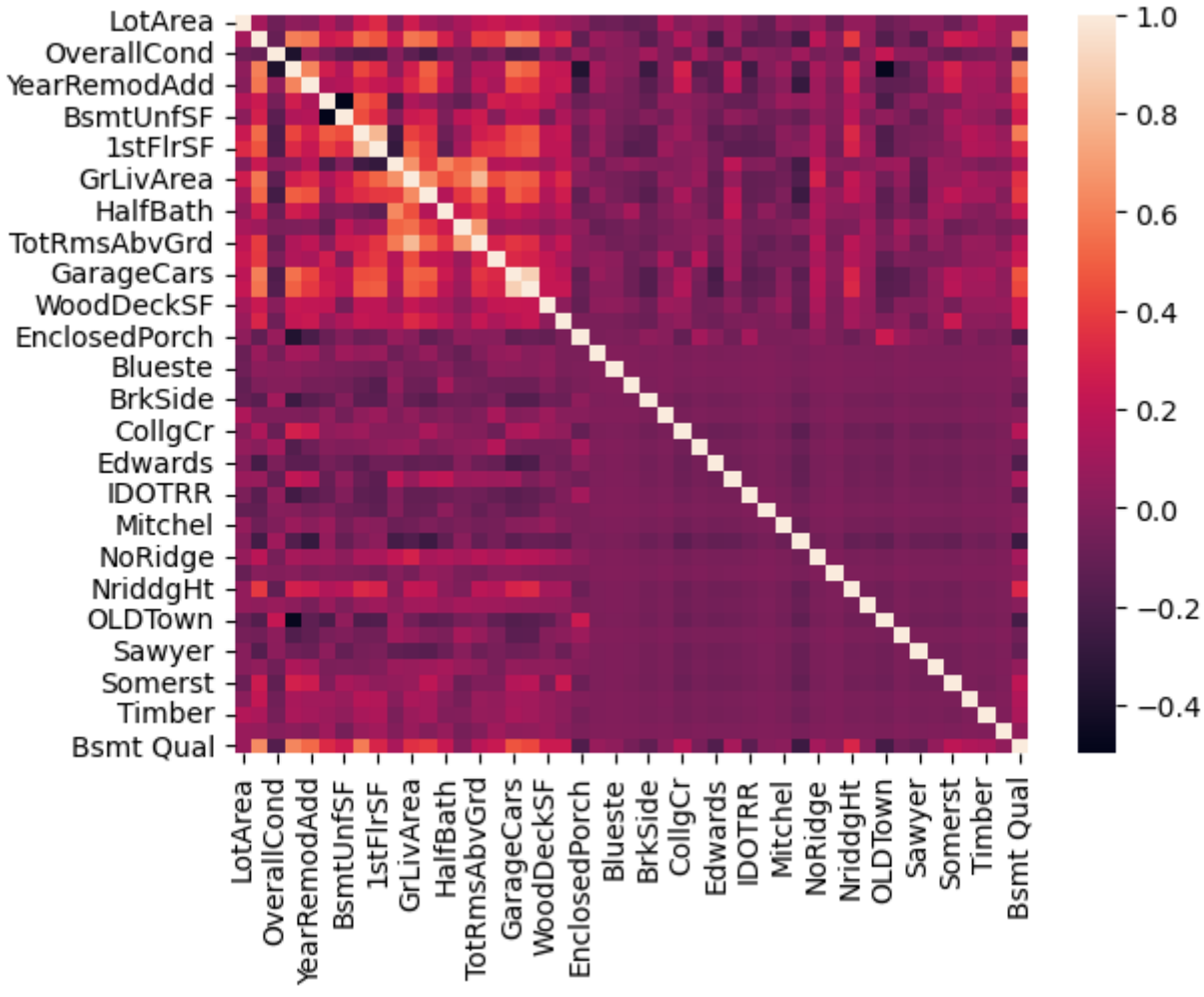
0	1
FullBath	0.000994
HalfBath	0.022358
BedroomAbvGr	-0.056027
TotRmsAbvGrd	0.051201
Fireplaces	0.0406
GarageCars	0.00161
GarageArea	0.112378
WoodDeckSF	0.017999
OpenPorchSF	0.023707
EnclosedPorch	-0.002801
Blmngtn	793967070484.795654
Blueste	365827957378.811523
BrDale	814364304631.275635
BrkSide	1470721404506.125488
ClearCr	1026620305422.403931
CollgCr	2198410806737.254639
Crawfor	1337127407481.94751
Edwards	1860944517918.066895
Gilbert	1853014352220.636719
IDOTRR	1145196626390.019043

0	1
MeadowV	834240657510.672485
Mitchel	1416819289966.038818
Names	2891476524648.777832
NoRidge	1238892481602.776123
NPkVill	751439227386.505615
NriddgHt	1837020903775.030762
NWAmes	1702101513915.983154
OLDTown	2060156147336.770752
SWISU	1042242971357.214355
Sawyer	1666218830014.025879
SawyerW	1591467269428.688965
Somerst	1945412775894.903809
StoneBr	978142231255.40332
Timber	1159093788628.891357
Veenker	706250014889.251221
Bsmt Qual	-0.004655

- Indeed, some correlations are high, as the heatmap indicates, which may cause multicollinearity and ill-fitting.

In [95]:
```
sns.heatmap(X_train.corr())
```

Out[95]:
```
<Axes: >
```


LotArea
OverallCond
YearRemodAdd
BsmtUnfSF
1stFlrSF
GrLivArea
HalfBath
TotRmsAbvGrd
GarageCars
WoodDeckSF
EnclosedPorch
Blueste
BrkSide
CollgCr
Edwards
IDOTRR
Mitchel
NoRidge
NriddgHt
OLDTown
Sawyer
Somerst
Timber
Bsmt Qual
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4

Ridge regression

In [96]:
```
# Importing Ridge
from sklearn.linear_model import Ridge
```

- Train on the training data set and use test data set to determine test MSE.

In [97]:
```
n=np.int(len(X)*.75) # choose 75% of data for training
# The alpha used by Python's ridge should be the lambda above times the
alphas=[0.01*n, 0.02*n, 0.03*n, 0.04*n, 0.05*n, 0.075*n,0.1*n,0.125*n,
mses=[]
for alpha in alphas:
    ridge=Ridge(alpha=alpha)
    ridge.fit(X_train,y_train)
    pred=ridge.predict(X_test)
    mses.append(mse(y_test,pred))
mses
```

```
/var/folders/46/b127yp714m71zfmt9j7_lhwh0000gq/T/ipykernel_5177
9/2053880030.py:1: DeprecationWarning: `np.int` is a deprecated
alias for the builtin `int`. To silence this warning, use `int`
by itself. Doing this will not modify any behavior and is safe.
When replacing `np.int`, you may wish to use e.g. `np.int64` or
`np.int32` to specify the precision. If you wish to review your
current use, check the release note link for additional informa
tion.
Deprecated in NumPy 1.20; for more details and guidance: http
```

```
s://numpy.org/devdocs/release/1.20.0-notes.html#deprecations
  n=np.int(len(X)*.75) # choose 75% of data for training
```

Out[97]:

```
[0.1290016969234628,
 0.1291714414174131,
 0.12941226532095346,
 0.129701057392689,
 0.13002471627663426,
 0.13093789509920922,
 0.13194883759388032,
 0.1330231986296970 8,
 0.13414251976000655,
 0.1364753768000092,
 0.1464571160986866 4]
```

- This is how to use cross-validation; just specify the number of folds (`cv`) and MSE as the `scoring` function:

In [98]:
```
# The alpha used by Python's ridge should be the lambda above times the
alphas=[0.01*n, 0.02*n, 0.03*n, 0.04*n, 0.05*n, 0.075*n,0.1*n,0.125*n,
mses=[]
for alpha in alphas:
    scores = cross_val_score(Ridge(alpha=alpha), X, y, cv=4, scoring="n
    mses.append(np.mean(scores))
#np.transpose([alphas, mses])
mses
```

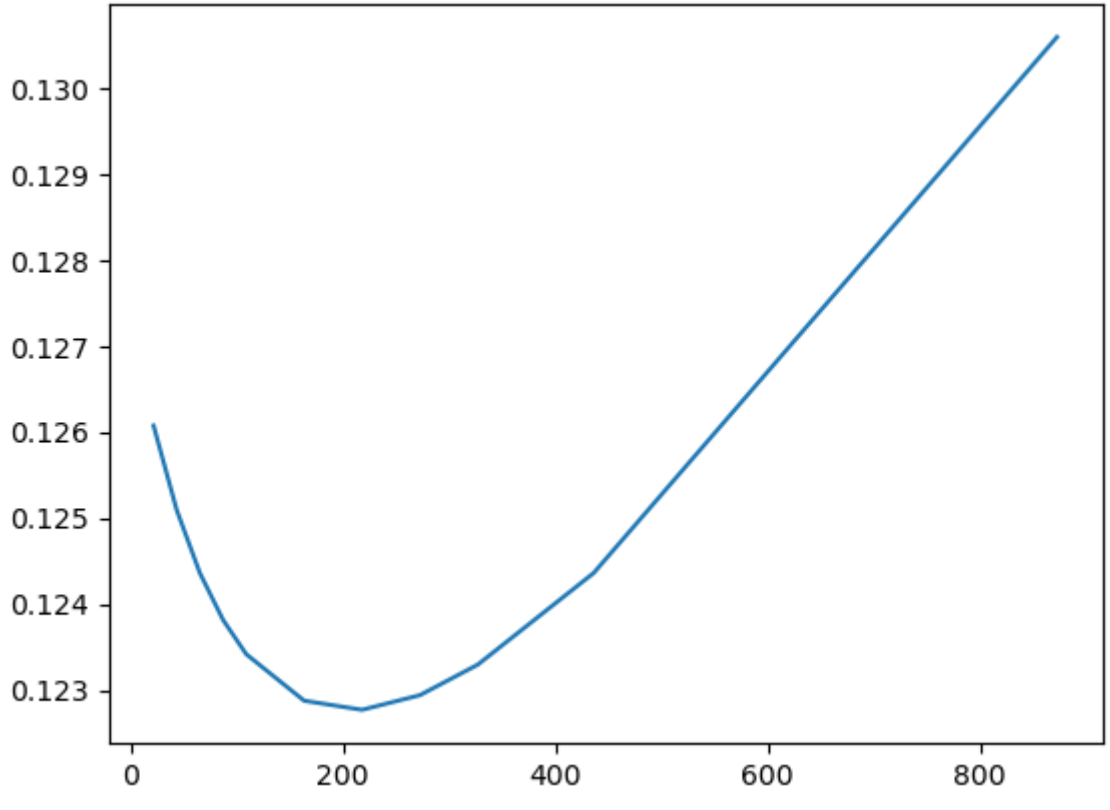
Out[98]:
```
[0.1260729550240368,
 0.12509029977210726,
 0.12435770257439965,
 0.12381364622001008,
 0.1234152803278201,
 0.12287421953596336,
 0.1227683115115362,
 0.1229385883925512,
 0.12329420458443228,
 0.12435934533876439,
 0.13059077561292226]
```

- Average test MSE varies with the hyperparameter α.
- The best model choice is at approximately $0.1 \cdot n = 218$

.

In [99]:
```
plt.plot(alphas, mses)
```

Out[99]:
```
[<matplotlib.lines.Line2D at 0x2a03bddd0>]
```



Lasso

In [100]:
```
# Import Lasso
from sklearn.linear_model import Lasso
```

In [101]:
```
# Here we produce results for alpha=0.05 which corresponds to lambda=0.
lasso = Lasso(alpha=0.05)
lasso.fit(X_train, y_train)
```

Out[101]:

▼ Lasso

```
Lasso(alpha=0.05)
```

- As Lasso acts as a variable selection method we would expect some coefficients to be set to zero:

In [102]:
```
# DataFrame with corresponding feature and its respective coefficients
coeffs = pd.DataFrame(
    [
        ['intercept'] + list(X_train.columns),
        [lasso.intercept_] + lasso.coef_.tolist()
    ]
).transpose().set_index(0)
coeffs
```

Out[102]:

	1
0	
intercept	0.012617
LotArea	0.033435
OverallQual	0.297123
OverallCond	0.0
YearBuilt	0.036746
YearRemodAdd	0.073493
BsmtFinSF1	0.105878
BsmtUnfSF	-0.0
TotalBsmtSF	0.058556
1stFlrSF	0.059207
2ndFlrSF	0.0

	1
0	
GrLivArea	0.290129
FullBath	0.0
HalfBath	0.0
BedroomAbvGr	-0.0
TotRmsAbvGrd	0.0
Fireplaces	0.023073
GarageCars	0.003054
GarageArea	0.09851
WoodDeckSF	0.003786
OpenPorchSF	0.0
EnclosedPorch	-0.0
Blmngtn	-0.0
Blueste	-0.0
BrDale	-0.0
BrkSide	0.0
ClearCr	0.0
CollgCr	-0.0
Crawfor	0.0
Edwards	-0.0
Gilbert	-0.0

	1
0	
IDOTRR	-0.0
MeadowV	-0.0
Mitchel	-0.0
Names	-0.0
NoRidge	0.008125
NPkVill	-0.0
NriddgHt	0.074678
NWAmes	-0.0
OLDTown	-0.0
SWISU	-0.0
Sawyer	-0.0
SawyerW	-0.0
Somerst	0.0
StoneBr	0.04425
Timber	0.0
Veenker	-0.0
Bsmt Qual	0.042183

- Now, let's find again the parameter with minimal average test MSE:

In [ ]:
```
# We now consider different lambda values. The alphas are half the lamb
alphas=[0.0025/2, 0.005/2, 0.01/2, 0.015/2, 0.02/2, 0.025/2, 0.03/2, 0.
mses=[]
for alpha in alphas:
    scores = cross_val_score(Lasso(alpha=alpha), X, y, cv=4, scoring="n
    mses.append(np.mean(scores))
#np.transpose([alphas,mses])
mses
```

- The optimal parameter is at $\alpha = 0.0075$
  :

In [ ]:
```
plt.plot(alphas, mses)
```

Logistic regression

- In a regression setting, numerical variables are predicted.
- Another application is classification, which is about predicting the category a new observation belongs to.
- In supervised learning, and with two categories, a variation of regression, called **logistic regression** can be used.
- Given features $X_1, \ldots, X_m$, suppose there are two classes to which observations can belong.
- An example is the prediction of a loan's default risk, given characteristics of the creditor such as age, education, marital status, etc.
- Another example is the classification of e-mails into junk or non-junk e-mails.

Logistic regression

- Logistic regression can be used to calculate the probability of a positive outcome via the **sigmoid function**

$$P(Y = 1|X) = \frac{1}{1 + \mathrm{e}^{-X}} = \frac{\mathrm{e}^{X}}{1 + \mathrm{e}^{X}},$$

where e is the Euler constant.

In [ ]:
```
x=np.arange(-7,7,0.25)
plt.plot(x, 1/(1+np.exp(-x)))
```

Logistic regression

- Setting $X = a + b_1 X_1 + b_2 X_2 + \cdots + b_m X_m$, the probability of a positive outcome is

  $$P(Y = 1 | X_1, \ldots, X_m) = \frac{1}{1 + \exp(-a - \sum_{j=1}^{m} b_j X_j)}.$$

- The objective is to find the coefficients $a, b_1, \ldots, b_m$ that best classify the given data.
- **Maximum likelihood** is a versatile method for this type of problem, when OLS does not apply.
- Without going into detail, the **log likelihood function** is given as

  $$\ell(a, b_1, \ldots, b_m | x_1, \ldots, x_n) = \sum_{k:y_k=1} \ln(p(x_k)) + \sum_{k:y_k=0} \ln(1 - p(x_k)),$$

  and the parameters are chosen that maximise this function.
- (Note: The likelihood function is derived by considering the observations to be independent outcomes of a Bernoulli random variable.)

Example: Credit risk

- The dataset in this example is taken from James et al.: An Introduction to Statistical Learning. Springer, 2013.
- It contains simulated data of defaults on credit card payments, on the basis of credit card balance (amongst other things).
- An excellent tutorial and examples on logistic regression in Python is available here: https://realpython.com/logistic-regression-python/.
- We will use the `sklearn` package below. Logistic regression can also be performed with the `statsmodels.api`, in which case p-values and other statistics are calculated.

In [ ]:
```
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import classification_report, confusion_matrix
from sklearn.model_selection import train_test_split
```

In [ ]:
```
data = pd.read_csv("./data/Default_JamesEtAl.csv")
# data = pd.read_csv("https://raw.githubusercontent.com/packham/Python_
```

In [ ]:
```
x=np.array(data["balance"]).reshape(-1,1) # array must be two-dimensior
y=np.array([True if x=="Yes" else False for x in data["default"]]) # li

x_train, x_test, y_train, y_test = train_test_split(x, y, test_size=0.2
```

In [ ]:
```
model = LogisticRegression(solver='liblinear', random_state=0)
model.fit(x_train,y_train)
```

In [ ]:
```
# fitted parameters
a=model.intercept_[0]
b=model.coef_[0,0]
[a,b]
```

- Scatter plot of data and fitted logistic function:

In [ ]:
```
plt.scatter(x,y,c='orange', marker="+")
plt.xlabel('balance')
xrange=range(0,3000,10)
plt.plot(xrange,1/(1+np.exp(-a-b *xrange)))
```

- Predictions:

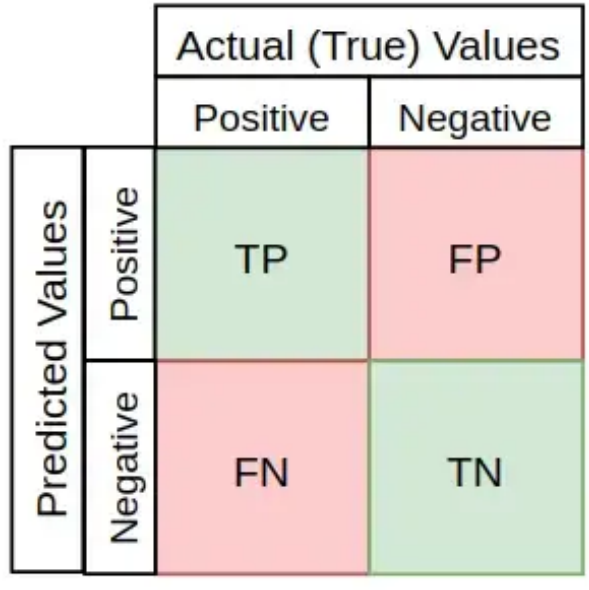
In [ ]:
```
model.predict_proba(x_train)[:5]
```

In [ ]:
```
model.predict(x_train)[:10]
```

- Mean accuracy of the model:

In [ ]:
```
[model.score(x_train,y_train), model.score(x_test,y_test)]
```

- Confusion matrix:



https://towardsdatascience.com/a-look-at-precision-recall-and-f1-score-36b5fd0dd3ec

In [ ]:
```
confusion_matrix(y_train,model.predict(x_train))
```

In [ ]:
```
confusion_matrix(y_test,model.predict(x_test))
```

- See link below for an explanation of the metrics:

https://towardsdatascience.com/a-look-at-precision-recall-and-f1-score-36b5fd0dd3ec

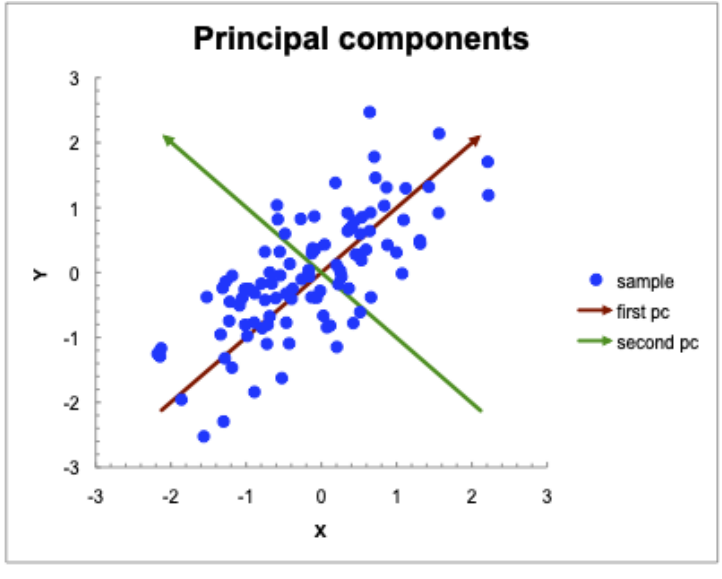
In [ ]:
```
print(classification_report(y_train, model.predict(x_train)))
```

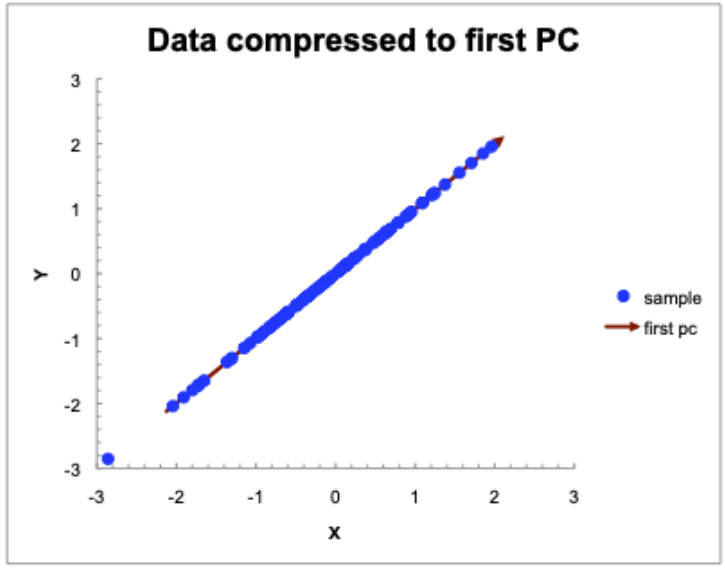
In [ ]:
```
print(classification_report(y_test, model.predict(x_test)))
```

Principal component analysis

- **Principal Component Analysis (PCA)** summarises a large set of correlated variables by smaller number of representative variables that explain most of the variability of the original data set.
- It is a standard method for reducing the dimension of high dimensional, highly correlated systems.
- The principal components are common factors that are unobservable (latent) and directly estimated from the data.
- While being explanatory in a statistical sense, the factors do not necessarily have an economic interpretation.

PCA


Principal components
3
2
1
0
-1
-2
-3
Y
-3
-2
-1
0
1
2
3
X
sample
first pc
second pc


Data compressed to first PC
3
2
1
0
-1
-2
-3
Y
-3
-2
-1
0
1
2
3
X
sample
first pc

PCA

- The objective is to take p random variables $X_1, X_2, \ldots, X_p$ and find (linear) combinations of these to produce random variables $Z_1, \ldots, Z_p$, the **principal components**, that are uncorrelated.
- Geometrically, expressing $X_1, \ldots, X_p$ through linear combinations $Z_1, \ldots, Z_p$ can be thought of as shifting and rotating the axes of the coordinate system (see left graph).
- Hence, the transform leaves the data points unchanged, but expresses them using different coordinates.
- The lack of correlation is a useful property because it means that the principal components are measuring different "dimensions" of the data.
- The principal components can be ordered according to their variance, that is, $\mathrm{Var}(Z_1) \geq \mathrm{Var}(Z_2) \geq \cdots \geq \mathrm{Var}(Z_p)$.
- If the variance captured in the higher dimensions is sufficiently small, then discarding those higher dimensions will retain most of the variability, so only little information is lost.

PCA

- Let (X, Y) be are standard normally distributed random variables with a correlation of 0.7.
- The left graph above shows a scatterplot of a sample of 100 random numbers $(x_1, y_1), \ldots, (x_{100}, y_{100})$.
- By shifting and rotating the axes, new variables Z_1, Z_2, the principal components, with $Z_i = a_i X + b_i Y$ are obtained.
- The data sample expressed in terms of Z_1, Z_2 is uncorrelated.
- Also, the variance of the data contribution from the first principal component Z_1 is much greater than the variance contribution from the second principal component.
- The graph on the right shows the data points when the data are onto the first principal component, discarding the second dimensions.

PCA

- The sample variance of $(x_1, \ldots, x_n)$ is 0.9244 and the sample variance of $(y_1, \ldots, y_n)$ is 0.9226, whereas the sample variance of the data in the first principal component is 1.6014 and of the second principal component is 0.2456.
- In other words, while the original axes each account for roughly 50% of the total variance, the first principal component accounts for 87% of the sample variance.
- Neglecting the second principal component and expressing the data in the first principal component only retains 87% of the variance (see right graph.
- In practice, the number of dimensions will be higher, and one will choose the number of principal components to reflect a certain variance contribution such as 99%.
- If the data are sufficiently correlated, then only few dimensions (principal components) will be required even for high-dimensional data.

PCA example for interest rate term structure

- Interest rates of different maturities are known to exhibit large correlations.
- Interest rate term structures are therefore a good candidate for a representation by a few factors only.
- What do you think are the main ways in which an interest term structure moves over time?

- The data below is taken from

  Hull: Machine Learning in Business. 3rd edition, independently published, 2021.

In [ ]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from scipy import linalg
```

In [ ]:
```
data=pd.read_excel("./data/Treasuries_Hull.xlsx", index_col=0, parse_da
#data=pd.read_excelI"https://raw.githubusercontent.com/packham/Python_C
data.head()
```

- Our interest lies in movements of interest rate term structures, therefore we take first differences of the data.

In [ ]:
```
d=100*data.diff();
d.dropna(inplace=True)
d.head()
```

- Correlations of term structure movements:

In [ ]:
```
sns.heatmap(d.corr())
```

- The principal components are the eigenvectors of the correlation matrix.
- These are also called factor loadings. They express the "weight" of each factor for each maturity.

In [ ]:
```
w, vr=linalg.eig(d.corr()) # eigenvalues, eigenvectors
```

- The first PC captures changes in the level the term structure.
- The second PC captures changes in the slope.
- The third PC can be interpreted as a hump in the term structure.

In [ ]:
```
plt.figure(figsize=(7,5))
plt.xticks(range(len(d.columns)), d.columns, rotation='vertical')
plt.plot(np.real(vr[:,0:3])); # first factor loadings
plt.legend(["PC 1", "PC 2", "PC 3"])
```

- The eigenvalues express the variance captured by each PC.

In [ ]:
```
np.real(100*w/w.sum()) # percentage variances
```

- The principal component scores are the original data expressed in the PC coordinate system, dimension by dimension.
- These are obtained by multiplying each PC vector with the original data (de-meaned).
- Since the principal components are determined from maximising the variance explained by the model, this can be used to interpret the principal components.
- The correlation of each score with the original data allow for an interpretation of what each principal component represents.

In [ ]:
```
pc1 = np.transpose(vr[:,0]*(d-d.mean())).sum()
pc2 = np.transpose(vr[:,1]*(d-d.mean())).sum()
pc3 = np.transpose(vr[:,2]*(d-d.mean())).sum()
pc4 = np.transpose(vr[:,3]*(d-d.mean())).sum()
pc5 = np.transpose(vr[:,4]*(d-d.mean())).sum()
```

- This gives the same result:

In [ ]:
```
pc = np.transpose(np.matmul(np.transpose(vr), np.transpose(d.values-d.v
```

In [ ]:
```
for i in range(5):
    d.insert(i, 'pc' + str(i), pc[:,i])
```

- The plot below shows the correlations of each interest rate with the first five PC's.
- Note how the interpretation is similar to the factor loadings above.

In [ ]:
```
plt.figure(figsize=(7,5))
sns.heatmap(np.transpose(d.corr().head(5)).iloc[5:])
```