Appendix

A. Transforming problem (13) into (14)

When all components of $g_{\min}$ and $g_{\max}$ are finite and $g_{\min} < g_{\max}$ we can write

$$g_{\min} \leq g(\boldsymbol{v}) \Leftrightarrow \begin{cases} g_{\min} + s_{\min} - g(\boldsymbol{v}) = 0 \\ s_{\min} \geq 0 \end{cases} \tag{18a}$$

$$g(\boldsymbol{v}) \leq g_{\max} \Leftrightarrow \begin{cases} g(\boldsymbol{v}) + s_{\max} - g_{\max} = 0 \\ s_{\max} \geq 0 \end{cases} \tag{18b}$$

Hence

$$x = \begin{bmatrix} \boldsymbol{v} \\ s_{\min} \\ s_{\max} \end{bmatrix}, \quad x_{\min} = \begin{bmatrix} v_{\min} \\ 0 \\ 0 \end{bmatrix}, \quad x_{\max} = \begin{bmatrix} v_{\max} \\ \infty \mathbf{1} \\ \infty \mathbf{1} \end{bmatrix} \tag{18c}$$

$$c(x) = \begin{bmatrix} g_{\min} + s_{\min} - g(\boldsymbol{v}) \\ g(\boldsymbol{v}) + s_{\max} - g_{\max} \end{bmatrix} \tag{18d}$$

When some components of $g_{\min}$ or $g_{\max}$ are unbounded, those constraints are discarded and no slack components are necessary. Similarly, when for some i, there holds $g_{\min}^{(i)} = g_{\max}^{(i)}$, the transformation of $g_{\min}^{(i)} \leq g(\boldsymbol{v}) \leq g_{\max}^{(i)}$ into a component of the equality constraint vector $c(x)$ is obvious, and again no slack component is necessary.

B. Further comments to the Interior-Point method discussed in V-B

We observe that the inner loop (Steps 2–4) is exited when the current iterate satisfies $E_{\mu_j}(\xi_{k+1}) \leq \kappa \mu_j$. Then the value of μ_j is reduced in Step 5 performing an iteration of the outer loop (Steps 2–5). This implies that the solution of the inner loop becomes more and more accurate as the outer loop is executed. We also notice that $E_0(\cdot)$, i.e. $E_\mu(\cdot)$ with $\mu = 0$, represents the error of the KKT system for NLP (14). Hence, the overall algorithm is terminated when the KKT system is satisfied within a tolerance ε, specified by the user.

A crucial part of an effective interior-point algorithm is the line search that is performed in Step 2 using backtracking, i.e. starting from a large candidate value for α_k and reducing it when suitable conditions for acceptance are not satisfied. In general a step is acceptable if it makes some progress towards feasibility, i.e. reducing $\|c(x)\|$, and/or achieves an improvement in the objective function $\varphi_{\mu_j}(x)$. This is typically achieved in IPOPT by also including, in the line search, a so-called second-order correction Newton step which aims to solve solely $c(x) = 0$. In particular situations, the achieved value of α_k is too small, and this forces the algorithm to enter a *restoration* phase. For details and reasoning of this phase the interested reader is referred to [43].

C. Symmetric linear system solved in place of (17)

System (17) is non-symmetric; IPOPT performs elimination of the last two row blocks (and elimination of $p_k^{\underline{z}}$ and $p_k^{\overline{z}}$) obtaining the following symmetric system to be solved

$$\begin{bmatrix} W_k + \underline{\Sigma}_k + \overline{\Sigma}_k & A_k \\ A_k^T & 0 \end{bmatrix} \begin{bmatrix} p_k^x \\ p_k^\lambda \end{bmatrix} = - \begin{bmatrix} \nabla \varphi_{\mu_j}(x_k) + A_k \lambda_k \\ c(x_k) \end{bmatrix} \tag{19}$$

in which $\underline{\Sigma}_k$ and $\overline{\Sigma}_k$ are diagonal matrices with entries

$$\sigma_{\min}^{(i)} = \begin{cases} \underline{z}^{(i)} / (x^{(i)} - x_{\min}^{(i)}) & \text{if } i \in I_{\min} \\ 0 & \text{if } i \notin I_{\min} \end{cases} \tag{20a}$$

$$\sigma_{\max}^{(i)} = \begin{cases} \overline{z}^{(i)} / (x_{\max}^{(i)} - x^{(i)}) & \text{if } i \in I_{\max} \\ 0 & \text{if } i \notin I_{\max} \end{cases} \tag{20b}$$