CO 487 Course Notes: Applied Cryptography

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Contents

1 Intro and History

Definition 1.1 (Cryptography)

Cryptography is about securing communications in the presence of malicious adversaries

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Remark

States of information

- Data at rest
- Data at transit
- Data while processing

Corollary 1.1.1 (Fundamental Goals of Cryptography)

- **Confidentiality** - Keeping data secret from all but those authorized to see it
- **Integrity** - Ensuring data has not been altered by unauthorized means
- **Authentication** - Corroborating the source of data or identity of an entity
- **Non-repudiation** - Preventing an entity from denying previous commitments or actions
- **Deniability** - Allowing an entity to deny previous commitments or actions
- **Consensus** - Ensuring a number of entities agree on the state of some data
- **Availability** - Ensuring a computer system works even if parts of it fail or are tampered with

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Remark

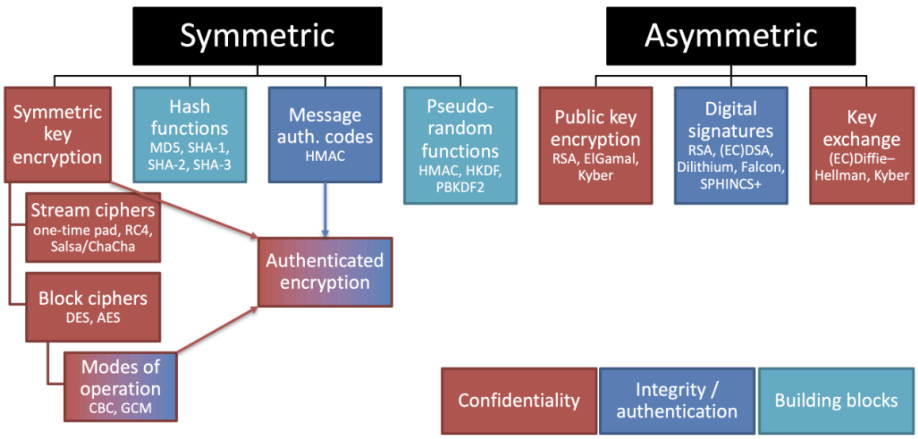

Figure 1: Cryptography building blocks

Symmetric cryptography: Uses a single shared secret to protect data. Both parties use the same secret, so it is simple and efficient, but requires the secret to be shared securely.

Asymmetric cryptography: Uses two related keys, one public and one private. This allows parties to communicate or verify identity without sharing a secret first, at the cost of being more complex and slower.

Definition 1.2 (Caesar Cipher)

Caesar cipher is a simple substitution cipher that encrypts text by shifting each letter a fixed number of positions forward in the alphabet, wrapping around at the end

Encrypt(m) where $m \in \{A, \dots, Z\}^*$

1 : **for** $i = 1, \dots, |m|$
2 : $x \leftarrow \text{Encode}(m_i)$
3 : $y \leftarrow x + 23 \bmod 26$
4 : (or equivalently $y \leftarrow x - 3 \bmod 26$)
5 : $c_i \leftarrow \text{Decode}(y)$
6 : **return** c

Decrypt(c) where $c \in \{A, \dots, Z\}^*$

1 : **for** $i = 1, \dots, |c|$
2 : $x \leftarrow \text{Encode}(c_i)$
3 : $y \leftarrow x - 23 \bmod 26$
4 : (or equivalently $y \leftarrow x + 3 \bmod 26$)
5 : $m_i \leftarrow \text{Decode}(y)$
6 : **return** m

Encode / Decode:

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	14	15	16	17	18	19	20	21	22	23	24	25

This scheme is **NOT** secure

♣

Remark

Note the difference between "encrypt" and "encode", and "decrypt" and "decode"

- **encode and decode** map letters to numbers without trying to add security – just mapping to a more convenient space
- **encrypt and decrypt** try to add security

Definition 1.3 (Shift Cipher)

Idea: Modify Caeser cipher by introducing a secret key

Key space

$\mathcal{K} = \{0, \dots, 25\}$
Randomly sample key $k \xleftarrow{\$} \mathcal{K}$

Encrypt(k, m) where $m \in \{A, \dots, Z\}^*$

1 : **for** $i = 1, \dots, |m|$
2 : $x \leftarrow \text{Encode}(m_i)$
3 : $y \leftarrow x + k \bmod 26$
4 : $c_i \leftarrow \text{Decode}(y)$
5 : **return** c

Decrypt(k, c) where $c \in \{A, \dots, Z\}^*$

1 : **for** $i = 1, \dots, |c|$
2 : $x \leftarrow \text{Encode}(c_i)$
3 : $y \leftarrow x - k \bmod 26$
4 : $m_i \leftarrow \text{Decode}(y)$
5 : **return** m

This scheme is **NOT** secure

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How might we break the shift cipher?

Assumptions

Definition 1.4 (Kerckhoff's Principal)

We know the encryption / decryption algorithm being used but not any secret keys

and,

We are given a cipher text to break

Two approaches

1. Try all 26 possible secret key values $k = 0, \ldots, 25$ (brute force / exhaustive key search)
2. Frequency Analysis

Definition 1.5 (Frequency Analysis)

Compare the distribution of letters in the cipher text with the distribution of letters in the underlying plain text space

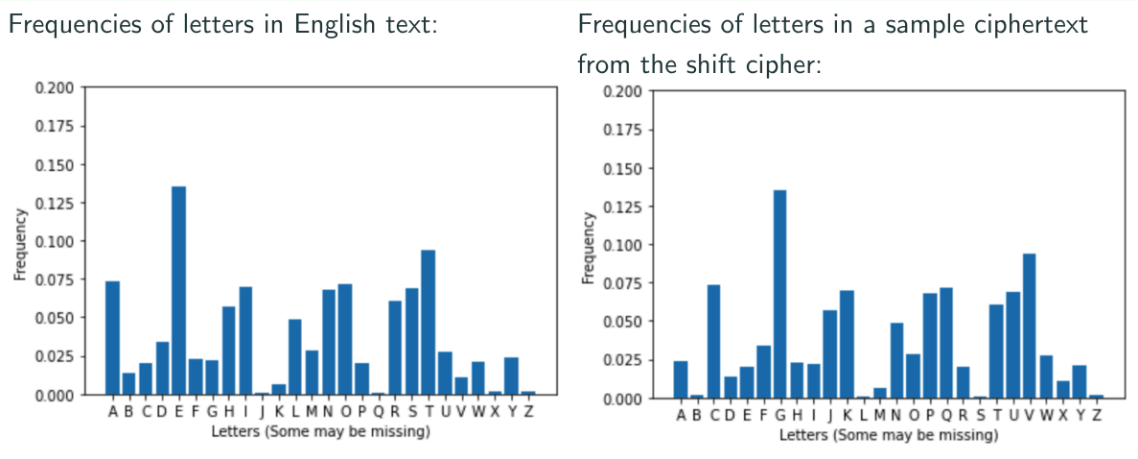


Figure 4: $k = 2$

Definition 1.6 (Vigenere Cipher)

Use different shift ciphers for different parts of the message to reduce effect of frequency analysis

Key space	**Message and ciphertext space**
$\mathcal{K} = \{A, \ldots, Z\}^B$	$\mathcal{M} = \mathcal{C} = \cup_{i \geq 0}\{A, \ldots, Z\}^{iB}$

Encrypt(k, m)

1 : **for** $i = 1, \ldots, |m|$
2 : $\quad c_i \leftarrow m_i + k[i \bmod B] \bmod 26$
3 : **return** c

Decrypt(k, c)

1 : **for** $i = 1, \ldots, |c|$
2 : $\quad m_i \leftarrow c_i - k[i \bmod B] \bmod 26$
3 : **return** m

Example with block length $B = 6$

$m =$	t	h	i	s		i	s		a	m	e	s	s	a	g	e
$+ k =$	C	R	Y	P		T	O		C	R	Y	P	T	O	C	R
$c =$	V	Y	G	H		B	G		C	D	C	H	L	O	I	V

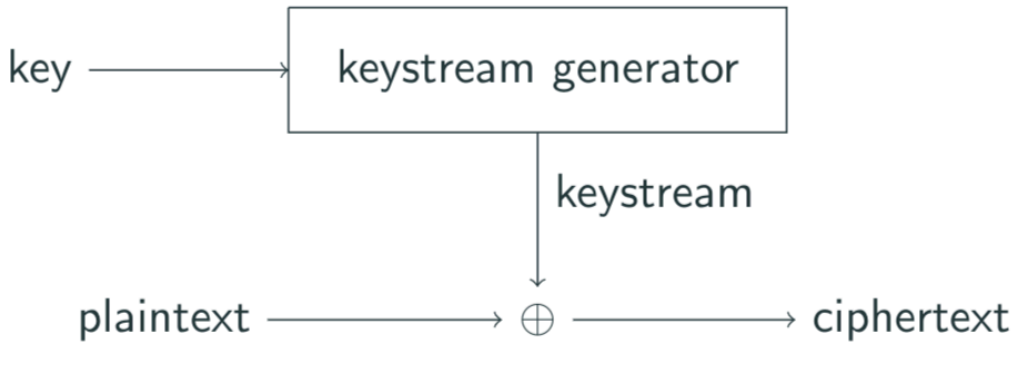
This scheme is **NOT** secure

2 Symmetric Encryption

2.1 Block Ciphers

Definition 2.1.1 (Stream Cipher)

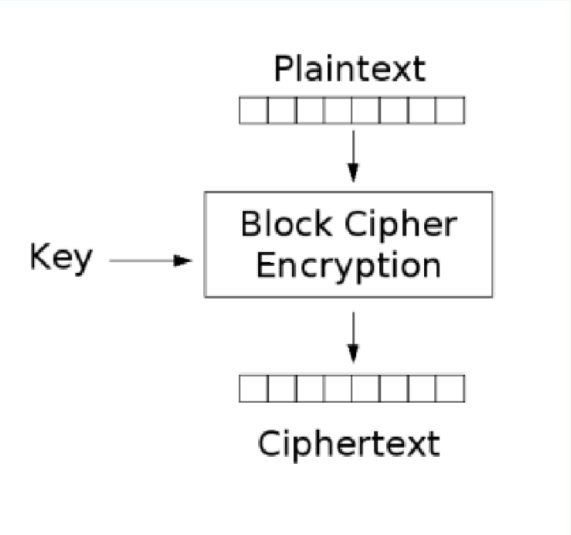
A stream cipher is a symmetric key encryption scheme in which each successive character of plaintext determines a single character of ciphertext



Definition 2.1.2 (Block Cipher)

A block cipher is a symmetric key encryption scheme in which a fixed length block of plaintext determines an equal sized block of ciphertext

e.g. AES, DES

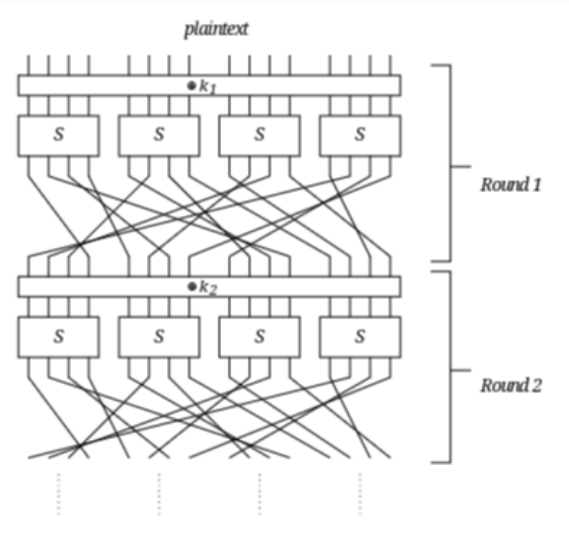


Corollary 2.1.2.1 (Desirable Properties of Block Ciphers)

- **Security**
  - **Diffusion:** Each ciphertext bit should depend on all plaintext and all key bits
  - **Confusion:** The relationship between key bits, plaintext bits, and ciphertext bits should be complicated
  - **Cascade or Avalanche Effect:** Changing one bit of plaintext or key should change each bit of ciphertext with 50% probability
  - **Key Length:** Should be small, but large enough to preclude exhaustive key search
- **Efficiency**
  - Simplicity (easier to implement and analyze)
  - High encryption and decryption rate
  - Suitability for hardware or software

Definition 2.1.3 (Substitution Permutation Networks)

A substitution permutation network (SPN) is a multiple round iterated block cipher where each round consist of a substitution operation followed by a permutation operation. During each round, a round key is `XOR`ed into the state. The round keys k_i are derived from the main key k using a key schedule function.



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Warning

Despite being called a Substitution Permutation Network, the permutation can be any invertible linear function.

Remark

To a mathematician, any bijective (one-to-one) function on a finite set can be considered a "permutation". To avoid confusion lets clarify permutation and substitution in a substitution permutation network.

1. **Bijection or Substitution**

The S-box will (often) be a one-to-one function on some small space

e.g. 4 or 8 bit strings

11110001 $\rightarrow$ 10100001

11110010 $\rightarrow$ 01000101

In this course this will be referred to as a bijection or substitution

2. **Permutation**

The permutation will be a one-to-one function of positions of bits.

e.g. A block of 128 bits

$p : \{0, 1, 2, ..., 127\} \rightarrow \{0, 1, 2, ..., 127\}$

This tells us to move the bit at position i to position $p(i)$

Remark

Two perspevtives on a permutation

1. As a function from bit position to bit position (i.e a one-to-one function on $\{0, 2, ...\}, 127)$
2. As a function from bit strings to bit strings (i.e take each bit in the bit string and rearrange it according to how the positions are permuted)

As an example, we could have a permutation like this:

Start Position	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
End Position	11	13	15	0	8	7	9	14	5	2	10	12	6	1	4	3

which would act on 16-bit strings as dictated above, which could also be done by a matrix:

$$\begin{pmatrix} 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

Definition 2.1.4 (The Advanced Encryption Standard (AES))

Requirements

- **Key Sizes:** 128, 192 and 256 bits
- **Block Sizes:** 128 bits
- Efficient on both hardware and software platforms
- Availability on a worldwide, non-exclusive, royalty-free basis
- AES is a SPN where the "permutation" operation consist of two linear transformations (one of which is a permutation)
- All operations are byte oriented

key length	number of rounds h
128	10
192	12
256	14