Mobile Communications
(ETE 4162)

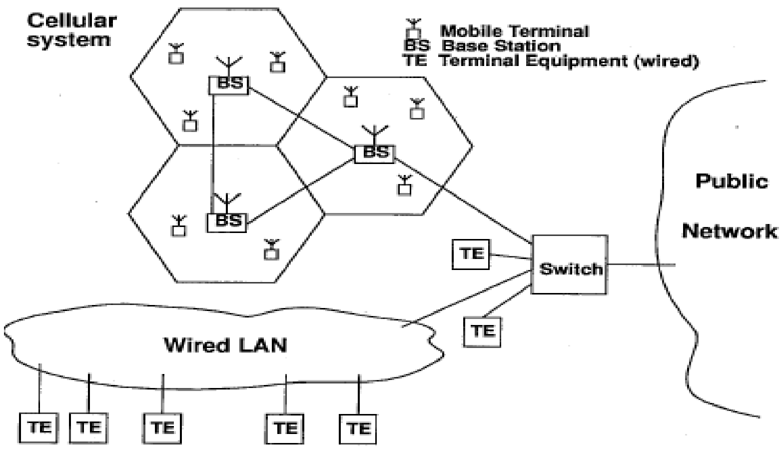
Mobile Radio Propagation

4th YEAR ETE
2021

Content

1- Introduction to Mobile Radio Propagation

1-a). Introduction: Radio Communication Concept



1.b). Introduction: Radio Frequency Bands

Classification Band	Initials	Frequency Range	Propagation Mode
Extremely low	ELF	< 300 Hz	Ground wave
Infra low	ILF	300 Hz - 3 kHz	
Very low	VLF	3 kHz - 30 kHz	
Low	LF	30 kHz - 300 kHz	
Medium	MF	300 kHz - 3 MHz	Ground/Sky wave
High	HF	3 MHz - 30 MHz	Sky wave
Very high	VHF	30 MHz - 300 MHz	Space wave
Ultra high	UHF	300 MHz - 3 GHz	
Super high	SHF	3 GHz - 30 GHz	
Extremely high	EHF	30 GHz - 300 GHz	
Tremendously high	THF	300 GHz - 3000 GHz	

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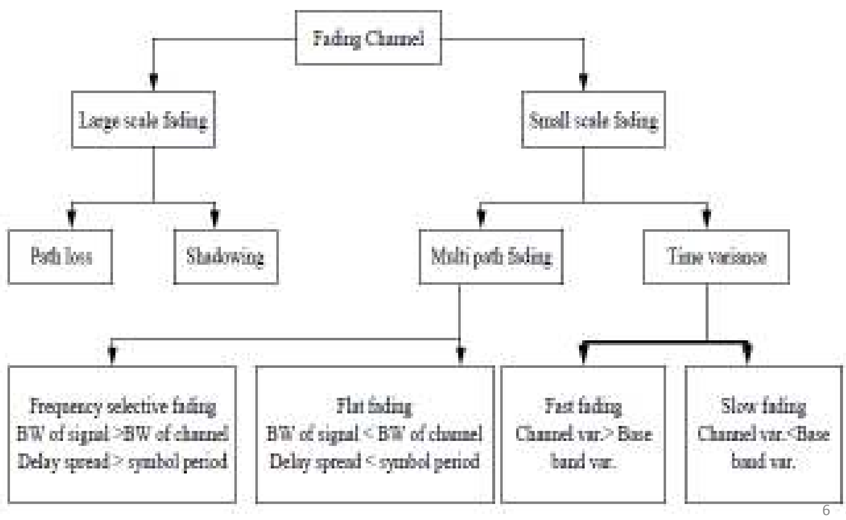
1.c). Introduction: Overview of fading channels



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1.d). Introduction: Terms and Definitions (1/2)

1) The term **fading** is used to describe rapid fluctuation of the amplitude of a radio signal over a short period of time or travel distance.
2) A **microcell** is a cell in a mobile phone network served by a low power cellular base station (tower), covering a limited area such as a mall, a hotel, or a transportation hub. A microcell is usually larger than a picocell, though the distinction is not always clear. A microcell uses power control to limit the radius of its coverage area.
3) **Base station:** A fixed station in a mobile radio system used for radio communication with mobile stations. Base stations are located at the center or on the edge of a coverage region and consist of radio channels and transmitter and receiver antennas mounted on a tower.
4) **Control channel:** Radio channels used for transmission of call setup, call request, call initiation, and other beacon or control purposes.
5) **Forward channel (also called downlink):** Radio channel used for transmission of information from the base station to the mobile.
6) **Mobile station:** A station in the cellular radio service intended for use while in motion at unspecified locations. Mobile stations may be hand-held personal units (portables) or installed in vehicles (mobiles).
7) **Mobile switching center:** Switching center which coordinates the routing of calls in a large service area. In a cellular radio system, the MSC connects the cellular base station and the mobiles to the PSTN. An MSC is also called a mobile telephone switching office (MTSO).

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1.d). Introduction: Terms and Definitions (2/2)

LARGE SCALE PROPAGATION MODELS

- Propagation models that predict the mean signal strength for an arbitrary transmitter –receiver (T-R) separation distance are useful in estimating the radio coverage area of a transmitter and are called large –scale propagation models, since they characterize signal strenght over large T-R separation distance (several hundreds or thousands of meters).

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2- LARGE SCALE PROPAGATION MODELS

2.a). Free Space Propagation Model

- Large Scale Model
- Goal:
  - To predict received signal strength when transmitter and receiver have a clear, unobstructed line-of-sight (LOS) between them.
- Examples:
  - Satellite systems,
  - LOS microwave systems

2.a). Free Space Propagation Model, Con't

- Friis Free Space Equation (valid for Far-field)

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}$$

$P_r(d)$ – Received signal strength at distance d
P_t – Transmitted power
G_t – Transmitter antenna gain
G_r – Receiver antenna gain
λ – Wavelength (m)
d – Distance between transmitter and receiver (m)
L – Systemloss not related to propagation (≥ 1; No LOSS CASE $L = 1$)

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2.a). Free Space Propagation Model, Con't

Antenna Gain:

$$G = \frac{4\pi A_e}{\lambda^2}$$ A_e – Effective antenna aperture

$$\lambda = \frac{c}{f}$$

c – speed of light (3×10^8 m/s)
f – carrier frequency (Hz)

Effective Isotropic Radiated Power (EIRP)

$$EIRP = P_t G_t$$

Far field condition (Fraunhofer Region)

d_f=*Far field*

$$d_f = \frac{2D^2}{\lambda}$$
$$d_f >> \lambda$$
$$d_f >> D$$

D – Largest physical linear dimension of the antenna

2.a). Free Space Propagation Model, Con't
FREE SPACE PATH LOSS MODEL

- Path Loss: Difference between effective transmitted power and received power.
- Free Space Path Loss (in dB)

$$PL[dB] = 10\log_{10}\left(\frac{P_t}{P_r}\right) = 10\log_{10}\left(\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2}\right)$$

- In far-field region

$$P_r(d) = P_r(d_0)\left(\frac{d_0}{d}\right)^2 \qquad d \geq d_0 \geq d_f$$

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2.a). Free Space Propagation Model, Con't

Example-1.

- Find the far-field distance for an antenna with maximum dimension of 1m and operating frequency of 900MHz.

Solution:

- Given the largest dimension of antenna, D=1m
- Operating frequency: f=900MHz

$$\lambda = c / f = \frac{3\times 10^8\, m/s}{900\times 10^6\, Hz} \qquad d_f = \frac{2D^2}{\lambda} = \frac{2(1)^2}{0.33} = 6m$$

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2.a). Free Space Propagation Model, Con't
EXAMPLE-2-

- If a transmitter produces 50 watts of power, express the transmit power in units of (a) dBm, and (b) dBW. If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency. Find the received power in dBm at a free space distance of 100m from the antenna. What is Pr(10km)? Assume unity gain for the receiver antenna.

Solution

- A) Transmitter power:
$$P_t(dBm) = 10\log[P_t(mW)/(1mW)] = 10\log[50\times10^3] = 47dBm$$
- B) Transmitter power:
$$P_t(dBm) = 10\log[P_t(W)/(1W)] = 10\log[50] = 17dBW$$
- The received power can be determined as follows:

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} = \frac{50(1)(1)(1/3)^2}{(4\pi)^2(100)^2(1)} = 3.5\times10^{-6}W = 3.5\times10^{-3}mW$$

$$P_r(dBm) = 10\log P_r(mW) = 10\log(3.5\times10^{-3}mW) = -24.5dBm$$

$$P_r(10km) = P_r(100) + 20\log\left[\frac{100}{10000}\right] = -24.5dBm - 40dB = -64.5dBm$$

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2.a). Free Space Propagation Model, Con't.
Free Space received power and path Loss
Example-3-

- What is the received power (in dBm) in the free space of a signal whose transmit power is 1W and carrier frequency is 2.4 GHz if the receiver is at a distance of 1 mile (1.6km) from the transmitter? Assume that the transmitter and receiver antenna gains are 1. What is the path loss in dB?
- **Solution:**
- 10log(P_t)=30dBm because 1W in dBm is 10 log(1000mW/1mW)=30dBm.
- Using equation $\frac{p_r}{p_t} = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2$,f_c=2.4GHz, antenna gains of 1,
- distance of 1 meter, we have 10log(P_0)=30-40.046dBm and
- P_r=-10.046-20log(1600)=-74.128dBm.
- The path loss is given by the difference between 10log (P_t) and 10log (P_r) (where both are in dBm). This is 104.128dBm.

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2.a). Free Space Propagation Model, Con't.
Free Space received power and path Loss
Example-3-

- What is the received power (in dBm) in the free space of a signal whose transmit power is 1W and carrier frequency is 2.4 GHz if the receiver is at a distance of 1 mile (1.6km) from the transmitter? Assume that the transmitter and receiver antenna gains are 1. What is the path loss in dB?
- **Solution:**

$$P_t(dBm) = 10\log(mW\ /1mW) = 10\log(10^3 mW\ /1mW) = 30dBm$$

$$\lambda = c\ /\ f = 3*10^8\ /\ 2.4*10^9 = 0.0125m$$

$$P_r = \frac{P_t * G_t * G_r * \lambda^2}{(4\pi)^2 d^2} = \frac{1*1*1*(0.0125)^2}{(4*3.14)^2*(1600)^2} = 3*10^{-11}W$$

$$P_r(dBm) = 10\log(3*10^{-11}) = -105dBm$$

- The path loss is given by the difference between 10log (P_t) and 10log (P_r) (where both are in dBm):

$$PL(dBm) = P_t(dBm) - P_r(dBm) = 30dBm - (-105dBm) = 135dBm$$

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2.b). Relating Power to Electric Field.

- In free space, the power flux density Pd (expressed in W/m^2) is given by:

$$P_d = \frac{EIRP}{4\pi d^2} = \frac{P_t G_t}{4\pi d^2} = \frac{E^2}{R_{fs}} = \frac{E^2}{\eta} W\ /\ m^2$$

- Where R_{fs} is the intrinsic impedance of free space given by η=120πΩ (377Ω). Thus, the power flux density is: $P_d = \frac{|E|^2}{377\Omega} W\ /\ m^2$
-
- Where $|E|$ represents the magnitude of the radiating portion of the electric field in the far field. Pd may be thought as the EIRP divided by the surface area of a sphere with radius d. The power received at distance d, Pr(d) is given by the power flux density times the effective aperture of the receiver antenna, and can be related to the electric field. E is the electric field inV/m.

$$P_r(d) = P_d A_e = \frac{|E|^2}{120\pi} A_e = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2} watts$$

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2.b). Relating Power to Electric Field, Con't.

- If the receiver is modeled as a matched resistive load to the receiver, then the receiver antenna will induce an rms voltage into the receiver which is half of the open circuit voltage at the antenna. Thus, if V is the rms voltage at the input of a receiver (measured by a high impedance voltmeter), and Rant is the resistance of the matched receiver, the received power is given by:

$$P_r(d) = \frac{V^2}{R_{ant}} = \frac{[V_{ant}/2]^2}{R_{ant}} = \frac{V_{ant}^2}{4R_{ant}}$$

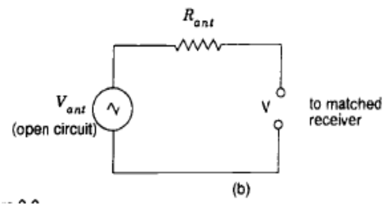

Fig. Model for voltage applied to the input of a receiver.

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2.b). Relating Power to Electric Field. Example-1-

- Assume a receiver is located 10 km from a 50W transmitter. The carrier frequency is 900MHz, free space propagation is assumed. Gt=1. and Gr=2, find (a) the power at the receiver, (b) the magnitude of the E-field at the receiver antenna (c , the rms voltage applied to the receiver input assuming that the receiver antenna has a purely real impedance of 50Ω and is matched to the receiver.
- Solutions:
- Given: Transmitted power: Pt=50W
- Carrier Frequency, fc= 900MHz
- Transmitter antenna gain; Gt=1
- Receiver antenna gain: Gr=2
- Receiver antenna resistance: 50Ω

$$P_r(d) = 10\log\left(\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2}\right) = 10\log\left(\frac{50*1*2*(1/3)^2}{(4\pi)^2 10000^2}\right) = -91.5dBW = -61.5dBm$$

- The magnitude of the received E-field is:

$$|E| = \sqrt{\frac{\Pr(d)120\pi}{Ae}} = \sqrt{\frac{\Pr(d)120\pi}{Gr\lambda^2/4\pi}} = \sqrt{\frac{7*10^{-10}*120\pi}{2*0.33^2/4\pi}} = 0.0039V/m$$

- The open circuit rms voltage at the receiver input is:

$$V_{ant} = \sqrt{P_r(d)*4R_{ant}} = \sqrt{7*10^{-10}*4*50} = 0.374mV$$

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2.c)- The Three Basic Propagation Mechanisms

- **Reflection**
  - Propagation electromagnetic wave impinges upon an object which is larger as compared to wavelength.
    - - e.g., the surface of the Earth, buildings, walls, etc.
- **Diffraction**
  - Radio path between transmitter and receiver obstructed by surface with sharp irregular edges.
  - Waves bend around the obstacle, even when LOS (line of sight) does not exist.
- **Scattering**
  - Scattered waves are produced by rough surfaces, small objects, or by other irregularities in the channel.
  - Objects smaller than the wavelength of the propagation wave.
    - - e.g. foliage, street signs, lamp posts,
- **Shadowing** is caused by obstacles between the transmitter and receiver that attenuate signal power through absorption, reflection, scattering, and diffraction.

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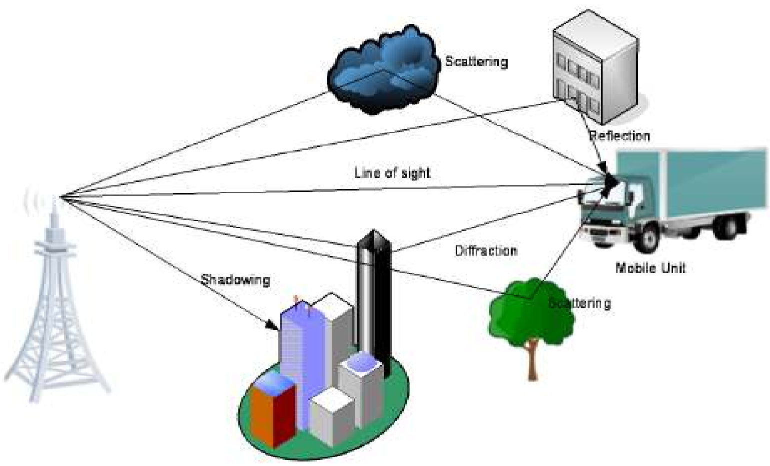
2.c)- The Three Basic Propagation Mechanisms, con't



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2.c)- The Three Basic Propagation Mechanisms, con’t
Reflection from Dielectrics, con’t

- The nature of reflection varies with the direction of polarization of the E-field.
- Two distinct cases:
- The E-field polarization is parallel with the plane of incidence (that is, the E-field has a **vertical polarization**, or normal component, with respect to the reflecting surface).
- The E-field polarization is perpendicular to the plane of incidence (that is, the incident E-field is pointing out of the page towards the reader, and is perpendicular to the page and parallel to the reflecting surface).

2.c)- The Three Basic Propagation Mechanisms, con’t
Reflection from Dielectrics, con’t

E-Field in plane of incidence

$$\Gamma_{\parallel} = \frac{E_r}{E_i} = \frac{\eta_2 \sin\theta_t - \eta_1 \sin\theta_i}{\eta_2 \sin\theta_t + \eta_1 \sin\theta_i}$$

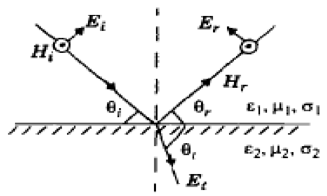
E-Field not in plane of incidence

$$\Gamma_{\perp} = \frac{E_r}{E_i} = \frac{\eta_2 \sin\theta_i - \eta_1 \sin\theta_t}{\eta_2 \sin\theta_i + \eta_1 \sin\theta_t}$$

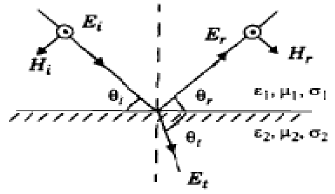
Case of 1st medium is free space: Vertical and horizontal polarization become:

$$\Gamma_{\parallel(V)} = \frac{-\varepsilon_r \sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i}}{\varepsilon_r \sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i}}$$

$$\Gamma_{\perp(H)} = \frac{\sin\theta_i - \sqrt{\varepsilon_r - \cos^2\theta_i}}{\sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i}}$$



(a) E-field in the plane of incidence



(b) E-field normal to the plane of incidence

2.c)- The Three Basic Propagation Mechanisms, con't
Reflection from Dielectrics, Con't.

- **Example 3.4:** demonstrate that if medium 1 is free space and medium 2 is a dielectric, both $|\Gamma_{\parallel}|$ and $|\Gamma_{\perp}|$ approach 1 as θ_i approaches 0^0 regardless of ε_r .

Solution:

Substituting $\theta_i = 0^0$ in equation (3.24)

$$\Gamma_{\parallel} = \frac{-\varepsilon_r \sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i}}{\varepsilon_r \sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i}}$$

$$\Gamma_{\parallel} = \frac{-\varepsilon_r \sin 0 + \sqrt{\varepsilon_r - \cos^2 0}}{\varepsilon_r \sin 0 + \sqrt{\varepsilon_r - \cos^2 0}} = \frac{\sqrt{\varepsilon_r - 1}}{\sqrt{\varepsilon_r - 1}} = 1$$

Substituting $\theta_i = 0^0$ in equation (3.25)

$$\Gamma_{\perp} = \frac{\sin\theta_i - \sqrt{\varepsilon_r - \cos^2\theta_i}}{\sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i}}$$

$$\Gamma_{\perp} = \frac{\sin\theta_i - \sqrt{\varepsilon_r - \cos^2\theta_i}}{\sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i}} = \frac{-\sqrt{\varepsilon_r - 1}}{\sqrt{\varepsilon_r - 1}} = -1$$

This example illustrates that ground may be modeled as a perfect reflector with a reflection coefficient of unit magnitude when an incident wave grazes the earth, regardless of polarization or ground dielectric properties.

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2.c)- The Three Basic Propagation Mechanisms, con't
Brewster Angle, con't

- The Brewster angle is the angle at which no reflection occurs in the medium of origin. It occurs when the incident angle θ_B is such that the reflection coefficient $\Gamma_{\parallel}$ is equal to zero. The Brewster angle is given by the value of θ_B which satisfies:

$$\sin(\theta_B) = \sqrt{\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}}$$

For the case when the first medium is free space and the second medium has a relative permitivity ε_r , the above equation can be expressed as:

$$\sin(\theta_B) = \sqrt{\frac{\varepsilon_r - 1}{\varepsilon_r^2 - 1}}$$

Note that the Brewster angle occurs only for verticle (i.e. Parallel) polarization.

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2.c)- The Three Basic Propagation Mechanisms, con't
Brewster Angle, con't

- Calculate the Brewster angle for a wave impinging on ground having a permitivity of $\varepsilon_r = 4$.
- **Solution:**
- The Brewster angle can be found by substituting the values for ε_r in equation:

$$\sin(\theta_i) = \frac{\sqrt{(4)-1}}{\sqrt{(4)^2-1}} = \sqrt{\frac{3}{15}} = \sqrt{\frac{1}{5}}$$

$$\theta_i = \sin^{-1}\sqrt{\frac{1}{5}} = 26.56^0$$

- Thus the Brewster angle for $\varepsilon_r = 4$ is equal to 26.56^0

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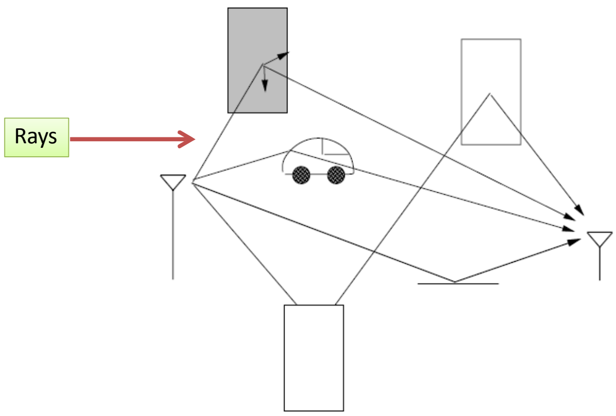
2.c)- The Three Basic Propagation Mechanisms, con't
Ray Tracing Models



Reflection, diffraction, and scattering effects on the wavefront are approximated using simple geometric equations instead of Maxwell's more complex wave equations.

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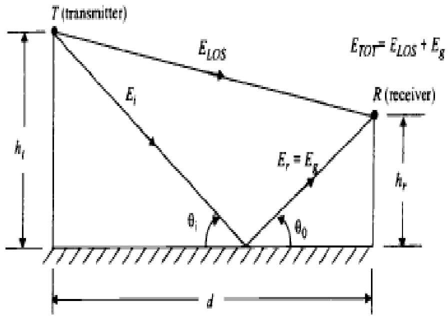
2.c)- The Three Basic Propagation Mechanisms, con't
Two Ray Model:

•The 2- ray ground reflection model is a useful propagation model that is based on geometric optics, and considers both the direct path and a ground reflected propagation path between transmitter and receiver.
•This model has been found to be reasonably accurate for predicting the large-scale signal strenght over distances of several kilometers for mobile radio systems that use tall towers as well as for line of sight microcell channels in urban environments.



$$PL[dB] \approx 10 \log_{10}\left(\frac{P_t}{P_r}\right) = 10 \log_{10}\left(\frac{G_t G_r \lambda^2}{(4\pi)^2 d^4}\right)$$

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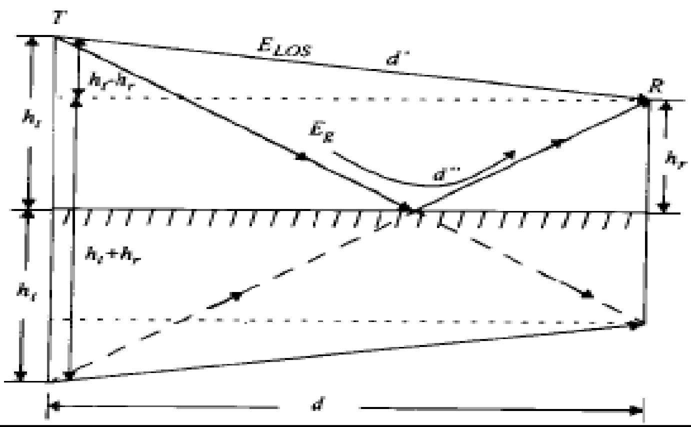
2.c)- The Three Basic Propagation Mechanisms, con't
Two Ray Model, Con't

- Using the method of images, The path Δ diferrence between the line of sight and the ground reflected paths can be expressed as:
- $$\Delta = d'' - d' = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$



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2.c)- The Three Basic Propagation Mechanisms, con't
Two Ray Model, Con't

- When the T-R separation distance d is very large compared to $h_t + h_r$, the path difference becomes:

$$\Delta = d'' - d' \approx \frac{2h_t h_r}{d}$$

- Once the path difference is known, the phase difference θ_Δ between the two E-field components and the time delay τ_d between the arrival of the two components can be easily computed using the following relations: $\theta_\Delta = \frac{2\pi\Delta}{\lambda} = \frac{\Delta\omega_c}{c}$ and $\tau_d = \frac{\Delta}{c} = \frac{\theta_\Delta}{2\pi f_c}$
- Then: $\frac{\theta_\Delta}{2} \approx \frac{2\pi h_t h_r}{\lambda d} < 0.3 rad$ and $d > \frac{20\pi h_t h_r}{3\lambda} \approx \frac{20 h_t h_r}{\lambda}$
- Therefore, the received E-Field can be approximated as: $E_{TOT}(d) \approx \frac{2E_0 d_0}{d}\frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} V/m$

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2.c)- The Three Basic Propagation Mechanisms, con't
Two Ray Model, Con't

- Where k is a constant related to Eo, the antenna heights, and the wavelength.
- We have seen that the power received at d is related to the square of the electric field. Therefore, the power at a distance d from the transmitter can be expressed as:

$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}$$

- The path loss for the 2-ray model (with antenna gains) can be expressed in dB as:

$$Pl(dB) = 40 \log d - (10 \log G_t + 10 \log G_r + 20 \log h_t + 20 \log h_r)$$

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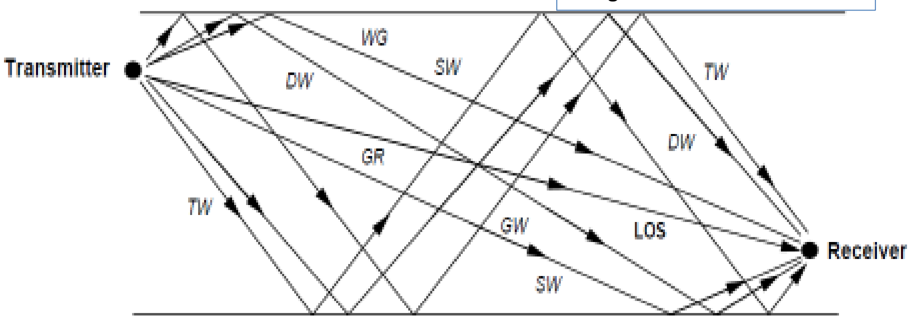
2.c)- The Three Basic Propagation Mechanisms, con't

Ten Ray Model, con't

Designed for urban microcells



$$P_r = P_t \left[\frac{\lambda}{4\pi}\right]^2 \left| \frac{\sqrt{G_l}}{l} + \sum_{i=1}^{9} \frac{\Gamma_i \sqrt{G_{x_i}} e^{j\Delta\phi_i}}{x_i} \right|^2,$$

$$with: \Delta\phi_i = 2\pi(x_i - l)/\lambda$$

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2.c)- The Three Basic Propagation Mechanisms, con't

Example

- A mobile is located 5km away from a base station and uses a verticle λ/4 monopole antenna with a gain of 2.55 dB to receive cellular radio signals. The E-field at 1km from the transmitter is measured to be 10^{-3} V/m. The carrier frequency used for this system is 900MHz. A) Find the length and the gain of the receiving antenna. B) Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50m and the receiving antenna is 1.5 m above ground.
- Solution: T-R separation distance=5 km, E-field at a distance of 1km= 10^{-3} V/m, Frequency of operation, f=900 MHz. $\lambda = \frac{c}{f} = \frac{3\times10^8}{900\times10^6} = 0.333m$
- Lenght of the antenna, $L = \lambda/4 = 0.333/4 = 0.0833m = 8.33cm$
- Gain of $\lambda/4$ monopole antenna can be obtained . Gain of antenna=1.8=2.55 dB
- B) since $d >> \sqrt{h_t h_r}$, the electric field is given by:

$$E_R(d) = \frac{2E_0 d_0 2\pi h_t h_r}{d*\lambda*d} \approx \frac{k}{d^2} v/m = \frac{2*10^{-3}*1*10^3}{5*10^3}\left[\frac{2\pi(50)(1.5)}{0.333(5*10^3)}\right] = 113.1*10^{-6} V/m$$

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2.c)- The Three Basic Propagation Mechanisms, con't
Example, Cont.

- The received power at a distance d can be obtained as follows:

$$P_r(d) = \frac{(113.1 \times 10^{-6})^2}{377} \times \left[\frac{1.8 \times (0.333)^2}{4\pi}\right]$$

$$P_r(d = 5km) = 5.4 \times 10^{-13} W = -122.68 dBW$$

- Or -92.68dBm.

$$p_r(d) = \frac{|E|^2}{120\pi} A_e$$

$$A_e = \frac{G\lambda^2}{4\pi}$$

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2.c)- The Three Basic Propagation Mechanisms, con't
Diffraction

- Diffraction allows radio signals to propagate around the curved surface of the earth, beyond the horizon, and to propagate behind obstructions.
- Although the received field strength decreases rapidly as a receiver moves deeper into the obstructed (shadowed) region, the diffraction field still exists and often has sufficient strenght to produce a useful signal.
- The phenomenon of diffraction can be explained by Huygen's principle, which states that ' 'All points on a wavefront can be considered as point sources for the production of a secondary wavelets, and that these wavelets combine to produce a new wavefront in the direction of propagation.
- The diffraction is caused by the the propagation of a secondary wavelets into a shadowed region.
- The field strenght of a diffracted wave in the shadowed region is the vector sum of the electric field components of all the secondary wavelets in the space around the obstacle.

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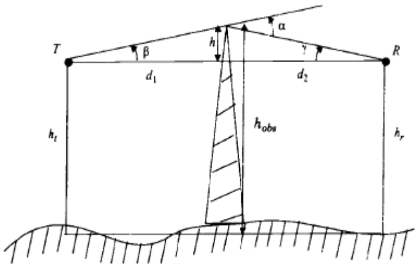
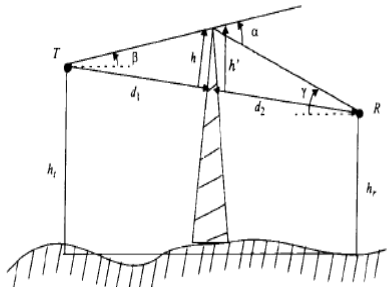
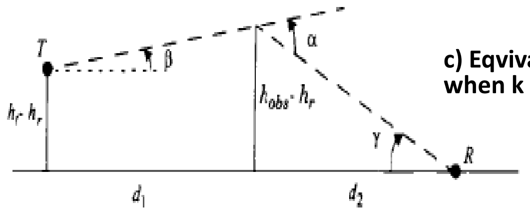
2.c)- The Three Basic Propagation Mechanisms, con't
Fresnel Zone Geometry.

a) Knife-edge diffraction geometry.



b) Knife-edge diffraction geometry. Different heights





c) Eqvivalent Knife-edge diffraction when k and k' are virtually identical.

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2.c)- The Three Basic Propagation Mechanisms, con't
Fresnel Zone Geometry

- Consider a transmitter and receiver separated in free space. Let an obstructing screen of effective height h with infinite width be placed between them at a distance d1 from the transmitter as d from the receiver. Assuming h<<d1, d2 and h>>λ , then the difference between the direct path and the diffracted path, called the excess path length (Δ (D)), can be obtained from the geometry of the above figure (b) as:
- $$\Delta_D = \frac{h^2}{2}\frac{(d_1 + d_2)}{d_1 d_2}$$
- The corresponding phase difference is given by:
- $$\phi = \frac{2\pi\Delta}{\lambda} \approx \frac{2\pi}{\lambda}\frac{h^2}{2}\frac{(d_1 + d_2)}{d_1 d_2}$$, then $\alpha = \beta + \gamma$,
  then from the above figure (c), we have: $$\alpha \approx h\left(\frac{d_1 + d_2}{d_1 d_2}\right)$$
- Using the dimensionless Fresnel-Kirchoff diffraction parameter v which is given by:
- $\alpha \equiv radians$ and $\upsilon = h\sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}} = \alpha\sqrt{\frac{2 d_1 d_2}{\lambda(d_1 + d_2)}}, then, \phi = \frac{\pi}{2}\upsilon^2$

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2.c)- The Three Basic Propagation Mechanisms, con't
Fresnel Zone Geometry

- It is clear that the phase difference between a direct line-of-sight and diffracted path is a function of height and position of the obstruction, as well as the transmitter and receiver location.
- The concept of diffraction loss as a function of the path difference around an obstruction is explained by Fresnel zones. Fresnel zones represent successive regions where secondary waves have a path length from the transmitter to receiver which are nλ/2 greater than the total path lenght of a line of sight path.
- The concentric circles on the plane represent the loci of the origins of secondary wavelets which propagate to the receiver such that the total path length increases by λ/2 for successive circles. These circles are called Fresnel zones. The radius of the nth Fresnel zone circle is denoted by rn and can be expressed in terms of n, λ, d1 and d2 by:

$$r_n = \sqrt{\frac{n \lambda d_1 d_2}{d_1 + d_2}}$$

- This approximation is valid for d1,d2>>rn

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2.c)- The Three Basic Propagation Mechanisms, con't
Knife-edge Diffraction Model

- Estimating the signal attenuation caused by diffraction of radio waves over hills and buildings is essential in predicting the field strength in a given service area.
- The electric field strength Ed of a knife-edge diffracted wave is given by: $\frac{E_d}{E_o} = F(\upsilon) = \frac{(1+j)}{2} \int_{\upsilon}^{\infty} \exp((-j\pi t^2)/2)\,dt$
- Where Eo is the free space field strength in the absence of both the ground and the knife edge, and F(v) is the complex Fresnel integral.
- The diffraction gain due to the presence of a knife edge, as compared to the free space E-field, is given by: $G_d(dB) = 20\log|F(\upsilon)|$
- An approximate solution is:

$$G_d(dB) = 0, when, \upsilon \le -1.....(a)$$
$$G_d(dB) = 20\log(0.5 - 0.62\upsilon), when, -1 \le \upsilon \le 0...(b)$$
$$G_d(dB) = 20\log(0.5\exp(-0.95\upsilon)), when, 0 \le \upsilon \le 1...(c)$$
$$G_d(dB) = 20\log(0.4 - \sqrt{0.1184 - (0.38 - 0.1\upsilon)^2}), when, 1 \le \upsilon \le 2.4,....(d)$$
$$G_d(dB) = 20\log\left(\frac{0.225}{\upsilon}\right), when, \upsilon > 2.4,....(e)$$

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2.c)- The Three Basic Propagation Mechanisms, con't Example

- Compute the diffraction loss for the three cases shown in next slide (i.e. Figure 3.12). Assume

$$\lambda = 1/3m, d_1 = 1km, d_2 = 1km, (a)h = 25m, (b)h = 0, (c)h = -25m.$$

- Compare your answers using values from figure 3.14, as well as the approximate solution given by equation (3.61a) –(3.61.e). For each of these cases, identify the Fresnel zone within which the tip of the obstruction lies.
- **Solution:**
- **Given:**

$$\lambda = 1/3m, d_1 = 1km, d_1 = 1km, (a)h = 25m, (b)h = 0, (c)h = -25m.$$

- **A) the fresnel diffraction parameter is obtained as:**

$$v = h\sqrt{\frac{2(d_1 + d_2}{\lambda d_1 d_2}} = 25\sqrt{\frac{2(1000+1000}{(1/3)*1000*1000}} = 2.74$$

- **The diffraction loss is:**

$$G_d(dB) = 20\log\left(\frac{0.225}{v}\right) = 20\log\left(\frac{0.225}{2.74}\right) = -21.7dB$$

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2.c)- The Three Basic Propagation Mechanisms, con't *DİFFRACTİON LOSS*

- Using the figure below, the diffraction loss is obtained approximately as 22dB.

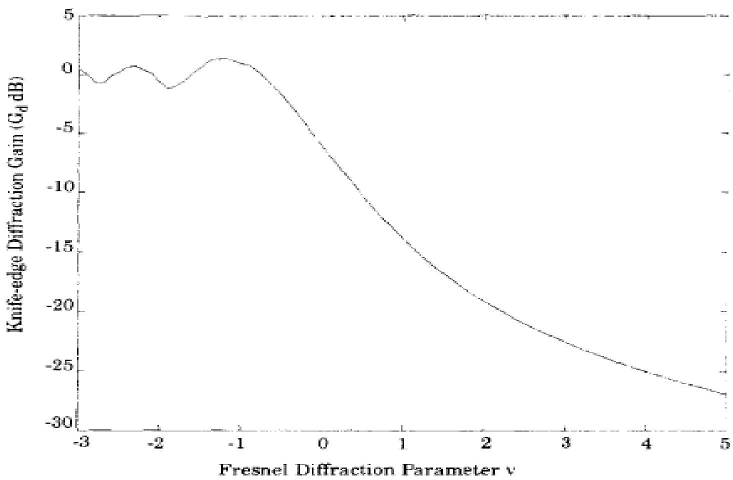

Figure 3.14
Knife-edge diffraction gain as a function of Fresnel diffraction parameter v

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- The path length difference between the direct and diffracted rays is derived as follows:

$$\Delta \approx \frac{h^2}{2}\frac{(d_1 + d_2)}{d_1 d_2} = \frac{25^2}{2}\frac{(1000+1000)}{1000+1000} = 0.625m$$

- To find the Fresnel zone in which the tip of the obstruction lies we need to compute n which satisfies the relation:

$$\Delta = n\lambda / 2. For, \lambda = 1/3m, and\ \Delta = 0.625m$$
$$n = \frac{2\Delta}{\lambda} = \frac{2 \times 0.625}{0.3333} = 3.75$$

- Therefore, the tip of the obstruction completely blocks the first three fresnel zones.
- h=0, v=0, diffraction loss is -6dB, n=0.5
- h=-25, v=-2.74, the diffraction loss is 1dB, n=1

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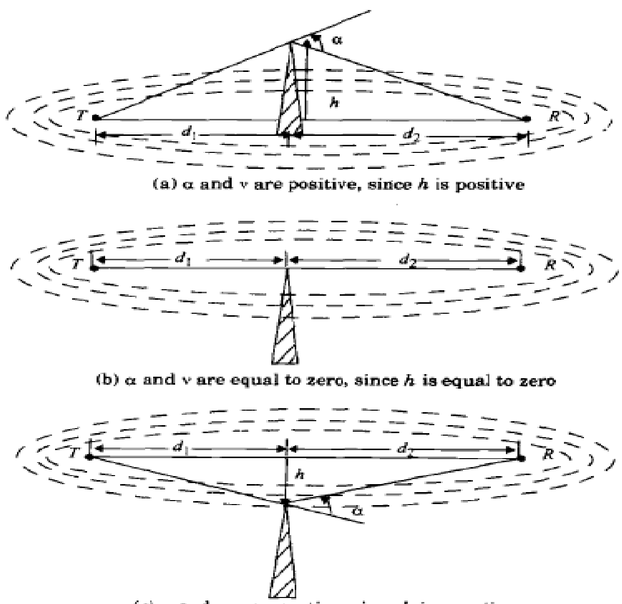

(a) α and v are positive, since h is positive

(b) α and v are equal to zero, since h is equal to zero

(c) α and v are negative, since h is negative

Figure 3.12
Illustration of Fresnel zones for different knife-edge diffraction scenarios.

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2.c)- The Three Basic Propagation Mechanisms, con't
Example 3.8

- Given the geometry below next slide, determine (a) the loss due to knife-edge diffraction and (b) the height of the obstacle required to induce 6dB diffraction loss. Assume f=900MHz.
- Solution:
- (a) The wavelength $\lambda = \frac{c}{f} = \frac{3*10^8}{900*10^6} = \frac{1}{3}m$

$$\beta = \tan^{-1}\left(\frac{75-25}{10000}\right) = 0.2865^0$$
$$\gamma = \tan^{-1}\left(\frac{75}{2000}\right) = 2.15^0$$
$$\alpha = \beta + \gamma = 2.434^0 = 0.0424\,rad$$
$$v = 0.0424\sqrt{\frac{2*10000*2000}{(1/3)*(10000+2000)}} = 4.24$$

- Therefore, the diffraction loss is 25.5 dB.
- B) for 6dB diffraction loss, v=0, h is found using similar triangles $(\beta = -\gamma)$ as shown below

$$\frac{h}{2000} = \frac{25}{12000}, h = 4.16m$$

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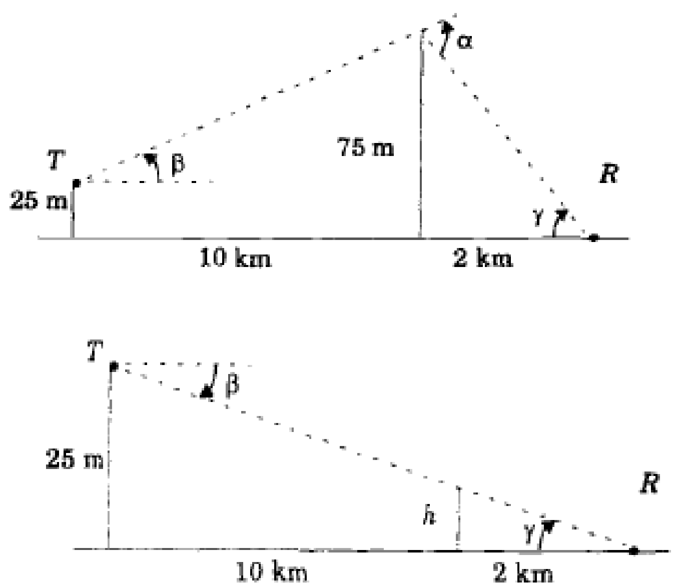

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2.d). Practical link Budget Design Using Path Loss Models

- Most radio propagation models are derived using a combination of analytical and empirical methods.
- The empirical approach is based on fitting curves or analytical expressions that recreate a set of measured data.
- Over time, some classical propagation models have emerged, which are now used to predict large-scale coverage for mobile communication systems design.
- By using path loss models to estimate the received signal level as a function of distance, it becomes possible to predict the SNR for a mobile communication system.

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2.d). Practical link Budget Design Using Path Loss Models
LOG-DİSTANCE PATH LOSS MODEL

- Received power decreases logaritmically with distance.
- Valid for both indoor and outdoor channels.
- Average large-scale path loss:

$$\overline{PL}(d) \propto \left(\frac{d}{d_0}\right)^n \qquad n - \text{Pathloss exponent}$$

$$\overline{PL}(d) = \overline{PL}(d_0) + 10n\log_{10}\left(\frac{d}{d_0}\right)$$

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2.d). Practical link Budget Design Using Path Loss Models

TYPİCAL LARGE-SCALE PATH LOSS

Table 4.2 Path Loss Exponents for Different Environments

Environment	Path Loss Exponent, n
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

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2.d). Practical link Budget Design Using Path Loss Models

LOG-NORMAL SHADOWİNG:

- Measurements have shown that at any distance, path loss is random, causing deviation from average distance.
- This can be modelled by log-normal distribution (i.e. Gaussian distribution in dB)

$$\overline{PL}(d)[dB] = \overline{PL}(d_0) + 10n\log_{10}\left(\frac{d}{d_0}\right) + X_\sigma$$

X_σ – Zero- mean Gaussian distributed random variable with st.dev.σ in dB

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2.d). Practical link Budget Design Using Path Loss Models

LOG-NORMAL SHADOWİNG

- Gaussian Probability Distribution Function

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/2\sigma^2}$$

- Q-Function & Error function (erf)

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^{\infty} e^{-x^2/2} dx = \frac{1}{2}\left[1 - erf\left(z/\sqrt{2}\right)\right]$$

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2.d). Practical link Budget Design Using Path Loss Models

LOG-NORMAL SHADOWİNG

- Probability that the received signal exceeds certain value γ

$$\Pr\left[P_r(d) > \gamma\right] = Q\left(\frac{\gamma - \overline{P}_r(d)}{\sigma}\right)$$

- Probability that the received signal is below certain value γ (OUTAGE PROBABILITY)

$$\Pr\left[P_r(d) < \gamma\right] = Q\left(\frac{\overline{P}_r(d) - \gamma}{\sigma}\right)$$

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2.d). Practical link Budget Design Using Path Loss Models

DETERMINATION OF PERCENTAGE OF COVERAGE AREA

- Percentage of useful service area $U(\gamma)$ (The percentage of service area where received signal strength exceeds γ)

$$U(\gamma) = \frac{1}{\pi R^2}\int \Pr\left[P_r(d) > \gamma\right] dA = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^{R} \Pr\left[P_r(d) > \gamma\right] r\,dr\,d\vartheta$$

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2.d). Practical link Budget Design Using Path Loss Models

DETERMINATION OF PERCENTAGE OF COVERAGE AREA

$$U(\gamma) = \frac{1}{2}\left[1 - erf(a) + \exp\left(\frac{1-2ab}{b^2}\right) \times \left(1 - erf\left(\frac{1-ab}{b}\right)\right)\right]$$

$$a = \frac{\gamma - P_t + \overline{PL}(d_0) + 10n\log_{10}(R/d_0)}{\sigma\sqrt{2}}$$

$$b = \frac{10n\log_{10}(e)}{\sigma\sqrt{2}}$$

Derivation availabe @ Rappaport Ch4, p.141-142

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2.d). Practical link Budget Design Using Path Loss Models

AREA VERSUS DISTANCE COVERAGE MODEL WITH SHADOWING MODEL

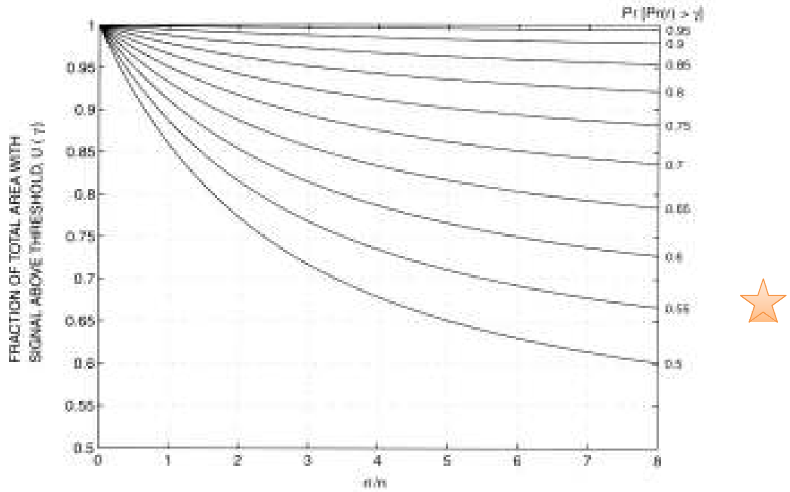

Figure 4.18 Family of curves relating fraction of total area with signal above threshold, $U(\gamma)$ as a function of probability of signal above threshold on the cell boundary.

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2.d). Practical link Budget Design Using Path Loss Models

AREA VERSUS DISTANCE COVERAGE MODEL -EXAMPLE

- Four received power measurements were taken at distances of 100m, 200m, 1km, and 3km from the transmitter. These measured values are given in the table below. For do=100m, a) find the minimum mean square error (MMSE) estimate for the path loss exponent, n; b) calculate the standard deviation about the mean value c) predict the likelihood that the received signal level at 2km will be greater than -60dBm; and (e) predict the percentage of the area within a 2 km radius cell that receives signals greater than -60dBm, given the results in (d).

Distance from Transmitter	Received Power
100m	0
200m	-20dBm
1000m	-35dBm
3000m	-70dBm

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2.d). Practical link Budget Design Using Path Loss Models

AREA VERSUS DISTANCE COVERAGE MODEL -EXAMPLE

- The MMSE estimate may be found using the following method. Let P_i be the received power at a distance d_i and let $\acute{P}_t$ be the estimate of p_t using the $(d/d_o)^n$ path loss of equation (3.67). The sum of squared errors between the measured and estimated values is given by:
$$J(n) = \sum_{i}^{k} (p_i - \hat{p}_i)^2$$
- The value of n which minimizes the mean square error can be obtained by equating the derivative of $J(n)$ to zero, and then solving for n.
- A) using the equation (3.68), we find $\hat{p}_i = p_i(d_0) - 10n\log\left(\frac{d_i}{100m}\right)$
- Recognizing that $p(d_0) = 0dBm$, we find the following estimates for $\hat{p}_i$ in dBm.
- $$\hat{p}_{1=0}, \hat{p}_2 = -3n, \hat{p}_3 = -10n, \hat{p}_4 = -14.77n$$

The sum of squared errors is then given by:

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$$J(n) = (0-0)^2 + (-20-(-3n))^2 + (-35-(-10n))^2 + (-70-(-14.77n))^2$$
$$= 6525 - 2887.8n + 327.153n^2$$
$$\frac{dJ(n)}{dn} = 654.306n - 2887.8.$$

- Setting this equal to zero, the value of n is obtained as n=4.4
- (b) The sample variance $\sigma^2 = J(n)/4$ at $n = 4.4$ can be obtained as follows:

$$J(n) = (0+0) + (-20+13.2)^2 + (-35+44)^2 + (-70+64.988)^2$$
$$= 152.36.$$
$$\sigma^2 = 152.36/4 = 38.09$$

- Therefore $\sigma = 6.17dB$ which is a biased estimate. In general, a greater number of measurements are needed to reduce σ^2
- C) the estimate of the received power at d=2km is given by:

$$\hat{p}(d = 2km) = 0 - 10(4.4)\log(2000/100) = -57.24dBm$$

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- A Gaussian random variable having zero mean and $\sigma = 6.17$ could be added to this value to simulate random shadowing effects at d=2km.
- D) The probability that the received signal level will be greater than -60dBm is given by:

$$P_r\left[p_r(d) > -60dBm\right] = Q\left(\frac{\gamma - \overline{p}_r(d)}{\sigma}\right) = Q\left(\frac{-60+57.24}{6.17}\right) = 67.4\%$$

- E) If 67.4% of the users on the boundary receive signals greater than -60dBm, then equation (3.78) or Figure 3.18 may be used to determine that 92% of the cell area receives coverage above -60dBm.

2.e). Outdoor Propagation Models

- Radio transmission in a mobile communications system often takes place over irregular terrain.
- The terrain profile of a particular area needs to be taken into account for estimating the path loss.
- The terrain profile may vary from a simple curved earth profile to a highly mountainous profile.
- The presence of trees, buildings, and other obstacles also must be taken into account.
- A number of propagation models are available to predict path loss over irregular terrain.
- While all these models aim to predict signal strength at a particular receiving point or in a specific local area (called a sector), the methods vary widely in their approach, complexity, and accuracy.
- Most of these models are based on a systematic interpretation of measurement data obtained in the service area.
- Some of the commonly used outdoor propagation models are now discussed.

2.e). Outdoor Propagation Models
i.) Longley-Rice Model

- The Longley-Rice model [Ric67], [Lon68] is applicable to point-to-point communication systems in the frequency range from 40 MHz to 100 GHz, over different kinds of terrain.
- The median transmission loss is predicted using the path geometry of the terrain profile and the refractivity of the troposphere.
- Geometric optics techniques (primarily the 2-ray ground reflection model) are used to predict signal strengths within the radio horizon. Diffraction losses over isolated obstacles are estimated using the Fresnel-Kirchoff knife-edge models.
- Forward scatter theory is used to make troposcatter predictions over long distances, and far field diffraction losses in double horizon paths are predicted using a modified Van der Pol-Bremmer method.
- The Longley-Rice propagation prediction model is also referred to as the ITS irregular terrain model.

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2.e). Outdoor Propagation Models
ii. Okumura Model

- Okumura's model is one of the most widely used models for signal prediction in urban areas.
- This model is applicable for frequencies in the range 150 MHz to 1920 MHz (although it is typically extrapolated up to 3000 MHz) and distances of 1 km to 100 km.
- It can be used for base station antenna heights ranging from 30 m to 1000 m. The model can be expressed as:

$$L_{50}(dB) = L_F + A_{mu}(f, d) - G(h_{te}) - G(h_{re}) - G_{AREA}$$

- where L50 is the 50th percentile (i.e., median) value of propagation path loss, LF is the free space propagation loss, Amu is the median attenuation relative to free space, G(hte) is the base station antenna height gain factor, G(hre) is the mobile antenna height gain factor, and GAREA is the gain due to the type of environment.

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2.e). Outdoor Propagation Models
ii. Okumura Model, con't.

- Okumura found that G(hte) varies at a rate of 20dB/decade and G(hre) varies at a rate of 10dB/decade for heights less than 3m.

$$G(h_{te}) = 20\log\left(\frac{h_{te}}{200}\right),\ for: 1000m > h_{te} > 30m$$
$$G(h_{re}) = 10\log\left(\frac{h_{re}}{3}\right),\ for: h_{re} \le 3m$$
$$G(h_{re}) = 20\log\left(\frac{h_{re}}{3}\right),\ for: 10m > h_{re} > 3m$$

- Okumura's model is considered to be among the simplest and best in terms of accuracy in path loss prediction for mature cellular and land mobile radio systems in cluttered environments.
- It is very practical and has become a standard for system planning in modern land mobile radio systems in Japan.
- The major disadvantage with the model is its slow response to rapid changes in terrain, therefore the model is fairly good in urban and suburban areas, but not as good in rural areas.

2.e). Outdoor Propagation Models
ii. Okumura Model, con't.-Example-

- Find the media path loss using Okumura's model for d=50km, h_{te}=100m, h_{re}=10m in a suburban environment. If the base station transmitter radiates an EIRP of 1kW at a carrier frequency of 900MHz. Find the power at the receiver (assume a unitsy gain receiving antenna).
- Solution:
- The free space path loss L_f can be calculated using equation as:

$$L_F = 10\log\left[\frac{\lambda^2}{(4\pi)^2 d^2}\right] = 10\log\left[\frac{(3\times10^8/900\times10^6)^2}{(4\pi)^2\times(50\times10^3)^2}\right] = 125.5dB$$

$$From\,Okumura\,Curves$$
$$A_{mu}(900MHz(50km)) = 43dB$$
$$G_{ARFA} = 9dB$$

Using equation (3.81.a) and (3.81.c) we have:

$$G(h_{te}) = 20\log\left(\frac{h_{te}}{200}\right) = 20\log\left(\frac{100}{200}\right) = -6dB$$
$$G(h_{re}) = 20\log\left(\frac{h_{re}}{3}\right) = 20\log\left(\frac{10}{3}\right) = 10.46dB$$

2.e). Outdoor Propagation Models

ii. Okumura Model, con't.-Example-

- Using equation (3.80) the total mean path loss is

$$L_{50}(dB) = L_F + A_{mu}(f,d) - G(h_{te}) - G(h_{re}) - G_{AREA}$$
$$= 125.5dB + 43dB - (-6)dB - 10.46dB - 9dB = 155.04dB$$

- Therefore, the median received power is:

$$P_r(d) = EIRP(dBm) - L_{50}(dB) + G_r(dB)$$
$$= 60dBm - 155.04dB + 0dB = -95.04dBm$$

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2.e). Outdoor Propagation Models
iii. Hata Model

- The Hata model is an empirical formulation of the graphical path loss data provided by Okumura, and is valid from 150 MHz to 1500 MHz. Hata presented the urban area propagation loss as a standard formula and supplied correction equations for application to other situations.
- The standard formula for median path loss in urban areas is given by:

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2.e). Outdoor Propagation Models
iii. Hata Model

$$PL_{Urban}[dB] = 69.55 + 26.16\log_{10}(f_c) - 13.82\log_{10}(h_t) - a(h_r) + (44.9 - 6.55\log_{10}(h_t))\log_{10}(d)$$ $150\text{MHz} < f_c < 1500\text{MHz}$

$$30m \le h_{te} \le 200m, 1m \le h_{re} \le 10m, d\,(in, km)$$

$$a(h_{re}) = (1.1\log f_c - 0.7)h_{re} - (1.56\log f_c - 0.8)dB$$ Small to mid-size cities

$$a(h_{re}) = 8.29(\log 1.54h_{re})^2 - 1.1dB, for: f_c \le 300MHz$$
$$a(h_{re}) = 3.2(\log 11.75h_{re})^2 - 4.97dB, for: f_c \ge 300MHz$$ Larger cities, $300\text{MHz} < f_c$

$$PL_{Suburban}[dB] = PL_{Urban}[dB] - 2(\log_{10}(f_c/28))^2 - 5.4$$

$$PL_{Rural}[dB] = PL_{Urban}[dB] - 4.78(\log_{10}(f_c))^2 - 18.33\log_{10}(f_c) - K$$

$K = 35.94$ → Countryside $K = 40.94$ → Desert

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2.e). Outdoor Propagation Models
iv. COST231 Extension to Hata Model

$$PL_{Urban}[dB] = 46.3 + 33.9\log_{10}(f_c) - 13.82\log_{10}(h_t) - a(h_r) + (44.9 - 6.55\log_{10}(h_t))\log_{10}(d) + C_M$$

$$C_M = \begin{cases} 0 & \text{midsize cities, suburbs} \\ 3 & \text{metropolitan areas} \end{cases}$$

$1500\text{MHz} < f_c < 2000\text{MHz}$

$30m < h_t < 200m$

$1m < h_r < 10m$

$1km < d < 20km$

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2.e). Outdoor Propagation Models

Okumura-Hata Model –Example-2-

- Problem : Using the Okumura-Hata Model
  - Determine the path loss of a 900 MHz cellular system operating in a large city from a base station with the height of 100 m and mobile station installed in a vehicle with antenna height of 2 m. **The distance between the mobile and the base station is 4 km**.
- Solution:
  - We calculate the terms in the Okumura-Hata model as follows:
  - $f_c = 900MHz; h_b = 100m; h_m = 2m; d=4km.$
  - $a(h_m) = 3.2[lg(11.75h_m)]^2 - 4.79 = 1.225dB$
  - $L_p = 69.55 + 26.16lgf_c - 13.82lgh_b - a(h_m) + [44.9 - 6.55lgh_b]lgd = 137.3dB$

2.f). Indoor Propagation Models

- With the advent of Personal Communication Systems (PCS), there is a great deal of interest in characterizing radio propagation inside buildings.
- The indoor radio channel differs from the traditional mobile radio channel in two aspects — the distances covered are much smaller, and the variability of the environment is much greater for a much smaller range of T-R separation distances.
- It has been observed that propagation within buildings is strongly influenced by specific features such as the layout of the building, the construction materials, and the building type.
- This section outlines models for path loss within buildings.
- Indoor radio propagation is dominated by the same mechanisms as outdoor: reflection, diffraction, and scattering. However, conditions are much more variable.
- For example, signal levels vary greatly depending on whether interior doors are open or closed inside a building. Where antennas are mounted also impacts large-scale propagation. Antennas mounted at desk level in a partitioned office receive vastly different signals than those mounted on the ceiling.
- Also, the smaller propagation distances make it more difficult to insure far-field radiation for all receiver locations and types of antennas.

2.f). Indoor Propagation Models

i. Log-distance Path Loss Model

- Indoor path loss has been shown by many researchers to obey the distance power law in equation.

$$PL(dB) = PL(d_0) + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma$$

- where the value of n depends on the surroundings and building type, and Xa represents a normal random variable in dB having a standard deviation of a dB.

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2.f). Indoor Propagation Models

ii. Attenuation Factor Model

- An in-building propagation model that includes the effect of building type as well as the variations caused by obstacles.
- This model provides flexibility and was shown to reduce the standard deviation between measured and predicted path loss to around 4 dB, as compared to 13 dB when only a log-distance model was used in two different buildings.
- The attenuation factor model is given by:

$$\overline{P}L(d)[dB] = \overline{P}L(d)[dB] + 10n_{SF}\log\left(\frac{d}{d_0}\right) + FAF[dB]$$

- Where n_{SF} represents the exponent value for the "same floor" measurement. Then, for multiple floors, we have:

$$\overline{P}L(d)[dB] = \overline{P}L(d_o) + 10n_{MF}\log\left(\frac{d}{d_0}\right)$$

- Where n_{MF} denotes a path loss exponent based on measurements through multiple floors.

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2.f). Indoor Propagation Models
iii. *Indoor Attenuation Factors*

Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminium Siding	20.4
All Metal	26

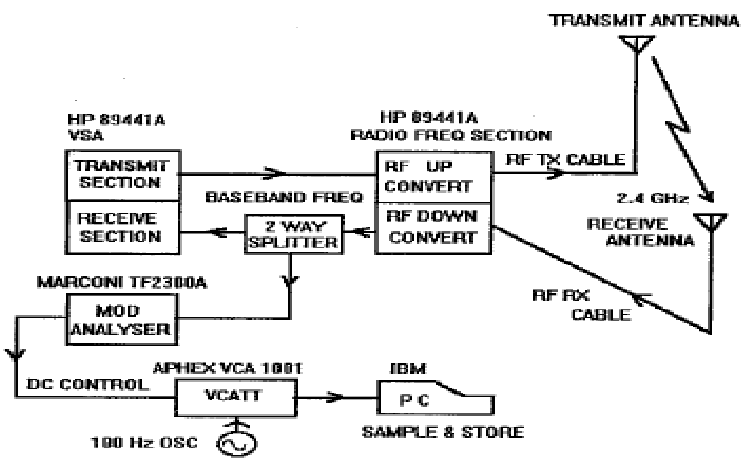
$$\overline{PL}_{INDOOR}(d) = \overline{PL}(d) + \sum_{i=1}^{N_f} FAF_i + \sum_{i=1}^{N_p} PAF_i$$

Floor attenuation factor

Partition attenuation factor

2.f). Indoor Propagation Models

Example-: Measurement system.



2.f). Indoor Propagation Models

Example

- This example demonstrates how to use equations (3.94) and (3.95) to predict the mean path loss 30m from the transmitter, through three floors of office building 1 (see table 3.5). From table 3.5, the mean path loss exponent for same-floor measurements in a building is n=3.27, the mean path loss exponent for three floor measurements is n=5.22, and the average floor attenuation factor is FAF=24.4dB for three floors between the transmitter and receiver.
- **Solution:**
- The mean path loss using equation (3.94) is:

$$\overline{P}L(30m)[dB] = \overline{P}L(1m)[dB] + 10 \times 3.27 \times \log(30) + 24.4 = 104.2dB$$

$equ(3.95)$

$$\overline{P}L(30m)[dB] = \overline{P}L(1m)[dB] + 10 \times 5.22 \times \log(30) = 108.6dB$$

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3- Small-scale fading and multipath

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3. a) Small-scale fading and multipath

- Small-scale fading, or simply fading, is used to describe the rapid fluctuation of the amplitude of a radio signal over a short period of time or travel distance, so that large-scale path loss effects may be ignored.
- Fading is caused by interference between two or more versions of the transmitted signal which arrive at the receiver at slightly different times.
- These waves, called multipath waves, combine at the receiver antenna to give a resultant signal which can vary widely in amplitude and phase, depending on the distribution of the intensity and relative propagation time of the waves and the bandwidth of the transmitted signal.
- Small-Scale Multipath Propagation in the radio channel creates small-scale fading effects. The three most important effects are:
  - Rapid changes in signal strength over a small travel distance or time interval
  - Random frequency modulation due to varying Doppler shifts on different multipath signals
  - Time dispersion (echoes) caused by multipath propagation delays.

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3. a)Small-scale fading and multipath

- In built-up urban areas, fading occurs because the height of the mobile antennas are well below the height of surrounding structures, so there is no single line-of-sight path to the base station. Even when a line-of-sight exists, multipath still occurs due to reflections from the ground and surrounding structures.
- The incoming radio waves arrive from different directions with different propagation delays. The signal received by the mobile at any point in space may consist of a large number of plane waves having randomly distributed amplitudes, phases, and angles of arrival.
- These multipath components combine vectorially at the receiver antenna, and can cause the signal received by the mobile to distort or fade.
- Even when a mobile receiver is stationary, the received signal may fade due to movement of surrounding objects in the radio channel.
- Due to the relative motion between the mobile and the base station, each multipath wave experiences an apparent shift in frequency.
- The shift in received signal frequency due to motion is called the Doppler shift, and is directly proportional to the velocity and direction of motion of the mobile with respect to the direction of arrival of the received multipath wave.

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3. b) Factors Influencing Small-Scale Fading

- **Multipath propagation** — The presence of reflecting objects and scatterers in the channel creates a constantly changing environment that dissipates the signal energy in amplitude, phase, and time. These effects result in multiple versions of the transmitted signal that arrive at the receiving antenna, displaced with respect to one another in time and spatial orientation. Multipath propagation often lengthens the time required for the baseband portion of the signal to reach the receiver which can cause signal smearing due to intersymbol interference.
- **Speed of the mobile** — The relative motion between the base station and the mobile results in random frequency modulation due to different Doppler shifts on each of the multipath components. Doppler shift will be positive or negative depending on whether the mobile receiver is moving toward or away from the base station.
- **Speed of surrounding objects** — If objects in the radio channel are in motion, they induce a time varying Doppler shift on multipath components. If the surrounding objects move at a greater rate than the mobile, then this effect dominates the small-scale fading. Otherwise, motion of surrounding objects may be ignored, and only the speed of the mobile need be considered.
- **The transmission bandwidth of the signal** — If the transmitted radio signal bandwidth is greater than the "bandwidth" of the multipath channel, the received signal will be distorted, but the received signal strength will not fade much over a local area (i.e., the small-scale signal fading will not be significant).

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3. c) Doppler Shift

- Consider a mobile moving at a constant velocity v, along a path segment having length d between points X and Y, while it receives signals from a remote source S, as illustrated in Fig. 4.1.
- The difference in path lengths traveled by the wave from source S to the mobile at points X and Y is:

$$\Delta l = d\cos\theta = \upsilon \Delta t \cos\theta$$

- The phase change in the received signal due to the difference in path lengths is therefore: $$\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi\upsilon\Delta l}{\lambda}\cos\theta$$
- $f_d = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{\upsilon}{\lambda}\cos\theta$,Doppler shift, is given by f_d where
- It relates the Doppler shift to the mobile velocity and the spatial angle between the direction of motion of the mobile and the direction of arrival of the wave.

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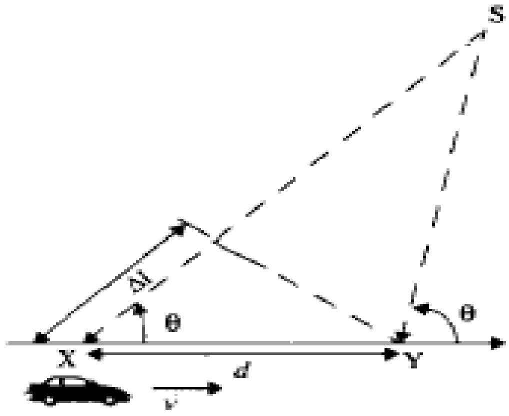
3. c)Doppler Shift and effect



3. c) Doppler Shift and effect-Example

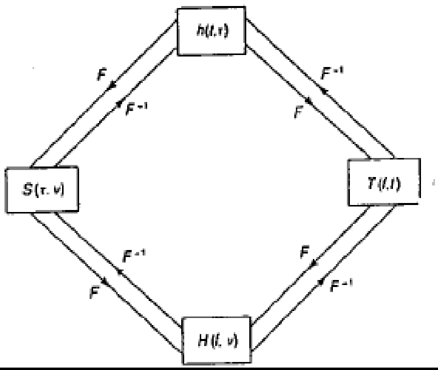
- Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving (a) directly towards the transmitter, (b) directly away from the transmitter, (c) in a direction which is perpendicular to the direction of arrival of the transmitted signal.
- Solution: $f_c = 1850MHz$
- Carrier frequency:
- Therefore, wavelength: $\lambda = \frac{c}{f_c} = \frac{3\times10^8}{1850\times10^6} = 0.162m$
- Vehicle speed: $\upsilon = 60mph = 26.82m/s$
- (a)The vehicle is moving towards the Tr, and the doppler shift is positive. The received frequency is given by:
$$f = f_c + f_d = 1850\times10^6 + \frac{26.82}{0.162} = 1850.00016MHz$$
- (b) The vehicle is moving away from the Tr. The Doppler shift is negative.
$$f = f_c - f_d = 1850\times10^6 - \frac{26.82}{0.162} = 1849.999834MHz$$
- (c) The vehicle is moving peripendicular to the angle of arrival of the Transmitted signal. In this case θ=90^0 and cosθ=0 and no doppler shift and the received signal frequency is the same as the transmitted f=1850MHz.

3.d) Relationship Between System Functions

- System functions:

  $h(t,\tau)$: Input Delay Spread Function
  $H(f,\upsilon)$: Output Doppler Spread Function
  $T(t,f)$: Time – Variant Transfer Function
  $S(\tau,\upsilon)$: Delay Doppler Spread Function

- Relationship via Fourier F and inverse Fourier transform



3.e) Impulse Response Model of a Multipath Channel

- The small-scale variations of a mobile radio signal can be directly related to the impulse response of the mobile radio channel. The impulse response is a wideband channel characterization and contains all information necessary to simulate or analyze any type of radio transmission through the channel.
- The impulse response is a useful characterization of the channel, since it may be used to predict and compare the performance of many different mobile communication systems and transmission bandwidths for a particular mobile channel condition.
- lb show that a mobile radio channel may be modeled as a linear filter with a time varying impulse response, consider the case where time variation is due strictly to receiver motion in space.

3.e) Impulse Response Model of a Multipath Channel

- Let x(t) represent the transmitted signal, then the received signal y(d,t) at position d can be expressed as a convolution of x(t) with h(d,t).

$$y(d,t) = x(t) \otimes h(d,t) = \int_{-\infty}^{\infty} x(\tau) h(d, t-\tau) d\tau$$

- For d=vt, we have:

$$y(t) = \int_{-\infty}^{t} x(\tau) h(\upsilon t, t-\tau) d\tau = x(t) \otimes h(\upsilon t, t) = x(t) \otimes h(d,t)$$

- Since v may be assumed constant over a short time (or distance) interval, we may let x(t) represent the transmitted bandpass waveform, y(t) the received waveform, and h(t,τ) completely characterizes the channel and is a function of both t and τ. The variable t represents the time variation due to motion, whereas τ represents the channel multipath delay for a fixed value of t. The received signal y(t) can be expressed as a convolution of the transmitted signal x(t) with the channel impulse response.

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t,\tau) d\tau = x(t) \otimes h(t,\tau)$$

3.e) Impulse Response Model of a Multipath Channel

- It is useful to discretize the multipath delay axis τ of the impulse response into equal time delay segments called excess delay bins, where each bin has a time delay width equal to τ_{i+1} where τ_0 is equal to 0, and represents the first arriving signal at the receiver.
- This technique of quantizing the delay bins determines the time delay resolution of the channel model, and the useful frequency span of the model can be shown to be $1/(2\Delta\tau)$, That is, the model may be used to analyze transmitted signals having bandwidths which are less than $1/(2\Delta\tau)$.
- Note that $\tau_0 = 0$ is the excess time delay of the first arriving multipath component, and neglects the propagation delay between the transmitter and receiver Excess delay is the relative delay of the ith multipath component as compared to the first arriving component and is given by τ_i. The maximum excess delay of the channel is given by $N\Delta\tau$.

Example:

- Assume a discrete channel impulse response is used to model urban radio channels with excess delays as large as 100 μs and microcellular channels with excess delays no larger than 4μs. If the number of multipath bins is fixed at 64, find (a) $\Delta\tau$, and (b) the maximum bandwidth which the two models can accurately represent. Repeat the exercise for an indoor channel model with excess delays as large as 500 ns.
- **Solution:**
- The maximum excess delay of the channel model is given by $\tau_N = N\Delta\tau$. Therefore, for τ_N=100μs, and N = 64 we obtain $\Delta\tau = \tau_N/N$ = 1.5625μs. The maximum bandwidth that the SMRCIM model can accurately represent is equal to $1/(2\Delta\tau)$ = 1/(2(l15625μs)) =0.32MHz. For the SMRCJM urban microcell model, τ_N= 4μs, $\Delta\tau = \tau_N/N$= 62.5 ns. The maximum bandwidth that can be represented is $1/(2\Delta\tau)$ = 1/(2(62.Sns)) =8MHz. Similarly, for indoor channels,

$$\Delta\tau = \tau_\Delta / N = \frac{500\times10^{-9}}{64} = 7.8125ns$$

- The max. Bandwidth for the indoor channel model is:

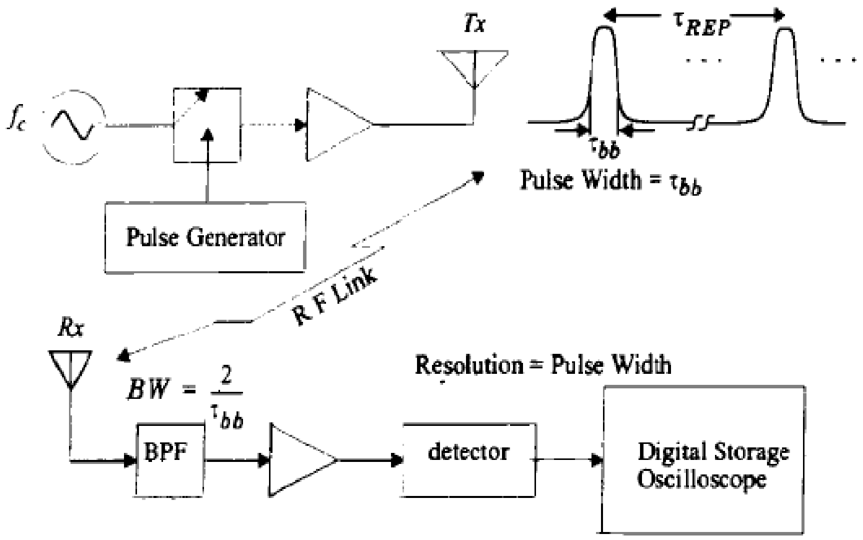
$$1/(2\Delta\tau) = 1/(2(7.8125ns)) = 64MHz$$

3.f). Small-Scale Multipath Measurements

- Because of the importance of the multipath structure in determining the small-scale fading effects, a number of wideband channel sounding techniques have been developed.
- These techniques may be classified as direct pulse measurements, spread spectrum sliding correlator measurements, and swept frequency measurements.

3.f). Small-Scale Multipath Measurements

Direct RF channel impulse response measurement system.



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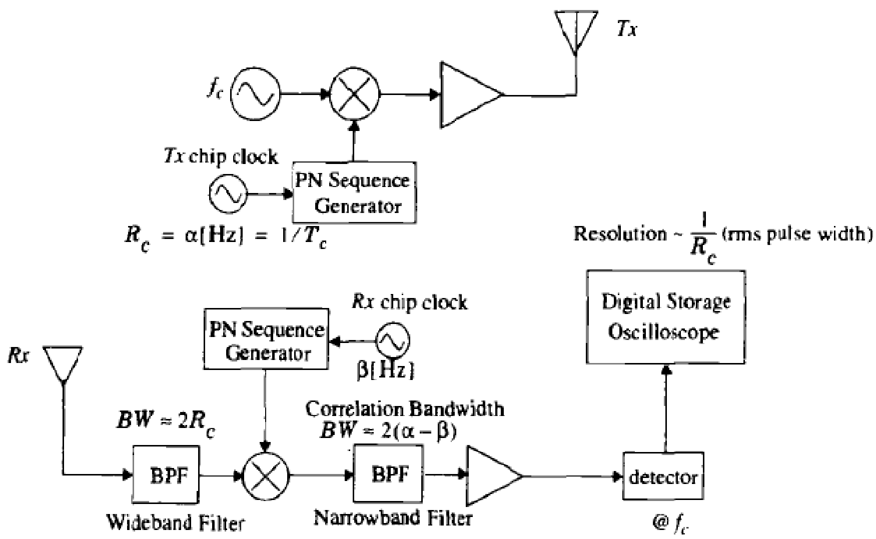
3.f). Small-Scale Multipath Measurements

Spread Spectrum channel impulse response measurement system.



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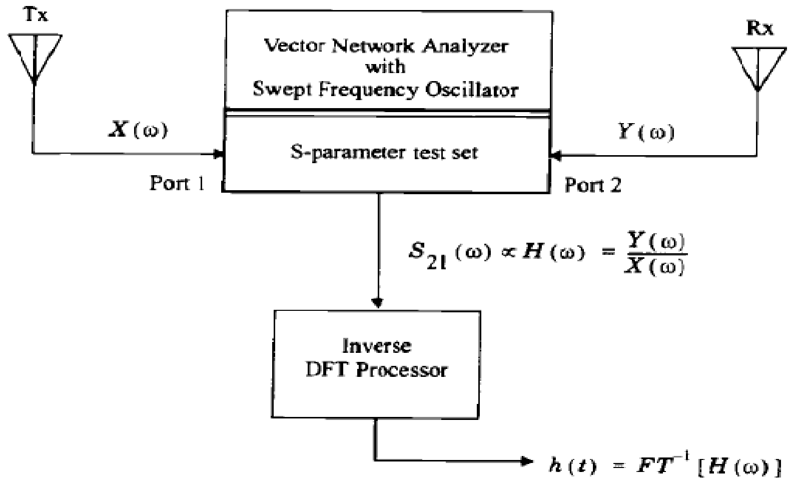
3.f). Small-Scale Multipath Measurements

Frequency domain channel impulse response measurement system.



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3.g)-Parameters of Mobile multipath channels

Time Dispersion Parameters:

- In order to compare different multipath channels and to develop some general design guidelines for wireless systems, parameters which grossly quantify the multipath channel are used.
- The mean excess delay, rms delay spread, and excess delay spread (X dB) are multipath channel parameters that can be determined from a power delay profile.
- The time dispersive properties of wide band multipath channels are most commonly quantified by their mean excess delay (τ)and rms delay spread (τ_t). The mean excess delay is the first moment of the power delay profile and is defined to be:
- $\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$ the rms delay spead is: $\sigma_t = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$
- Where:
- $\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$,

These delays are measured relative to the first detectable signal arriving at the receiver at τ_0=0

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3.g)-Parameters of Mobile multipath channels

Coherence Bandwidth

- Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered "flat" (i.e., a channel which passes all spectral components with approximately equal gain and linear phase);
- In other words, coherence bandwidth is the range of frequencies over which two frequency components have a strong potential for amplitude correlation.
- If the coherence bandwidth is defined as the bandwidth over which the frequency correlation function is above 0.9, then the coherence bandwidth is approximately:

$$B_c \approx \frac{1}{50\sigma_\tau}$$

- If the definition is relaxed so that the frequency correlation function is above 0.5, then the coherence bandwidth is approximately:

$$B_c \approx \frac{1}{5\sigma_\tau}$$

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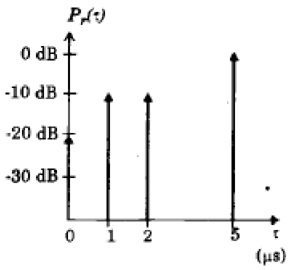
Example

- Calculate the mean excess delay, rms delay spread, and the maximum excess delay (10 dB) for the multipath profile given in the figure. Estimate the 50% coherence bandwidth of the channel. Would this channel be suitable for AMPS or GSM service without the use of an equalizer?
- **Solution:**



$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{(1)(5)+(0.1)(1)+(0.1)(2)+(0.01)(0)}{[0.01+0.1+0.1+1]} = 4.38\,\mu s$$

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{(1)(5)^2+(0.1)(1)^2+(0.1)(2)^2+(0.01)(0)}{1.21} = 21.07\,\mu s^2$$

$$\sigma_t = \sqrt{\overline{\tau^2}-(\bar{\tau})^2} = \sqrt{21.07-(4.38)^2} = 1.37\mu s$$

$$B_c \approx \frac{1}{5\sigma_\tau} = \frac{1}{5(1.37\,\mu s)} = 146kHz$$

Coherence bandwidth, since it is greater than 30kHz, AMPS will work without equalizer. GSM requires 200kHz BW, thus equalizer is needed.

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3. g) Doppler Spread and Coherence Time

- Delay spread and coherence bandwidth are parameters which describe the time dispersive nature of the channel in a local area.
- However, they do not offer information about the time varying nature of the channel caused by either relative motion between the mobile and base station, or by movement of objects in the channel.
- Doppler spread and coherence time are parameters which describe the time varying nature of the channel in a small-scale region.
- Doppler spread BD is a measure of the spectral broadening caused by the time rate of change of the mobile radio channel and is defined as the range of frequencies over which the received Doppler spectrum is essentially non-zero. If the baseband signal bandwidth is much greater than BD, the effects of Doppler spread are negligible at the receiver.
- Coherence time T, is the time domain dual of Doppler spread and is used to characterize the time varying nature of the frequency dispersiveness of the channel in the time domain. The Doppler spread and coherence time are inversely proportional to one another.

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3.g)-Parameters of Mobile multipath channels

Doppler Spread and Coherence Time

- Coherence Time:

$$T_C \approx \frac{1}{f_m}, for, f_m : \max DopplerShift; f_m = \upsilon / \lambda$$

$$T_C \approx \frac{9}{16\pi f_m}, for, T_{correlation} > 0.5$$

- A popular rule of thumb for modern digital communications is to define the coherence time as the geometric mean as follows:

$$T_c = \sqrt{\frac{9}{16\pi f_m^2}} = \frac{0.423}{f_m}$$

- The definition of coherence time implies that two signals arriving with a time separation greater than Tc are affected differently by the channel.

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Example:

- Determine the proper spatial sampling interval required to make small-scale propagation measurements which assume that consecutive samples are highly correlated in time. How many samples will be required over 10 m travel distance if fc = 1900 MHz and v = 50 m/s. How long would it take to make these measurements, assuming they could be made in real time from a moving vehicle? What is the Doppler spread B_D for the channel?

$$T_C \approx \frac{9}{16\pi f_m} = \frac{9\lambda}{16\pi \upsilon f_c} = \frac{9*3*10^8}{16*3.14*50*1900*10^6} = 565\mu s$$

Taking time samples at less than half Tc at 282.5 μs

$$\Delta x = \frac{\upsilon T_C}{2} = \frac{50*565\mu s}{2} = 0.014125m = 1.41cm$$

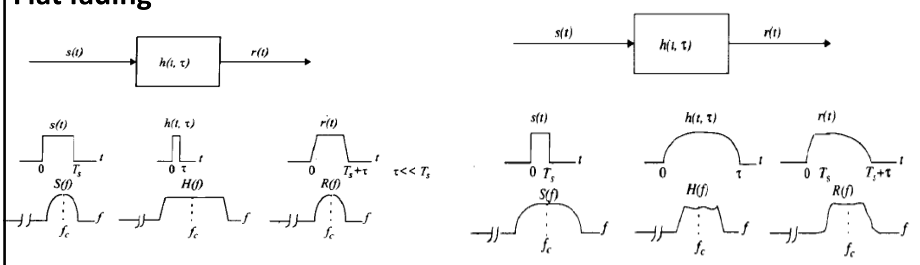
$$N_x = \frac{10}{\Delta x} = \frac{10}{0.014125} = 708 samples$$

The time taken to make this measurement is equal to 10m/50m/s=0.2s

$$B_D = f_m = \frac{vf_c}{c} = \frac{50*1900*10^6}{3*10^8} = 316.66Hz$$

3.g) Parameters of mobile multipath channels
Flat fading Vs Frequency Selective Fading

Flat fading **Frequency Selective Fading**



It can be seen from Figure above that if the channel gain changes over time, a change of amplitude occurs in the received signal. Over time, the received signal r (t) varies in gain, but the spectrum of the transmission is preserved. In a flat fading channel, the reciprocal bandwidth of the transmitted signal is much larger than the multipath time delay spread of the channel, For frequency selective fading, the spectrum S (I) of the transmitted signal has a bandwidth which is greater than the coherence bandwidth Bc of the channel. Viewed in the frequency domain, the channel becomes frequency selective, where the gain is different for different frequency components. frequency selective fading is caused by multipath delay & which approach or exceed the symbol period of the transmitted symbol. Frequency selective fading channels are also known as wideband channels since the bandwidth of the signal s(t) is wider than the bandwidth of the channel impulse response.

3.g) Parameters of mobile multipath channels
Rayleigh and Rician Distributions

- **Rayleigh Fading Distribution:** In mobile radio channels, the Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component. It is well known that the envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution. The Rayleigh distribution has a probability density function (pdf) given by:

$$p(r) = \begin{cases} \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), (0 \le r \le \infty) \\ 0, (r < 0) \end{cases}$$

$$P(R) = \Pr(r \le R) = \int_0^R p(r)dr$$
$$= 1 - \exp\left(-\frac{R^2}{2\sigma^2}\right)$$

- Where σ is the rms value of the received voltage signal before envelope detection, and σ^2 is the time-average power of the received signal before envelope detection. The mean value is:

$$r_{mean} = E[r] = \int_0^\infty rp(r)dr = \sigma\sqrt{\frac{\pi}{2}} = 1.2533\sigma$$

3.g) Parameters of mobile multipath channels
Rayleigh and Rician Distributions

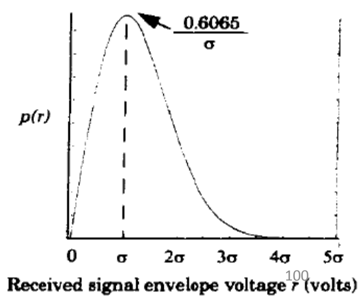
- The variance of the Rayleigh distribution is given by σ_r^2 which represents the ac power in the signal envelope:

$$\sigma_r^2 = E[r^2] - E^2[r] = \int_0^\infty r^2 p(r)dr - \frac{\sigma^2\pi}{2}$$
$$= \sigma^2\left(2 - \frac{\pi}{2}\right) = 0.4292\sigma^2$$

- The rms value of the envelope is the square root of the mean square, or $\sqrt{2}\sigma$, the median value of r is found by solving:

$$\frac{1}{2} = \int_0^{r_{median}} p(r)dr, and, r_{median} = 1.177\sigma$$

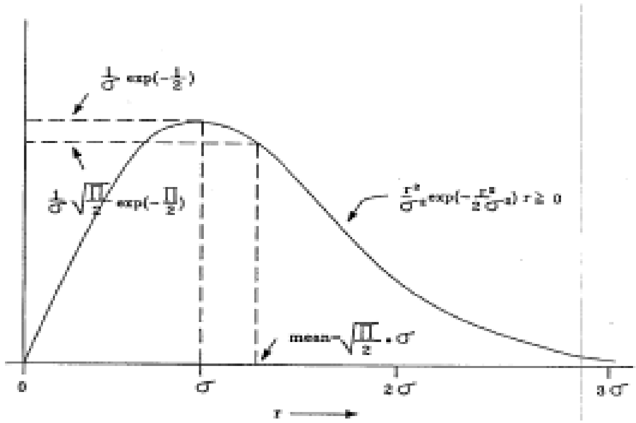
- The pdf is:



3.g) Parameters of mobile multipath channels

Rayleigh Distributions

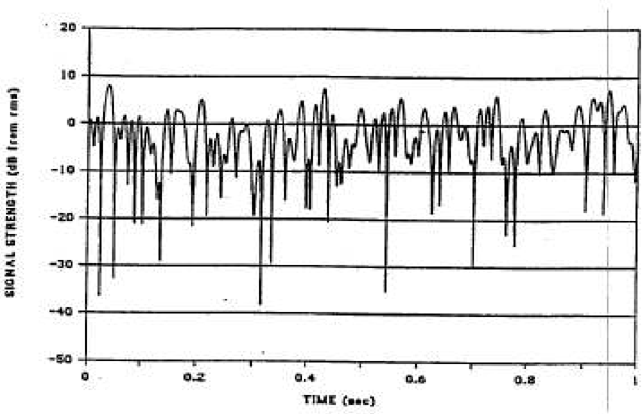
- Rayleigh PDF



3.g) Parameters of mobile multipath channels

Typical Profile of Received Signal's Rayleigh Fading Envelope

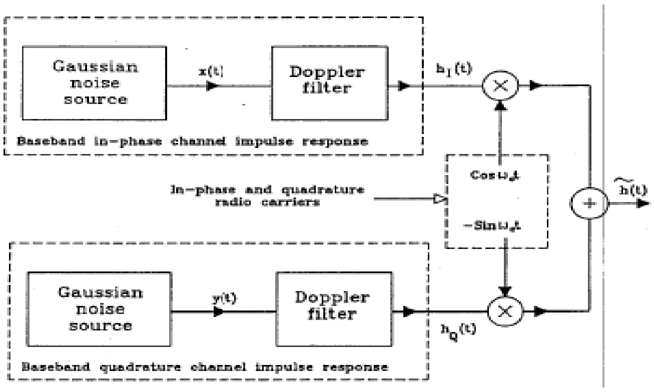
- MS speed of 50km/h, $f_c = 900$MHz



3.g) Parameters of mobile multipath channels

Model to Generate a Rayleigh Fading Profile

- Block diagram



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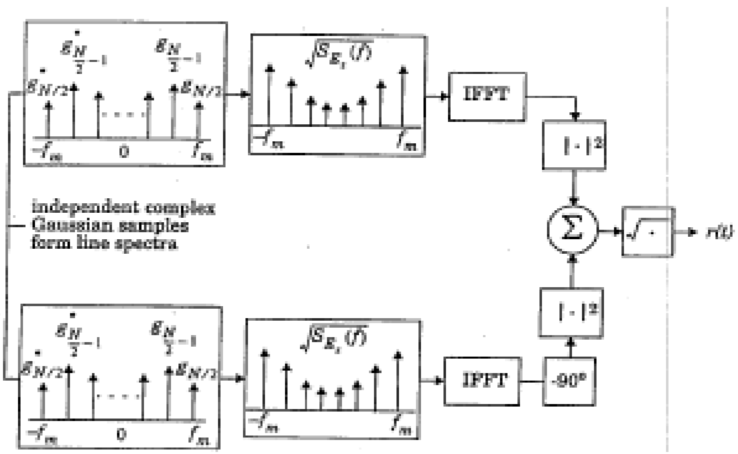
3.g) Parameters of mobile multipath channels

Frequency Domain Implementation of a Rayleigh Fading Simulator

- Block diagram



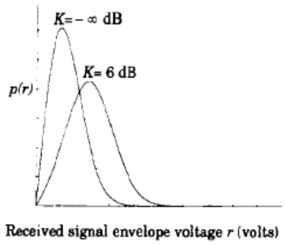
3.g) Parameters of mobile multipath channels

Ricean Fading Distribution

- When there is a dominant stationary (nonfading) signal component present, such as a line-of-sight propagation path, the small-scale fading envelope distribution is Ricean. In such a situation, random multipath components arriving at different angles are superimposed on a stationary dominant signal. At the output of an envelope detector, this has the effect of adding a dc component to the random multipath.

$$p(r) = \begin{cases} \frac{r}{\sigma^2}\exp\left(-\frac{(r^2+A^2)}{2\sigma^2}\right) I_0\left(\frac{Ar}{\sigma^2}\right), for: (A \geq 0, r \geq 0) \\ 0, for (r < 0) \end{cases}$$

$$K = A^2 / (2\sigma^2)$$

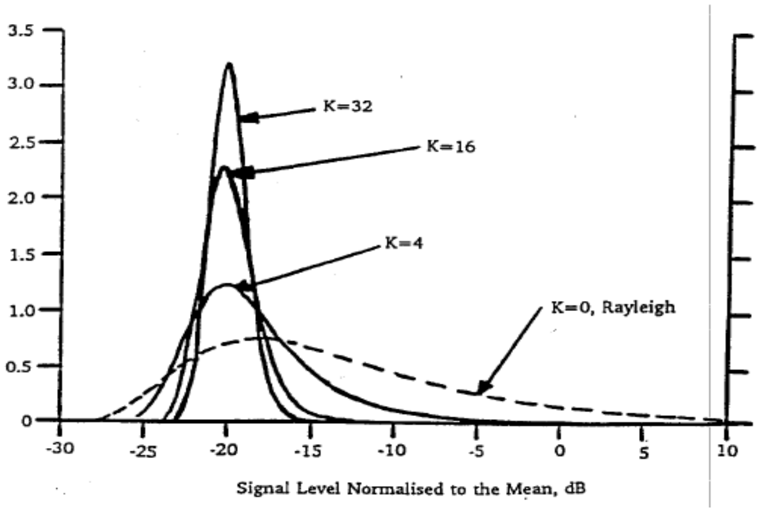


- The parameter A denotes the peak amplitude of the dominant signal and I(.) is the modified Bessel function of the first kind and zero-order.

Probability density function of Ricean distributions: $K = -\infty$ dB (Rayleigh) and $K = 6$ dB. For K>>1, the Ricean pdf is approximately Gaussian about the mean.

3.g) Parameters of mobile multipath channels

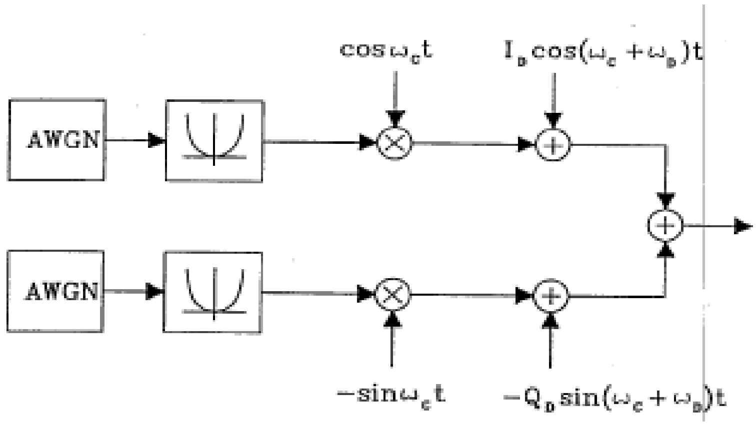
Rician Pdf



3.g) Parameters of mobile multipath channels

Model to Generate a Rician Channel

- Block diagram



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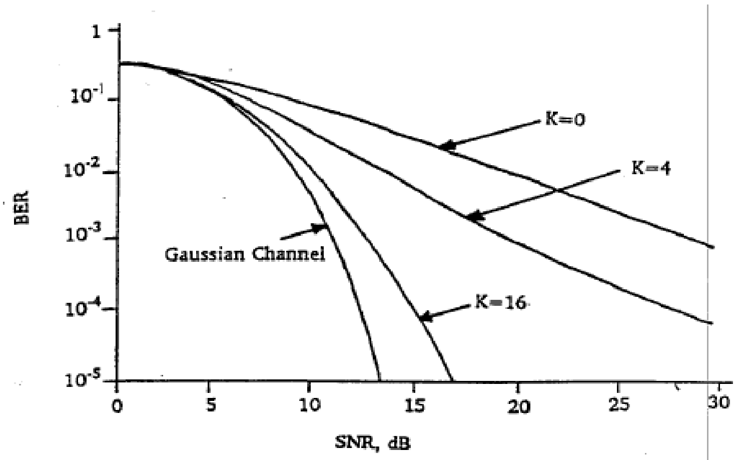
3.g) Parameters of mobile multipath channels

BER vs Channel SNR for Different K-factors and Non-coherent FSK

- BER performance



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3.g) Parameters of mobile multipath channels

Nakagami-m Fading Distribution

- Nakagami-m is another useful fading model that is utilised in multipath fading channels to characterise the statistics of signals transmission.
- It is very useful since some data realizations do not work well with Rayleigh or Rician fading distribution.
- Therefore, Nakagami-m becomes a more general method to define some fading distribution with adjustable parameters. For random variables, the probability density function (PDF) of the Nakagami-m is defined as:

$$f_X(x) = \frac{2}{\Gamma(m)}\left(\frac{m}{\eta}\right)^m x^{2m-1} e^{\left(-\frac{mx^2}{\eta}\right)}, with: m = \frac{\eta^2}{E[(X-\eta)^2]}, m \geq 1/2$$

- Where η is the mean square of X defined as: $\eta = E[X^2]$ and the parameter m is defined as the expectation operator.
- For m=1, the distribution reduces to Rayleigh fading. For $m=(K+1)^2/(2K+1)$ the distribution is approximately Rician fading with parameter K. For $m=\infty$, there is no fading and P_r is a constant.

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3. h) Types of Small Scale Fading

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

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3.i) Statistical Models for Multipath Fading Channels

- Several multipath models have been suggested to explain the observed statistical nature of a mobile channel.

Level Crossing and Fading Statistics:

- .The level crossing rate (LCR) and average fade duration of a Rayleigh fading signal are two important statistics which are useful for designing error control codes and diversity schemes to be used in mobile communication systems, since it becomes possible to relate the time rate of change of the received signal to the signal level and velocity of the mobile.
- The level crossing rate (LCR) is defined as the expected rate at which the Rayleigh fading envelope, normalized to the local rms signal level, crosses a specified level in a positive-going direction. The number of level crossings per second is given by:

$$N_R = \int_0^{\infty} r p(R, \dot{r}) d\dot{r} = \sqrt{2\pi} f_m \rho e^{-\rho^2}$$

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3.i) Statistical Models for Multipath Fading Channels

- where r' is the time derivative of r (t) (i.e., the slope), p (R, r') is the joint density function of r and f at r' = R, fm is the maximum Doppler frequency and p = R/Rrms is the value of the specified level R, normalized to the local rms amplitude of the lading envelope.
- **Example:**
- For a Rayleigh fading signal, compute the positive going level crossing rate for ρ=1, when the maximum Doppler Frequency (fm) is 20Hz. What is the maximum velocity of the mobile for this Doppler Frequency if the carrier frequency is 900MHz.
- **Solutions:**
- The number of zero level crossings is:
  $N_R = \sqrt{2\pi}(20)(1)e^{-1} = 18.44 cros\sin gs / \sec ond$
- The maximum velocity of the mobile can be obtained using the Doppler relation $f_{d,\max} = \upsilon / \lambda$
- Therefore velocity of the mobile at $f_m = 20Hz$
- $\upsilon = f_d \lambda = 20Hz(1/3m) = 6.66m/s = 24km/hr$

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3.i) Statistical Models for Multipath Fading Channels

- The average fade duration is defined as the average period of time for which the received signal is below a specified level R. For a Rayleigh fading signal, this is given by: $\bar{\tau} = \frac{1}{N_R} P_r[r \le R]$
- Where Pr[r<=R] is the probability that the received signal r is less than R and is given by: $P_r[r \le R] = \frac{1}{T}\sum_i \tau_i$
- Where τ_i is the duration of the fade and T is the observation interval of the fading signal. The probability that the received signal r is less than the threshold R is found from the Rayleigh distribution as:

$$P_r[r \le R] = \int_0^R p(r)dr = 1 - \exp(-\rho^2)$$

- Where p(r) is the pdf of a Rayleigh Distribution. The average fade duration as a function of ρ and f_m can be expressed as:

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

- The average duration of a signal fade helps determine the most likely number of signaling bits that may be lost during a fade.

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3.i) Statistical Models for Multipath Fading Channels

- Example: Find the average fade duration for threshold levels ρ=0.01, ρ=0.1, and ρ=1. when the Doppler frequency is 200Hz.

Solution: Average fade duration can be found by substituting the given values in equation:

$$\bar{\tau} = \frac{e^{0.01^2} - 1}{(0.01)200\sqrt{2\pi}} = 19.9\mu s, for : \rho = 0.01$$

$$\bar{\tau} = \frac{e^{0.01^2} - 1}{(0.01)200\sqrt{2\pi}} = 19.9\mu s, for, \rho = 0.1$$

$$\bar{\tau} = \frac{e^{0.01^2} - 1}{(0.01)200\sqrt{2\pi}} = 19.9\mu s, for, \rho = 1$$

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Example

- Find the average fade duration for a threshold level of ρ = 0.707 when the Doppler frequency is 20 Hz. For a binary digital modulation with bit duration of 50 bps, is the Rayleigh fading slow or fast? What is the average number of bit errors per second for the given data rate. Assume that a bit error occurs whenever any portion of a bit encounters a fade for which ρ<0.1.
- **Solution:**

$$\overline{\tau} = \frac{e^{0.707^2} - 1}{(0.707)20\sqrt{2\pi}} = 18.3ms$$

- For a data rate of 50 bps, the bit period is 20 ms. Since the bit period is greater than the average fade duration, for the given data rate the signal undergoes fast Rayleigh fading. The average fade duration for ρ=0.1 is equal to 0.002 s. This is less than the duration of one bit. Therefore, only one bit on average will be lost during a fade. The number of level crossings for ρ= 0.1 is Nr= 4.96 crossings per seconds. Since a bit error is assumed to occur whenever a portion of a bit encounters a fade, and since average fade duration spans only a fraction of a bit duration, the total number of bits in error is 5 per second, resulting in a BER = (5/50) = 0.1.

4. Diversity Techniques

- Diversity is a powerful communication receiver technique that provides wireless link improvement at relatively low cost. Unlike equalization, diversity requires no training overhead since a training sequence is not required by the transmitter.
- Furthermore, there are a wide range of diversity implementations, many which are very practical and provide significant link improvement with little added cost.
- Diversity exploits the random nature of radio propagation by finding independent (or at least highly uncorrelated) signal paths for communication.
- In virtually all applications, diversity decisions are made by the receiver, and are unknown to the transmitter.
- The diversity concept can be explained simply. If one radio path undergoes a deep fade, another independent path may have a strong signal.
- By having more than one path to select from, both the instantaneous and average SNRs at the receiver may be improved, often by as much as 20 dB to 30 dB.

4. Diversity Techniques, con't.

- There are two types of fading — small-scale and large-scale fading.
- Small-scale fades are characterized by deep and rapid amplitude fluctuations which occur as the mobile moves over distances of just a few wavelengths. These fades are caused by multiple reflections from the surroundings in the vicinity of the mobile. Small-scale fading typically results in a Rayleigh fading distribution of signal strength over small distances. By selecting the best signal at all times, a receiver can mitigate small-scale fading effects (this is called antenna diversity or space diversity).
- Large-scale fading is caused by shadowing due to variations in both the terrain profile and the nature of the surroundings. In deeply shadowed conditions, the received signal strength at a mobile can drop well below that of free space.

4. Diversity techniques

- In wireless networks, diversity techniques are used to mitigate the effects of multipath fading without additional bandwidth ressources or the transmission power. The basic idea of this technique is that, in multipath propagation environment, signals are faded independently and some signals are highly faded while others are less attenuated. Thus, the use of a proper combination technique decreases the severity of fading and consequently it improves the network performance and reliable transmission.

4. a). Derivation of selection Diversity Improvement

- Consider M independent Rayleigh fading channels available at a receiver. Each channel is called a diversity branch. Further assume that each branch has the same average SNR given by:
- $$SNR = \Gamma = \frac{E_b}{N_o}\overline{\alpha}^2$$
- If each branch has instantaneous SNR=γ_i then from the pdf of γ_i is given:
$$p(\gamma_i) = \frac{1}{\Gamma}e^{\frac{-\gamma_i}{\Gamma}}, \gamma_i \geq 0$$
- Where Γ is the mean SNR of each branch. The probability that a single branch has SNR less than some threshold γ is:
$$P_r[\gamma_i \leq \gamma] = \int_0^{\gamma} p(\gamma_i)\,d\gamma_i = \int_0^{\gamma} \frac{1}{\Gamma}e^{\frac{-\gamma_i}{\Gamma}}\,d\gamma_i$$
- Now, the probability that all M independent diversity branches receive signals which are simultaneously less than some specific SNR threshold γ is:
$$P_r[\gamma_1, \ldots, \gamma_M \leq \gamma] = \left(1 - e^{-\gamma/\Gamma}\right)^M = P_M(\gamma)$$

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4.a). Derivation of selection Diversity Improvement

- $P_M(\boldsymbol{\gamma})$ is the probability of all branches failing to acheive SNR=γ. If a single branch acheives SNR>γ then the probability that SNR>γ for one or more branches is given by:
$$P_r[\gamma_i > \gamma] = 1 - P_M(\gamma) = 1 - (1 - e^{-\gamma/\Gamma})^M$$
- This is as expression for the probability of exceeding a threshold when selection diversity is used.
- To determine the average SNR ratio of the received signal when diversity is used, it is first necessary to find the pdf of the fading signal.
$$p_M(\gamma) = \frac{d}{d\gamma}P_M(\gamma) = \frac{M}{\Gamma}(1 - e^{-\gamma/\Gamma})^{M-1}e^{-\gamma/\Gamma}$$
- Then, the mean SNR (γ), may be expressed as:
$$\overline{\gamma} = \int_0^{\infty} \gamma\, p_M(\gamma)d\gamma = \Gamma\int_0^{\infty} M\,x(1 - e^{-x})^{M-1}e^{-x}$$
$$where, x = \gamma/\Gamma$$
- The average SNR improvement offered by selection diversity. $\frac{\overline{\gamma}}{\Gamma} = \sum_{k=1}^{M}\frac{1}{k}$

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4.a) Example

- Assume four branch diversity is used, where each branch receives an independent Rayleigh fading signal. If the average SNR is 20dB, determine the probability that the SNR will drop below 10dB. Compare this with the case of a single receiver without diversity.
- **Solution:**
- For this example the specified threshold γ=10dB, Γ=20dB, and there are four branches. Thus γ/Γ=0.1 , then we have:
$$P_4(10dB) = (1 - e^{-0.1})^4 = 0.000082$$
- When diversity is not used and M=1, we have:
$$P_1(10dB) = (1 - e^{-0.1})^1 = 0.095$$
- Notice that without diversity the SNR drops below the specified threshold with a probability that is three orders of magnitude greater than if four branch diversity is used.

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4.b) Derivation of Maximal Ratio Combining Improvement

- In maximum ratio combining, the voltage signals r_i from each of the M diversity branches are co-phased to provide coherent voltage addition and are individually weighted to provide optimal SNR. If each branch has gain Gi, then the resulting signal envelope applied to the detector is:
$$r_M = \sum_{i=1}^{M} G_i r_i$$
- The total noise power applied to the detector is given: N is the noise power: $N_T = N \sum_{i=1}^{M} G_i^2$
- SNR applied to the detector is: $\gamma_M = \dfrac{r_M^2}{2N_T}$
- Using Chebychev’s inequality: SNR is maximised as follows:
$$\gamma_M = \frac{1}{2}\frac{\sum (r_i^2 / N)^2}{\sum (r_i^2 / N^2)} = \frac{1}{2}\sum_{i=1}^{M}\frac{r_i^2}{N} = \sum_{i=1}^{M}\gamma_i$$
- The SNR out of the diversity is simply the sum of the SNRs in each branch.

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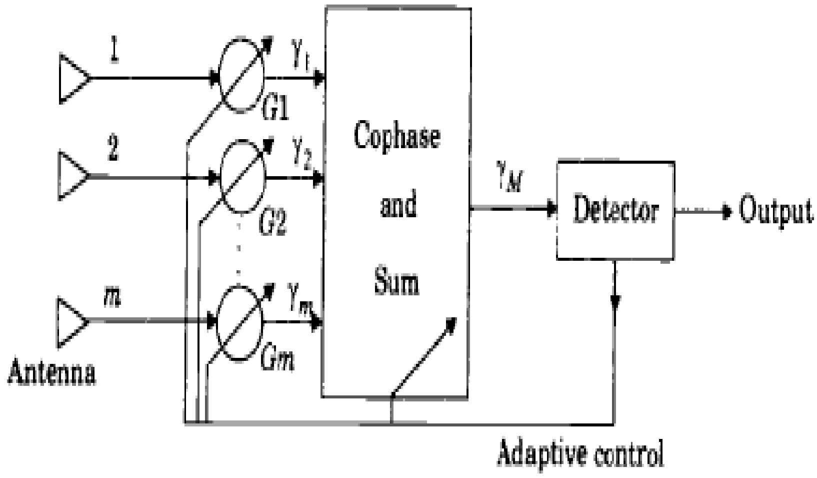
4.c). Maximal ratio combiner.



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4.d) Equal Gain Combining

- This allows the receiver to exploit signals that are simultaneously received on each branch. The possibility of producing an acceptable signal from a number of unacceptable inputs is still retained, and performance is only marginally inferior to maximal ratio combining and superior to selection diversity.

Frequency Diversity

- Frequency diversity transmits information on more than one carrier frequency. The rationale behind this technique is that frequencies separated by more than the coherence bandwidth of the channel will not experience the same fades. Frequency diversity is often employed in microwave line-of-sight links which carry several channels in a frequency division multiplex mode (FDM). Due to tropospheric propagation and resulting refraction, deep fading sometimes occurs.

Time Diversity

- Time diversity repeatedly transmits information at time spacings that exceed the coherence time of the channel, so that multiple repetitions of the signal will be received with independent fading conditions, thereby providing for diversity. One modem implementation of time diversity involves the use of the RAKE receiver.

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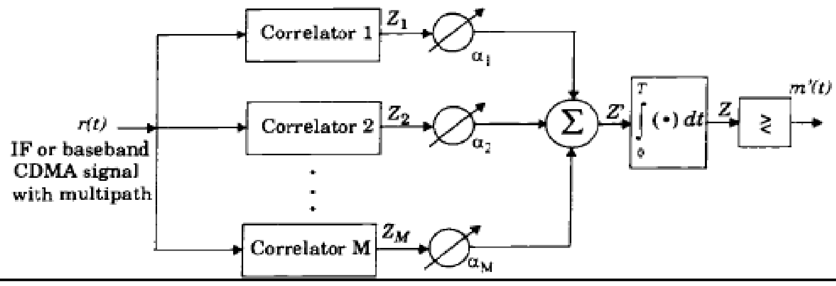
4.e). RAKE Receiver

- The RAKE receiver, shown in Figure below, is essentially a diversity receiver designed specifically for CDMA, where the diversity is provided by the fact that the multipath components are practically uncorrelated from one another when their relative propagation delays exceed a chip period. A RAKE receiver utilizes multiple correlators to separately detect the M strongest multipath components. The outputs of each correlator are weighted to provide a better estimate of the transmitted signal than is provided by a single component. Demodulation and bit decisions are then based on the weighted outputs of the M correlators.



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4.f). MIMO (Multiple Input Multiple Output

- In wireless communication networks, diversity techniques have been explored for their advantages to improve the wireless communication system.
- In this direction, multiple input multiple output (MIMO) systems have attracted too much attention in research as one of the techniques exploring the diversity gain.
- MIMO systems are considered as a key technique in modern wireless cellular networks that employ multiple antennas at both the transmitter and receiver stations.
- In MIMO systems, base stations (BSs) and mobile stations (MSs) are equipped with multiple antennas that make MIMO systems to be available at both the uplink and the downlink sides as represented in Figure .
- MIMO techniques can be employed in different ways such as transmit or receive diversity for enhancing the transmission reliability, spatial multiplexing for improving the system data rates and beamforming for increasing the network coverage .

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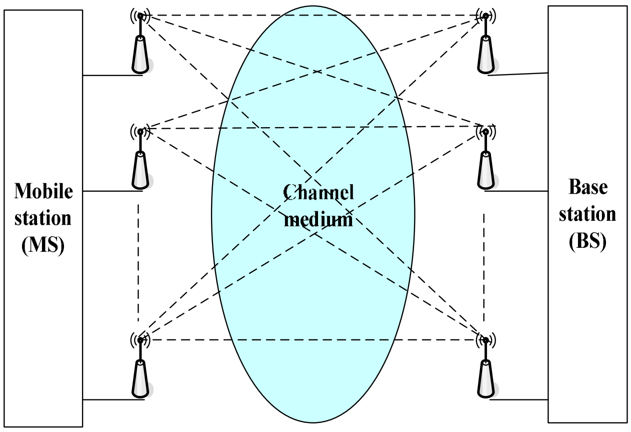
4.f). MIMO Communications



4.f). i) Diversity: SIMO: Single Input Multiple Output

$$Y = h\,x + n$$
$$y_1 = h_1 x + n_1$$
$$y_2 = h_2 x + n_2$$
$$\cdots\cdots$$
$$y_L = h_L x + n_L$$

$$\Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_L \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \cdots \\ h_L \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ \cdots \\ n_L \end{bmatrix}$$

$$W^H y = w_1^* y_1 + w_2^* y_2 + \ldots.. + w_L^* y_L, MRC$$

$$\underline{y} = \underline{hx} + \underline{n}$$

$$W^H \underline{y} = W^H(\underline{hx} + \underline{n}) = W^H \underline{hx} + W^H \underline{n}$$

-Beamformer output

-Signal Components

-Noise Components

$$E\left[\left|W^H .\underline{n}\right|^2\right] = E\left\{(W^H .n)(W^H .n)^*\right\}$$

$$E\left[(W^H .n)(W^H .n)^*\right] = E\left[\sum_{i=1}^{L} |W_i|^2 |n_i|^2 + \sum_i \sum_j W_i W_j^* n_i^* n_j\right]$$

$$= \sigma_n^2 W^H W$$

4.f). i). Diversity: SIMO: Single Input Multiple Output

$$E\left[(W^H hx)^2\right] = E\left\{(W^H hx)(W^H hx\right\} = \left|W^H h\right|^2 P$$

$$SNR = \frac{\left|W^H h\right|^2 p}{\sigma_n^2 W^H W} = \left|W^H h\right|^2 \frac{p}{\sigma_n^2}, with: \left|W^2\right| = 1 = W^H W$$

$$SNR = \left|\frac{h^H}{\|h\|} h\right| \frac{p}{\sigma_n^2} = \left|\frac{|h|^2}{\|h\|}\right|^2 \frac{p}{\sigma_n^2} = \frac{\|h\|^2}{\|h\|^2} \frac{p}{\sigma_n^2} = \|h\|^2 \frac{p}{\sigma_n^2}, with: W_{opt} = \frac{1}{\|h\|} h$$

$$SNR = \|h\|^2 \frac{p}{\sigma_n^2} = \underbrace{\left(|h_1|^2 + |h_2|^2 + \ldots + |h_n|^2\right)}_{g = sum\ of\ Rayleigh\ distance} \frac{p}{\sigma_n^2} = g . \frac{p}{\sigma_n^2}$$

$$AvgBER = \int_0^\infty Q(\sqrt{g^{SNR_r}}) f(g) dg$$

$$= \left(\frac{1-\lambda}{2}\right)^L \sum_{l=0}^{L-1} \binom{L+l-1}{l} \left(\frac{1+\lambda}{2}\right)^l, with: \lambda = \sqrt{\frac{SNR}{2+SNR}}$$

$$L = 1; AvgBER = \left(\frac{1-\lambda}{2}\right)^1 \sum_{l=0}^{0} \binom{0}{0.1} \left(\frac{1+\lambda}{2}\right)^{0.1} = \frac{1}{2} - \frac{\lambda}{2} = \frac{1}{2}(1-\lambda) = \frac{1}{2SNR}$$

$$AvgBER = \left(\frac{1}{2SNR}\right)^L \sum_{l=0}^{L-1} \binom{L+l-1}{l} = \frac{1}{2^L}\left(\frac{1}{SNR}\right)^L \binom{2L-1}{L}, at: high(SNR)$$

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4.f). i) Example

- What is the SNR to acheive BER=10^{-6} with L=2?

$$AvgBER = \frac{1}{2^L}\left(\frac{1}{SNR}\right)^L \binom{2L-1}{L}$$

$$10^{-6} = \frac{1}{2^6} \frac{1}{(SNR)^2} \binom{3}{2}$$

$$SNR = 23.35 dB, for: L = 2$$
$$SNR = 47 dB, for: L = 1$$

- Probability of being in deep fade:

$$\|h\|^2 p < \sigma_n^2$$

$$g < \frac{1}{SNR}$$

$$P(g < \frac{1}{SNR}) = \int_0^{1/SNR} \frac{g^{L-1}}{(L-1)!} e^{-g} dg \approx \frac{1}{L!}\left(\frac{1}{SNR}\right)^L$$

$$Diversity order; d = -\lim_{SNR \to \infty} \frac{\log P_e(SNR)}{\log(SNR)}$$

- In case of multipath antenna and independent channel, the spacing required should be: λ/2.

$$for: GSM: f_c = 900MHz \Rightarrow spc = \frac{\lambda}{2} = \frac{3*10^8}{2*900*10^6} = 16.6cm$$

$$for: 4G: f_c = 2.3GHz \Rightarrow spc = \frac{\lambda}{2} = \frac{3*10^8}{2*2.3*10^9} = 6.5cm$$

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4.f). ii). MIMO Channel Capacity

- Consider the channel is known at the transmitter, the capacity equals the sum of capacities on each of the independent channels with the transmit power optimally allocated between these channels. Usind SVD (single value decomposition), we have:

$$H = U\Sigma V^H = [\underline{u}_1 : \underline{u}_2 : \ldots : \underline{u}_t] \begin{bmatrix} \sigma_1 & 0 & 0 & \ldots & 0 \\ 0 & \sigma_1 & & & 0 \\ .. & & & & \\ .. & & & & \\ 0 & & & & \sigma_n \end{bmatrix} \begin{bmatrix} \underline{v}_1^H \\ \underline{v}_2^H \\ \ldots \\ \ldots \\ \underline{v}_t^H \end{bmatrix}$$

$$\|u_i\|^2 = 1, \underline{u}_i^H \underline{u}_j = 0, if, i \neq j$$

$$\|v_i\|^2 = 1, \underline{v}_i^H \underline{v}_j = 0, if, i \neq j$$

$$\sigma_1, \sigma_2, \ldots, \sigma_t \Rightarrow \sin gularvalues$$

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4.f) ii). MIMO Channel Capacity

Simplified equation of MIMO using SDV:

$$H = U\Sigma V^H$$

$$\underline{y} = H\underline{x} + \underline{n}; decompose\ H$$

$$\underline{y} = U\Sigma V^H \underline{x} + \underline{n}$$

$$U^H \underline{y} = U^H U\Sigma V^H \underline{x} + U^H \underline{n}$$

$$\underline{\tilde{y}} = \Sigma V^H \underline{x} + \underline{\tilde{n}}$$

$$\underline{\tilde{y}} = \Sigma V^H V\underline{\tilde{x}} + \underline{\tilde{n}}; \underline{x} = V\underline{\tilde{x}}$$

$$\underline{\tilde{y}} = \Sigma\underline{\tilde{x}} + \underline{\tilde{n}}$$

$$\begin{bmatrix} \tilde{y}_1 \\ \ldots \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} \sigma_1 & \ldots & 0 \\ \ldots & \ldots & \ldots \\ 0 & \ldots & \sigma_t \end{bmatrix} \begin{bmatrix} \tilde{n}_1 \\ \ldots \\ \tilde{n}_t \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \ldots \\ \tilde{n}_t \end{bmatrix}$$

$$SNR_i = \frac{p_i \sigma_i^2}{\sigma_n^2}$$

$$c = \log_2\left(1 + \frac{p_i \sigma_i^2}{\sigma_n^2}\right) = \log_2(1 + SNR)$$

$$c_1 = \log_2\left(1 + \frac{p_1 \sigma_1^2}{\sigma_n^2}\right)$$

....

$$c_t = \log_2\left(1 + \frac{p_t \sigma_t^2}{\sigma_n^2}\right)$$

$$C = \sum_{i=1}^{t} c_i = \sum_{i=1}^{t} \log_2\left(1 + \frac{p_i \sigma_i^2}{\sigma_n^2}\right)$$

$$p_1 + p_2 + \ldots + p_t \leq p$$

$$\sum_{i=1}^{t} p_i = P; \max\ MIMO\ Capacity$$

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4.f) ii). MIMO Channel Capacity

- Contraint optimization problem:

$$\max \sum_{i=1}^{t} \log_2\left(1+\frac{p_i\sigma_i^2}{\sigma_i^2}\right)$$

$$f=\sum_{i=1}^{t} \log_2\left(1+\frac{p_i\sigma_i^2}{\sigma_n^2}\right)+\lambda\left(p-\sum p_i\right)$$

$$\frac{\partial f}{\partial p_1}=0 \Rightarrow \frac{\frac{\sigma_1^2}{\sigma_n^2}}{1+\frac{p_1\sigma_n^2}{\sigma_n^2}}+\lambda(-1)=0$$

$$p_1=\left(\frac{1}{\lambda}-\frac{\sigma_n^2}{\sigma_1^2}\right),\ldots,p_t=\left(\frac{1}{\lambda}-\frac{\sigma_n^2}{\sigma_t^2}\right)$$

$$P=\sum_{i=1}^{t} p_i=\sum_{i=1}^{t}\left(\frac{1}{\lambda}-\frac{\sigma_n^2}{\sigma_t^2}\right)$$

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4.f) ii.) Example: MIMO Channel Capacity

$$H=\begin{bmatrix}2 & -6 & 0\\ 3 & 4 & 0\\ 0 & 0 & 2\end{bmatrix}, p_0=-1.25dB;\sigma_n^2=3dB$$

Compute the maximum capacity.

$$H=\begin{bmatrix}-6/\sqrt{3} & 2/\sqrt{13} & 0\\ 4/\sqrt{52} & 3/\sqrt{13} & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\sqrt{52} & 0 & 0\\ 0 & \sqrt{13} & 0\\ 0 & 0 & 2\end{bmatrix}\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\sigma_1^2=52;\sigma_2^2=13;\sigma_3^2=4$$

$$c=\sum_{i=1}^{t} c_i=\sum_{i=1}^{t}\log_2\left(1+\frac{p_i\sigma_i^2}{\sigma_n^2}\right)$$

$$c=\log_2\left(1+\frac{p_1\sigma_1^2}{\sigma_n^2}\right)+\log_2\left(1+\frac{p_2\sigma_2^2}{\sigma_n^2}\right)+\log_2\left(1+\frac{p_3\sigma_3^2}{\sigma_n^2}\right)$$

$$=\log_2\left(1+\frac{p_1*52}{2}\right)+\log_2\left(1+\frac{p_1*13}{2}\right)+\log_2\left(1+\frac{p_1*4}{2}\right)$$

$$p_1+p_2+p_3\le 0.75$$

$$\left(\frac{1}{\lambda}-\frac{1}{26}\right)+\left(\frac{1}{\lambda}-\frac{2}{13}\right)+\left(\frac{1}{\lambda}-\frac{1}{2}\right)=0.75$$

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4.f) ii.) Example: MIMO Channel Capacity

$$N = 1; \Rightarrow \frac{1}{\lambda} = 0.48$$

$$p_3 = 0.48 - \frac{1}{2} = -0.02 < 0, \; p_3 = 0$$

$$N = 2;$$

$$\left(\frac{1}{\lambda} - \frac{1}{26}\right) + \left(\frac{1}{\lambda} - \frac{2}{13}\right) = 0.75$$

$$\frac{1}{\lambda} = 0.4712$$

$$p_1 = 0.4327 > 0 \Rightarrow -3.63dB$$

$$p_2 = 0.3174 > 0 \Rightarrow -4.98dB$$

$$p_3 = 0$$

$$C = \log_2\left(\frac{1 + 52 * 0.4327}{2}\right) + \log_2\left(\frac{1 + 13 * 0.3174}{2}\right)$$

$$= 5.23 \; b/s/Hz$$

Problems

1) Find the capacity and optimal power allocation(s) for the MIMO channel, assuming that SNR=10dB and BW=1Hz, and channel gains are σ_1=1.333, σ_2=0.5129, σ_3=0.0965. $(h\text{int}.\gamma_i = 10\sigma_i^2)$
2) Consider the set of empirical measurements of *Pr/Pt given in the table below for an indoor* system at 2 GHz. Find the path loss exponent *that minimizes the MSE between the simplified* model (2.29) and the empirical dB power measurements, assuming that *d0 = 1 m and K is determined* from the free space path loss formula at this *d0. Find the received power at 150 m for the simplified path* loss model with this path loss exponent and a transmit power of 1 mW (0 dBm).

Distance from Transmitter	**M=Pr/Pt**
10 m	-70 dB
20 m	-75 dB
50 m	-90 dB
100 m	-110 dB
300 m	-125 dB
Table: Path loss measurements	

3) Assume a SNR of 25dB is desired at the receiver. If a 900MHz cellular transmitter has an EIRP of 100W, and the AMPS receiver uses a 0dB gain antenna and has a 10dB noise figure, find the percentage of time that the desired SNR is acheived at a distance of 10km from the transmitter. Assume n=4, σ=8dB and d_o=1km.

4) Consider an indoor wireless LAN with fc = 900 MHz, cells of radius 10 m, and nondirectional antennas. Under the free-space path loss model, what transmit power is required at the access point such that all terminals within the cell receive a minimum power of 10 µW. How does this change if the system frequency is 5 GHz?

5) Determine the critical distance for the two-ray model in an urban microcell (*ht = 10m, hr = 3 m) and an indoor microcell (ht = 3m, hr = 2 m) for fc = 2 GHz.*

6) Consider a channel with Rayleigh fading and average received power *Pr = 20 dB. Find* the probability that the received power is below 10 dB.

7) Consider a wideband channel with multipath intensity profile:

$$A_c(\tau) = \begin{cases} e^{-\tau}, 0 \le \tau \le 20\ \mu\text{sec} \\ 0, \quad else \end{cases}$$

Find the mean and rms delay spreads of the channel and find the maximum symbol period such that a linearly-modulated signal transmitted through this channel does not experience ISI.

8) In indoor channels σ_{Tm}≈50 ns whereas in outdoor microcells σ_{Tm}≈30µs. Find the maximum symbol rate Rs=1/Ts for these environments such that a linearly-modulated signal transmitted through these environments experiences negligible ISI.

9) For a channel with Doppler spread *Bd* = 80 Hz, what time separation is required in samples of the received signal such that the samples are approximately independent.
10) Show that the Brewster angle (case where Γ_υ=0) is given by θ_i where:

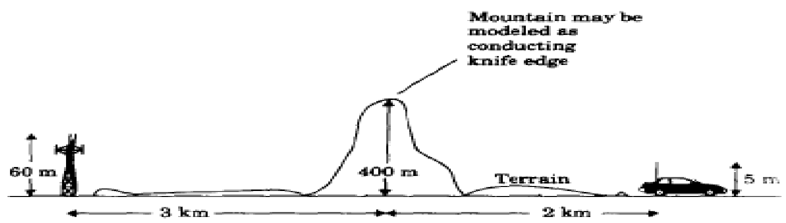
$$\sin\theta_i = \sqrt{\frac{\varepsilon_r^2 - \varepsilon_r}{\varepsilon_r^2 - 1}}$$

11) (a) Explain the advantages and disadvantages of the 2-ray ground reflection model in the analysis of path loss. (b) In the following cases, tell whether the 2-ray model could be applied, and explain why or why not:

$$h_t = 35m, h_r = 3m, d = 250m$$
$$h_t = 30m, h_r = 1.5m, d = 450m$$

12) (c)What insight does the 2-ray model provide about large-scale path loss that was disregarded when cellular systems used very large cells?
13) In a 2-ray ground reflected model, assume that $\theta_\Delta$must be kept below 6.261 radians for phase cancellation reasons. Assuming a receiver height of 2m, and given a requirement that $\theta_i$be less than, 5^0 what are the minimum allowable values for the T-R separation distance and the height of the transmitter antenna?. The carrier frequency is 900MHz.

14) If P_t=10W, G_t=10dB, G_r=3dB, and L=1dB at 900MHz, Compute the received power for the knife-edge geometry shown in figure below. Compare this value with the theoretical free space received power if an obstruction did not exist. What is the path loss due to diffraction for this case?



Knife-edge geometry

15) Assume that local average signal strenght field measurements were made inside a building, and post processing revealed that the measured data fit a distant-dependant mean power law model having a log-normal distribution about the mean. Assume the mean power law found to be $p_r(d) \propto d^{-3.5}$.If a signal of

- 1mW was received at do=1m from the transmitter, and at a distance of 10m, 10% of the measurements were stronger than -25dBm, define the standard deviation, σ, for the path loss model at d=10m.

16) If the received power at a reference distance do=1 km is equal to 1 microwatt, find the received powers at distance of 2km, 5km, 10km and 20km from the same transmitter for the following path loss models: (a) free space, b) n=3; c) n=4; d) 2-ray ground reflection using the exact expression; e) extended hata model. Assume f=1800Mhz, ht=40m, hr=3m, Gt=Gr=0 dB. Plot each of these models on the same graph over the range of 1km to 20km.

17) A transmitter provides 15W to an antenna having 12dB gain. The receiver antenna has a gain of 3dB and the receiver bandwidth is 30kHz. If the receiver system noise figure is 8dB and the carrier frequency is 1800 MHz, find the maximum T-R separation that will ensure that a SNR of 20dB is provided for 95% of the time. Assume n=4, σ=8dB and d_o=1km.

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18) Approximately how large can the rms delay spread be in order for a binary modulated signal with a bit rate of 25kbps to operate without an equalizer? What about an 8-PSK system with a bit rate of 75kbps?.

19) Given that the probability density fucntion of a Rayleigh distributed envelope is given by:

$$p(r) = \frac{r}{\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2}\right), where: \sigma^2 is\ variance$$

Show that the cumulative distribution function is given as:

$$p(r < R) = 1 - \exp\left(\frac{-R^2}{2\sigma^2}\right).$$

Find the % of time that a signal is 10dB or more below the rms value for a Rayleigh fading signal.

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20) The fading characteristics of a CW carrier in an urban area are to be measured. The following assumptions are made:

(1) The mobile receiver uses a simple verticle monopole

(2) Large-scale fading due to path loss is ignored.

(3) The mobile has no line-of-sight path to the base station.

(4) The pdf of the received signal follows a Rayleigh distribution.

(a) Derive the ratio of the desired signal level to the rms signal level that maximizes the level crossing rate. Express your enswer in dB.

(b) Assuming the maximum velocity of the mobile is 50km/hr, and the carrier frequency is 900MHz, determine the maximum number of times the signal envelope will fade below the level found in (a) during a 1 minute test.

(c) How long, on average, will each fade in (b) last.

20. An automobile moves with velocity v(t) shown in figure below. The received mobile signal experiences multipath Rayleigh fading on a 900MHz CW carrier. What is the average crossing rate and fade duration over the 100 s interval?.

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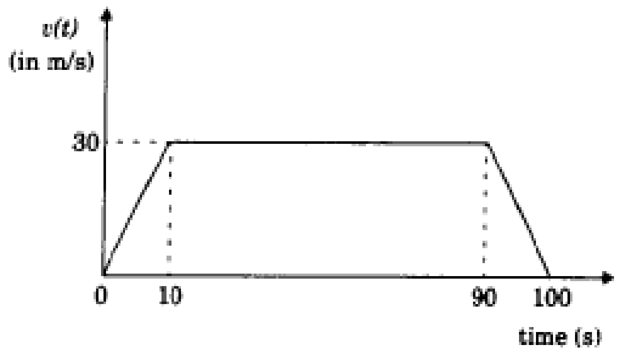
- Assume ρ=0.1 and ignore large-scale fading effects.



Graph of velocity of mobile.

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-END-