Open CASCADE Technology
7.2.0

BRep Format

Contents

1 Introduction

BREP format is used to store 3D models and allows to store a model which consists of vertices, edges, wires, faces, shells, solids, compsolids, compounds, edge triangulations, face triangulations, polylines on triangulations, space location and orientation. Any set of such models may be stored as a single model which is a compound of the models.

The format is described in an order which is convenient for understanding rather than in the order the format parts follow each other. BNF-like definitions are used in this document. Most of the chapters contain BREP format descriptions in the following order:

- format file fragment to illustrate the part;
- BNF-like definition of the part;
- detailed description of the part.

Note that the format is a part of Open CASCADE Technology (OCCT).

Some data fields of the format have additional values, which are used in OCCT.

Some data fields of the format are specific for OCCT.

2 Format Common Structure

ASCII encoding is used to read/write BREP format from/to file. The format data are stored in a file as text data.

BREP format uses the following BNF terms:

- $<$\n$>$: It is the operating-system-dependent ASCII character sequence which separates ASCII text strings in the operating system used;

- $<$_\n$>$: = " "$*<$\n$>$;

- $<$_$>$: = " "+; It is a not empty sequence of space characters with ASCII code 21h;

- $<$flag$>$: = "0" $|$ "1";

- $<$int$>$: It is an integer number from -2^{31} to 2^{31}-1 which is written in denary system;

- $<$real$>$: It is a real from -1.7976931348623158 · 10^{308} to 1.7976931348623158 · 10^{308} which is written in decimal or E form with base 10.The point is used as a delimiter of the integer and fractional parts;

- $<$2D point$>$: = $<$real$><$_$><$real$>$;

- $<$3D point$>$: = $<$real$>(<$_$><$real$>)^2$;

- $<$2D direction$>$: It is a $<$2D point$>$ *x y* * *so that* $*x^2 + y^2 = 1$;

- $<$3D direction$>$: It is a $<$3D point$>$ *x y z* * *so that* $*x^2 + y^2 + z^2 = 1$;

- $<$+$>$: It is an arithmetic operation of addition.

The format consists of the following sections:

- $<$content type$>$;

- $<$version$>$;

- $<$locations$>$;

- $<$geometry$>$;

- $<$shapes$>$.

$<$content type$>$ = "DBRep_DrawableShape" $<$_\n$><$_\n$>$; $<$content type$>$ have other values [1].

$<$version$>$ = ("CASCADE Topology V1, (c) Matra-Datavision" $|$ "CASCADE Topology V2, (c) Matra-Datavision")$<$←_\n$>$; The difference of the versions is described in the document.

Sections $<$locations$>$, $<$geometry$>$ and $<$shapes$>$ are described below in separate chapters of the document.

3 Locations

Example

```
Locations  3
1
            0           0           1           0
            1           0           0           0
            0           1           0           0
1
            1           0           0           4
            0           1           0           5
            0           0           1           6
2   1 1 2 1 0
```

BNF-like Definition

```
<locations> = <location header> <_\n> <location records>;
<location header> = "Locations" <_> <location record count>;
<location record count> = <int>;
<location records> = <location record> ^ <location record count>;
<location record> = <location record 1> | <location record 2>;
<location record 1> = "1" <_\n> <location  data 1>;
<location record 2> = "2" <_> <location  data 2>;
<location data 1> = ((<_> <real>) ^ 4 <_\n>) ^ 3;
<location data 2> = (<int> <_> <int> <_>)* "0" <_\n>;
```

Description

$<$location data 1$>$ is interpreted as a 3 x 4 matrix $Q = \begin{pmatrix} q_{1,1} & q_{1,2} & q_{1,3} & q_{1,4} \\ q_{2,1} & q_{2,2} & q_{2,3} & q_{2,4} \\ q_{3,1} & q_{3,2} & q_{3,3} & q_{3,4} \end{pmatrix}$ which describes transformation of 3 dimensional space and satisfies the following constraints:

- $d \neq 0$ where $d = |Q_2|$ where $Q_2 = \begin{pmatrix} q_{1,1} & q_{1,2} & q_{1,3} & q_{1,4} \\ q_{2,1} & q_{2,2} & q_{2,3} & q_{2,4} \\ q_{3,1} & q_{3,2} & q_{3,3} & q_{3,4} \end{pmatrix}$;

- $Q_3^T = Q_3^{-1}$ where $Q_3 = Q_2/d^{1/3}$.

The transformation transforms a point (x, y, z) to another point (u, v, w) by the rule:

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = Q \cdot (x\, y\, z\, 1)^T = \begin{pmatrix} q_{1,1} \cdot x + q_{1,2} \cdot y + q_{1,3} \cdot z + q_{1,4} \\ q_{2,1} \cdot x + q_{2,2} \cdot y + q_{2,3} \cdot z + q_{2,4} \\ q_{3,1} \cdot x + q_{3,2} \cdot y + q_{3,3} \cdot z + q_{3,4} \end{pmatrix}.$$

Q may be a composition of matrices for the following elementary transformations:

- parallel translation $-\begin{pmatrix} 1 & 0 & 0 & q_{1,4} \\ 0 & 1 & 0 & q_{2,4} \\ 0 & 0 & 1 & q_{3,4} \end{pmatrix}$;

- rotation around an axis with a direction $*D(D_x, D_y, D_z)*$ by an angle φ −

$$\begin{pmatrix} D_x^2 \cdot (1-cos(\varphi))+cos(\varphi) & D_x \cdot D_y \cdot (1-cos(\varphi))-D_z \cdot sin(\varphi) & D_x \cdot D_z \cdot (1-cos(\varphi))+D_y \cdot sin(\varphi) & 0 \\ D_x \cdot D_y \cdot (1-cos(\varphi))+D_z \cdot sin(\varphi) & D_y^2 \cdot (1-cos(\varphi))+cos(\varphi) & D_y \cdot D_z \cdot (1-cos(\varphi))-D_x \cdot sin(\varphi) & 0 \\ D_x \cdot D_z \cdot (1-cos(\varphi))-D_y \cdot sin(\varphi) & D_y \cdot D_z \cdot (1-cos(\varphi))+D_x \cdot sin(\varphi) & D_z^2 \cdot (1-cos(\varphi))+cos(\varphi) & 0 \end{pmatrix};$$

- scaling $-\begin{pmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \end{pmatrix}$ where $S \in (-\infty, \infty)/\{0\}$;

- central symmetry –
$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix};$$

- axis symmetry –
$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix};$$

- plane symmetry –
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}.$$

<location data 2> is interpreted as a composition of locations raised to a power and placed above this <location data 2> in the section <locations>. <location data 2> is a sequence $l_1 p_1 ... l_n p_n$ of $n \geq 0$ integer pairs $l_i p_i$ $(1 \leq i \leq n)$. <flag> 0 is the indicator of the sequence end. The sequence is interpreted as a composition $L_{l_1}^{p_1} \cdot ... \cdot L_{l_n}^{p_n}$ where L_{l_i} is a location from l_i-th <location record> in the section locations. <location record> numbering starts from 1.

4 Geometry

```
<geometry> =
<2D curves>
<3D curves>
<3D polygons>
<polygons on triangulations>
<surfaces>
<triangulations>;
```

4.1 3D curves

Example

```
Curves 13
1 0 0 0 0 0 1
1 0 0 3 -0 1 0
1 0 2 0 0 0 1
1 0 0 0 -0 1 0
1 1 0 0 0 0 1
1 1 0 3 0 1 0
1 1 2 0 0 0 1
1 1 0 0 -0 1 0
1 0 0 0 1 0 -0
1 0 0 3 1 0 -0
1 0 2 0 1 0 -0
1 0 2 3 1 0 -0
1 1 0 0 1 0 0
```

BNF-like Definition

```
<3D curves> = <3D curve header> <_\n> <3D curve records>;

<3D curve header> = "Curves" <_> <3D curve count>;

<3D curve count> = <int>;

<3D curve records> = <3D curve record> ^ <3D curve count>;

<3D curve record> =
<3D curve record 1> |
<3D curve record 2> |
<3D curve record 3> |
<3D curve record 4> |
<3D curve record 5> |
<3D curve record 6> |
<3D curve record 7> |
<3D curve record 8> |
<3D curve record 9>;
```

4.1.1 Line - <3D curve record 1>

Example

```
1 1 0 3 0 1 0
```

BNF-like Definition

```
<3D curve record 1> = "1" <_> <3D point> <_> <3D direction> <_\n>;
```

Description

<3D curve record 1> describes a line. The line data consist of a 3D point $P*$ *and a 3D direction* $*D$. The line passes through the point P, has the direction $*D*$ and is defined by the following parametric equation:

$$C(u) = P + u \cdot D, \; u \in (-\infty, \; \infty).$$

The example record is interpreted as a line which passes through a point *P*=(1, 0, 3), has a direction *D*=(0, 1, 0) and is defined by the following parametric equation: $C(u) = (1,0,3) + u \cdot (0,1,0)$.

4.1.2 Circle - <3D curve record 2>

Example

```
2 1 2 3 0 0 1 1 0 -0 -0 1 0 4
```

BNF-like Definition

```
<3D curve record 2> = "2" <_> <3D circle center> <_> <3D circle N> <_> <3D circle Dx> <_> <3D circle Dy>
      <_> <3D circle radius> <_\n>;

<3D circle center> = <3D point>;

<3D circle N> = <3D direction>;

<3D circle Dx> = <3D direction>;

<3D circle Dy> = <3D direction>;

<3D circle radius> = <real>;
```

Description

<3D curve record 2> describes a circle. The circle data consist of a 3D point *P*, pairwise orthogonal 3D directions *N*, D_x* *and* $*D_y*$ *and a non-negative real* $*r*$. The circle has a center *P* and is located in a plane with a normal *N*. The circle has a radius $*r*$ and is defined by the following parametric equation:

$$C(u) = P + r \cdot (cos(u) \cdot D_x + sin(u) \cdot D_y), \; u \in [o, \; 2 \cdot \pi].$$

The example record is interpreted as a circle which has its center *P*=(1, 2, 3), is located in plane with a normal *N*=(0, 0 ,1). Directions for the circle are D_x=(1, 0 ,0) and D_y=(0, 1 ,0). The circle has a radius *r*=4 and is defined by the following parametric equation: $C(u) = (1,2,3) + 4 \cdot (cos(u) \cdot (1,0,0) + sin(u) \cdot (0,1,0))$.

4.1.3 Ellipse - <3D curve record 3>

Example

```
3 1 2 3 0 0 1 1 0 -0 -0 1 0 5  4
```

BNF-like Definition

```
<3D curve record 3> = "3" <_> <3D ellipse center> <_> <3D ellipse N> <_> <3D ellipse Dmaj> <_> <3D ellipse
      Dmin> <_> <3D ellipse Rmaj> <_> <3D ellipse Rmin> <_\n>;

<3D ellipse center> = <3D point>;

<3D ellipse N> = <3D direction>;

<3D ellipse Dmaj> = <3D direction>;

<3D ellipse Dmin> = <3D direction>;

<3D ellipse Rmaj> = <real>;

<3D ellipse Rmin> = <real>;
```

Description

<3D curve record 3> describes an ellipse. The ellipse data consist of a 3D point *P*, pairwise orthogonal 3D directions *N*, $*D_{maj}*$ and $*D_{min}*$ and non-negative reals $*r_{maj}*$ and $*r_{min}*$ so that $*r_{min}* \leq r_{maj}$. The ellipse has its center *P*, is located in plane with the normal *N*, has major and minor axis directions $D_{maj}*$ *and* $*D_{min}$, major and minor radii $*r_{maj}*$ and $*r_{min}*$ and is defined by the following parametric equation:

$$C(u) = P + r_{maj} \cdot cos(u) \cdot D_{maj} + r_{min} \cdot sin(u) \cdot D_{min}, u \in [0, 2 \cdot \pi].$$

The example record is interpreted as an ellipse which has its center *P*=(1, 2, 3), is located in plane with a normal *N*=(0, 0, 1), has major and minor axis directions D_{maj}=(1, 0, 0) and D_{min}=(0, 1, 0), major and minor radii r_{maj}=5 and r_{min}=4 and is defined by the following parametric equation: $C(u) = (1,2,3) + 5 \cdot cos(u) \cdot (1,0,0) + 4 \cdot sin(u) \cdot (0,1,0)$.

4.1.4 Parabola - <3D curve record 4>

Example

```
4 1 2 3 0 0 1 1 0 -0 -0 1 0  16
```

BNF-like Definition

```
<3D curve record 4> = "4" <_> <3D parabola origin> <_> <3D parabola N> <_> <3D parabola Dx> <_> <3D
    parabola Dy> <_> <3D parabola focal length> <_\n>;

<3D parabola origin> = <3D point>;

<3D parabola N> = <3D direction>;

<3D parabola Dx> = <3D direction>;

<3D parabola Dy> = <3D direction>;

<3D parabola focal length> = <real>;
```

Description

<3D curve record 4> describes a parabola. The parabola data consist of a 3D point *P*, pairwise orthogonal 3D directions *N*, D_x* *and* $*D_y*$ *and a non-negative real* $*f$. The parabola is located in plane which passes through the point $P*$ *and has the normal* $*N$. The parabola has a focus length $*f*$ and is defined by the following parametric equation:

$$C(u) = P + \frac{u^2}{4 \cdot f} \cdot D_x + u \cdot D_y, u \in (-\infty, \infty) \Leftarrow f \neq 0;$$

$$C(u) = P + u \cdot D_x, u \in (-\infty, \infty) \Leftarrow f = 0 \ (degenerated \ case).$$

The example record is interpreted as a parabola in plane which passes through a point *P*=(1, 2, 3) and has a normal *N*=(0, 0, 1). Directions for the parabola are D_x=(1, 0, 0) and D_y=(0, 1, 0). The parabola has a focus length *f*=16 and is defined by the following parametric equation: $C(u) = (1,2,3) + \frac{u^2}{64} \cdot (1,0,0) + u \cdot (0,1,0)$.

4.1.5 Hyperbola - <3D curve record 5>

Example

```
5 1 2 3 0 0 1 1 0 -0 -0 1 0 5  4
```

BNF-like Definition

```
<3D curve record 5> = "5" <_> <3D hyperbola origin> <_> <3D hyperbola N> <_> <3D hyperbola Dx> <_> <3D
    hyperbola Dy> <_> <3D hyperbola Kx> <_> <3D hyperbola Ky> <_\n>;

<3D hyperbola origin> = <3D point>;

<3D hyperbola N> = <3D direction>;
```

```
<3D hyperbola Dx> = <3D direction>;

<3D hyperbola Dy> = <3D direction>;

<3D hyperbola Kx> = <real>;

<3D hyperbola Ky> = <real>;
```

Descripton

<3D curve record 5> describes a hyperbola. The hyperbola data consist of a 3D point *P*, pairwise orthogonal 3D directions *N*, *Dx∗* and *∗Dy∗* and non-negative reals *∗kx∗* and *∗ky*. The hyperbola is located in plane which passes through the point *P∗ and has the normal ∗N*. The hyperbola is defined by the following parametric equation:

$$C(u) = P + k_x \cdot cosh(u) \cdot D_x + k_y \cdot sinh(u) \cdot D_y, u \in (-\infty, \ \infty).$$

The example record is interpreted as a hyperbola in plane which passes through a point *P*=(1, 2, 3) and has a normal *N*=(0, 0, 1). Other hyperbola data are *Dx*=(1, 0, 0), *Dy*=(0, 1, 0), *kx*=5 and *ky*=4. The hyperbola is defined by the following parametric equation: $C(u) = (1,2,3) + 5 \cdot cosh(u) \cdot (1,0,0) + 4 \cdot sinh(u) \cdot (0,1,0)$.

4.1.6 Bezier Curve - <3D curve record 6>

Example

```
6 1 2 0 1 0  4 1 -2 0  5 2 3  0  6
```

BNF-like Definition

```
<3D curve record 6> = "6" <_> <3D Bezier rational flag> <_> <3D Bezier degree>
<3D Bezier weight poles> <_\n>;

<3D Bezier rational flag> = <flag>;

<3D Bezier degree> = <int>;

3D Bezier weight poles> = (<_> <3D Bezier weight pole>) ^ (<3D Bezier degree> <+> "1");

<3D Bezier weight pole> = <3D point> [<_> <real>];
```

Description

<3D curve record 6> describes a Bezier curve. The curve data consist of a rational *r*, a degree $m \leq 25$ and weight poles.

The weight poles are *m*+1 3D points *B0 ... Bm∗ if the flag ∗r* is 0. The weight poles are *m*+1 pairs *∗B0h0 ... Bmhm∗* if flag *∗r∗* is 1. Here *∗Bi∗* is a 3D point and *∗hi∗* is a positive real $(0 \leq i \leq m)$. $h_i = 1$ $(0 \leq i \leq m)$ if the flag *∗r∗* is 0.

The Bezier curve is defined by the following parametric equation:

$$C(u) = \frac{\sum_{i=0}^{m} B_i \cdot h_i \cdot C_m^i \cdot u^i \cdot (1-u)^{m-i}}{\sum_{i=0}^{m} h_i \cdot C_m^i \cdot u^i \cdot (1-u)^{m-i}}, \ u \in [0, \ 1]$$

where $0^0 \equiv 1$.

The example record is interpreted as a Bezier curve with a rational flag *r*=1, degree *m*=2 and weight poles *B0*=(0, 1, 0), *h0*=4, *B1*=(1, -2, 0), *h1*=5 and *B2*=(2, 3, 0), *h2*=6. The Bezier curve is defined by the following parametric equation:

$$C(u) = \frac{(0,1,0) \cdot 4 \cdot (1-u)^2 + (1,-2,0) \cdot 5 \cdot 2 \cdot u \cdot (1-u) + (2,3,0) \cdot 6 \cdot u^2}{4 \cdot (1-u)^2 + 5 \cdot 2 \cdot u \cdot (1-u) + 6 \cdot u^2}.$$

4.1.7 B-spline Curve - <3D curve record 7>

Example

```
7 1 0  1 3 5  0 1 0  4 1 -2  0  5 2 3 0  6
 0 1 0.25 1 0.5 1 0.75 1 1 1
```

BNF-like Definition

```
<3D curve record 7> = "7" <_> <3D B-spline rational flag> <_> "0" <_> <3D B-spline degree> <_>
<3D B-spline pole count> <_> <3D B-spline multiplicity knot count> <3D B-spline weight poles>
<_\n> <3D B-spline multiplicity knots> <_\n>;

<3D B-spline rational flag> = <flag>;

<3D B-spline degree> = <int>;

<3D B-spline pole count> = <int>;

<3D B-spline multiplicity knot count> = <int>;

<3D B-spline weight poles> = (<_> <3D B-spline weight pole>) ^ <3D B-spline pole count>;

<3D B-spline weight pole> = <3D point> [<_> <real>];

<3D B-spline multiplicity knots> = (<_> <3D B-spline multiplicity knot>) ^ <3D B-spline multiplicity knot
      count>;

<3D B-spline multiplicity knot> = <real> <_> <int>;
```

Description

<3D curve record 7> describes a B-spline curve. The curve data consist of a rational flag r, a degree $m \leq 25$, pole count $n \geq 2$, multiplicity knot count k, weight poles and multiplicity knots.

The weight poles are n 3D points $*B_1 ... B_n*$ if the flag $*r*$ is 0. The weight poles are $*n*$ pairs $*B_1 h_1 ... B_n h_n*$ if the flag $*r*$ is 1. Here $*B_i*$ is a 3D point and $*h_i*$ is a positive real $(1 \leq i \leq n)$. $h_i = 1$ $(1 \leq i \leq n)$ if the flag $*r*$ is 0.

The multiplicity knots are $k*$ *pairs* $*u_1 q_1 ... u_k q_k*$. Here $*u_i*$ is a knot with a multiplicity $q_i \geq 1$ $(1 \leq i \leq k)$ so that

$$u_i < u_{i+1}(1 \leq i \leq k-1),$$

$$q_1 \leq m+1, \ q_k \leq m+1, \ q_i \leq m \ (2 \leq i \leq k-1), \sum_{i=1}^{k} q_i = m+n+1.$$

The B-spline curve is defined by the following parametric equation:

$$C(u) = \frac{\sum_{i=1}^{n} B_i \cdot h_i \cdot N_{i,m+1}(u)}{\sum_{i=1}^{n} h_i \cdot N_{i,m+1}(u)}, \ u \in [u_1, \ u_k]$$

where functions $N_{i,j}$ have the following recursion definition by j:

$$N_{i,1}(u) = \begin{cases} 1 \Leftarrow \bar{u}_i \leq u \leq \bar{u}_{i+1} \\ 0 \Leftarrow u < \bar{u}_i \vee \bar{u}_{i+1} \leq u \end{cases}, \ N_{i,j}(u) = \frac{(u - \bar{u}_i) \cdot N_{i,j-1}(u)}{\bar{u}_{i+j-1} - \bar{u}_i} + \frac{(\bar{u}_{i+j} - u) \cdot N_{i+1,j-1}(u)}{\bar{u}_{i+j} - \bar{u}_{i+1}}, \ (2 \leq j \leq m+1)$$

where

$$\bar{u}_i = u_j, \ (1 \leq j \leq k, \ \sum_{l=1}^{j-1} q_l + 1 \leq i \leq \sum_{l=1}^{j} q_l).$$

The example record is interpreted as a B-spline curve with a rational flag r=1, a degree m=1, pole count n=3, multiplicity knot count k=5, weight poles B_1=(0,1,0), h_1=4, B_2=(1,-2,0), h_2=5 and B_3=(2,3,0), h_3=6, multiplicity knots u_1=0, q_1=1, u_2=0.25, q_2=1, u_3=0.5, q_3=1, u_4=0.75, q_4=1 and u_5=1, q_5=1. The B-spline curve is defined by the following parametric equation:

$$C(u) = \frac{(0,1,0) \cdot 4 \cdot N_{1,2}(u) + (1,-2,0) \cdot 5 \cdot N_{2,2}(u) + (2,3,0) \cdot 6 \cdot N_{3,2}(u)}{4 \cdot N_{1,2}(u) + 5 \cdot N_{2,2}(u) + 6 \cdot N_{3,2}(u)}.$$

4.1.8   Trimmed Curve - <3D curve record 8>

Example

```
8  -4 5
1 1 2 3 1 0 0
```

BNF-like Definition

```
<3D curve record 8> = "8" <_> <3D trimmed curve u min> <_> <3D trimmed curve u max> <_\n> <3D curve
    record>;

<3D trimmed curve u min> = <real>;

<3D trimmed curve u max> = <real>;
```

Description

<3D curve record 8> describes a trimmed curve. The trimmed curve data consist of reals $u_{min}*$ and $*u_{max}*$ and <3D curve record> so that $*u_{min} < u_{max}$. The trimmed curve is a restriction of the base curve $*B*$ described in the record to the segment $[u_{min}, u_{max}] \subseteq domain(B)$. The trimmed curve is defined by the following parametric equation:

$$C(u) = B(u), \ u \in [u_{min}, u_{max}].$$

The example record is interpreted as a trimmed curve with u_{min}=-4 and u_{max}=5 for the base curve $B(u) = (1,2,3) + u \cdot (1,0,0)$. The trimmed curve is defined by the following parametric equation: $C(u) = (1,2,3) + u \cdot (1,0,0)$, $u \in [-4, 5]$.

4.1.9   Offset Curve - <3D curve record 9>

Example

```
9 2
0 1 0
1 1 2 3 1 0 0
```

BNF-like Definition

```
<3D curve record 9> = "9" <_> <3D offset curve distance> <_\n>;
<3D offset curve direction> <_\n>;
<3D curve record>;

<3D offset curve distance> = <real>;

<3D offset curve direction> = <3D direction>;
```

Description

<3D curve record 9> describes an offset curve. The offset curve data consist of a distance d, a 3D direction $*D*$ and a <3D curve record>. The offset curve is the result of offsetting the base curve $*B*$ described in the record to the distance $*d*$ along the vector $[B'(u), D] \neq \vec{0}$. The offset curve is defined by the following parametric equation:

$$C(u) = B(u) + d \cdot \frac{[B'(u), D]}{|[B'(u), D]|}, \ u \in domain(B).$$

The example record is interpreted as an offset curve with a distance $d=2$, direction $D=(0, 1, 0)$, base curve $B(u) = (1,2,3) + u \cdot (1,0,0)$ and defined by the following parametric equation: $C(u) = (1,2,3) + u \cdot (1,0,0) + 2 \cdot (0,0,1)$.

4.2 Surfaces

Example

```
Surfaces 6
1 0 0 0 1 0 -0 0 0 1 0 -1 0
1 0 0 0 -0 1 0 0 0 1 1 0 -0
1 0 0 3 0 0 1 1 0 -0 -0 1 0
1 0 2 0 -0 1 0 0 0 1 1 0 -0
1 0 0 0 0 0 1 1 0 -0 -0 1 0
1 1 0 0 1 0 -0 0 0 1 0 -1 0
```

BNF-like Definition

```
<surfaces> = <surface header> <_\n> <surface records>;

<surface header> = "Surfaces" <_> <surface count>;

<surface records> = <surface record> ^ <surface count>;

<surface record> =
<surface record 1> |
<surface record 2> |
<surface record 3> |
<surface record 4> |
<surface record 5> |
<surface record 6> |
<surface record 7> |
<surface record 8> |
<surface record 9> |
<surface record 10> |
<surface record 11>;
```

4.2.1 Plane - $<$ surface record 1 $>$

Example

```
1 0 0 3 0 0 1 1 0 -0 -0 1 0
```

BNF-like Definition

```
<surface record 1> = "1" <_> <3D point> (<_> <3D direction>) ^ 3 <_\n>;
```

Description

$<$surface record 1$>$ describes a plane. The plane data consist of a 3D point $P*$ *and pairwise orthogonal 3D directions* $*N$, D_u* *and* $*D_v$. The plane passes through the point P, has the normal $*N*$ and is defined by the following parametric equation:

$$S(u,v) = P + u \cdot D_u + v \cdot D_v, \ (u, \ v) \in (-\infty, \ \infty) \times (-\infty, \ \infty).$$

The example record is interpreted as a plane which passes through a point $P=(0, 0, 3)$, has a normal $N=(0, 0, 1)$ and is defined by the following parametric equation: $S(u,v) = (0,0,3) + u \cdot (1,0,0) + v \cdot (0,1,0)$.

4.2.2 Cylinder - $<$ surface record 2 $>$

Example

```
2 1 2 3 0 0 1 1 0 -0 -0 1 0 4
```

BNF-like Definition

```
<surface record 2> = "2" <_> <3D point> (<_> <3D direction>) ^ 3 <_> <real> <_\n>;
```

Description

<surface record 2> describes a cylinder. The cylinder data consist of a 3D point *P*, pairwise orthogonal 3D directions D_v, D_X and D_{Y*} *and a non-negative real* $*r$. *The cylinder axis passes through the point* $P*$ *and has the direction* $*D_v$. The cylinder has the radius $*r*$ and is defined by the following parametric equation:

$$S(u,v) = P + r \cdot (cos(u) \cdot D_x + sin(u) \cdot D_y) + v \cdot D_v, \ (u,v) \in [0, \ 2 \cdot \pi) \times (-\infty, \ \infty).$$

The example record is interpreted as a cylinder which axis passes through a point *P*=(1, 2, 3) and has a direction D_v=(0, 0, 1). Directions for the cylinder are D_X=(1,0,0) and D_Y=(0,1,0). The cylinder has a radius *r*=4 and is defined by the following parametric equation: $S(u,v) = (1,2,3) + 4 \cdot (cos(u) \cdot D_X + sin(u) \cdot D_Y) + v \cdot D_v$.

4.2.3 Cone - < surface record 3 >

Example

```
3 1 2 3 0 0 1 1 0 -0 -0 1 0 4
0.75
```

BNF-like Definition

```
<surface record 3> = "3" <_> <3D point> (<_> <3D direction>) ^ 3 (<_> <real>) ^ 2 <_\n>;
```

Description

<surface record 3> describes a cone. The cone data consist of a 3D point *P*, pairwise orthogonal 3D directions D_Z, D_X and D_Y, a non-negative real $*r*$ and a real $\varphi \in (-\pi/2, \ \pi/2)/\{0\}$. The cone axis passes through the point $P*$ *and has the direction* $*D_Z$. The plane which passes through the point $P*$ *and is parallel to directions* $*D_X*$ *and* $*D_{Y*}$ *is the cone referenced plane. The cone section by the plane is a circle with the radius* $*r$. The direction from the point $*P*$ to the cone apex is $-sgn(\varphi) \cdot D_Z$. The cone has a half-angle $|\varphi|$ and is defined by the following parametric equation:

$$S(u,v) = P + (r + v \cdot sin(\varphi)) \cdot (cos(u) \cdot D_X + sin(u) \cdot D_Y) + v \cdot cos(\varphi) \cdot D_Z, (u,v) \in [0, \ 2 \cdot \pi) \times (-\infty, \ \infty).$$

The example record is interpreted as a cone with an axis which passes through a point *P*=(1, 2, 3) and has a direction D_Z=(0, 0, 1). Other cone data are D_X=(1, 0, 0), D_Y=(0, 1, 0), *r*=4 and $\varphi = 0.75$. The cone is defined by the following parametric equation:

$$S(u,v) = (1,2,3) + (4 + v \cdot sin(0.75)) \cdot (cos(u) \cdot (1,0,0) + sin(u) \cdot (0,1,0)) + v \cdot cos(0.75) \cdot (0,0,1).$$

4.2.4 Sphere - < surface record 4 >

Example

```
4 1 2 3 0 0 1 1 0 -0 -0 1 0 4
```

BNF-like Definition

```
<surface record 4> = "4" <_> <3D point> (<_> <3D direction>) ^ 3 <_> <real> <_\n>;
```

Description

<surface record 4> describes a sphere. The sphere data consist of a 3D point *P*, pairwise orthogonal 3D directions D_Z, D_X and D_Y* *and a non-negative real* $*r$. The sphere has the center *P*, radius $*r*$ and is defined by the following parametric equation:

$$S(u,v) = P + r \cdot cos(v) \cdot (cos(u) \cdot D_x + sin(u) \cdot D_y) + r \cdot sin(v) \cdot D_Z, \ (u,v) \in [0, \ 2 \cdot \pi) \times [-\pi/2, \ \pi/2].$$

The example record is interpreted as a sphere with its center *P*=(1, 2, 3). Directions for the sphere are D_Z=(0, 0, 1), D_X=(1, 0, 0) and D_Y=(0, 1, 0). The sphere has a radius *r*=4 and is defined by the following parametric equation:

$$S(u,v) = (1,2,3) + 4 \cdot cos(v) \cdot (cos(u) \cdot (1,0,0) + sin(u) \cdot (0,1,0)) + 4 \cdot sin(v) \cdot (0,0,1).$$

4.2.5 Torus - < surface record 5 >

Example

```
5 1 2 3 0 0 1 1 0 -0 -0 1 0 8  4
```

BNF-like Definition

```
<surface record 5> = "5" <_> <3D point> (<_> <3D direction>) ^ 3 (<_> <real>) ^ 2 <_\n>;
```

Description

<surface record 5> describes a torus. The torus data consist of a 3D point *P*, pairwise orthogonal 3D directions D_Z, D_X and D_Y* *and non-negative reals* $*r_1*$ *and* $*r_2$. The torus axis passes through the point $P*$ *and has the direction* $*D_Z$. r_1* *is the distance from the torus circle center to the axis. The torus circle has the radius* $*r_2$. The torus is defined by the following parametric equation:

$$S(u,v) = P + (r_1 + r_2 \cdot cos(v)) \cdot (cos(u) \cdot D_x + sin(u) \cdot D_y) + r_2 \cdot sin(v) \cdot D_Z, \ (u,v) \in [0, \ 2 \cdot \pi) \times [0, \ 2 \cdot \pi).$$

The example record is interpreted as a torus with an axis which passes through a point *P*=(1, 2, 3) and has a direction D_Z=(0, 0, 1). D_X=(1, 0, 0), D_Y=(0, 1, 0), r_1=8 and r_2=4 for the torus. The torus is defined by the following parametric equation:

$$S(u,v) = (1,2,3) + (8 + 4 \cdot cos(v)) \cdot (cos(u) \cdot (1,0,0) + sin(u) \cdot (0,1,0)) + 4 \cdot sin(v) \cdot (0,0,1).$$

4.2.6 Linear Extrusion - < surface record 6 >

Example

```
6 0 0.6 0.8
2 1 2 3 0 0 1 1 0 -0 -0 1 0 4
```

BNF-like Definition

```
<surface record 6> = "6" <_> <3D direction> <_\n> <3D curve record>;
```

Description

<surface record 6> describes a linear extrusion surface. The surface data consist of a 3D direction D_v* *and a* <3D curve record>. *The linear extrusion surface has the direction* $*D_v$, the base curve $*C*$ described in the record and is defined by the following parametric equation:

$$S(u,v) = C(u) + v \cdot D_v, \ (u,v) \in domain(C) \times (-\infty, \infty).$$

The example record is interpreted as a linear extrusion surface with a direction D_v=(0, 0.6, 0.8). The base curve is a circle for the surface. The surface is defined by the following parametric equation:

$$S(u,v) = (1,2,3) + 4 \cdot (cos(u) \cdot (1,0,0) + sin(u) \cdot (0,1,0)) + v \cdot (0,0.6,0.8), \ (u,v) \in [0, 2 \cdot \pi] \times (-\infty, \infty).$$

4.2.7 Revolution Surface - $<$ surface record 7 $>$

Example

```
7 -4 0 3 0 1 0
2 1 2 3 0 0 1 1 0 -0 -0 1 0 4
```

BNF-like Definition

```
<surface record 7> = "7" <_> <3D point> <_> <3D direction> <_\n> <3D curve record>;
```

Description

$<$surface record 7$>$ describes a revolution surface. The surface data consist of a 3D point *P*, a 3D direction *D* and a $<$3D curve record$>$. The surface axis passes through the point *P* and has the direction *D*. The base curve *C* described by the record and the axis are coplanar. The surface is defined by the following parametric equation:

$$S(u,v) = P + V_D(v) + cos(u) \cdot (V(v) - V_D(v)) + sin(u) \cdot [D, V(v)], \ (u,v) \in [0, 2 \cdot \pi] \times domain(C)$$

where $V(v) = C(v) - P, V_D(v) = (D, V(v)) \cdot D$.

The example record is interpreted as a revolution surface with an axis which passes through a point *P*=(-4, 0, 3) and has a direction *D*=(0, 1, 0). The base curve is a circle for the surface. The surface is defined by the following parametric equation:

$$S(u,v) = (-4,0,3) + V_D(v) + cos(u) \cdot (V(v) - V_D(v)) + sin(u) \cdot [(0,1,0), V(v)], \ (u,v) \in [0, 2 \cdot \pi] \times [0, 2 \cdot \pi]$$

where $V(v) = (5,2,0) + 4 \cdot (cos(v) \cdot (1,0,0) + sin(v) \cdot (0,1,0)), V_D(v) = ((0,1,0), V(v)) \cdot (0,1,0)$.

4.2.8 Bezier Surface - $<$ surface record 8 $>$

Example

```
8 1 1 2 1 0 0 1  7 1 0 -4  10
0 1 -2  8 1 1 5  11
0 2 3  9 1 2 6  12
```

BNF-like Definition

```
<surface record 8> = "8" <_> <Bezier surface u rational flag> <_> <Bezier surface v rational flag> <_>
     <Bezier surface u degree> <_> <Bezier surface v degree> <_>
<Bezier surface weight poles>;

<Bezier surface u rational flag> = <flag>;

<Bezier surface v rational flag> = <flag>;
```

```
<Bezier surface u degree> = <int>;

<Bezier surface v degree> = <int>;

<Bezier surface weight poles> =
(<Bezier surface weight pole group> <_\n>) ^ (<Bezier surface u degree> <+> "1");

<Bezier surface weight pole group> = <Bezier surface weight pole>
(<_> <Bezier surface weight pole>) ^ <Bezier surface v degree>;

<Bezier surface weight pole> = <3D point> [<_> <real>];
```

Description

$<$surface record 8$>$ describes a Bezier surface. The surface data consist of a u rational flag r_u, v rational flag r_v, u degree $m_u \leq 25$, v degree $m_v \leq 25$ and weight poles.

The weight poles are $(m_u + 1) \cdot (m_v + 1)$ 3D points $B_{i,j}$ $((i,j) \in \{0,...,m_u\} \times \{0,...,m_v\})$ if $r_u + r_v = 0$. The weight poles are $(m_u + 1) \cdot (m_v + 1)$ pairs $B_{i,j} h_{i,j}$ $((i,j) \in \{0,...,m_u\} \times \{0,...,m_v\})$ if $r_u + r_v \neq 0$. Here $B_{i,j}$ is a 3D point and $h_{i,j}$ is a positive real $((i,j) \in \{0,...,m_u\} \times \{0,...,m_v\})$. $h_{i,j} = 1$ $((i,j) \in \{0,...,m_u\} \times \{0,...,m_v\})$ if $r_u + r_v = 0$.

The Bezier surface is defined by the following parametric equation:

$$S(u,v) = \frac{\sum_{i=0}^{m_u}\sum_{j=0}^{m_v} B_{i,j} \cdot h_{i,j} \cdot C_{m_u}^i \cdot u^i \cdot (1-u)^{m_u-i} \cdot C_{m_v}^j \cdot v^j \cdot (1-v)^{m_v-j}}{\sum_{i=0}^{m_u}\sum_{j=0}^{m_v} h_{i,j} \cdot C_{m_u}^i \cdot u^i \cdot (1-u)^{m_u-i} \cdot C_{m_v}^j \cdot v^j \cdot (1-v)^{m_v-j}}, (u,v) \in [0,1] \times [0,1]$$

where $0^0 \equiv 1$.

The example record is interpreted as a Bezier surface with a u rational flag r_u=1, v rational flag r_v=1, u degree m_u=2, v degree m_v=1, weight poles $B_{0,0}$=(0, 0, 1), $h_{0,0}$=7, $B_{0,1}$=(1, 0, -4), $h_{0,1}$=10, $B_{1,0}$=(0, 1, -2), $h_{1,0}$=8, $B_{1,1}$=(1, 1, 5), $h_{1,1}$=11, $B_{2,0}$=(0, 2, 3), $h_{2,0}$=9 and $B_{2,1}$=(1, 2, 6), $h_{2,1}$=12. The surface is defined by the following parametric equation:

$$S(u,v) = [(0,0,1) \cdot 7 \cdot (1-u)^2 \cdot (1-v) + (1,0,-4) \cdot 10 \cdot (1-u)^2 \cdot v + (0,1,-2) \cdot 8 \cdot 2 \cdot u \cdot (1-u) \cdot (1-v) +$$
$$(1,1,5) \cdot 11 \cdot 2 \cdot u \cdot (1-u) \cdot v + (0,2,3) \cdot 9 \cdot u^2 \cdot (1-v) + (1,2,6) \cdot 12 \cdot u^2 \cdot v] \div [7 \cdot (1-u)^2 \cdot (1-v) +$$
$$10 \cdot (1-u)^2 \cdot v + 8 \cdot 2 \cdot u \cdot (1-u) \cdot (1-v) + 11 \cdot 2 \cdot u \cdot (1-u) \cdot v + 9 \cdot u^2 \cdot (1-v) + 12 \cdot u^2 \cdot v]$$

4.2.9 B-spline Surface - $<$ surface record 9 $>$

Example

```
9  1 1 0 0 1 1 3 2 5 4 0 0 1  7 1 0 -4  10
0  1 -2  8 1 1 5  11
0  2 3  9 1 2 6  12

0  1
0.25  1
0.5  1
0.75  1
1  1

0  1
0.3  1
0.7  1
1 1
```

BNF-like Definition

```
<surface record 9> = "9" <_> <B-spline surface u rational flag> <_>
<B-spline surface v rational flag> <_> "0" <_> "0" <_> <B-spline surface u degree> <_>
<B-spline surface v degree> <_> <B-spline surface u pole count> <_>
<B-spline surface v pole count> <_> <B-spline surface u multiplicity knot count> <_>
```

```
<B-spline surface v multiplicity knot count> <_> <B-spline surface weight poles> <_\n>
<B-spline surface u multiplicity knots> <_\n> <B-spline surface v multiplicity knots>;

<B-spline surface u rational flag> = <flag>;

<B-spline surface v rational flag> = <flag>;

<B-spline surface u degree> = <int>;

<B-spline surface v degree> = <int>;

<B-spline surface u pole count> = <int>;

<B-spline surface v pole count> = <int>;

<B-spline surface u multiplicity knot count> = <int>;

<B-spline surface v multiplicity knot count> = <int>;

<B-spline surface weight poles> =
(<B-spline surface weight pole group> <_\n>) ^ <B-spline surface u pole count>;

<B-spline surface weight pole group> =
(<B-spline surface weight pole> <_>) ^ <B-spline surface v pole count>;

<B-spline surface weight pole> = <3D point> [<_> <real>];

<B-spline surface u multiplicity knots> =
(<B-spline surface u multiplicity knot> <_\n>) ^ <B-spline surface u multiplicity knot count>;

<B-spline surface u multiplicity knot> = <real> <_> <int>;

<B-spline surface v multiplicity knots> =
(<B-spline surface v multiplicity knot> <_\n>) ^ <B-spline surface v multiplicity knot count>;

<B-spline surface v multiplicity knot> = <real> <_> <int>;
```

Description

<surface record 9> describes a B-spline surface. The surface data consist of a u rational flag r_u, v rational flag r_v, u degree $m_u \leq 25$, v degree $m_v \leq 25$, u pole count $n_u \geq 2$, v pole count $n_v \geq 2$, u multiplicity knot count k_u, v multiplicity knot count k_v, weight poles, u multiplicity knots, v multiplicity knots.

The weight poles are $n_u \cdot n_v$ 3D points $B_{i,j}$ $((i,j) \in \{1,...,n_u\} \times \{1,...,n_v\})$ if $r_u + r_v = 0$. The weight poles are $n_u \cdot n_v$ pairs $B_{i,j}h_{i,j}$ $((i,j) \in \{1,...,n_u\} \times \{1,...,n_v\})$ if $r_u + r_v \neq 0$. Here $B_{i,j}$ is a 3D point and $h_{i,j}$ is a positive real $((i,j) \in \{1,...,n_u\} \times \{1,...,n_v\})$. $h_{i,j} = 1$ $((i,j) \in \{1,...,n_u\} \times \{1,...,n_v\})$ if $r_u + r_v = 0$.

The u multiplicity knots are $*k_u*$ pairs $u_1 q_1 ... u_{k_u} q_{k_u}$. Here u_i is a knot with multiplicity $q_i \geq 1$ $(1 \leq i \leq k_u)$ so that

$$u_i < u_{i+1} \ (1 \leq i \leq k_u - 1), q_1 \leq m_u + 1, \ q_{k_u} \leq m_u + 1, \ q_i \leq m_u \ (2 \leq i \leq k_u - 1), \ \sum_{i=1}^{k_u} q_i = m_u + n_u + 1.$$

The v multiplicity knots are $*k_v*$ pairs $v_1 t_1 ... v_{k_v} t_{k_v}$. Here v_j is a knot with multiplicity $t_i \geq 1$ $(1 \leq i \leq k_v)$ so that

$$v_j < v_{j+1} \ (1 \leq j \leq k_v - 1), t_1 \leq m_v + 1, \ t_{k_v} \leq m_v + 1, \ t_j \leq m_v \ (2 \leq j \leq k_v - 1), \ \sum_{j=1}^{k_v} t_j = m_v + n_v + 1.$$

The B-spline surface is defined by the following parametric equation:

$$S(u,v) = \frac{\sum_{i=1}^{n_u} \sum_{j=1}^{n_v} B_{i,j} \cdot h_{i,j} \cdot N_{i,m_u+1}(u) \cdot M_{j,m_v+1}(v)}{\sum_{i=1}^{n_u} \sum_{j=1}^{n_v} h_{i,j} \cdot N_{i,m_u+1}(u) \cdot M_{j,m_v+1}(v)}, (u,v) \in [u_1, u_{k_u}] \times [v_1, v_{k_v}]$$

where functions $N_{i,j}*$ and $*M_{i,j}*$ have the following recursion definition by $*j$:

$$N_{i,1}(u) = \left\{ 1 \Leftarrow \bar{u}_i \leq u \leq \bar{u}_{i+1} 0 \Leftarrow u < \bar{u}_i \lor \bar{u}_{i+1} \leq u \right.,$$

$$N_{i,j}(u) = \frac{(u - \bar{u}_i) \cdot N_{i,j-1}(u)}{\bar{u}_{i+j-1} - \bar{u}_i} + \frac{(\bar{u}_{i+j} - u) \cdot N_{i+1,j-1}(u)}{\bar{u}_{i+j} - \bar{u}_{i+1}}, \ (2 \leq j \leq m_u + 1),$$

$$M_{i,1}(v) = \left\{ \begin{array}{l} 1 \Leftarrow \bar{v}_i \leq v \leq \bar{v}_{i+1} \\ 0 \Leftarrow v < \bar{v}_i \lor \bar{v}_{i+1} \leq v \end{array} \right.,$$

$$M_{i,j}(v) = \frac{(v - \bar{v}_i) \cdot M_{i,j-1}(v)}{\bar{v}_{i+j-1} - \bar{v}_i} + \frac{(\bar{v}_{i+j} - v) \cdot M_{i+1,j-1}(v)}{\bar{v}_{i+j} - \bar{v}_{i+1}}, \ (2 \leq j \leq m_v + 1);$$

where

$$\bar{u}_i = u_j \ \left(1 \leq j \leq k_u, \ \sum_{l=1}^{j-1} q_l \leq i \leq \sum_{l=1}^{j} q_l\right), \bar{v}_i = v_j \ \left(1 \leq j \leq k_v, \ \sum_{l=1}^{j-1} t_l \leq i \leq \sum_{l=1}^{j} t_l\right);$$

The example record is interpreted as a B-spline surface with a u rational flag r_u=1, v rational flag r_v=1, u degree m_u=1, v degree m_v=1, u pole count n_u=3, v pole count n_v=2, u multiplicity knot count k_u=5, v multiplicity knot count k_v=4, weight poles $B_{1,1}$=(0, 0, 1), $h_{1,1}$=7, $B_{1,2}$=(1, 0, -4), $h_{1,2}$=10, $B_{2,1}$=(0, 1, -2), $h_{2,1}$=8, $B_{2,2}$=(1, 1, 5), $h_{2,2}$=11, $B_{3,1}$=(0, 2, 3), $h_{3,1}$=9 and $B_{3,2}$=(1, 2, 6), $h_{3,2}$=12, u multiplicity knots u_1=0, q_1=1, u_2=0.25, q_2=1, u_3=0.5, q_3=1, u_4=0.75, q_4=1 and u_5=1, q_5=1, v multiplicity knots v_1=0, r_1=1, v_2=0.3, r_2=1, v_3=0.7, r_3=1 and v_4=1, r_4=1. The B-spline surface is defined by the following parametric equation:

$$S(u,v) = [(0,0,1) \cdot 7 \cdot N_{1,2}(u) \cdot M_{1,2}(v) + (1,0,-4) \cdot 10 \cdot N_{1,2}(u) \cdot M_{2,2}(v) +$$
$$(0,1,-2) \cdot 8 \cdot N_{2,2}(u) \cdot M_{1,2}(v) + (1,1,5) \cdot 11 \cdot N_{2,2}(u) \cdot M_{2,2}(v) +$$
$$(0,2,3) \cdot 9 \cdot N_{3,2}(u) \cdot M_{1,2}(v) + (1,2,6) \cdot 12 \cdot N_{3,2}(u) \cdot M_{2,2}(v)] \div$$
$$[7 \cdot N_{1,2}(u) \cdot M_{1,2}(v) + 10 \cdot N_{1,2}(u) \cdot M_{2,2}(v) + 8 \cdot N_{2,2}(u) \cdot M_{1,2}(v) +$$
$$11 \cdot N_{2,2}(u) \cdot M_{2,2}(v) + 9 \cdot N_{3,2}(u) \cdot M_{1,2}(v) + 12 \cdot N_{3,2}(u) \cdot M_{2,2}(v)]$$

4.2.10 Rectangular Trim Surface - < surface record 10 >

Example

```
10 -1 2 -3 4
1 1 2 3 0 0 1 1 0 -0 -0 1 0
```

BNF-like Definition

```
<surface record 10> = "10" <_> <trim surface u min> <_> <trim surface u max> <_>
<trim surface v min> <_> <trim surface v max> <_\n> <surface record>;

<trim surface u min> = <real>;

<trim surface u max> = <real>;

<trim surface v min> = <real>;

<trim surface v max> = <real>;
```

Description

<surface record 10> describes a rectangular trim surface. The surface data consist of reals u_{min}, u_{max}, $v_{min}*$ *and* $*v_{max}*$ *and a* <surface record> *so that* $*u_{min} < u_{max}*$ *and* $*v_{min} < v_{max}*$. The rectangular trim surface is a restriction of the base surface $*B*$ described in the record to the set $[u_{min}, u_{max}] \times [v_{min}, v_{max}] \subseteq domain(B)$. The rectangular trim surface is defined by the following parametric equation:

$$S(u,v) = B(u,v), \ (u,v) \in [u_{min}, u_{max}] \times [v_{min}, v_{max}].$$

The example record is interpreted as a rectangular trim surface to the set [-1, 2]x[-3, 4] for the base surface $B(u,v) = (1,2,3) + u \cdot (1,0,0) + v \cdot (0,1,0)$. The rectangular trim surface is defined by the following parametric equation: $B(u,v) = (1,2,3) + u \cdot (1,0,0) + v \cdot (0,1,0), \ (u,v) \in [-1,2] \times [-3,4]$.

4.2.11 Offset Surface - < surface record 11 >

Example

```
11 -2
1 1 2 3 0 0 1 1 0 -0 -0 1 0
```

BNF-like Definition

```
<surface record 11> = "11" <_> <surface record distance> <_\n> <surface record>;

<surface record distance> = <real>;
```

Description

<surface record 11> describes an offset surface. The offset surface data consist of a distance *d∗ and a <surface record>. The offset surface is the result of offsetting the base surface ∗B∗ described in the record to the distance ∗d∗ along the normal ∗N∗ of surface ∗B*. The offset surface is defined by the following parametric equation:

$$S(u,v) = B(u,v) + d \cdot N(u,v), \ (u,v) \in domain(B).N(u,v) = [S'_u(u,v), S'_v(u,v)]$$

if $[S'_u(u,v), S'_v(u,v)] \neq \vec{0}$.

The example record is interpreted as an offset surface with a distance *d*=-2 and base surface $B(u,v) = (1,2,3) + u \cdot (1,0,0) + v \cdot (0,1,0)$. The offset surface is defined by the following parametric equation: $S(u,v) = (1,2,3) + u \cdot (1,0,0) + v \cdot (0,1,0) - 2 \cdot (0,0,1)$.

4.3 2D curves

Example

```
Curve2ds 24
1 0 0 1 0
1 0 0 1 0
1 3 0 0 -1
1 0 0 0 1
1 0 -2 1 0
1 0 0 1 0
1 0 0 0 -1
1 0 0 0 1
1 0 0 1 0
1 0 1 1 0
1 3 0 0 -1
1 1 0 0 1
1 0 -2 1 0
1 0 1 1 0
1 0 0 0 -1
1 1 0 0 1
1 0 0 0 1
1 0 0 1 0
1 3 0 0 1
1 0 0 1 0
1 0 0 0 1
1 0 2 1 0
1 3 0 0 1
1 0 2 1 0
```

BNF-like Definition

```
<2D curves> = <2D curve header> <_\n> <2D curve records>;

<2D curve header> = "Curve2ds" <_> <2D curve count>;

<2D curve count> = <int>;
```

```
    <2D curve records> = <2D curve record> ^ <2D curve count>;

    <2D curve record> =
    <2D curve record 1> |
    <2D curve record 2> |
    <2D curve record 3> |
    <2D curve record 4> |
    <2D curve record 5> |
    <2D curve record 6> |
    <2D curve record 7> |
    <2D curve record 8> |
    <2D curve record 9>;
```

4.3.1 Line - <2D curve record 1>

Example

```
    1 3 0 0 -1
```

BNF-like Definition

```
    <2D curve record 1> = "1" <_> <2D point> <_> <2D direction> <_\n>;
```

Description

<2D curve record 1> describes a line. The line data consist of a 2D point $P*$ *and a 2D direction* $*D$. The line passes through the point P, has the direction $*D*$ and is defined by the following parametric equation:

$$C(u) = P + u \cdot D, \ u \in (-\infty, \ \infty).$$

The example record is interpreted as a line which passes through a point P=(3,0), has a direction D=(0,-1) and is defined by the following parametric equation: $C(u) = (3,0) + u \cdot (0,-1)$.

4.3.2 Circle - <2D curve record 2>

Example

```
    2 1 2 1 0 -0 1 3
```

BNF-like Definition

```
<2D curve record 2> = "2" <_> <2D circle center> <_> <2D circle Dx> <_> <2D circle Dy> <_> <2D circle
     radius> <_\n>;

<2D circle center> = <2D point>;

<2D circle Dx> = <2D direction>;

<2D circle Dy> = <2D direction>;

<2D circle radius> = <real>;
```

Description

<2D curve record 2> describes a circle. The circle data consist of a 2D point P, orthogonal 2D directions D_x* *and* $*D_y*$ *and a non-negative real* $*r$. The circle has a center P. The circle plane is parallel to directions D_x* and $*D_y$. The circle has a radius $*r*$ and is defined by the following parametric equation:

$$C(u) = P + r \cdot (cos(u) \cdot D_x + sin(u) \cdot D_y), \ u \in [0, \ 2 \cdot \pi).$$

The example record is interpreted as a circle which has a center P=(1,2). The circle plane is parallel to directions D_x=(1,0) and D_y=(0,1). The circle has a radius r=3 and is defined by the following parametric equation: $C(u) = (1,2) + 3 \cdot (cos(u) \cdot (1,0) + sin(u) \cdot (0,1))$.

4.3.3 Ellipse - <2D curve record 3>

Example

```
3 1 2 1 0 -0 1 4 3
```

BNF-like Definition

```
<2D curve record 3> = "3" <_> <2D ellipse center> <_> <2D ellipse Dmaj> <_>
<2D ellipse Dmin> <_> <2D ellipse Rmaj> <_> <2D ellipse Rmin> <_\n>;

<2D ellipse center> = <2D point>;

<2D ellipse Dmaj> = <2D direction>;

<2D ellipse Dmin> = <2D direction>;

<2D ellipse Rmaj> = <real>;

<2D ellipse Rmin> = <real>;
```

Description

<2D curve record 3> describes an ellipse. The ellipse data are 2D point P, orthogonal 2D directions $*D_{maj}*$ and $*D_{min}*$ and non-negative reals $*r_{maj}*$ and $*r_{min}*$ that $*r_{maj}* \leq r_{min}$. The ellipse has a center P, major and minor axis directions $D_{maj}*$ and $*D_{min}$, major and minor radii $*r_{maj}*$ and $*r_{min}*$ and is defined by the following parametric equation:

$$C(u) = P + r_{maj} \cdot cos(u) \cdot D_{maj} + r_{min} \cdot sin(u) \cdot D_{min}, \, u \in [0, \, 2 \cdot \pi).$$

The example record is interpreted as an ellipse which has a center P=(1,2), major and minor axis directions D_{maj}=(1,0) and D_{min}=(0,1), major and minor radii r_{maj}=4 and r_{min}=3 and is defined by the following parametric equation: $C(u) = (1,2) + 4 \cdot cos(u) \cdot (1,0) + 3 \cdot sin(u) \cdot (0,1)$.

4.3.4 Parabola - <2D curve record 4>

Example

```
4 1 2 1 0 -0 1 16
```

BNF-like Definition

```
<2D curve record 4> = "4" <_> <2D parabola origin> <_> <2D parabola Dx> <_>
<2D parabola Dy> <_> <2D parabola focal length> <_\n>;

<2D parabola origin> = <2D point>;

<2D parabola Dx> = <2D direction>;

<2D parabola Dy> = <2D direction>;

<2D parabola focal length> = <real>;
```

Description

<2D curve record 4> describes a parabola. The parabola data consist of a 2D point P, orthogonal 2D directions D_x* and $*D_y*$ and a non-negative real $*f$. The parabola coordinate system has its origin $P*$ and axis directions $*D_x$ and $*D_y$. The parabola has a focus length $*f*$ and is defined by the following parametric equation:

$$C(u) = P + \frac{u^2}{4 \cdot f} \cdot D_x + u \cdot D_y, \, u \in (-\infty, \, \infty) \Leftarrow f \neq 0; C(u) = P + u \cdot D_x, \, u \in (-\infty, \, \infty) \Leftarrow f = 0 \, (degenerated \, case).$$

The example record is interpreted as a parabola in plane which passes through a point P=(1,2) and is parallel to directions D_x=(1,0) and D_y=(0,1). The parabola has a focus length f=16 and is defined by the following parametric equation: $C(u) = (1,2) + \frac{u^2}{64} \cdot (1,0) + u \cdot (0,1)$.

4.3.5   Hyperbola - <2D curve record 5>

Example

5 1 2 1 0 -0 1 3 4

BNF-like Definition

```
<2D curve record 5> = "5" <_> <2D hyperbola origin> <_> <2D hyperbola Dx> <_>
<2D hyperbola Dy> <_> <2D hyperbola Kx> <_> <2D hyperbola Ky> <_\n>;

<2D hyperbola origin> = <2D point>;

<2D hyperbola Dx> = <2D direction>;

<2D hyperbola Dy> = <2D direction>;

<2D hyperbola Kx> = <real>;

<2D hyperbola Ky> = <real>;
```

Description

<2D curve record 5> describes a hyperbola. The hyperbola data consist of a 2D point P, orthogonal 2D directions D_x* and $*D_y*$ and non-negative reals $*k_x*$ and $*k_y$. The hyperbola coordinate system has origin $P*$ and axis directions $*D_x*$ and $*D_y$. The hyperbola is defined by the following parametric equation:

$$C(u) = P + k_x \cdot cosh(u)D_x + k_y \cdot sinh(u) \cdot D_y, \ u \in (-\infty, \infty).$$

The example record is interpreted as a hyperbola with coordinate system which has origin P=(1,2) and axis directions D_x=(1,0) and D_y=(0,1). Other data for the hyperbola are k_x=5 and k_y=4. The hyperbola is defined by the following parametric equation: $C(u) = (1,2) + 3 \cdot cosh(u) \cdot (1,0) + 4 \cdot sinh(u) \cdot (0,1)$.

4.3.6   Bezier Curve - <2D curve record 6>

Example

6 1 2 0 1 4 1 -2 5 2 3 6

BNF-like Definition

```
<2D curve record 6> = "6" <_> <2D Bezier rational flag> <_> <2D Bezier degree>
<2D Bezier weight poles> <_\n>;

<2D Bezier rational flag> = <flag>;

<2D Bezier degree> = <int>;

<2D Bezier weight poles> = (<_> <2D Bezier weight pole>) ^ (<2D Bezier degree> <+> "1");

<2D Bezier weight pole> = <2D point> [<_> <real>];
```

Description

<2D curve record 6> describes a Bezier curve. The curve data consist of a rational flag r, a degree $m \leq 25$ and weight poles.

The weight poles are *m*+1 2D points B_0 ... B_m* if the flag *r* is 0. The weight poles are *m*+1 pairs *B_0h_0 ... B_mh_m* if the flag *r* is 1. Here *B_i* is a 2D point and *h_i* is a positive real $(0 \le i \le m)$. h_i=1 $(0 \le i \le m)$ if the flag *r* is 0.

The Bezier curve is defined by the following parametric equation:

$$C(u) = \frac{\sum_{i=0}^{m} B_i \cdot h_i \cdot C_m^i \cdot u^i \cdot (1-u)^{m-i}}{\sum_{i=0}^{m} h_i \cdot C_m^i \cdot u^i \cdot (1-u)^{m-i}}, \, u \in [0,1]$$

where $0^0 \equiv 1$.

The example record is interpreted as a Bezier curve with a rational flag *r*=1, a degree *m*=2 and weight poles B_0=(0,1), h_0=4, B_1=(1,-2), h_1=5 and B_2=(2,3), h_2=6. The Bezier curve is defined by the following parametric equation:

$$C(u) = \frac{(0,1) \cdot 4 \cdot (1-u)^2 + (1,-2) \cdot 5 \cdot 2 \cdot u \cdot (1-u) + (2,3) \cdot 6 \cdot u^2}{4 \cdot (1-u)^2 + 5 \cdot 2 \cdot u \cdot (1-u) + 6 \cdot u^2}.$$

4.3.7   B-spline Curve - <2D curve record 7>

Example

```
7 1 0  1 3 5  0 1  4 1 -2  5  2 3  6
 0 1 0.25 1 0.5 1 0.75 1 1 1
```

BNF-like Definition

```
<2D curve record 7> = "7" <_> <2D B-spline rational flag> <_> "0" <_> <2D B-spline degree> <_> <2D B-spline
    pole count> <_> <2D B-spline multiplicity knot count> <2D B-spline weight poles> <_\n> <2D B-spline
    multiplicity knots> <_\n>;

<2D B-spline rational flag> = <flag>;

<2D B-spline degree> = <int>;

<2D B-spline pole count> = <int>;

<2D B-spline multiplicity knot count> = <int>;

<2D B-spline weight poles> = <2D B-spline weight pole> ^ <2D B-spline pole count>;

<2D B-spline weight pole> = <_> <2D point> [<_> <real>];

<2D B-spline multiplicity knots> =
<2D B-spline multiplicity knot> ^ <2D B-spline multiplicity knot count>;

<2D B-spline multiplicity knot> = <_> <real> <_> <int>;
```

Description

<2D curve record 7> describes a B-spline curve. The curve data consist of a rational flag *r*, a degree $m \le 25$, a pole count $n \ge 2$, a multiplicity knot count *k*, weight poles and multiplicity knots.

The weight poles are *n* 2D points *B_1 ... B_n* if the flag *r* is 0. The weight poles are *n* pairs *B_1h_1 ... B_nh_n* if the flag *r* is 1. Here *B_i* is a 2D point and *h_i* is a positive real $(1 \le i \le n)$. h_i=1 $(1 \le i \le n)$ if the flag *r* is 0.

The multiplicity knots are *k* *pairs* *u_1q_1 ... u_kq_k*. Here *u_i* is a knot with multiplicity $q_i \ge 1$ $(1 \le i \le k)$ so that

$$u_i < u_{i+1} \, (1 \le i \le k-1), q_1 \le m+1, \, q_k \le m+1, \, q_i \le m \, (2 \le i \le k-1), \, \sum_{i=1}^{k} q_i = m+n+1.$$

The B-spline curve is defined by the following parametric equation:

$$C(u) = \frac{\sum_{i=1}^{n} B_i \cdot h_i \cdot N_{i,m+1}(u)}{\sum_{i=1}^{n} h_i \cdot N_{i,m+1}(u)}, \, u \in [u_1, \, u_k]$$

where functions $N_{i,j}*$ have the following recursion definition by $*j$

$$N_{i,1}(u) = \begin{cases} 1 \Leftarrow \bar{u}_i \leq u \leq \bar{u}_{i+1} \\ 0 \Leftarrow u < \bar{u}_i \vee \bar{u}_{i+1} \leq u \end{cases}, \quad N_{i,j}(u) = \frac{(u - \bar{u}_i) \cdot N_{i,j-1}(u)}{\bar{u}_{i+j-1} - \bar{u}_i} + \frac{(\bar{u}_{i+j} - u) \cdot N_{i+1,j-1}(u)}{\bar{u}_{i+j} - \bar{u}_{i+1}}, \quad (2 \leq j \leq m+1)$$

where

$$\bar{u}_i = u_j \left(1 \leq j \leq k, \sum_{l=1}^{j-1} q_l + 1 \leq i \leq \sum_{l=1}^{j} q_l\right).$$

The example record is interpreted as a B-spline curve with a rational flag r=1, a degree m=1, a pole count n=3, a multiplicity knot count k=5, weight poles B_1=(0,1), h_1=4, B_2=(1,-2), h_2=5 and B_3=(2,3), h_3=6 and multiplicity knots u_1=0, q_1=1, u_2=0.25, q_2=1, u_3=0.5, q_3=1, u_4=0.75, q_4=1 and u_5=1, q_5=1. The B-spline curve is defined by the following parametric equation:

$$C(u) = \frac{(0,1) \cdot 4 \cdot N_{1,2}(u) + (1,-2) \cdot 5 \cdot N_{2,2}(u) + (2,3) \cdot 6 \cdot N_{3,2}(u)}{4 \cdot N_{1,2}(u) + 5 \cdot N_{2,2}(u) + 6 \cdot N_{3,2}(u)}.$$

4.3.8   Trimmed Curve - <2D curve record 8>

Example

```
8 -4 5
1 1 2 1 0
```

BNF-like Definition

```
<2D curve record 8> = "8" <_> <2D trimmed curve u min> <_> <2D trimmed curve u max> <_\n>
<2D curve record>;

<2D trimmed curve u min> = <real>;

<2D trimmed curve u max> = <real>;
```

Description

<2D curve record 8> describes a trimmed curve. The trimmed curve data consist of reals $u_{min}*$ and $*u_{max}*$ and a <2D curve record> so that $*u_{min}* < *u_{max}$. The trimmed curve is a restriction of the base curve $*B*$ described in the record to the segment $[u_{min}, u_{max}] \subseteq domain(B)$. The trimmed curve is defined by the following parametric equation:

$$C(u) = B(u), \ u \in [u_{min}, u_{max}].$$

The example record is interpreted as a trimmed curve with u_{min}=-4, u_{max}=5 and base curve $B(u) = (1,2) + u \cdot (1,0)$. The trimmed curve is defined by the following parametric equation: $C(u) = (1,2) + u \cdot (1,0), \ u \in [-4,5]$.

4.3.9   Offset Curve - <2D curve record 9>

Example

```
9 2
1 1 2 1 0
```

BNF-like Definition

```
<2D curve record 9> = "9" <_> <2D offset curve distance> <_\n> <2D curve record>;

<2D offset curve distance> = <real>;
```

Description

<2D curve record 9> describes an offset curve. The offset curve data consist of a distance ∗d∗ and a <2D curve record>. The offset curve is the result of offsetting the base curve ∗B∗ described in the record to the distance ∗d∗ along the vector $(B'_Y(u), -B'_X(u)) \neq \vec{0}$ where $B(u) = (B'_X(u), B'_Y(u))$. The offset curve is defined by the following parametric equation:

$$C(u) = B(u) + d \cdot (B'_Y(u), -B'_X(u)), u \in domain(B).$$

The example record is interpreted as an offset curve with a distance d=2 and base curve $B(u) = (1,2) + u \cdot (1,0)$ and is defined by the following parametric equation: $C(u) = (1,2) + u \cdot (1,0) + 2 \cdot (0,-1)$.

4.4 3D polygons

Example

```
Polygon3D 1
2 1
0.1
1 0 0 2 0 0
0 1
```

BNF-like Definition

```
<3D polygons> = <3D polygon header> <_\n> <3D polygon records>;

<3D polygon header> = "Polygon3D" <_> <3D polygon record count>;

<3D polygon records> = <3D polygon record> ^ <3D polygon record count>;

<3D polygon record> =
<3D polygon node count> <_> <3D polygon flag of parameter presence> <_\n>
<3D polygon deflection> <_\n>
<3D polygon nodes> <_\n>
[<3D polygon parameters> <_\n>];

<3D polygon node count> = <int>;

<3D polygon flag of parameter presence> = <flag>;

<3D polygon deflection> = <real>;

<3D polygon nodes> = (<3D polygon node> <_>) ^ <3D polygon node count>;

<3D polygon node> = <3D point>;

<3D polygon u parameters> = (<3D polygon u parameter> <_>) ^ <3D polygon node count>;

<3D polygon u parameter> = <real>;
```

Description

<3D polygons> record describes a 3D polyline *L∗ which approximates a 3D curve ∗C*. The polyline data consist of a node count $m \geq 2$, a parameter presence flag *p*, a deflection $d \geq 0$, nodes N_i $(1 \leq i \leq m)$ and parameters u_i $(1 \leq i \leq m)$. The parameters are present only if *p*=1. The polyline *L∗ passes through the nodes. The deflection ∗d∗ describes the deflection of polyline ∗L∗ from the curve ∗C*:

$$\max_{P \in C} \min_{Q \in L} |Q - P| \leq d.$$

The parameter u_i $(1 \le i \le m)$ is the parameter of the node N_i* *on the curve* $*C$:

$$C(u_i) = N_i.$$

The example record describes a polyline from m=2 nodes with a parameter presence flag p=1, a deflection d=0.1, nodes N_1=(1,0,0) and N_2=(2,0,0) and parameters u_1=0 and u_2=1.

4.5 Triangulations

Example

```
Triangulations 6
4 2 1 0
0 0 0 0 0 3 0 2 3 0 2 0 0 0 3  0 3 -2 0 -2 2 4 3 2 1 4
4 2 1 0
0 0 0 1 0 0 1 0 3 0 0 3 0 0 0  1 3 1 3 0 3 2 1 3 1 4
4 2 1 0
0 0 3 0 2 3 1 2 3 1 0 3 0 0 0  2 1 2 1 0 3 2 1 3 1 4
4 2 1 0
0 2 0 1 2 0 1 2 3 0 2 3 0 0 0  1 3 1 3 0 3 2 1 3 1 4
4 2 1 0
0 0 0 0 2 0 1 2 0 1 0 0 0 0 0  2 1 2 1 0 3 2 1 3 1 4
4 2 1 0
1 0 0 1 0 3 1 2 3 1 2 0 0 0 3  0 3 -2 0 -2 2 4 3 2 1 4
```

BNF-like Definition

```
<triangulations> = <triangulation header> <_\n> <triangulation records>;

<triangulation header> = "Triangulations" <_> <triangulation count>;

<triangulation records> = <triangulation record> ^ <triangulation count>;

<triangulation record> = <triangulation node count> <_> <triangulation triangle count> <_> <triangulation
      parameter presence flag> <_> <triangulation deflection> <_\n>
<triangulation nodes> [<_> <triangulation u v parameters>] <_> <triangulation triangles> <_\n>;

<triangulation node count> = <int>;

<triangulation triangle count> = <int>;

<triangulation parameter presence flag> = <flag>;

<triangulation deflection> = <real>;

<triangulation nodes> = (<triangulation node> <_>) ^ <triangulation node count>;

<triangulation node> = <3D point>;

<triangulation u v parameters> =
(<triangulation u v parameter pair> <_>) ^ <triangulation node count>;

<triangulation u v parameter pair> = <real> <_> <real>;

<triangulation triangles> = (<triangulation triangle> <_>) ^ <triangulation triangle count>;

<triangulation triangle> = <int> <_> <int> <_> <int>.
```

Description

$<$triangulation record$>$ describes a triangulation $T*$ *which approximates a surface* $*S$. The triangulation data consist of a node count $m \ge 3$, a triangle count $k \ge 1$, a parameter presence flag p, a deflection $d \ge 0$, nodes N_i $(1 \le i \le m)$, parameter pairs u_i v_i $(1 \le i \le m)$, triangles $n_{j,1}$ $n_{j,2}$ $n_{j,3}$ $(1 \le j \le k, n_{j,l} \in \{1,...,m\}$ $(1 \le l \le 3))$. The parameters are present only if p=1. The deflection describes the triangulation deflection from the surface:

$$\max_{P \in S} \min_{Q \in T} |Q - P| \le d.$$

The parameter pair u_i v_i $(1 \le i \le m)$ describes the parameters of node $*N_i*$ on the surface:

$$S(u_i, v_i) = N_i.$$

The triangle $n_{j,1}\ n_{j,2}\ n_{j,3}$ $(1 \le j \le k)$ is interpreted as a triangle of nodes $N_{n_{j,1}}\ N_{n_{j,2}}$ and $N_{n_{j,3}}$ with circular traversal of the nodes in the order $N_{n_{j,1}}\ N_{n_{j,2}}$ and $N_{n_{j,3}}$. From any side of the triangulation $*T*$ all its triangles have the same direction of the node circular traversal: either clockwise or counterclockwise.

Triangulation record

```
4 2 1 0
0 0 0 0 0 3 0 2 3 0 2 0 0 0 3  0 3 -2 0 -2 2 4 3 2 1 4
```

describes a triangulation with m=4 nodes, k=2 triangles, parameter presence flag p=1, deflection d=0, nodes N_1=(0,0,0), N_2=(0,0,3), N_3=(0,2,3) and N_4=(0,2,0), parameters (u_1, v_1)=(0,0), (u_2, v_2)=(3,0), (u_3, v_3)=(3,-2) and (u_4, v_4)=(0,-2), and triangles $(n_{1,1}, n_{1,2}, n_{1,3})$=(2,4,3) and $(n_{2,1}, n_{2,2}, n_{2,3})$=(2,1,4). From the point (1,0,0) ((-1,0,0)) the triangles have clockwise (counterclockwise) direction of the node circular traversal.

4.6 Polygons on triangulations

Example

```
PolygonOnTriangulations 24
2 1 2
p 0.1 1 0 3
2 1 4
p 0.1 1 0 3
2 2 3
p 0.1 1 0 2
2 1 2
p 0.1 1 0 2
2 4 3
p 0.1 1 0 3
2 1 4
p 0.1 1 0 3
2 1 4
p 0.1 1 0 2
2 1 2
p 0.1 1 0 2
2 1 2
p 0.1 1 0 3
2 2 3
p 0.1 1 0 3
2 2 3
p 0.1 1 0 2
2 4 3
p 0.1 1 0 2
2 4 3
p 0.1 1 0 3
2 2 3
p 0.1 1 0 3
2 1 4
p 0.1 1 0 2
2 4 3
p 0.1 1 0 2
2 1 2
p 0.1 1 0 1
2 1 4
p 0.1 1 0 1
2 4 3
p 0.1 1 0 1
2 1 4
p 0.1 1 0 1
2 1 2
p 0.1 1 0 1
2 2 3
p 0.1 1 0 1
2 4 3
```

```
p 0.1 1 0 1
2 2 3
p 0.1 1 0 1
```

BNF-like Definition

```
<polygons on triangulations> = <polygons on triangulations header> <_\n>
<polygons on triangulations records>;

<polygons on triangulations header> =
"PolygonOnTriangulations" <_> <polygons on triangulations record count>;

<polygons on triangulations record count> = <int>;

<polygons on triangulations records> =
<polygons on triangulations record> ^ <polygons on triangulations record count>;

<polygons on triangulations record> =
<polygons on triangulations node count> <_> <polygons on triangulations node numbers> <_\n>
"p" <_> <polygons on triangulations deflection> <_>
<polygons on triangulations parameter presence flag>
[<_> <polygons on triangulations u parameters>] <_\n>;

<polygons on triangulations node count> = <int>;

<polygons on triangulations node numbers> =
<polygons on triangulations node number> ^ <polygons on triangulations node count>;

<polygons on triangulations node number> = <int>;

<polygons on triangulations deflection> = <real>;

<polygons on triangulations parameter presence flag> = <flag>;

<polygons on triangulations u parameters> =
(<polygons on triangulations u parameter> <_>) ^ <polygons on triangulations node count>;

<polygons on triangulations u parameter> = <real>;
```

Description

<polygons on triangulations> describes a polyline *L∗ on a triangulation which approximates a curve ∗C*. The polyline data consist of a node count $m \geq 2$, node numbers $n_i \geq 1$, deflection $d \geq 0$, a parameter presence flag ∗p∗ and parameters u_i $(1 \leq i \leq m)$. The parameters are present only if *p*=1. The deflection *d∗ describes the deflection of polyline ∗L∗ from the curve ∗C*:

$$\max_{P \in C} \min_{Q \in L} |Q - P| \leq d.$$

Parameter u_i $(1 \leq i \leq m)$ is n_i-th node *C(u_i)∗ parameter on curve ∗C*.

4.7 Geometric Sense of a Curve

Geometric sense of curve ∗C∗ described above is determined by the direction of parameter ∗u∗ increasing.

5 Shapes

An example of section shapes and a whole ∗.brep file are given in chapter 7 Appendix.

BNF-like Definition

```
<shapes> = <shape header> <_\n> <shape records> <_\n> <shape final record>;

<shape header> = "TShapes" <_> <shape count>;

<shape count> = <int>;

<shape records> = <shape record> ^ <shape count>;

<shape record> = <shape subrecord> <_\n> <shape flag word> <_\n> <shape subshapes> <_\n>;

<shape flag word> = <flag> ^ 7;

<shape subshapes> = (<shape subshape> <_>)* "*";

<shape subshape> =
<shape subshape orientation> <shape subshape number> <_> <shape location number>;

<shape subshape orientation> = "+" | "-" | "i" | "e";

<shape subshape number> = <int>;

<shape location number> = <int>;

<shape final record> = <shape subshape>;

<shape subrecord> =
("Ve" <_\n> <vertex data> <_\n>) |
("Ed" <_\n> <edge data> <_\n>) |
("Wi" <_\n> <_\n>) |
("Fa" <_\n> <face data>) |
("Sh" <_\n> <_\n>) |
("So" <_\n> <_\n>) |
("CS" <_\n> <_\n>) |
("Co" <_\n> <_\n>);
```

Description

<shape flag word> f_1 f_2 f_3 f_4 f_5 f_6 f_7 <flag>s f_i $(1 \leq i \leq 7)$ are interpreted as shape flags in the following way:

- f_1 – free;

- f_2 – modified;

- f_3 – IGNORED(version 1) \ checked (version 2);

- f_4 – orientable;

- f_5 – closed;

- f_6 – infinite;

- f_7 – convex.

The flags are used in a special way [1].

<shape subshape orientation> is interpreted in the following way:

- + – forward;

- - – reversed;

- i – internal;

   • e – external.

<shape subshape orientation> is used in a special way [1].

<shape subshape number> is the number of a <shape record> which is located in this section above the <shape subshape number>. <shape record> numbering is backward and starts from 1.

<shape subrecord> types are interpreted in the following way:

   • "Ve" – vertex;

   • "Ed" – edge;

   • "Wi" – wire;

   • "Fa" – face;

   • "Sh" – shell;

   • "So" – solid;

   • "CS" – compsolid;

   • "Co" – compound.

<shape final record> determines the orientation and location for the whole model.

5.1   Common Terms

The terms below are used by <vertex data>, <edge data> and <face data>.

BNF-like Definition

```
<location number> = <int>;

<3D curve number> = <int>;

<surface number> = <int>;

<2D curve number> = <int>;

<3D polygon number> = <int>;

<triangulation number> = <int>;

<polygon on triangulation number> = <int>;

<curve parameter minimal and maximal values> = <real> <_> <real>;

<curve values for parameter minimal and maximal values> =
real> <_> <real> <_> <real> <_> <real>;
```


Description

<location number> is the number of <location record> from section locations. <location record> numbering starts from 1. <location number> 0 is interpreted as the identity location.

<3D curve number> is the number of a <3D curve record> from subsection <3D curves> of section <geometry>. <3D curve record> numbering starts from 1.

<surface number> is the number of a <surface record> from subsection <surfaces> of section <geometry>. <surface record> numbering starts from 1.

<2D curve number> is the number of a <2D curve record> from subsection <2D curves> of section <geometry>. <2D curve record> numbering starts from 1.

<3D polygon number> is the number of a <3D polygon record> from subsection <3D polygons> of section <geometry>. <3D polygon record> numbering starts from 1.

<triangulation number> is the number of a <triangulation record> from subsection <triangulations> of section <geometry>. <triangulation record> numbering starts from 1.

<polygon on triangulation> number is the number of a <polygons on triangulations record> from subsection <polygons on triangulations> of section <geometry>. <polygons on triangulations record> numbering starts from 1.

<curve parameter minimal and maximal values> $u_{min}*$ and $*u_{max}*$ are the curve parameter $*u$ bounds: $u_{min} \leq u \leq u_{max}$.

<curve values for parameter minimal and maximal values> $u_{min}*$ and $*u_{max}*$ are real pairs $*x_{min}\ y_{min}*$ and $*x_{max}\ y_{max}*$ that $(*x_{min}, y_{min})= C\ (u_{min})$ and $(x_{max}, y_{max})= C\ (u_{max})$ where $*C*$ is a parametric equation of the curve.

5.2 Vertex data

BNF-like Definition

```
<vertex data> = <vertex data tolerance> <_\n> <vertex data 3D representation> <_\n>
<vertex data representations>;

<vertex data tolerance> = <real>;

<vertex data 3D representation> = <3D point>;

<vertex data representations> = (<vertex data representation> <_\n>)* "0 0";

<vertex data representation> = <vertex data representation u parameter> <_>
<vertex data representation data> <_> <location number>;

<vertex data representation u parameter> = <real>;

<vertex data representation data> =
("1" <_> <vertex data representation data 1>) |
("2" <_> <vertex data representation data 2>) |
("3" <_> <vertex data representation data 3>);

<vertex data representation data 1> = <3D curve number>;

<vertex data representation data 2> = <2D curve number> <_> <surface number>;

<vertex data representation data 3> =
<vertex data representation v parameter> <_> <surface number>;

<vertex data representation v parameter> = <real>;
```

Description

The usage of <vertex data representation u parameter> $*U*$ is described below.

<vertex data representation data 1> and parameter $U*$ *describe the position of the vertex* $*V*$ *on a 3D curve* $*C$. Parameter $U*$ *is a parameter of the vertex* $*V*$ *on the curve* $*C$: $C(u)=V$.

<vertex data representation data 2> and parameter $U*$ *describe the position of the vertex* $*V*$ *on a 2D curve* $*C*$ *which is located on a surface. Parameter* $*U*$ *is a parameter of the vertex* $*V*$ *on the curve* $*C$: $C(u)=V$.

<vertex data representation data 3> and parameter $u*$ *describe the position of the vertex* $*V*$ *on a surface* $*\hookleftarrow$ $S*$ *through* <vertex data representation v parameter> $*v$: $S(u,v)=V$.

<vertex data tolerance> $t*$ *describes the maximum distance from the vertex* $*V*$ *to the set* $*R$ *of vertex* $*\hookleftarrow$ $V*$ *representations:*

$$\max_{P \in R}|P - V| \leq t.$$

5.3 Edge data

BNF-like Definition

```
<edge data> = <_> <edge data tolerance> <_> <edge data same parameter flag> <_> edge data same range flag>
        <_> <edge data degenerated flag> <_\n> <edge data representations>;

<edge data tolerance> = <real>;

<edge data same parameter flag> = <flag>;

<edge data same range flag> = <flag>;

<edge data degenerated flag> = <flag>;

<edge data representations> = (<edge data representation> <_\n>)* "0";

<edge data representation> =
"1" <_> <edge data representation data 1>
"2" <_> <edge data representation data 2>
"3" <_> <edge data representation data 3>
"4" <_> <edge data representation data 4>
"5" <_> <edge data representation data 5>
"6" <_> <edge data representation data 6>
"7" <_> <edge data representation data 7>;

<edge data representation data 1> = <3D curve number> <_> <location number> <_>
<curve parameter minimal and maximal values>;

<edge data representation data 2> = <2D curve number> <_> <surface number> <_>
<location number> <_> <curve parameter minimal and maximal values>
[<_\n> <curve values for parameter minimal and maximal values>];

<edge data representation data 3> = (<2D curve number> <_>) ^ 2 <continuity order> <_> <surface number> <_>
        <location number> <_> <curve parameter minimal and maximal values> <\n> <curve values for parameter minimal
        and maximal values>];

<continuity order> = "C0" | "C1" | "C2" | "C3" | "CN" | "G1" | "G2".

<edge data representation data 4> =
<continuity order> (<_> <surface number> <_> <location number>) ^ 2;

<edge data representation data 5> = <3D polygon number> <_> <location number>;

<edge data representation data 6> =
<polygon on triangulation number> <_> <triangulation number> <_> <location number>;

<edge data representation data 7> = (<polygon on triangulation number> <_>) ^ 2
<triangulation number> <_> <location number>;
```

Description

Flags <edge data same parameter flag>, <edge data same range flag> and <edge data degenerated flag> are used in a special way [1].

<edge data representation data 1> describes a 3D curve.

<edge data representation data 2> describes a 2D curve on a surface. <curve values for parameter minimal and maximal values> are used only in version 2.

<edge data representation data 3> describes a 2D curve on a closed surface. <curve values for parameter minimal and maximal values> are used only in version 2.

<edge data representation data 5> describes a 3D polyline.

<edge data representation data 6> describes a polyline on a triangulation.

<edge data tolerance> *t* *describes the maximum distance from the edge* *E* to the set *R* of edge *←E* representations:

$$\max_{C \in R} \max_{P \in E} \min_{Q \in C} |Q - P| \leq t$$

5.4 Face data

BNF-like Definition

```
<face data> = <face data natural restriction flag> <_> <face data tolerance> <_> <surface number> <_>
      <location number> <\n> ["2" <_> <triangulation number>];

<face data natural restriction flag> = <flag>;

<face data tolerance> = <real>;
```

Description

<face data> describes a surface *S∗ of face ∗F∗ and a triangulation ∗T∗ of face ∗F*. The surface *S∗* may be empty: <surface number> = 0.

<face data tolerance> *t∗ describes the maximum distance from the face ∗F∗ to the surface ∗S*:

$$\max_{P \in F} \min_{Q \in S} |Q - P| \leq t$$

Flag <face data natural restriction flag> is used in a special way [1].

6 Appendix

This chapter contains a ∗.brep file example.

```
DBRep_DrawableShape

CASCADE Topology V1, (c)  Matra-Datavision
Locations 3
1
            0               0               1               0
            1               0               0               0
            0               1               0               0
1
            1               0               0               4
            0               1               0               5
            0               0               1               6
2  1 1 2 1 0
Curve2ds 24
1 0 0 1 0
1 0 0 1 0
1 3 0 0 -1
1 0 0 0 1
1 0 -2 1 0
1 0 0 1 0
1 0 0 0 -1
1 0 0 0 1
1 0 0 1 0
1 0 1 1 0
1 3 0 0 -1
1 1 0 0 1
1 0 -2 1 0
1 0 1 1 0
1 0 0 0 -1
1 1 0 0 1
1 0 0 0 1
1 0 0 1 0
1 3 0 0 1
1 0 0 1 0
1 0 0 0 1
1 0 2 1 0
1 3 0 0 1
1 0 2 1 0
Curves 13
1 0 0 0 0 0 1
1 0 0 3 -0 1 0
1 0 2 0 0 0 1
1 0 0 0 -0 1 0
1 1 0 0 0 0 1
1 1 0 3 0 1 0
1 1 2 0 0 0 1
1 1 0 0 -0 1 0
1 0 0 0 1 0 -0
1 0 0 3 1 0 -0
1 0 2 0 1 0 -0
1 0 2 3 1 0 -0
1 1 0 0 1 0 0
Polygon3D 1
2 1
0.1
1 0 0 2 0 0
0 1
PolygonOnTriangulations 24
2 1 2
p 0.1 1 0 3
2 1 4
p 0.1 1 0 3
2 2 3
p 0.1 1 0 2
2 1 2
p 0.1 1 0 2
2 4 3
p 0.1 1 0 3
```

```
2 1 4
p 0.1 1 0 3
2 1 4
p 0.1 1 0 2
2 1 2
p 0.1 1 0 2
2 1 2
p 0.1 1 0 3
2 2 3
p 0.1 1 0 3
2 2 3
p 0.1 1 0 2
2 4 3
p 0.1 1 0 2
2 4 3
p 0.1 1 0 3
2 2 3
p 0.1 1 0 3
2 1 4
p 0.1 1 0 2
2 4 3
p 0.1 1 0 2
2 1 2
p 0.1 1 0 1
2 1 4
p 0.1 1 0 1
2 4 3
p 0.1 1 0 1
2 1 4
p 0.1 1 0 1
2 1 2
p 0.1 1 0 1
2 2 3
p 0.1 1 0 1
2 4 3
p 0.1 1 0 1
2 2 3
p 0.1 1 0 1
Surfaces 6
1 0 0 0 1 0 -0 0 0 1 0 -1 0
1 0 0 0 -0 1 0 0 0 1 1 0 -0
1 0 0 3 0 0 1 1 0 -0 -0 1 0
1 0 2 0 -0 1 0 0 0 1 1 0 -0
1 0 0 0 0 0 1 1 0 -0 -0 1 0
1 1 0 0 1 0 -0 0 0 1 0 -1 0
Triangulations 6
4 2 1 0
0 0 0 0 0 3 0 2 3 0 2 0 0 0 3  0 3 -2 0 -2 2 4 3 2 1 4
4 2 1 0
0 0 0 1 0 0 1 0 3 0 0 3 0 0 0  1 3 1 3 0 3 2 1 3 1 4
4 2 1 0
0 0 3 0 2 3 1 2 3 1 0 3 0 0 0  2 1 2 1 0 3 2 1 3 1 4
4 2 1 0
0 2 0 1 2 0 1 2 3 0 2 3 0 0 0  1 3 1 3 0 3 2 1 3 1 4
4 2 1 0
0 0 0 0 2 0 1 2 0 1 0 0 0 0 0  2 1 2 1 0 3 2 1 3 1 4
4 2 1 0
1 0 0 1 0 3 1 2 3 1 2 0 0 0 3  0 3 -2 0 -2 2 4 3 2 1 4

TShapes 39
Ve
1e-007
0 0 3
0 0

0101101
*
Ve
1e-007
0 0 0
0 0

0101101
```

```
*
Ed
 1e-007 1 1 0
1  1 0 0 3
2  1 1 0 0 3
2  2 2 0 0 3
6  1 1 0
6  2 2 0
0

0101000
-39 0 +38 0 *
Ve
1e-007
0 2 3
0 0

0101101
*
Ed
 1e-007 1 1 0
1  2 0 0 2
2  3 1 0 0 2
2  4 3 0 0 2
6  3 1 0
6  4 3 0
0

0101000
-36 0 +39 0 *
Ve
1e-007
0 2 0
0 0

0101101
*
Ed
 1e-007 1 1 0
1  3 0 0 3
2  5 1 0 0 3
2  6 4 0 0 3
6  5 1 0
6  6 4 0
0

0101000
-36 0 +34 0 *
Ed
 1e-007 1 1 0
1  4 0 0 2
2  7 1 0 0 2
2  8 5 0 0 2
6  7 1 0
6  8 5 0
0

0101000
-34 0 +38 0 *
Wi

0101000
-37 0 -35 0 +33 0 +32 0 *
Fa
0  1e-007 1 0
2  1
0101000
+31 0 *
Ve
1e-007
1 0 3
0 0
```

```
0101101
*
Ve
1e-007
1 0 0
0 0

0101101
*
Ed
 1e-007 1 1 0
1  5 0 0 3
2  9 6 0 0 3
2  10 2 0 0 3
6  9 6 0
6  10 2 0
0

0101000
-29 0 +28 0 *
Ve
1e-007
1 2 3
0 0

0101101
*
Ed
 1e-007 1 1 0
1  6 0 0 2
2  11 6 0 0 2
2  12 3 0 0 2
6  11 6 0
6  12 3 0
0

0101000
-26 0 +29 0 *
Ve
1e-007
1 2 0
0 0

0101101
*
Ed
 1e-007 1 1 0
1  7 0 0 3
2  13 6 0 0 3
2  14 4 0 0 3
6  13 6 0
6  14 4 0
0

0101000
-26 0 +24 0 *
Ed
 1e-007 1 1 0
1  8 0 0 2
2  15 6 0 0 2
2  16 5 0 0 2
6  15 6 0
6  16 5 0
0

0101000
-24 0 +28 0 *
Wi

0101000
-27 0 -25 0 +23 0 +22 0 *
Fa
0  1e-007 6 0
```

```
2  6
0101000
+21 0 *
Ed
 1e-007 1 1 0
1  9 0 0 1
2  17 2 0 0 1
2  18 5 0 0 1
6  17 2 0
6  18 5 0
0

0101000
-28 0 +38 0 *
Ed
 1e-007 1 1 0
1  10 0 0 1
2  19 2 0 0 1
2  20 3 0 0 1
6  19 2 0
6  20 3 0
0

0101000
-29 0 +39 0 *
Wi

0101000
-19 0 -27 0 +18 0 +37 0 *
Fa
0  1e-007 2 0
2  2
0101000
+17 0 *
Ed
 1e-007 1 1 0
1  11 0 0 1
2  21 4 0 0 1
2  22 5 0 0 1
6  21 4 0
6  22 5 0
0

0101000
-24 0 +34 0 *
Ed
 1e-007 1 1 0
1  12 0 0 1
2  23 4 0 0 1
2  24 3 0 0 1
6  23 4 0
6  24 3 0
0

0101000
-26 0 +36 0 *
Wi

0101000
-15 0 -23 0 +14 0 +33 0 *
Fa
0  1e-007 4 0
2  4
0101000
+13 0 *
Wi

0101000
-32 0 -15 0 +22 0 +19 0 *
Fa
0  1e-007 5 0
2  5
0101000
```

```
+11 0 *
Wi

0101000
-35 0 -14 0 +25 0 +18 0 *
Fa
0  1e-007 3 0
2   3
0101000
+9 0 *
Sh

0101100
-30 0 +20 0 -16 0 +12 0 -10 0  +8 0 *
So

0100000
+7 0 *
CS

0101000
+6 3 *
Ve
1e-007
1 0 0
0 0

0101101
*
Ve
1e-007
2 0 0
0 0

0101101
*
Ed
 1e-007 1 1 0
1  13 0 0 1
5  1 0
0

0101000
+4 0 -3 0 *
Co

1100000
+5 0 +2 0 *

+1 0
0
```