# Summary

In this course we have introduced the main ideas of classical thermodynamics, including the definition of temperature, the phenomena of thermal expansion and phase changes, the basics of kinetic theory of gases, the laws of thermodynamics and the analysis of heat engines. This section contains a brief summary of what you should know in order to do questions that you may be asked in an exam.

## What you need to know

Having completed this course, you should **know**:

* the laws of thermodynamics;
* the definition of **coefficient of thermal expansion** (linear and volume) and how to use it;
* the definitions of **heat capacity, specific heat capacity** and **molar heat capacity**;
* the definition of **latent heat**;
* the three ways in which heat can be transferred, and how they work;
* the definition of **thermal conductivity**;
* **Stefan’s law** of blackbody radiation;
* the definition of an **ideal gas**;
* what is meant by **equipartition of energy**, and how this relates to specific heat;
* what is meant by a **function of state**;
* the definitions of **isobaric, isothermal, isochoric** and **adiabatic** processes, and how to use them in calculations;
* what is meant by the **thermal efficiency** of a heat engine, and how to calculate it;
* the relationship between heat engines and heat pumps or refrigerators;
* the (thermodynamic) definition of **entropy**.

## What you should be able to do

You should **be able to**:

* do calculations involving thermal expansion;
* do calculations involving specific and latent heat;
* calculate heat transfer by conduction (using thermal conductivity) and radiation (using Ste­fan’s law);
* derive the ideal gas law;
* relate the molar heat capacity of a substance to the number of active degrees of freedom;
* do calculations involving heat engines, in particular the amount of work done and the thermal efficiency;
* calculate entropy changes in thermodynamic processes.

## What you do *not* need to know

You do not need to know the mathematical details of the Maxwell-Boltzmann distribution as worked out in section 3.5 and Appendix A.

You do not need to know details of the Stirling cycle in section 6.3.

You do not need to know details of two-phase heat engines like the Rankine cycle in section 6.5.

You do not need to be able to do calculations like the one in example 7.3.

# Appendix A

## from the Maxwell-Boltzmann equation

Using equation 3.8, we write

Writing for convenience, this becomes

To solve this, we need to know the **Gaussian integral**

This is given on the exams constant sheet, but let’s work it out for ourselves. It’s actually not difficult once you know the trick, but the trick is not at all obvious (it apparently originates with Poisson). We start by squaring the integral, which does not look like an improvement. However, since this is a definite integral, it does not have any dependence on the integration variable, so we can give that a different label in the two cases:

As 𝑥 and *y* are not dependent on each other, we can rewrite this as a double integral,

Now comes the clever trick. We can think of this double integral as being the integral of over the whole plane, *and we can do this integral using polar coordinates!* As there is no dependence on *θ*, take the element of area as being the thin ring with radius *r* and thickness d*r*, which has area . Then we have

Change variables to , , and we get

So, since this started out as the square of the integral we actually wanted, we conclude that

How does this help with equation A.1? The first thing to note is that is an **even function**, i.e. it doesn’t change if you exchange 𝑥 for –𝑥, so

The next step is to differentiate *with respect to a*:

So we can write equation A.1 as

But an integral is just a generalised sum, and we know that the derivative of a sum is the sum of the derivatives, *so we can swap round the differentiation and the integration[[1]](#footnote-1):*

We know from equation A.2 that . If we differentiate this twice with respect to *α*, we get

So finally we have

1. Note: this only works because the limits of the integral over 𝑥 are not functions of *a*. The method can still be used if the limits *are* functions of *a*, but it becomes much more complicated. [↑](#footnote-ref-1)