A Comparitive Analysis of Source Identification Algorithms

Pablo A. Curiel, Richard C. Tillquist

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Introduction

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- Applications of modeling spread using networks
  - diseases in a physical-contact network
  - electricity in a power grid network
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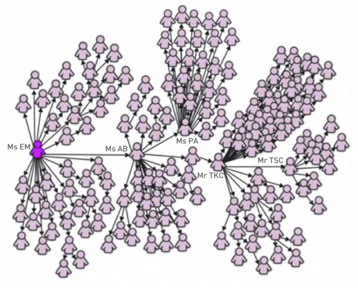

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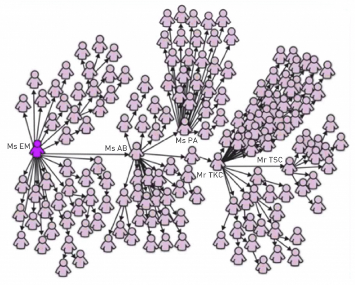

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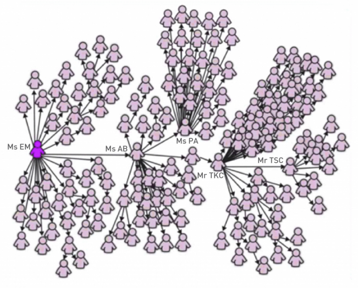
- Focus: comparing four state-of-the-art methods of identifying the source
  - Rumor centrality (2011)
  - Jordan centrality (2017)
  - NETSLEUTH (2014)
  - LISN (2019)



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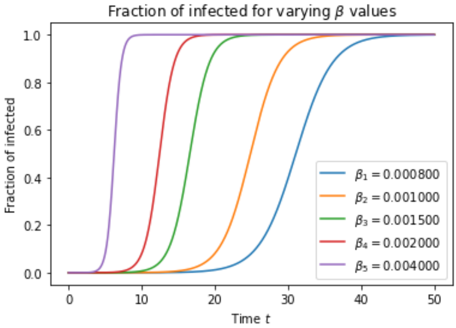
Classical Spread Models

- Susceptible-Infected (SI) model
  - Relies on two assumptions
    1. Compartmentalization
    2. Homogeneous mixing
  - Homogeneous mixing disregards the true contact network

$$\underbrace{\frac{dS}{dt}}_{Susceptible} = \underbrace{-\beta SI}_{\text{Susceptible become infected}}$$

$$\underbrace{\frac{dI}{dt}}_{Infected} = \underbrace{\beta SI}_{\text{Susceptible become infected}}$$

$\Longrightarrow$



Modeling Spread on Networks

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Graphic from Medium "Social Network Analytics"

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- Transmission rate (or speed of the spread) $\beta\langle k \rangle$
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- Transmission rate (or speed of the spread) $\beta\langle k\rangle$
  - $\langle k\rangle$ is the average degree of the network
- Transmission rate is analogous to basic reproductive rate R_0
  - $R_0 = \frac{\beta\langle k\rangle}{\mu}$, where μ is a recovery/removal rate
  - $R_0 > 1 \implies$ spread, $R_0 < 1 \implies$ spread dies out

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Related Work

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  - First paper to analytically study source identification
  - Rumor centrality: the number of permitted permutations of a graph
  - Permitted permutation: a possible ordering of nodes that led to a given subgraph
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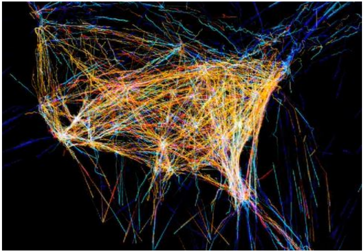
Methods

- Simulated spread for 100 time-steps
  - Applied source algorithms for $t \in \{10, 20, 30\}$



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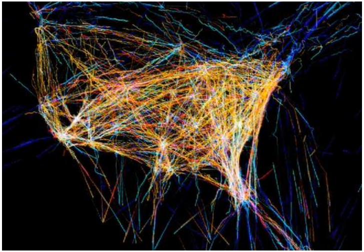

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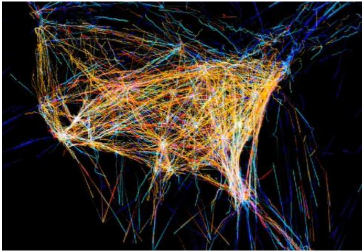
- Infection rates: $\beta \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$



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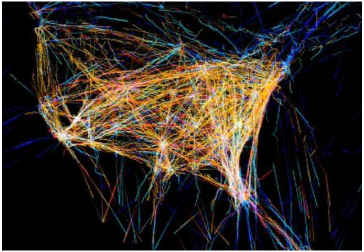
- Infection rates: $\beta \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$
- $6,000$ experiments for random graphs
  - $1,500$ for each graph type
  - 100 for each set of: graph type, graph size, β-value
- 100 experiments for airport network (20 per β)



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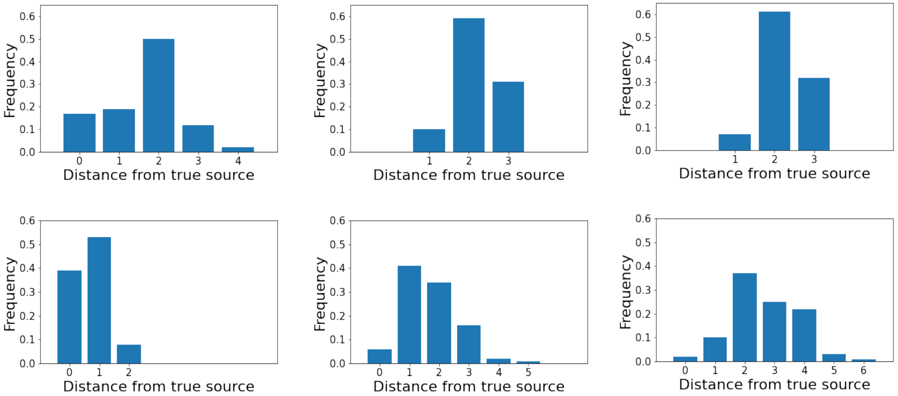
- Substantial improvements on Barabási-Albert trees ($m = 1$)
- Watts-Strogatz random graphs performed exceptionally well
- Minor changes when lowering only infection rate on:
  - Barabási-Albert random graphs ($m = 3$)
  - Erdős-Rényi random graphs
  - U.S. airport network
- Results obtained reflect those in literature

Results and Discussion (cont.)



- Top figures: LISN, Barabási-Albert ($m = 3$), $N = 500$, $\beta = 0.1$, $t \in \{10, 20, 30\}$
- Bottom figures: Rumor centrality, Barabási-Albert ($m = 1$), $N = 500$, $t = 10$, $\beta \in \{0.1, 0.3, 0.5\}$

Results and Discussion (cont.)

Graph Type	Method	a.) $\beta = 0.5, t = 30$		b.) $\beta = 0.1, t = 10$	
		$\frac{\langle R \rangle}{N}$	$\frac{\langle d \rangle}{\langle diam(G) \rangle}$	$\frac{\langle R \rangle}{N}$	$\frac{\langle d \rangle}{\langle diam(G) \rangle}$
Barabási-Albert ($m = 1$)	Rumor	0.50	0.25	**0.02**	0.06
	Jordan	0.51	0.25	**0.02**	0.10
	NETSLEUTH	**0.48**	0.29	**0.02**	0.13
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Erdős-Rényi ($p = \frac{\ln(N)+1}{N}$)	Rumor	0.55	0.48	**0.06**	0.32
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	NETSLEUTH	**0.45**	0.48	0.14	0.40
	LISN	0.53	0.40	0.09	0.32
Watts-Strogatz ($p = 0.01$, $k = 4$)	Rumor	0.12	0.15	**0.01**	0.02
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	NETSLEUTH	0.26	0.19	0.02	0.02
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US Airport Network	Rumor	**0.43**	0.25	0.32	0.25
	Jordan	0.44	0.25	0.36	0.25
	NETSLEUTH	0.47	0.25	0.47	0.25
	LISN	0.46	0.25	**0.28**	0.13

Future Work and Conclusion

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  - Experimenting on more source identification algorithms
  - Applying algorithms to more network structures
  - Extracting metrics in a statistical manner
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  - Graph types
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- Algorithm performance shown to depend on:
  - Observation time
  - Infection rate
  - Graph structure
  - Graph size

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Appendix: Graphs

- Barabási-Albert random graphs ($m = 1$ and $m = 3$)
- Erdős-Rényi random graphs ($p = \frac{\ln(N)+1}{N}$)
- Watts-Strogatz random graphs ($p = 0.01, k = 4$)
- real-world U.S. airport network ($N = 1,572, |E| = 17,214$)

Appendix: Rumor Centrality

- First paper to analytically study source identification
- Rumor centrality $R(v, G_I)$ is the number of permitted permutations of a graph G_I centered at some node v
  - Permitted permutation describes a possible ordering of nodes that led to G_I
- Example using Figure 1 assuming source node 1
  - Permitted permutation: $\{1, 2, 3, 4\}$
  - Unpermitted permutation: $\{1, 3, 2, 4\}$
- Source is estimated to be v^* such that $R(v^*, G_I) \geq R(v, G_I)\ \forall v \in G_I$
- Method is designed for tree-structured data
- For general graphs, method uses a BFS-tree rooted at each node
- Complexity: $O(|V_I|)$ for tree graphs, $O(|V_I|^2)$ for general graphs
  - $|V_I|$ is the set of infected nodes

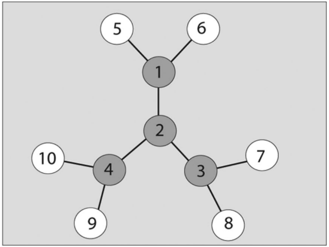
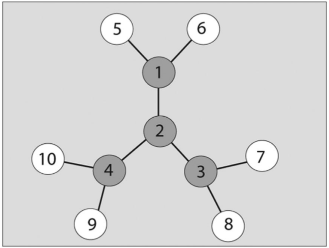


Figure 1: Tree graph where gray nodes make up infected subgraph I (from Shah and Zaman 2011)

Appendix: Jordan Centrality

- Measures a node's eccentricity in the graph
  - Eccentricity: max hop-distance to all other nodes
  - $J(v, I) = \max_{u \in I} d(v, u)$, where $d(v, u)$ is the hop-distance between nodes v, u
- Jordan infection center: infected node with minimum Jordan centrality
  - Estimated to be source node
  - For a Jordan infection center v^*, $v^* = \arg\min_{v \in I} J(v, I)$
- Example using Figure 1
  - $J(1, I) = J(3, I) = J(4, I) = 2$, $J(2, I) = 1$
  - Node 2 would be estimated to be source
- Complexity: $O(|V_I||E_I|)$, where $|E_I|$ is the set of edges in the infected subgraph



Graphic from Shah and Zaman (2011)

Appendix: NETSLEUTH

- Makes use of minimum description length principle
  - Referred to as minimal infection description
- Goal is to minimize total description length $\mathcal{L}(G_I, S, R)$
  - G_I is the infected subgraph
  - S is the set of seed nodes (possible sources)
  - R is a valid spread propagation ripple
- $\mathcal{L}(G_I, S, R) = \mathcal{L}(S) + \mathcal{L}(R|S)$
  - $\mathcal{L}(S)$ is the encoded length of the seed set S
  - $\mathcal{L}(R|S)$ is the encoded length of a ripple R starting at a seed set S
- Able to estimate multiple sources
- Complexity: $O(|E_I| + |E_F| + |V_I|)$
  - E_F is the set of edges connecting susceptible nodes to infected nodes

Appendix: LISN

- Assumes time for spread to propagate is exponentially distributed
- Method sums exponential random variables
  - Results in a gamma distrubtion
- Cumulative distribution function (CDF) used to update probabilities
- Node with max probability is estimated to be source
- Complexity: $O(|V_I|(|V| + |E|))$
- $F(t; n, \beta) = \frac{\gamma(n,\beta t)}{\Gamma(n)}$ is CDF of the gamma distribution
  - $\Gamma(n)$ is the standard gamma function
  - $\gamma(n, \beta t)$ is the lower-incomplete gamma function
  - n is the shortest distance between the two nodes
  - t is the observation time
  - β is the infection rate

Algorithm 1: A source detection algorithm

Input: G (network graph), I (infected nodes), T (total propagation time)
Output: rumor source estimate

```
initialization;
p ← {};
source ← v ∈ I;
forall v ∈ I do
    p(v) ← 1;
    forall u ∈ G.nodes do
        n ← ShortestPath(v, u);
        if u ∈ I then
            p(v) ← p(v) * F(T; n, λ);
            /* F is the cdf of gamma distribution */
        else
            p(v) ← p(v) * (1 − F(T; n, λ));
        end
    end
end
return source ← arg max_{v∈I} ℙ(v)
```

Appendix: Full table

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