Mathematical Modeling of Honeybee Colonies Infested with Mites and Virus

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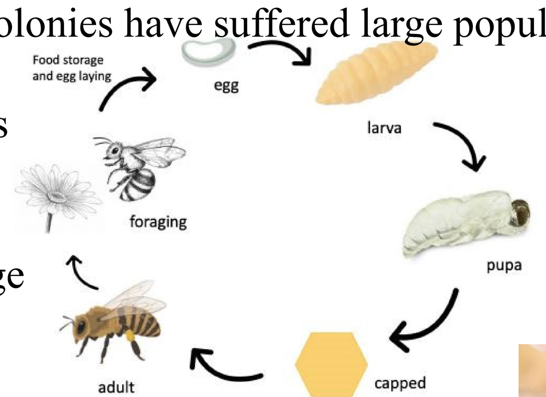
Abstract

Using a mathematical model of a honeybee colony infested with mites and virus, sensitivity analysis is conducted to determine what parameters the model is most sensitive to. The model consists of a system of five first-order ordinary differential equations and was developed by Dr. Vardayani Ratti. The sensitivity analysis suggests that the model is most sensitive to variations in: max eclosion rate, natural death rates of healthy bees, brood-maintenance coefficient, rate of forager mortality due to homing failure, and recruitment rate of hive bees to forager bees. Also, honeybees are highly dependent on temperature. The model's current parameter values are based on seasonal averages. Extending the model's parameters to match patterns in climate data is in-progress.

Background

Western honeybees (*Apis Mellifera*):

- Essential both agriculturally and economically [1]
- Two types of division of labor
  1. Reproduction (queen, drones, brood, workers)
  2. Colony growth/development (workers)
- Worker division of labor based on tasks performed
  - Hive bees: younger adults who work within hive
  - Forager bees: older adults are recruited to be foragers
- Since early 2000s, colonies have suffered large population losses
- Possible factors:
  - Parasitic mites
  - Viruses
  - Pesticides
  - Climate change



Mites

- Ectoparasitic mites (*Varroa destructor*)
- Feed on honeybees during development/adult stages
  - Can lead to death or virus infection
- Dependent on the availability of brood in hive
  - Female mites enter uncapped brood cells to lay eggs
  - Once cell is capped, mite lays eggs and feeds on developing bee
  - Feeding can lead to infection and death of developing bee
  - Once cell is uncapped, mites leave on newly-developed host
- Believed to be primary cause of colony losses
- Able to coexist with honeybees in absence of virus
- Beekeepers apply Varroacide treatment to try to control the mites

Virus

- More than 20 honeybee viruses are known
  - **Acute Bee Paralysis Virus (ABPV)**, Deformed Wing Virus (DWV), Kashmir Bee Virus (KBV), etc.
- Twelve are carried and transmitted by mites
- ABPV is common cause of colony failure
- *Varroa* mites are a mechanical vector of ABPV

Mathematical Model

$$\underbrace{\dot{x}_h}_{\text{Hive bees}} = \underbrace{\mu g(x_h + x_f)h(m)}_{\text{Bee eclosion (birth)}} - \underbrace{\beta_1 m \frac{x_h}{x_h + y + x_f}}_{\text{Bee virus infection}} - \underbrace{(d_1 + \delta_1)x_h}_{\text{Hive bee death}} - \underbrace{\gamma_1(m+n)x_h}_{\text{Mite-induced death}} - \underbrace{x_h R(x_h, x_f)}_{\text{Forager recruitment}} \quad (1)$$

$$\underbrace{\dot{x}_f}_{\text{Forager bees}} = \underbrace{x_h R(x_h, x_f)}_{\text{Forager recruitment}} - \underbrace{\beta_1 m \frac{x_f}{x_h + y + x_f}}_{\text{Bee virus infection}} - \underbrace{(p + d_2 + \delta_2)x_f}_{\text{Forager bee death}} - \underbrace{\gamma_2(m+n)x_f}_{\text{Mite-induced death}} \quad (2)$$

$$\underbrace{\dot{y}}_{\text{Infected bees}} = \underbrace{\beta_1 m \frac{x_h + x_f}{x_h + y + x_f}}_{\text{Bee virus infection}} - \underbrace{(d_3 + \delta_3)y}_{\text{Infected bee death}} - \underbrace{\gamma_3(m+n)y}_{\text{Mite-induced death}} \quad (3)$$

$$\underbrace{\dot{m}}_{\text{Virus-carrying mites}} = \underbrace{rm\left(1 - \frac{m+n}{\alpha(x_h + y + x_f)}\right)}_{\text{Logistic mite model}} + \underbrace{\beta_2 n \frac{y}{x_h + y + x_f}}_{\text{Mites acquire virus}} - \underbrace{\beta_3 m \frac{x_h + x_f}{x_h + y + x_f}}_{\text{Mites lose virus}} - \underbrace{\delta_4 m}_{\text{Treatment}} \quad (4)$$

$$\underbrace{\dot{n}}_{\text{Virus-free mites}} = \underbrace{rn\left(1 - \frac{m+n}{\alpha(x_h + y + x_f)}\right)}_{\text{Logistic mite model}} - \underbrace{\beta_2 n \frac{y}{x_h + y + x_f}}_{\text{Mites acquire virus}} + \underbrace{\beta_3 m \frac{x_h + x_f}{x_h + y + x_f}}_{\text{Mites lose virus}} - \underbrace{\delta_5 n}_{\text{Treatment}} \quad (5)$$

- Parameter values based on seasonal averages
- Division of labor among workers modeled by equations 1 and 2
- $g(x_h + x_f) = \frac{(x_h+x_f)^i}{K^i+(x_h+x_f)^i}$, where the integer exponent i > 1
  - Represents brood-maintenance in colony
- $h(m) = e^{-mk}$, where k > 0
  - Represents how birthrate of bees is affected by presence of mites
- $R(x_h, x_f) = \sigma_1 - \sigma_2\left(\frac{x_f}{x_h+x_f}\right)$
  - Describes effect of social inhibition on forager recruitment

Goals of Study

- Reproduce figures from 2017 research paper by Dr. Ratti [2]
- Perform sensitivity analysis to determine what parameters the model is most sensitive to
- Implement refined weather data for more accurate parameter values (in-progress)
  - Physical parameters are highly dependent on temperature

Sensitivity Analysis Methods

- Calculated two outputs:
  - Percent change in average total population
  - Colony lifespan
- Individually varied parameters +/- 10% of their seasonal avgs.
- Simulated colony for 10 years (t = 3650 days)

Results

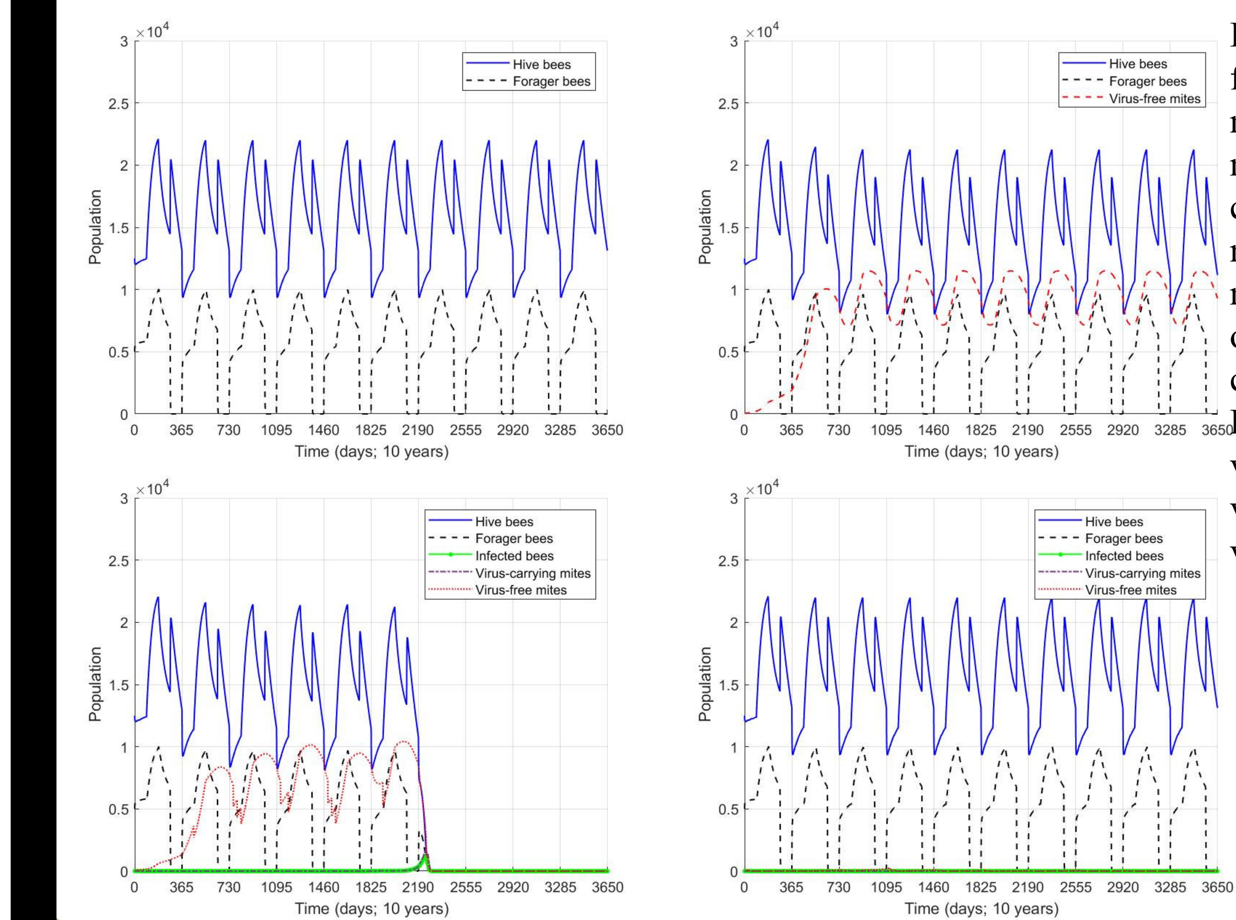


Figure 1: Reproduced simulation figures. **Top-left:** bee-only model. **Top-right:** bee-mite model in absence of virus shows coexistence between bees and mites. **Bottom-left:** full bee-mite-virus model with low levels of varroacide treatment; colony collapses after start of 7th year. **Bottom-right:** full bee-mite-virus model with high levels of varroacide treatment; mites and virus are eradicated.

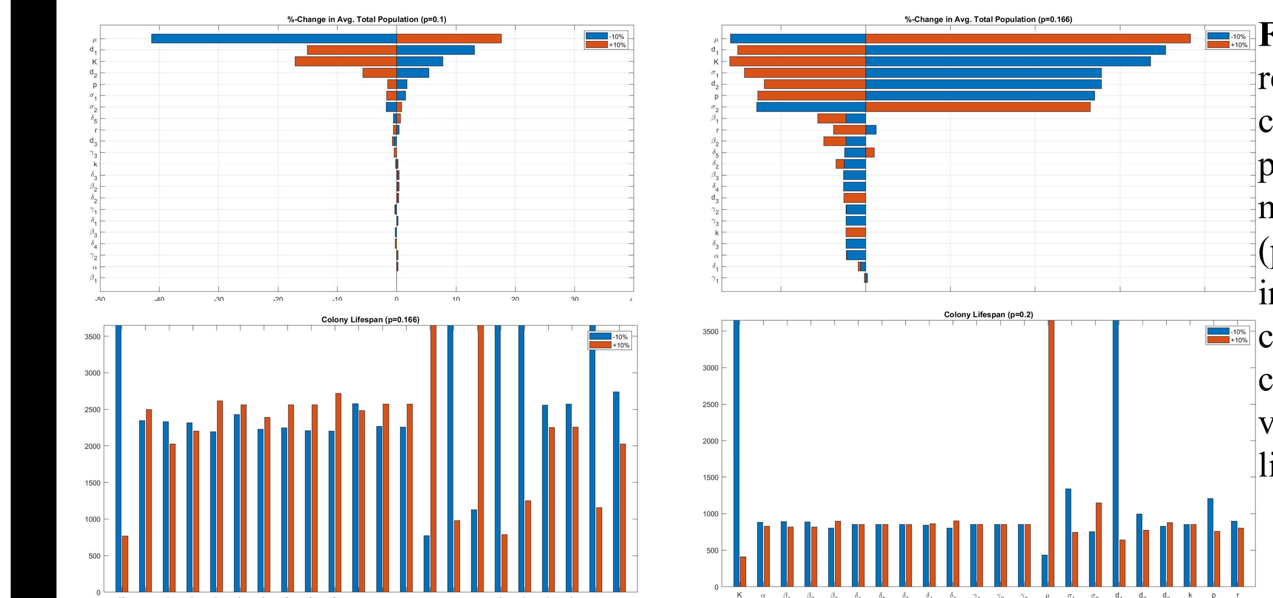


Figure 2: Sensitivity analysis results. **Top-left:** percent-change in average total population; low rate of forager mortality due to homing failure (p). **Top-right:** percent-change in average total population; critical p-value. **Bottom-left:** colony lifespan; critical p-value. **Bottom-right:** colony lifespan; high p-value.

- In presence of mites and virus, colony depends on level of varroacide treatment (Figure 2, bottom)
- Max eclosion rate μ has greatest effect on population size (Figure 2, top)
- Colony's long-term survival depends on variation of specific parameters (Figure 2, bottom)

Conclusions and Future Work

- Successfully reproduced figures in 2017 research paper
- According to sensitivity analysis, most sensitive parameters:
  - μ: Max eclosion (birth) rate
  - $d_{1,2}$: Natural death rates of hive and forager bees
  - p: Rate of forager mortality due to homing failure
  - σ_1: Max recruitment rate of hive bees to foragers
- Sensitivity analysis results agree with prior literature [2,3]
- Future work: extend model parameter to match patterns in climate data
  - Realistically, parameter values vary during season
  - Few models found using refined weather data [4]
- Implementation of climate change could provide more realistic scenario (Figure 3)

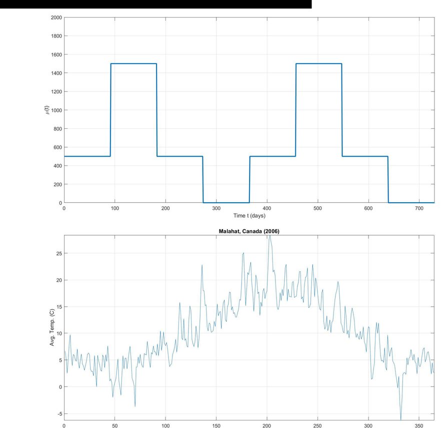


Figure 3: Top: plot of max eclosion rate μ over two years. **Bottom:** daily climate data from Malahat, Canada (2006) [5].

References and Acknowledgements

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[4] Becher, M.A., Grimm, V., Thorbek, P., Horn, J., Kennedy, P.J. and Osborne, J.L. (2014), BEEHAVE: a systems model of honeybee colony dynamics and foraging to explore multifactorial causes of colony failure. J Appl Ecol, 51: 470-482. https://doi.org/10.1111/1365-2664.12222
[5] Daily weather data: https://open.canada.ca/data/en/dataset/2ac87744-928b-4fd7-aa8e-48 225f408b67

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