

CSCI 6110.H001

Applied Combinatorics & Graph Theory

Dr. N. Adlai A. DePano

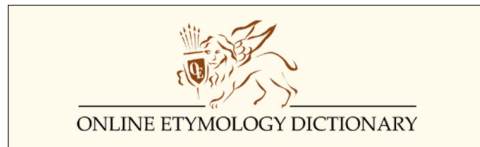
Fall 2021


ndepano@uno.edu



1

What is combinatorics?



combine (v.)  early 15c., from Middle French *combiner* (14c.), from Late Latin *combinare* "to unite, yoke together," from Latin *com-* "together" (see **com-**) + *bini* "two by two," adverb from *bi-* "twice" (see **binary**). Related: *Combinative*; *combined*; *combining*.



2

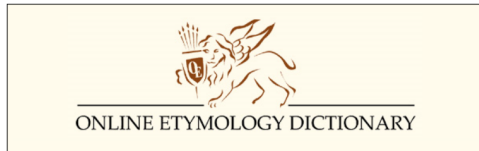
What is combinatorics?

- Concerned with study of arrangements, patterns, designs, assignments, schedules, connections, and configurations
- Almost all areas of activity give rise to problems of a combinatorial nature



3

What is graph theory?



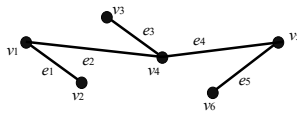
graph (n.)  1878, shortening of *graphic formula* (see **graphic**). The verb meaning "to chart on a graph" is from 1889. Related: *Graphed*; *graphing*.



4

What is graph theory?

- Concerned with study of graphs
- Graph G is a triple consisting of a vertex set $V(G)$, an edge set $E(G)$, and a relation that associates with each edge two vertices called its endpoints



5

What is combinatorics?

- Concerned with study of arrangements, patterns, designs, assignments, schedules, connections, and configurations
- Almost all areas of activity give rise to problems of a combinatorial nature
- Three basic problems of combinatorics
 - existence problem
 - counting problem
 - optimization problem



6

Existence problem

- Is there at least one arrangement of a particular kind?

Given n courses, m rooms, and k time slots, is it possible to schedule all the courses in the rooms at particular time slots such that certain criteria are satisfied (e.g., each course's students will all be accommodated in the room assigned to it)

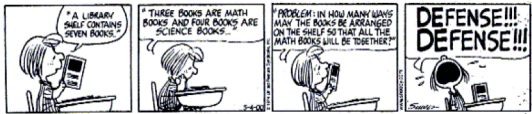


7

Counting problem

- How many possible arrangements are there?

CLASSIC PEANUTS



8

Optimization problem

- Of all the possible arrangements, which is the best according to some criterion?

Natural gas pipeline design involves choosing which pipes to use and when:

- Which diameter pipe?
- What length of pipe?
- Which pipe for the gas pressure involved?
- Which pipe for the temperature involved?

Which design that satisfies all requirements minimizes the cost and meets deadlines?



9

Computers & combinatorics

- Progress in solving combinatorial optimization problems has gone hand in hand with the development of computers
- Modern combinatorics is largely concerned with developing algorithms for solving existence, counting, or optimization problems
- Often, “brute force” solutions are the only known technique to solve problems



10

History & Applications of Combinatorics

- Combinatorics has achieved greatest impetus in modern times
- Nonetheless it is an old branch of mathematics
 - Permutations or arrangements in order known in China before 1100 B.C.
 - Euclid knew about the binomial expansion $(a+b)^n$ for $n=2$ about 300 B.C.
 - Formula for permutations of n sets known at least 2500 years ago



11

History & Applications of Combinatorics

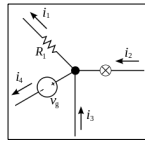
- More recently, Pascal and Fermat studied combinatorial problems associated with gambling odds
- Euler invented graph theory in the 18th century
- Hamilton applied combinatorial techniques in the study of games and puzzles in the 19th century



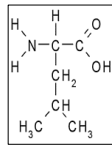
12

History & Applications of Combinatorics

- Kirchoff developed a graph theoretical approach to electrical networks in the 19th century



- Cayley developed enumeration techniques in studying organic chemistry



13

History & Applications of Combinatorics

- In modern times, combinatorics has impacted
 - computer science
 - transportation
 - industrial planning
 - electrical engineering
 - experimental design
 - genetics
 - political science



14
