CSCI 6110

Applied Combinatorics & Graph Theory

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Generating combinatorial objects

- Some problem may be so hard as to require a “brute force” solution
- How to produce all combinatorial objects needed
- Generated stream may be of various types
  - subsets of a set (power set)
  - *k*-subsets of a set
  - permutations of a set

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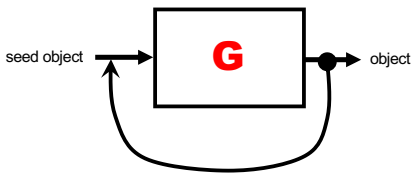
Generating combinatorial objects

- General set-up:



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Generating combinatorial objects

- **G**: Generating subsets of a given set

Algorithm: Subsets of $\{1,\ldots,n\}$
FIRST SUBSET Y is $\varnothing$.
NEXT SUBSET *after* Y:
- *Find the last element i not in Y (working back from the end).*
- *If there's no such element, then Y was the last subset.*
- *Remove from Y all elements after i, and add i to Y. Return this set.*

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Generating combinatorial objects

- Example: **Subsets of** $\{1,2,3\}$ are generated in the following order –

1: $\varnothing$	5: $\{1\}$
2: $\{3\}$	6: $\{1,3\}$
3: $\{2\}$	7: $\{1,2\}$
4: $\{2,3\}$	8: $\{1,2,3\}$

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Generating combinatorial objects

- **G**: Generating k-subsets of a given set

Algorithm: k-Subsets of $\{1,\ldots,n\}$
FIRST SUBSET Y is $\{1,\ldots,k\}$.
NEXT SUBSET *after* $Y=\{y_1,\ldots,y_k\}$, *where* $y_1<\ldots<y_k$:
- *Find the first i such that $y_i+1\notin Y$;*
- *Increase y_i by 1, set $y_j=j$ for $j<i$, and return the new set Y;*
- *This fails if $i=k$, $y_k=n$, in which case $Y=\{n-k+1,\ldots,n\}$ is the last set.*

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Generating combinatorial objects

- Example: **2-Subsets of** {1,2,3} are generated in the following order –

  1: {1,2}
  2: {1,3}
  3: {2,3}

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Generating combinatorial objects

- **G**: Generating permutations of a given set

Algorithm: Permutations of $\{1,\ldots,n\}$
FIRST PERMUTATION is given by $x_i = i$ for $i=1,\ldots,n$.
NEXT PERMUTATION *after* $\{x_1,\ldots,x_n\}$:

- *Find the largest j for which $x_j < x_{j+1}$ (working back from the end);*
- *If no such j exists, then the current permutation is the last.*
- *Interchange the value of x_j with the least x_k greater than x_j with $k>j$; then reverse the sequence of values of $\{x_{j+1},\ldots,x_n\}$; return this permutation.*

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Generating combinatorial objects

- Example: **Permutations of** {1,2,3} are generated in the following order –

  1: {1,2,3} 4: {2,3,1}
  2: {1,3,2} 5: {3,1,2}
  3: {2,1,3} 6: {3,2,1}

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