CSCI 6110

Applied Combinatorics & Graph Theory

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A Review of Basic Counting Rules

- PRODUCT RULE

  If something can happen in n_1 ways, *and* no matter how the first thing happens, a second thing can happen in n_2 ways, then the two things together can happen in $n_1 \times n_2$ ways.

  Generalizing, if something can happen in n_1 ways, *and* no matter how the first thing happens, a second thing can happen in n_2 ways, *and* no matter how the first two things happen, a third thing can happen in n_3 ways, *and* ..., then all the things together can happen in

  $$n_1 \times n_2 \times n_3 \times \ldots$$

  ways

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A Review of Basic Counting Rules

- How many different license plates?

vs.

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A Review of Basic Counting Rules

- SUM RULE

  If one event can occur in n_1 ways and a second event in n_2 (different) ways, then there are $n_1 + n_2$ ways in which *either* the first event *or* the second event can occur *(but not both)*.

  Generalizing, if one event can occur in n_1 ways, a second event can occur in n_2 (different) ways, a third event can occur in n_3 (still different ways), ..., then there are

  $$n_1 + n_2 + n_3 + \cdots$$

  ways in which (exactly) one of the events can occur.

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A Review of Basic Counting Rules

- How many different variables of length 5?

BASIC	vs.	**Java**
Variable names may start with a letter or underscore, but can otherwise contain any letter or number, and the underscore character.		The first character of a variable name *must* be a letter, underscore or dollar sign. Subsequent characters can also include digits. Letters are `A` to `Z`, `a` to `z`, digits are `0` to `9`.

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A Review of Basic Counting Rules

- PERMUTATIONS

  A permutation of an n-set is an arrangement of the elements of the set in order.

  The number of permutations of an n-set is given by

  $$n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1 = n!$$

- r-PERMUTATIONS

  An r-permutation of an n-set is an arrangement of r elements of the set in order.

  The number of r-permutations of an n-set is given by

  $$P(n,r) = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1) = n!/(n-r)!$$

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A Review of Basic Counting Rules

- SUBSETS

  The number of subsets of an n-set is given by
  $$2 \times 2 \times 2 \times \cdots \times 2 \times 2 = 2^n$$

- r-COMBINATIONS

  An r-combination of an n-set is a selection of r elements of the set, which means that order does not matter.

  The number of r-combinations of an n-set is given by
  $$C(n,r) = n!/(r!(n-r)!)$$

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What is "guided discovery"?

- Falls under the pedagogical methodology called ***constructivism***
- More recently has evolved into ***authentic learning***
- Activities are done in groups and facilitated by a "guide-on-the-side" rather than a "sage-on-the-stage"
- Not all educators subscribe to the methodology

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Combinatorics via "guided discovery"

- Make use of notes produced by a team led by Prof. Kenneth P. Bogart from Dartmouth
- Based on philosophy that –

> "You learn the most about a subject when you are figuring it out directly for yourself, and learn the least when you are trying to figure out what someone else is saying about it."

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Group activities

- Main activity will be solving problems designed to lead one to discover the basic principles of combinatorial mathematics
- Here is a sample problem from the notes:

> 3. One of the schools sending its team to the tournament has to send its players from some distance, and so it is making sandwiches for team members to eat along the way. There are three choices for the kind of bread and five choices for the kind of filling. How many different kinds of sandwiches are available?

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"Twentyfold Way"

- Distribution problems

The Twentyfold Way: A Table of Distribution Problems		
k objects and conditions on how they are received	n recipients and mathematical model for distribution	
	Distinct	Identical
1. Distinct no conditions		
2. Distinct Each gets at most one		
3. Distinct Each gets at least one		
4. Distinct Each gets exactly one		
5. Distinct, order matters		
6. Distinct, order matters Each gets at least one		
7. Identical no conditions		
8. Identical Each gets at most one		
9. Identical Each gets at least one		
10. Identical Each gets exactly one		

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"Twentyfold Way"

- Distribution problems

The Twentyfold Way: A Table of Distribution Problems		
k objects and conditions on how they are received	n recipients and mathematical model for distribution	
	Distinct	Identical
1. Distinct no conditions	n^k functions	
2. Distinct Each gets at most one		1 if $k \leq n$; 0 otherwise
3. Distinct Each gets at least one		
4. Distinct Each gets exactly one	$k! = n!$ bijections	1 if $k = n$; 0 otherwise
5. Distinct, order matters		
6. Distinct, order matters Each gets at least one		
7. Identical no conditions		
8. Identical Each gets at most one	$\binom{n}{k}$ subsets	1 if $k \leq n$; 0 otherwise
9. Identical Each gets at least one		
10. Identical Each gets exactly one	1 if $k = n$; 0 otherwise	1 if $k = n$; 0 otherwise

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"Twentyfold Way"

- Distribution problems

The Twentyfold Way: A Table of Distribution Problems		
k objects and conditions on how they are received	n recipients and mathematical model for distribution	
	Distinct	Identical
1. Distinct no conditions	n^k functions	
2. Distinct Each gets at most one	$k!$ k-element permutations	1 if $k \leq n$; 0 otherwise
3. Distinct Each gets at least one		
4. Distinct Each gets exactly one	$k! = n!$ bijections	1 if $k = n$; 0 otherwise
5. Distinct, order matters		
6. Distinct, order matters Each gets at least one		
7. Identical no conditions		
8. Identical Each gets at most one	$\binom{n}{k}$ subsets	1 if $k \leq n$; 0 otherwise
9. Identical Each gets at least one		
10. Identical Each gets exactly one	1 if $k = n$; 0 otherwise	1 if $k = n$; 0 otherwise

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