Your Name ______________________________ **CSCI 6110**

Test No. 1

(Thursday, September 28, 2017)

<u>GENERAL INSTRUCTIONS</u>: Attempt all questions; do not spend too much time on any one — the point distribution is marked clearly — budget your time accordingly. No penalties will be assessed for wrong answers (no points would be awarded either). Keep your work neat and clearly indicate your answers. You may use the back of the sheets if you need additional space.

Part I. MINI-ESSAYS. Answer the questions as completely as you can without sacrificing conciseness. The space provided should be a good indication of the length of answer required.

1. The three types of problems in combinatorics was discussed in the textbook and in class. Supply examples of each ***from the field of computer science*** to illustrate the relevance of the subject matter in a computer science curriculum. Some possibilities might be from algorithm design, programming languages, computer security, bioinformatics, *etc*. *(10 pts.)*

2. Briefly describe the ***pigeonhole principle***. Include an example of how it is involved in any of the three "types" of combinatorial problems. *(5 pts.)*

3. What is a ***derangement***? Give an example of a problem involving derangements in combinatorics. *(5 pts.)*

Part II. *Multiple Choice.* Choose the best answer from among the choices supplied. Record the answers on the scantron sheet provided. *(2 pts. each)*

1. Combinatorics is concerned, among others, with the study of this activity:
   a. arrangements and patterns
   b. designs and assignments
   c. schedules and connections
   d. configurations
   e. All of these are areas of concern

2. Of the three basic problems of combinatorics, this might be the most difficult, since it requires an additional element – a metric that is the basis of the calculated answer:
   a. existence problem
   b. counting problem
   c. optimization problem
   d. All are equally difficult.
   e. There is no valid basis for comparison.

3. The sum rule for counting applies to events that are mutually exclusive. Assess the validity of this statement.
   a. It is a valid statement.
   b. It is non a valid statement.

4. The sum rule for counting applies only to events where the results are <u>mutually exclusive</u>, that is, we are counting results that cannot occur in both events. Assess the validity of this statement.
   a. It is a valid statement.
   b. It is non a valid statement.

5. In an n-set, there are this many <u>duplicated</u> elements.
   a. 0
   b. 1
   c. n
   d. $n-1$
   e. $n+1$

6. An n-set has this many <u>permutations</u> that begin with the number 1.
   a. 0
   b. 1
   c. $n!$
   d. $(n-1)!$
   e. $(n+1)!$

7. The value of $n!$ can be approximated by computing $s_n = \sqrt{2\pi n}\,(n/e)^n$. This is known as:
   a. Turing's approximation
   b. Gauss' approximation
   c. Stirling's approximation
   d. Lagrange's approximation
   e. Laplace's approximation

8. An r-permutation of an n-set is denoted in the textbook by $P(n,r)$. It supplies the number of ways one can arrange r elements from the n-*set* when order is important. The correct formula for $P(n,r)$ is:
   a. $n!r!$
   b. $(n+r)!$
   c. $r!/n!$
   d. $n!+r!$
   e. $n!/(n-r)!$

9. The number of subsets of the n-set is given by this formula:
   a. $n!$
   b. $(n-1)!$
   c. $n(n-1)/2$
   d. 2^n
   e. None of the above

10. An r-combination of an n-set is denoted in the textbook by $C(n,r)$. It supplies the number of r-element subsets of the n-set. The correct formula for $C(n,r)$ is:
    a. $n(n-r)/2$
    b. $(n-r)!$
    c. $n!/r!(n-r)!$
    d. $n!/(n+r)!$
    e. $n!/(n-r)!$

11. The relationship between $P(n,r)$ and $C(n,r)$ is:
    a. $P(n,r) = C(n,r)$
    b. $P(n,r) \geq C(n,r)$
    c. $P(n,r) \leq C(n,r)$
    d. $P(n,r) = C(n,r) \times P(r,r)$
    e. All of the above

12. $C(n,r)$ can be computed in a recursive manner. The double recurrence relation involving $C(n,r)$ is valid: $C(n,r) = C(n–1,r–1) + C(n–1,r)$
As with all recurrences, the basis case(s) must be defined. This is one of the basis cases –

    a. $C(1,1) = 0$
    b. $C(1,1) = 1$
    c. $C(1,1) = 2$
    d. $C(n,n) = 0$
    e. $C(n,n) = n$

13. $P(n,r)$ can likewise be computed in a recursive manner. A recurrence equation involving $P(n,r)$ is given by:

    a. $P(n,r) = P(n–1,r–1) + P(n–1,r)$
    b. $P(n,r) = n\ P(n–1,r–1)$
    c. $P(n,r) = P(n–1,r–1) / P(n–1,r)$
    d. All of the above
    e. None of the above

14. The number of r-permutations of an m-set where replacement or repetition is allowed is denoted in the textbook by $P^R(m,r)$. The correct formula for $P^R(m,r)$ is:

    a. $P^R(m,r) = m\ P(m,r)$
    b. $P^R(m,r) = r^m$
    c. $P^R(m,r) = m^r$
    d. $P^R(m,r) = P(m+r–1,r)$
    e. There is no formula for this.

15. Similarly, the number of r-combinations of an m-set where replacement or repetition is allowed is denoted in the textbook by $C^R(m,r)$. The correct formula for $C^R(m,r)$ is:

    a. $C^R(m,r) = m\ C(m,r)$
    b. $C^R(m,r) = m^r$
    c. $C^R(m,r) = r^m$
    d. $C^R(m,r) = C(m+r–1,r)$
    e. There is no formula for this.

16. According to the textbook, combinatorics is one of the fastest-growing areas of mathematics (and may we add thanks in part to the rise of computers). Assess the validity of this statement.

    a. This is a valid statement.
    b. This is not a valid statement.

17. The algorithms presented in Chapter 2 of the textbook generated this/these kind(s) of combinatorial objects:

    a. permutations of the set $\{1,2,\ldots,n\}$
    b. bit strings of length n
    c. r-combinations of the set $\{1,2,\ldots,n\}$
    d. All of these were generated.
    e. None of these were generated.

18. There are these many binary functions on n variables: $\{0,1\}^n \rightarrow \{0,1\}$

    a. n^2
    b. $P(n,2) = n(n–1)$
    c. $C(n,2) = n(n–1)/2$
    d. 2^n
    e. 2^{2^n}

19. Most, if not all, of the counting formulas in combinatorics are derived from the basic counting rules (product rule, sum rule). Assess the validity of this statement

    a. This is a valid statement.
    b. This is not a valid statement.

20. "And" and "or" are key words that usually indicate whether the sum rule or the product rule is appropriate. The word ______ suggests the ***product*** rule; the word ______ suggests the ***sum*** rule.

    a. and/or
    b. or/and
    c. and/and
    d. or/or
    e. None of these is a valid answer.

Part III. PROBLEM-SOLVING. Supply the answers as requested in the problem description.

1. How many car owners can be accommodated with the current policy of assigning plate numbers in the Philippines (see the example to the right)? How does this compare to that of another Asian country India (see the example to the right)? Does this reflect the relative population size of both countries? *(5 pts.)*

2. How many numbers between 100 and 1000 are ***odd*** and have distinct digits? *(5 pts.)*

3. How many different playlists can be formed from a collection of 5000 tracks if no repetitions are allowed and the playlist must contain 50 tracks? Assume that the order in which the tracks are played ***does*** make a difference. *(10 pts.)*

4. National Basketball Association (NBA) plays an 82-game season where each game is either won or lost (all "ties" with overtime, double-overtime, triple-overtime, etc.). How many different 82-game seasons are possible with this setup ("seasons" are the number of ways a team can get to a certain "record")? *(10 pts.)*

5. In Group Activity No. 3, we worked on $S(k,n)$, the number of partitions of a set with k elements into n blocks. One of the activities asked for the value of $S(k,1)$, which is the number of partitions of a set with k elements into 1 block. We determined that this was equal to 1.

   a. What about $S(k,2)$, the number of partitions of a set with k elements into 2 blocks? Explain convincingly why the following recurrence accurately represents this value:

   $$S(k,2) = 2 \times S(k-1,2) + 1$$

   (5 pts.)

   b. Determine the value of $S(k,2)$ by solving the recurrence above. How does your answer compare to the number of subsets one can form from a set with $k-1$ elements? Can you explain why this is the case? *(5 pts.)*