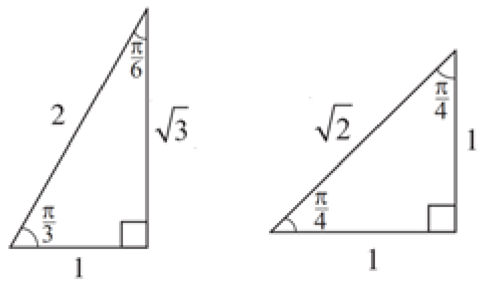
1 Exact trig values

To calculate the exact trig value, we can use a combination of the ratio triangles and compound angle rules.
The ratio triangles are provided in the formula sheet:



The compound angle rules are:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan A \pm B = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

For example, calculate the exact value of $\sin\left(\frac{\pi}{12}\right)$
Consider that $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$
$\sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$
From the ratio triangles, we can work out the exact values of each part:
$\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \sin\frac{\pi}{4}\right) = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$
$\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$
Rationalising by multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$:
$\sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{4}$

Harder: using algebra and trig identities

For angles that where we can't simply use the ratio triangles, we can calculate exact values by forming a quadratic. When the angle we are finding is a factor of 90 or 180, we can rewrite the equation to be sine or cosine of $n\theta$, where $n\theta$ multiplies to 90 or 180.
This then enables us to rearrange using identities and simplify by evaluating sine or cosine of 90 or 180 (or π or 2π).
For example, find the exact value of $\sin 18$.
Since 18 is a factor of 90, we can rewrite this as below. (Note that while it is also a factor of 180, we use the lower value as that requires less working):
$5\theta = 90$
$2\theta + 3\theta = 90$
$2\theta = 90 - 3\theta$
$\sin(2\theta) = \sin(90 - 3\theta)$
$2\sin\theta\cos\theta = \sin 90 \cos 3\theta - \cos 90 \sin 3\theta$
Evaluating $\sin(90) = 1$ and $\cos(90) = 0$, we get:
$2\sin\theta\cos\theta = \cos 3\theta$
Splitting the 3θ into a sum:
$2\sin\theta\cos\theta = \cos(2\theta + \theta)$
$2\sin\theta\cos\theta = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$
Using identities to change the equation so each term has a common factor of $\cos\theta$:
$2\sin\theta\cos\theta = (1 - 2\sin^2\theta)\cos\theta - 2\sin^2\theta\cos\theta$
Divide through by $\cos\theta$
$2\sin\theta = (1 - 2\sin^2\theta) - 2\sin^2\theta$
$2\sin\theta = 1 - 4\sin^2\theta$
Turn into a quadratic and solve using the quadratic equation:
$4\sin^2\theta + 2\sin\theta - 1 = 0$
$\sin\theta = \frac{-2\pm\sqrt{20}}{8} = \frac{-2\pm2\sqrt{5}}{8} = \frac{-1\pm\sqrt{5}}{4}$
Since we know $\sin 18$ is positive (from out knowledge of the graph), $\sin 18 = \frac{-1+\sqrt{5}}{4}$

Questions

(Answers - page **??**)

1. $\cos 45$
2. $\sin 105$
3. $\tan 60$
4. $\cos \frac{7\pi}{12}$
5. $\cos \frac{\pi}{12}$
6. $\tan \frac{2\pi}{3}$
7. $\cos \frac{5\pi}{12}$
8. $\sin \frac{4\pi}{3}$
9. $\sin \frac{7\pi}{4}$
10. $\tan \frac{3\pi}{4}$

Using algebra and compound angle rules, find the exact values of the following:

11. $\cos 18$
12. $\sin 36$
13. $\sin \frac{2\pi}{5}$