Answers - Integration by parts - DI method (page ??)

1. $\int x^2 \sin(2x)\,dx$

	D	I
+	x^2	$\sin(2x)$
-	$2x$	$-\frac{1}{2}\cos(2x)$
+	2	$-\frac{1}{4}\sin(2x)$
-	0	$\frac{1}{8}\cos(2x)$

Stop is reached when we get zero in the D row.

$\int x^2 \sin(2x)\,dx = -\frac{x^2}{2}\cos(2x) + \frac{x}{2}\sin(2x) + \frac{1}{4}\cos(2x) + c$

2. $\int e^x \cos(x)\,dx$

	D	I
$+$	e^x	$\cos x$
-	e^x	$\sin x$
$+$	e^x	$-\cos x$

The third row is a "repeat" of the first, so we can stop now. The integral is diagonal products plus the integral of the final row product.

$\int e^x \cos(x)\,dx = e^x \sin(x) + e^x \cos(x) - \int e^x \cos(x)\,dx$

$2\int e^x \cos(x)\,dx = e^x \sin(x) + e^x \cos(x)$

$\int e^x \cos(x)\,dx = \frac{e^x \sin(x) + e^x \cos(x)}{2} + c$

3. $\int (\ln(x))^2\,dx$

	D	I
$+$	$\ln(x))^2$	1
-	$\frac{2\ln x}{x}$	x

Since the product of the second row can (relatively) easily be integrated, the integral will be:

$\int (\ln(x))^2\,dx = x\ln(x))^2 - \int 2\ln x\,dx$

Using the DI method again for this:

	D	I
+	$2\ln x$	1
-	$\frac{2}{x}$	x

The product of the second row can be integrated so we stop, giving us:

$2\ln x\, dx = 2x\ln x - \int 2\, dx = 2x\ln x - 2x$

Therefore, our final integral is:

$\int (\ln(x))^2\, dx = x(\ln(x))^2 - 2x\ln x + 2x + c$

4. $\int \sin^3(x)\, dx$

	D	I
+	$\sin^2(x)$	$\sin(x)$
-	$2\sin(x)\cos(x)$	$-\cos(x)$

The product of the second row integrates easily so we stop:

$\int 2\sin(x)\cos^2(x)\, dx = -\frac{2}{3}\cos^3(x)$

Therefore, our final integral is:

$\int \sin^3(x)\, dx = -\sin^2(x)\cos(x) - \frac{2}{3}\cos^3(x) + c$

5. $\int \frac{\ln(x)}{x^2}\, dx$

	D	I
+	$\ln x$	$\frac{1}{x^2}$
-	$\frac{1}{x}$	$-\frac{1}{x}$

The product of the second row is easy to integrate so we stop:

$\int \frac{\ln(x)}{x^2}\, dx = -\frac{\ln}{x} - \int -\frac{1}{x^2}\, dx$

$\int \frac{\ln(x)}{x^2}\,dx = -\frac{\ln}{x} + \int \frac{1}{x^2}\,dx$

$\int \frac{\ln(x)}{x^2}\,dx = -\frac{\ln}{x} - \frac{1}{x} + c$

6. $\int 4x\cos(2-3x)\,dx$

	D	I
+	$4x$	$\cos(2-3x$
-	4	$-\frac{1}{3}\sin(2-3x)$
+	0	$-\frac{1}{9}\cos(2-3x)$

Stop because we reach zero in the D column, so the integral is:

$\int 4x\cos(2-3x)\,dx = -\frac{4x}{3}\sin(2-3x) + \frac{4}{9}\cos(2-3x) + c$

7. $\int e^{-x}\cos(x)\,dx$

	D	I
+	e^{-x}	$\cos(x)$
-	$-e^{-x}$	$\sin(x)$
+	e^{-x}	$-\cos(x)$

The third row repeats, so we stop:

$\int e^{-x}\cos(x)\,dx = e^{-x}\sin(x) - e^{-x}\cos(x) + \int e^{-x} \times -\cos(x)\,dx$

$\int e^{-x}\cos(x)\,dx = e^{-x}\sin(x) - e^{-x}\cos(x) - \int e^{-x}\cos(x)\,dx$

$2\int e^{-x}\cos(x)\,dx = e^{-x}\sin(x) - e^{-x}\cos(x)x$

$\int e^{-x}\cos(x)\,dx = \frac{e^{-x}}{2}(\sin(x) - \cos(x)) + c$