

# 1 Turning equations into quadratics

When there are three terms in an equation, we can often turn them into a quadratic, where the subject is not  $x$  but another expression that we substitute in.

For example,  $e^{4x} - 5e^{2x} + 6 = 0$  can be solved by making it a quadratic in terms of  $e^{2x}$ .

$$u = e^{2x}$$

$$u^2 - 5u + 6 = 0$$

$$u = 2, 3$$

Then we just back-substitute and solve:

$$e^{2x} = 2$$

$$2x = \ln 2$$

$$x = \frac{\ln 2}{2}$$

$$e^{2x} = 3$$

$$2x = \ln 3$$

$$x = \frac{\ln 3}{2}$$

If all three terms contain a variable, we can also divide the equation through by something to turn one of those into a constant, enabling us to then solve it as a quadratic.

For example,  $3(2^{3x}) - 11(2^{2x}) - 2^{x+2} = 0$

If we divide each term by a common factor of  $2^x$ , the equation changes to:

$$\frac{3(2^{3x})}{2^x} - \frac{11(2^{2x})}{2^x} - \frac{2^{x+2}}{2^x} = 0$$

$$3(2^{2x}) - 11(2^x) - 2^2 = 0$$

We can now make the substitution  $u = 2^x$  to solve the equation:

$$3u^2 - 11u - 4 = 0$$

$$u = -\frac{1}{3}, 4$$

Since  $2^x$  can clearly never be negative, we can disregard the first solution.

$$2^x = 4$$

$$x = 2$$

## Questions

1. Solve  $2^x + 4^x = 24$
2. Solve  $4^x + 6^x = 9^x$
3. Solve  $8(9^x) + 3(6^x) - 81(4^x) = 0$
4. Solve  $25^x + 2(15^x) - 24(9^x) = 0$