Answers - Euler's formula (page ??)

1. $(-i)^i = e^{-\frac{i\pi}{2}i}$
   $= e^{-\frac{i^2\pi}{2}}$
   $= e^{\frac{\pi}{2}}$

2. Since $-1 = e^{i\pi}$, we can write this expression as $\ln\left(e^{i\pi}\right) = i\pi$

3. $e^{i(A-B)} = e^{iA}e^{-iB}$

   This means that:

   $$\begin{aligned}\cos(A-B) + i\sin(A-B) &= (\cos(A) + i\sin(A))(\cos(-B) + i\sin(-B)) \\ &= (\cos(A) + i\sin(A))(\cos(B) - i\sin(B))\end{aligned}$$

   Equating real and imaginary parts:

   $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$

   $\sin(A-B) = \cos(B)\sin(A) - \cos(A)\sin(B)$

   Substituting $-B$ for B in the second equation:

   $\sin(A+B) = \sin(A)\cos(-B) - \cos(A)\sin(-B) = \sin(A)\cos(B) + \cos(A)\sin(B)$

4. Since $i = e^{\frac{i\pi}{2}}$, we can write the expression as $((e^{\frac{i\pi}{2}})^i)^2 = (e^{-\frac{\pi}{2}})^2 = e^{-\pi}$

5. Separating the expression into three terms:

   $\ln\left(-25e^{i^i}\right) = \ln(-1) + \ln(25) + \ln(e^{i^i})$

   Since $-1 = e^{i\pi}$, we can simplify the expression:

   $\ln(e^{i\pi}) + \ln(25) + \ln e^{i^i}$

   $i\pi + \ln(25) + i^i$

   $i^i = e^{\frac{i\pi}{2}i} = e^{-\frac{\pi}{2}}$

   So the expression simplifies to $i\pi + \ln(25) + e^{-\frac{\pi}{2}}$

6. $e^{i\theta} = \cos\theta + i\sin\theta$

   $e^{-i\theta} = \cos(-\theta) + i\sin(-\theta) = \cos\theta - i\sin\theta$

$\therefore e^{i\theta} + e^{-i\theta} = 2\cos\theta$

$\therefore \cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$ As required

7. $e^{i\theta} = \cos\theta + i\sin\theta$

   $e^{-i\theta} = \cos(-\theta) + i\sin(-\theta) = \cos\theta - i\sin\theta$

   $\therefore e^{i\theta} - e^{-i\theta} = 2i\sin\theta$

   $\therefore \sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$ As required

8. $\cos\theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$

   $\therefore \cos(i) = \frac{1}{2}(e^{i^2} + e^{-i^2}) = \frac{1}{2}(e^{-1} + e^{1}) = \frac{1}{2e} + \frac{e}{2}$

9. $\frac{1}{2}(\sqrt{3} + i) = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} = e^{\frac{i\pi}{6}}$

   $\therefore -i\ln\left(\frac{1}{2}(\sqrt{3} + i)\right) = -i\ln\left(e^{\frac{i\pi}{6}}\right)$

   $-i \times \frac{i\pi}{6} = \frac{\pi}{6}$

10. Multiply by e^x

    $e^{2x} + e^0 = 0$

    $e^{2x} + 1 = 0$

    $e^{2x} = -1$

    $\ln e^{2x} = \ln(-1)$

    $2x = \ln(e^{i(\pi+2n\pi)})$

    $2x = i(\pi + 2n\pi)$

    $x = \frac{i}{2}(\pi + 2n\pi)$