Term 1 Week 5

1. $\int (\frac{1-\sin x}{1+\sin x} \times \frac{1-\sin x}{1-\sin x})dx$

   $\int \frac{1-2\sin x+\sin^2 x}{1-\sin^2 x}dx$

   $\int \frac{1-2\sin x+\sin^2 x}{\cos^2 x}dx$

   $\int \frac{1}{\cos^2 x}dx - \int \frac{2\sin x}{\cos^2 x}dx + \int \frac{\sin^2 x}{\cos^2 x}$

   $\int \sec^2{(x)}dx - \int \frac{2\sin x}{\cos x} \times \frac{1}{\cos x}dx + \int \tan^2{(x)}dx$

   $\int \sec^2{(x)}dx - \int 2\tan{(x)}\sec{(x)}dx + \int (\sec^2{(x)} - 1)dx$

   $= \tan{(x)} - \sec{(x)} + \tan{(x)} - x + c$

   $= 2\tan{(x)} - \sec{(x)} - x + c$

2. Using Pythagoras:
   $\sin^2 x + \sin^2 2x = \sin^2 3x$

   We need to rewrite each term so that they are all in terms of $\sin x$
   $\sin^2 2x = (\sin 2x)^2 = (2\sin x\cos x)^2 = 4\sin^2 x\cos^2 x$
   $= 4\sin^2 x(1-\sin^2 x)$
   $= 4\sin^2 x - 4\sin^4 x$

   $\sin 3x = \sin{(2x+x)} = \sin 2x\cos x + \cos 2x\sin x$
   $= 2\sin x\cos^2 x + (1-2\sin^2 x)\sin x$
   $= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x$
   $= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$
   $= 3\sin x - 4\sin^3 x$

   Therefore, $\sin^3{(3x)} = (3\sin x - 4\sin^3 x)^3 = 16\sin^6 x - 24\sin^4 x + 9\sin^2 x$

   Substituting everything back into the original equation:
   $\sin^2(x) + 4\sin^2(x) - 4\sin^4(x) = 16\sin^6(x) - 24\sin^4(x) + 9\sin^2(x)$
   $16\sin^6(x) - 20\sin^4(x) + 4\sin^2(x) = 0$

   Simplifying and factorising:
   $4\sin^6(x) - 5\sin^4(x) + \sin^2(x) = 0$

$\sin^2(x)(4\sin^4(x) - 5\sin^2(x) + 1) = 0$
$\sin^2(x)(4\sin^2(x) - 1)(\sin^2(x) - 1) = 0$
Difference of two squares:
$\sin^2(x)(2\sin(x) + 1)(2\sin(x) - 1)(\sin(x) + 1)(\sin(x) - 1) = 0$

From here, we just examine each factor and see which gives valid solutions for our triangle.
$sin^2(x) = 0$
$x = 0$ (Not valid)

$2\sin(x) + 1 = 0$
$\sin(x) = -\frac{1}{2}$ (Not valid)

$2\sin(x) - 1 = 0$
$\sin(x) = \frac{1}{2}$
$x = \frac{\pi}{6}$ (valid)

$\sin(x) + 1 = 0$
$\sin(x) = -1$ (Not valid)

$\sin(x) - 1 = 0$
$\sin(x) = 1$
$x = \frac{\pi}{2}$ (Not valid as the side with $\sin(2x)$ will have length of $\sin(\pi) = 0$)

Therefore, the only valid solution is $x = \frac{\pi}{6}$.

3. Separate the RHS into two factors:
   $x^x = 5^x \times 5^{25}$

   Divide by 5^x:
   $\frac{x^x}{5^x} = 5^{25}$

   Simplifying the LHS:
   $(\frac{x}{5})^x = 5^{25}$

   Take the fifth root:
   $(\frac{x}{5})^{\frac{x}{5}} = 5^5$

   Therefore, $\frac{x}{5} = 5$
   $x = 25$