Answers - Sum of Roots (page ??)

1. $z^{11} = 1 = \cos 0 + i \sin 0$

   $z = \cos\left(\frac{2\pi k}{11}\right) + i \sin\left(\frac{2\pi k}{11}\right), k = 0, \pm 1, \pm 2, \pm 3, \pm 4$

   Since $z^{11} = 1$ is the same as $z^{11} + z^{10} + ... - 1 = 0$, we know the sum of the roots is zero.

   Also, since $\cos x$ is an even function, we know that $\cos\left(-\frac{2\pi k}{11}\right) = \cos\left(\frac{2\pi k}{11}\right)$.

   This means that the sum of the roots is:

   $$\begin{aligned}
   \cos 0 + 2\cos\left(\frac{2\pi}{11}\right) + 2\cos\left(\frac{4\pi}{11}\right) + 2\cos\left(\frac{6\pi}{11}\right) + 2\cos\left(\frac{8\pi}{11}\right) + 2\cos\left(\frac{10\pi}{11}\right) &= 0 \\
   1 + 2\cos\left(\frac{2\pi}{11}\right) + 2\cos\left(\frac{4\pi}{11}\right) + 2\cos\left(\frac{6\pi}{11}\right) + 2\cos\left(\frac{8\pi}{11}\right) + 2\cos\left(\frac{10\pi}{11}\right) &= 0 \\
   2\cos\left(\frac{2\pi}{11}\right) + 2\cos\left(\frac{4\pi}{11}\right) + 2\cos\left(\frac{6\pi}{11}\right) + 2\cos\left(\frac{8\pi}{11}\right) + 2\cos\left(\frac{10\pi}{11}\right) &= -1 \\
   \cos\left(\frac{2\pi}{11}\right) + \cos\left(\frac{4\pi}{11}\right) + \cos\left(\frac{6\pi}{11}\right) + \cos\left(\frac{8\pi}{11}\right) + \cos\left(\frac{10\pi}{11}\right) &= -\frac{1}{2}
   \end{aligned}$$

2. $z^5 - 1 = 0$

   $\alpha^5 - 1 = 0$

   $(\alpha - 1)(\alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1) = 0$

   But α is complex, so:

   $\alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1 = 0$

   $\alpha^4 + \alpha^3 + \alpha^2 + \alpha = -1$

   As required.

3. Sum of the roots is $\sin\theta + \cos\theta$

$$
\begin{aligned}
\frac{\sin\theta}{1-\cot\theta}+\frac{\cos\theta}{1-\tan\theta} &= \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{1-\frac{sin\theta}{\cos\theta}}\\
&= \frac{\sin\theta}{\frac{\sin\theta}{\sin\theta}-\frac{\cos\theta}{\sin\theta}}+\frac{\cos\theta}{\frac{\cos\theta}{\cos\theta}-\frac{sin\theta}{\cos\theta}}\\
&= \frac{\sin^2\theta}{\sin\theta-\cos\theta}+\frac{\cos^2\theta}{\cos\theta-\sin\theta}\\
&= \frac{\sin^2\theta-\cos^2\theta}{\sin\theta-\cos\theta}\\
&= \frac{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}{\sin\theta-\cos\theta}\\
&= \sin\theta+\cos\theta
\end{aligned}
$$

As required.