1 Sum of roots of polynomials

The sum of the roots of any polynomial in the form $ax^n + bx^{n-1} + cx^{n-2} + ... + z = 0$ will always be equal to $-\frac{b}{a}$.
We can see that this holds for quadratics in the form $ax^2 + bx + c = 0$ as we know from when we factorise we need to find two numbers that multiply to c and add to b. This gives us the factors, and since the roots are $(x - x_1)$, it means the sum will be $-b$ (which is $\frac{-b}{1}$ since $a = 1$ here).
We can also see this from the quadratic equation: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
If we add the two roots, we get:
$\frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} = -\frac{2b}{2a} = -\frac{b}{a}$
This holds for all polynomials. For example, in the polynomial $p(x) = 2x^4 - x^3 + 2x - 1 = 0$ we know the four roots will sum to $\frac{1}{2}$, since $-(-\frac{1}{2}) = \frac{1}{2}$.

Questions

(Answers - page **??**)

1. Find the roots of the equation $z^{11} = 1$. Use this to show that:
   $\cos\left(\frac{2\pi}{11}\right) + \cos\left(\frac{4\pi}{11}\right) + \cos\left(\frac{6\pi}{11}\right) + \cos\left(\frac{8\pi}{11}\right) + \cos\left(\frac{10\pi}{11}\right) = -\frac{1}{2}$
2. If α is a complex root of the equation $z^5 = 1$, show that $\alpha + \alpha^2 + \alpha^3 + \alpha^4 = -1$
3. The roots of the quadratic equation $ax^2 + bx + c = 0$ are $\sin\theta$ and $\cos\theta$.
   Show that: $\frac{\sin\theta}{1-\cot\theta} + \frac{\cos\theta}{1-\tan\theta} = -\frac{b}{a}$