

Answers - Implicit differentiation (page ??)

$$1. 8x + 4y \times \frac{dy}{dx} = 0$$

$$4y \times \frac{dy}{dx} = -8x$$

$$\frac{dy}{dx} = \frac{-2x}{y}$$

$$2. 6y^2 + 12xy \times \frac{dy}{dx} - 3 \frac{dy}{dx} = 0$$

$$(12xy - 3) \frac{dy}{dx} = -6y^2$$

$$\frac{dy}{dx} = \frac{-2y^2}{4xy - 1}$$

$$3. 10xy^2 + 10x^2y \frac{dy}{dx} - 3y - 3x \frac{dy}{dx} = 0$$

$$(10x^2y - 3x) \frac{dy}{dx} = 3y - 10xy^2$$

$$\frac{dy}{dx} = \frac{3y - 10xy^2}{10x^2y - 3x}$$

$$4. y + x \frac{dy}{dx} + e^y \frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{2-y}{x+e^y}$$

$$\frac{d^2y}{dx^2} = \frac{-(x+e^y) \frac{dy}{dx} - (2-y)(1+e^y \frac{dy}{dx})}{(x+e^y)^2}$$

$$\text{When } x = 0, e^y = 1 \Rightarrow y = 0 \text{ and } \frac{dy}{dx} = \frac{2-0}{0+1} = 2$$

Hence,

$$\frac{d^2y}{dx^2} = \frac{-(x+e^y) \frac{dy}{dx} - (2-y)(1+e^y \frac{dy}{dx})}{(x+e^y)^2}$$

$$= \frac{-(0+1)2 - (2-0)(1+2 \times 2)}{(0+1)^2}$$

$$= -8$$

$$5. \text{ Let } y = \sinh^{-1} x \Rightarrow \sinh y = x$$

$$x = \frac{1}{2}(e^y - e^{-y}) \Rightarrow$$

Differentiating implicitly:

$$1 = \frac{1}{2}(e^y \frac{dy}{dx} + e^{-y} \frac{dy}{dx})$$

$$\frac{dy}{dx} \left(\frac{1}{2}(e^y + e^{-y}) \right) = 1$$

$$\frac{dx}{dy} = \left(\frac{1}{2}(e^y + e^{-y})\right) \Rightarrow$$

$$\frac{dx}{dy} = \cosh y \Rightarrow \frac{dy}{dx} = \frac{1}{\cosh y}$$

$$\text{From the definition: } \sinh^2 x - \cosh^2 x = -1$$

$$\cosh y = \sqrt{(\sinh y)^2 + 1}$$

$$\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{(\sinh y)^2 + 1}} = \frac{1}{\sqrt{x^2 + 1}}$$

$$6. \ x^2 + y^2 = 25$$

$$2x + 2y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\left. \frac{dy}{dx} \right|_{(3,4)} = -\frac{3}{4}$$

$$\frac{dx}{dt} = \frac{dx}{dy} \cdot \frac{dy}{dt}$$

$$\frac{dx}{dt} = -\frac{4}{3} \times -2 = \frac{8}{3}$$