

1 Log problems

You should be familiar with all of the log rules:

$$y = \log_b(x) \iff x = b^y$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

$$\log_b(x^n) = n \log_b(x)$$

$$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$$

When faced with tricky problems involving logs, we use the above rules to manipulate the equations into something we can solve more easily. A common technique is to use the change of base formula to change a log term into a fraction with a different base. For example:

$$\log_8(x) + \log_{16}(x) = 1$$

Notice that both terms have bases which are powers of 2, therefore we will change the base to 2 for each term:

$$\frac{\log_2(x)}{\log_2(8)} = \frac{\log_2(x)}{3}$$

$$\frac{\log_2(x)}{\log_2(16)} = \frac{\log_2(x)}{4}$$

$$\text{Giving us an equation of: } \frac{\log_2(x)}{3} + \frac{\log_2(x)}{4} = 1$$

We can then easily solve:

$$4\log_2(x) + 3\log_2(x) = 12$$

$$7\log_2(x) = 12$$

$$\log_2(x) = \frac{12}{7}$$

$$x = 2^{\frac{12}{7}} = 3.28$$

Another technique is to take the log of both sides to help us rearrange the equation into something easier to solve. For example:

$$x^{\log_2(x)} = 256x^2$$

If we take \log_2 of both sides, we get:

$$\log_2(x^{\log_2(x)}) = \log_2(256x^2)$$

We can now move the power on the LHS out to the front, and also split the RHS into two terms.

$$\log_2(x) \log_2(x) = \log_2(256) + \log_2(x^2)$$

Simplifying:

$$(\log_2(x))^2 = 8 + 2\log_2(x)$$

This is a quadratic where the subject is $\log_2(x)$, so if we do a u-substitution where $u = \log_2(x)$ we get:

$$u^2 - 2u - 8 = 0$$

Solving, we have $u = -2, 4$.

Now we just reverse our substitution to find the value(s) of x:

$$\log_2(x) = -2 \rightarrow x = \frac{1}{4}$$

$$\log_2(x) = 4 \rightarrow x = 16$$

Questions

(Answers - page ??)

1. Solve for x:

$$x^{\log_3(x)} = 81x^3$$

2. Solve for x:

$$\log_4(2^x + 48) = x - 1$$

3. $\log_x(y) + \log_y(x) = 2$ Find the value of $\frac{x}{y} + \frac{y}{x}$

4. If $\sqrt{\log_a(b)} + \sqrt{\log_b(a)} = 2$, then find the value of $\log_{ab}(a) - \log_{\frac{1}{ab}}(b)$

5. If $2^{3x-5} = 3^{x+3}$ and $x = \log(864^{\log_{10}(y)})$, then find the value of $y^{\log_{10} \frac{8}{3}}$

6. Solve for x:

$$\log_7(\log_9(x^2 + \sqrt{x+1} + 8)) = 0$$

7. If $\log_{16}(x) + \log_8(y) = 11$ and $\log_8(x) + \log_{16}(y) = 10$ then find the value of $\frac{y}{x^2}$

8. Solve for x:

$$\log_{\log_2(x)}(4) = \log_2(\log_4(x))$$

9. Solve for x and y:

$$\log_4(x) + \log_9(y) = 2$$

$$\log_x(2) + \log_y(3) = 1$$

10. If $\log_5(4)$, $\log_5(2^x + \frac{1}{2})$ and $\log_5(2^x - \frac{1}{4})$ are in arithmetic progression, find the value of x and also find the common difference.

11. Solve the system:

$$\log_{10}(x^2 + y^2) = 1 + \log_{10}(13)$$

$$\log_{10}(x + y) - \log_{10}(x - y) = 3 \log_{10}(2)$$

12. Evaluate the expression:

$$\frac{1}{1+\log_a(bc)} + \frac{1}{1+\log_b(ac)} + \frac{1}{1+\log_c(ab)}$$