

# 1 Combinations and permutations

Both of these refer to various ways in which objects from a set may be selected, generally without replacement, to form subsets.

A Permutation refers to selecting a subset where the order of selection matters, while a Combination is when the order does not matter.

In other words, Combinations are counting the how many selections we can make from  $n$  objects, while Permutations count the number of arrangements of  $n$  objects.

The formulas for each are below, where  $n$  is the number of objects and  $r$  is the size of the subset:

Permutations:  ${}^nP_r = \frac{n!}{(n-r)!}$

Combinations:  ${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

E.g. If there are 20 people in a room and they all shake hands with each other, how many handshakes are there? In this case, we are asking how many different subsets of size 2 can we select from a group of 20?

Since the order doesn't matter, as person A shaking hands with person B is the same as person B shaking hands with person A, we use the *Combination* equation.

$$\binom{20}{2} = \frac{20!}{2!(20-2)!} = \frac{20!}{2 \times 18!} = \frac{20 \times 19}{2} = 190$$

Notice that we can cancel out parts of the factorials since they have common factors, so that:

$$\frac{20!}{18!} = \frac{20 \times 19 \times \dots \times 2 \times 1}{18 \times 17 \times \dots \times 2 \times 1} = 20 \times 19$$

E.g. If I want to select a Cantamaths team of 4 students from a class of 16, how many different teams are possible?

Again, since the order is not important (team ABCD is the same as team BADC), we use a combination.

$$\binom{16}{4} = \frac{16!}{4!(16-4)!} = \frac{16!}{4! \times 12!} = \frac{16 \times 15 \times 14 \times 13}{4 \times 3 \times 2 \times 1} = 1820$$

## Questions

(Answers - page ??)

1. If there are 10 different people in a room and they all shake each other's hands, how many handshakes are there?
2. (a) 5 boys stand in a line, posing for a photo. How many possible orders are there?  
(b) 3 girls then join the group. How many possible photos are there if the girls must stand next to each other?
3. We have 6 books to distribute to three students A, B and C.  
How many different ways are there of distributing these books if:  
(a) A is given 1 book, B is given 2 books, and C is given 3 books?  
(b) Each student is given 2 books?
4. A company has 20 male employees and 30 female employees. A grievance committee is to be established. If the committee will have 3 male employees and 2 female employees, how many ways can the committee be chosen?
5. Eight candidates are competing to get a job at a prestigious company. The company has the freedom to choose as many as two candidates. In how many ways can the company choose two or fewer candidates.
6. A committee of 5 members must be chosen from a track club. The club has 15 sprinters, 9 jumpers, and 7 long-distance runners. The committee must have exactly 1 jumper and 1 long-distance runner. How many ways can the committee be chosen?
7. There are 10 people forming a commission. Two of them are students from different colleges. The commission is composed of 6 members and if one of the students is in it the other must be as well. How many commissions like these can there be?
8. Using 3 sticks of 5 different colours, how many unique equilateral triangles can be made. Assume you have at least 3 sticks of each colour. Note: if a triangle can be rotated and/or flipped to create another, they are not different.

9. Given  ${}^pC_q = {}^pC_r, q \neq r$ , express  $p$  in terms of  $q$  and  $r$ .
10. There are many integer solutions to the equation  $\binom{n}{r} = \binom{n+1}{r-1}$ , including  $n = r = 1$   
Find an expression for  $n$  in terms of  $r$ , and hence find another of the integer solutions.
11. If  $k$  and  $n$  are positive integers, and  $k < n$ , prove that  $k\binom{n}{k} = n\binom{n-1}{k-1}$
12. Prove that  $\binom{n}{0} + \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} + \cdots + \frac{1}{n+1}\binom{n}{n} = \frac{2^{n+1}-1}{n+1}$