

Answers - Euler's Formula (page ??)

$$\begin{aligned} 1. \quad (-i)^i &= e^{-\frac{i\pi}{2}i} \\ &= e^{-\frac{i^2\pi}{2}} \\ &= e^{\frac{\pi}{2}} \end{aligned}$$

$$2. \quad \text{Since } -1 = e^{i\pi}, \text{ we can write this expression as } \ln(e^{i\pi}) = i\pi$$

$$3. \quad e^{i(A-B)} = e^{iA}e^{-iB}$$

This means that:

$$\begin{aligned} \cos(A-B) + i \sin(A-B) &= (\cos(A) + i \sin(A))(\cos(-B) + i \sin(-B)) \\ &= (\cos(A) + i \sin(A))(\cos(B) - i \sin(B)) \end{aligned}$$

Equating real and imaginary parts:

$$\begin{aligned} \cos(A-B) &= \cos(A)\cos(B) + \sin(A)\sin(B) \\ \sin(A-B) &= \cos(B)\sin(A) - \cos(A)\sin(B) \end{aligned}$$

Substituting $-B$ for B in the second equation:

$$\sin(A+B) = \sin(A)\cos(-B) - \cos(A)\sin(-B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$4. \quad \text{Since } i = e^{\frac{i\pi}{2}}, \text{ we can write the expression as } ((e^{\frac{i\pi}{2}})^i)^2 = (e^{-\frac{\pi}{2}})^2 = e^{-\pi}$$

5. Separating the expression into three terms:

$$\ln(-25e^{ii}) = \ln(-1) + \ln(25) + \ln(e^{ii})$$

Since $-1 = e^{i\pi}$, we can simplify the expression:

$$\begin{aligned} \ln(e^{i\pi}) + \ln(25) + \ln e^{ii} \\ i\pi + \ln(25) + i^i \\ i^i = e^{\frac{i\pi}{2}i} = e^{-\frac{\pi}{2}} \end{aligned}$$

So the expression simplifies to $i\pi + \ln(25) + e^{-\frac{\pi}{2}}$

$$6. \quad e^{i\theta} = \cos \theta + i \sin \theta$$

$$e^{-i\theta} = \cos(-\theta) + i \sin(-\theta) = \cos \theta - i \sin \theta$$

$$\therefore e^{i\theta} + e^{-i\theta} = 2 \cos \theta$$

$$\therefore \cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta}) \text{ As required}$$

$$7. e^{i\theta} = \cos \theta + i \sin \theta$$

$$e^{-i\theta} = \cos(-\theta) + i \sin(-\theta) = \cos \theta - i \sin \theta$$

$$\therefore e^{i\theta} - e^{-i\theta} = 2i \sin \theta$$

$$\therefore \sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta}) \text{ As required}$$

$$8. \cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta})$$

$$\therefore \cos(i) = \frac{1}{2}(e^{i^2} + e^{-i^2}) = \frac{1}{2}(e^{-1} + e^1) = \frac{1}{2e} + \frac{e}{2}$$

$$9. \frac{1}{2}(\sqrt{3} + i) = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} = e^{\frac{i\pi}{6}}$$

$$\therefore -i \ln \left(\frac{1}{2}(\sqrt{3} + i) \right) = -i \ln \left(e^{\frac{i\pi}{6}} \right)$$

$$-i \times \frac{i\pi}{6} = \frac{\pi}{6}$$

$$10. \text{ Multiply by } e^x$$

$$e^{2x} + e^0 = 0$$

$$e^{2x} + 1 = 0$$

$$e^{2x} = -1$$

$$\ln e^{2x} = \ln(-1)$$

$$2x = \ln(e^{i(\pi+2n\pi)})$$

$$2x = i(\pi + 2n\pi)$$

$$x = \frac{i}{2}(\pi + 2n\pi)$$