

## Term 3 Week 1

Solve for  $x$ :

1.  $\sqrt{x} + \sqrt{-x} = 2$

$$-x = i^2 x$$

$$\sqrt{x} + \sqrt{i^2 x} = 2$$

$$\sqrt{x} + i\sqrt{x} = 2$$

$$(1 + i)\sqrt{x} = 2$$

$$\sqrt{x} = \frac{2}{1+i}$$

$$x = \frac{4}{(1+i)^2} = \frac{4}{2i}$$

Rationalising:

$$x = \frac{4}{2i} \times \frac{2i}{2i} = \frac{8i}{-4} = -2i$$

Considering the original equation, we know that  $x = 2i$  is also a solution.

2.  $\ln(x) + \ln(-x) = 0$

$$\ln(-x^2) = 0$$

Using the log rule,  $e^0 = -x^2 \Rightarrow -x^2 = 1$

$$x^2 = -1$$

$$x^2 = i^2$$

$$x = \pm i$$

3.  $e^x + e^{-x} = 0$

Multiply by  $e^x$ :

$$e^{2x} + 1 = 0$$

$$e^{2x} = -1$$

Take natural log of both sides:

$$2x = \ln(-1)$$

Using Euler's Identity of  $e^{i\pi} = -1$ , we get:

$$2x = \ln e^{i\pi}$$

$$2x = i\pi$$

$$x = \frac{i\pi}{2}$$