

Answers - Turning problems into quadratics (page ??)

1. $(2^2)^x + 2^x - 24 = 0$

$$(2^x)^2 + 2^x - 24 = 0$$

Making the substitution $u = 2^x$

$$u^2 + 2u - 24 = 0$$

$$u = 4.42, -5.42$$

2^x can never be negative so we can ignore the -5.42 solution.

$$2^x = 4.42$$

$$\ln 2^x = \ln 4.42$$

$$x \ln 2 = \ln 4.42$$

$$x = \frac{\ln 4.42}{\ln 2}$$

$$x = 2.14 \text{ (1dp)}$$

2. Rearrange to $9^x - 6^x - 4^x = 0$

We need a constant so divide through by the lowest term.

$$\frac{9^x}{4^x} - \frac{6^x}{4^x} - \frac{4^x}{4^x} = 0$$

$$\left(\frac{9}{4}\right)^x - \left(\frac{6}{4}\right)^x - 1 = 0$$

$$\left(\left(\frac{3}{2}\right)^2\right)^x - \left(\frac{3}{2}\right)^x - 1 = 0$$

$$\left(\left(\frac{3}{2}\right)^x\right)^2 - \left(\frac{3}{2}\right)^x - 1 = 0$$

Use the substitution $u = \left(\frac{3}{2}\right)^x$

$$u^2 - u - 1 = 0$$

$$u = 1.618, -0.618$$

$\left(\frac{3}{2}\right)^x$ can never be negative so we ignore -0.618.

$$\left(\frac{3}{2}\right)^x = 1.618$$

$$\ln \left(\frac{3}{2}\right)^x = \ln 1.618$$

$$x \ln \left(\frac{3}{2}\right) = \ln 1.618$$

$$x = \frac{\ln 1.618}{\ln \frac{3}{2}}$$

$$x = 1.187$$

3. We need a constant so divide through by the lowest term.

$$8\left(\frac{9^x}{4^x}\right) + 3\left(\frac{6^x}{4^x}\right) - 81 = 0$$

$$8\left(\frac{9}{4}\right)^x + 3\left(\frac{6}{4}\right)^x - 81 = 0$$

$$8\left(\left(\frac{3}{2}\right)^2\right)^x + 3\left(\frac{3}{2}\right)^x - 81 = 0$$

$$8\left(\left(\frac{3}{2}\right)^x\right)^2 + 3\left(\frac{3}{2}\right)^x - 81 = 0$$

Use the substitution $u = \left(\frac{3}{2}\right)^x$

$$8u^2 + 3u - 81 = 0$$

$$u = 3, -3.375$$

Since $\left(\frac{3}{2}\right)^x$ can never be negative, we can ignore the -3.375 solution.

$$\left(\frac{3}{2}\right)^x = 3$$

$$\ln\left(\frac{3}{2}\right)^x = \ln 3$$

$$x \ln\left(\frac{3}{2}\right) = \ln 3$$

$$x = \frac{\ln 3}{\ln \frac{3}{2}} = 2.71$$

4. We need a constant so divide through by the lowest term.

$$\left(\frac{25^x}{9^x}\right) + 2\left(\frac{15^x}{9^x}\right) - 24 = 0$$

$$\left(\frac{25}{9}\right)^x + 2\left(\frac{15}{9}\right)^x - 24 = 0$$

$$\left(\left(\frac{5}{3}\right)^2\right)^x + 2\left(\frac{5}{3}\right)^x - 24 = 0$$

$$\left(\left(\frac{5}{3}\right)^x\right)^2 + 2\left(\frac{5}{3}\right)^x - 24 = 0$$

Use the substitution $u = \left(\frac{5}{3}\right)^x$

$$u^2 + 2u - 24 = 0$$

$$u = 4, -6$$

Since $\left(\frac{5}{3}\right)^x$ can never be negative, we can ignore the -6 solution.

$$\left(\frac{5}{3}\right)^x = 4$$

$$\ln\left(\frac{5}{3}\right)^x = \ln 4$$

$$x \ln\left(\frac{5}{3}\right) = \ln 4$$

$$x = \frac{\ln 4}{\ln \frac{5}{3}} = 2.714$$