Answers - Exact trig values (page ??)

1. $\cos 45 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

2. $\sin 105 = \sin(60+45) = \sin 60 \cos 45 + \cos 60 \sin 45$

   $= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$

   $= \frac{\sqrt{3}+1}{2\sqrt{2}}$

   Rationalising by multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$:

   $= \frac{\sqrt{6}+\sqrt{2}}{4}$

3. $\tan 60 = \sqrt{3}$

4. $\cos \frac{7\pi}{12} = \cos\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

   $= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

   $= \frac{1}{2} \times \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

   $= \frac{1-\sqrt{3}}{2\sqrt{2}}$

   Rationalise by multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$

   $= \frac{\sqrt{2}-\sqrt{6}}{4}$

5. $\cos \frac{\pi}{12} = \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right) = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

   $= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$

   $= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

   $= \frac{1+\sqrt{3}}{2\sqrt{2}}$

   Rationalise by multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$

   $= \frac{\sqrt{2}+\sqrt{6}}{4}$

6. $\tan\left(\frac{2\pi}{3}\right) = \tan\left(2 \times \frac{\pi}{3}\right)$

   $= \frac{2\tan\left(\frac{\pi}{3}\right)}{1-\tan^2\left(\frac{\pi}{3}\right)}$

   $= \frac{2\times\sqrt{3}}{1-(\sqrt{3})^2}$

   $= \frac{2\sqrt{3}}{1-3} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$

7. $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

   $= \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$

   $= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$

   $= \frac{\sqrt{3}-1}{2\sqrt{2}}$

   Rationalising by multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$:

   $= \frac{\sqrt{6}-\sqrt{2}}{4}$

8. $\sin\left(-\frac{4\pi}{3}\right) = -\sin\left(\frac{4\pi}{3}\right)$ (Since sine is an odd function)

   $= -\sin\left(\pi + \frac{\pi}{3}\right)$

   $= -\left(\sin(\pi)\cos\left(\frac{\pi}{3}\right) + \cos(\pi)\sin\left(\frac{\pi}{3}\right)\right)$

   $= -\left(0 - \frac{\sqrt{3}}{2}\right)$

   $= \frac{\sqrt{3}}{2}$

9. $\sin\left(\frac{7\pi}{12}\right) = \sin\left(2\pi - \frac{\pi}{4}\right)$

   $= \sin(2\pi)\cos\left(\frac{\pi}{4}\right) - \cos(2\pi)\sin\left(\frac{\pi}{4}\right)$

   $= 0 - \frac{1}{\sqrt{2}}$

   $= -\frac{1}{\sqrt{2}}$

   $= -\frac{\sqrt{2}}{2}$

10. $\tan\left(\frac{3\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right)$

    $= \frac{\tan(\pi) - \tan\left(\frac{\pi}{4}\right)}{1 - \tan(\pi)\tan\left(\frac{\pi}{4}\right)}$

    $= \frac{0-1}{1-0\times 1}$

    $= -1$

11. $\theta = 18$

    $5\theta = 90$

    $2\theta + 3\theta = 90$

    $2\theta = 90 - 3\theta$

    $\sin 2\theta = \sin(90 - 3\theta)$

    $2\sin\theta\cos\theta = \sin 90 \cos 3\theta - \cos 90 \sin 3\theta$

$2\sin\theta\cos\theta = \cos 3\theta$

$2\sin\theta\cos\theta = \cos(2\theta+\theta)$

$2\sin\theta\cos\theta = \cos 2\theta\cos\theta - \sin 2\theta\sin\theta$

Use double angle rules for both cosine and sine:

$2\sin\theta\cos\theta = (1-2\sin^2\theta)\cos\theta - 2\sin^2\theta\cos\theta$

Divide through by $\cos\theta$:

$2\sin\theta = (1-2\sin^2\theta) - 2\sin^2\theta$

$2\sin\theta = 1 - 4\sin^2\theta$

Form a quadratic and solve:

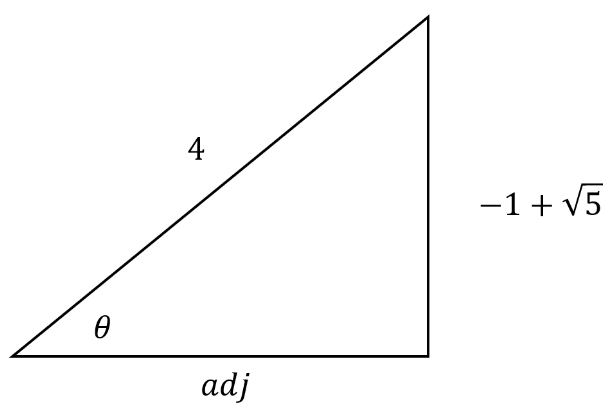
$4\sin^2\theta + 2\sin\theta - 1 = 0$

$\sin\theta = \frac{-2\pm\sqrt{20}}{8} = \frac{-2\pm2\sqrt{5}}{8} = \frac{-1\pm\sqrt{5}}{4}$

Because we know $\sin 18$ is positive we can disregard the negative solution:

$\sin 18 = \frac{-1+\sqrt{5}}{4}$

Using a right-angle triangle we can now find the value of $\cos 18$



$(adj)^2 = 4^2 - (-1+\sqrt{5})^2$

$(adj)^2 = 10 + 2\sqrt{5}$

$adj = \sqrt{10+2\sqrt{5}}$

$\cos 18 = \frac{\sqrt{10+2\sqrt{5}}}{4}$

12. $\theta = 36$

    $5\theta = 180$

    $2\theta + 3\theta = 180$

    $2\theta = 180 - 3\theta$

    $\sin 2\theta = \sin(180 - 3\theta)$

$2\sin\theta\cos\theta = \sin 180\cos 3\theta - \cos 180\sin 3\theta$

$2\sin\theta\cos\theta = \sin 3\theta$

$2\sin\theta\cos\theta = \sin(2\theta+\theta)$

$2\sin\theta\cos\theta = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

Use double angles rules for both sine and cosine:

$2\sin\theta\cos\theta = 2\sin\theta\cos^2\theta + (2\cos^2\theta - 1)\sin\theta$

Divide through by $\sin\theta$:

$2\cos\theta = 2\cos^2\theta + (2\cos^2\theta - 1)$

Form a quadratic:

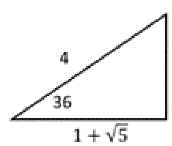
$4\cos^2\theta - 2\cos\theta - 1 = 0$

$\cos\theta = \frac{2\pm\sqrt{20}}{8}$

Since we know that $\cos 36$ is positive, we can ignore the negative:

$\cos\theta = \cos 36 = \frac{1+\sqrt{5}}{4}$

We can use this to find $\sin 36$ by substituting into a right-angle triangle:



Now we can use Pythagoras to find the opposite side, which then can be used to find $\sin 36$:

Opposite $= \sqrt{4^2 - (1+\sqrt{5})^2} = \sqrt{10-2\sqrt{5}}$

This means that $\sin 36 = \frac{O}{H} = \frac{\sqrt{10-2\sqrt{5}}}{4}$

13. $\theta = \frac{2\pi}{5}$

$5\theta = 2\pi$

$2\theta = 2\pi - 3\theta$

$\sin 2\theta = \sin(2\pi - 3\theta)$

$2\sin\theta\cos\theta = \sin 2\pi\cos 3\theta - \cos 2\pi\sin 3\theta$

$2\sin\theta\cos\theta = -\sin 3\theta$

$2\sin\theta\cos\theta = -\sin(2\theta+\theta)$

$2\sin\theta\cos\theta = -(\sin 2\theta\cos\theta + \cos 2\theta\sin\theta)$

$2\sin\theta\cos\theta = -2\sin\theta\cos^2\theta - (2\cos^2\theta - 1)\sin\theta$

Divide through by $\sin\theta$:

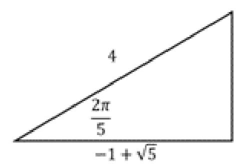
$2\cos\theta = -2\cos^2\theta - (2\cos^2\theta - 1)$

Form a quadratic:

$4\cos^{\theta} + 2\cos\theta - 1 = 0$

$\cos\theta = \cos\left(\frac{2\pi}{5}\right) = \frac{-2\pm\sqrt{20}}{8} = \frac{-1\pm\sqrt{5}}{4}$

We can use this to find $\sin\frac{2\pi}{5}$ by substituting it into a right-angle triangle:



Now we can use Pythagoras to find the opposite side, which then can be used to find $\sin\frac{2\pi}{5}$

Opposite $= \sqrt{4^2 - (-1+\sqrt{5})^2} = \sqrt{10+2\sqrt{5}}$

This means $\sin\frac{2\pi}{5} = \frac{O}{H} = \frac{\sqrt{10+2\sqrt{5}}}{4}$