Answers - Implicit differentiation (page ??)

1. $8x + 4y \times \dfrac{dy}{dx} = 0$

$$4y \times \frac{dy}{dx} = -8x$$

$$\frac{dy}{dx} = \frac{-2x}{y}$$

2. $6y^2 + 12xy \times \dfrac{dy}{dx} - 3\dfrac{dy}{dx} = 0$

$$(12xy - 3)\frac{dy}{dx} = -6y^2$$

$$\frac{dy}{dx} = \frac{-2y^2}{4xy - 1}$$

3. $10xy^2 + 10x^2y\dfrac{dy}{dx} - 3y - 3x\dfrac{dy}{dx} = 0$

$$(10x^2y - 3x)\frac{dy}{dx} = 3y - 10xy^2$$

$$\frac{dy}{dx} = \frac{3y - 10xy^2}{10x^2y - 3x}$$

4. $y + x\frac{dy}{dx} + e^y\frac{dy}{dx} = 2$

$\frac{dy}{dx} = \frac{2-y}{x+e^y}$

$\frac{d^2y}{dx^2} = \frac{-(x+e^y)\frac{dy}{dx} - (2-y)(1+e^y\frac{dy}{dx})}{(x+e^y)^2}$

When $x = 0, e^y = 1 \Rightarrow y = 0$ and $\frac{dy}{dx} = \frac{2-0}{0+1} = 2$

Hence,

$$\frac{d^2y}{dx^2} = \frac{-(x+e^y)\frac{dy}{dx} - (2-y)(1+e^y\frac{dy}{dx})}{(x+e^y)^2}$$

$$= \frac{-(0+1)2 - (2-0)(1+2\times 2)}{(0+1)^2}$$

$$= -8$$

5. Let$y = \sinh^{-1} x \Rightarrow \sinh y = x$

$x = \frac{1}{2}(e^y - e^{-y}) \Rightarrow$

Differentiating implicitly:

$1 = \frac{1}{2}(e^y\frac{dy}{dx} + e^{-y}\frac{dy}{dx})$

$\frac{dy}{dx}(\frac{1}{2}(e^y + e^{-y})) = 1$

$\frac{dx}{dy} = (\frac{1}{2}(e^y + e^{-y})) \Rightarrow$

$\frac{dx}{dy} = \cosh y \Rightarrow \frac{dy}{dx} = \frac{1}{\cosh y}$

From the definition: $\sinh^2 x - \cosh^2 x = -1$

$\cosh y = \sqrt{(\sinh y)^2 + 1}$

$\frac{dy}{dx} = \frac{1}{\cosh y} = \frac{1}{\sqrt{(\sinh y)^2+1}} = \frac{1}{\sqrt{x^2+1}}$

6. $x^2 + y^2 = 25$

$2x + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{x}{y}$

$\frac{dy}{dx}|_{(3,4)} = -\frac{3}{4}$

$\frac{dx}{dt} = \frac{dx}{dy}.\frac{dy}{dt}$

$\frac{dx}{dt} = -\frac{4}{3} \times -2 = \frac{8}{3}$