Answers - Camel principle (page ??)

1. $\int \frac{x}{x+1}\,dx = \int \frac{x+1-1}{x+1}\,dx$
   $= \int \frac{x+1}{x+1}\,dx - \int \frac{1}{x+1}\,dx$
   $= \int 1\,dx - \int \frac{1}{x+1}\,dx$
   $= x - \ln|x+1| + c$

2. $\int \frac{1}{1+e^x}\,dx = \int \frac{1+e^x-e^x}{1+e^x}\,dx$
   $= \int \frac{1+e^x}{1+e^x}\,dx - \int \frac{e^x}{1+e^x}\,dx$
   $= \int 1\,dx - \int \frac{e^x}{1+e^x}\,dx$
   $= x - \ln|1+e^x| + c$

3. $\int \frac{2}{2+e^{2x}}\,dx = \int \frac{2+e^{2x}-e^{2x}}{2+e^{2x}}\,dx$
   $= \int \frac{2+e^{2x}}{2+e^{2x}}\,dx - \int \frac{e^{2x}}{2+e^{2x}}\,dx$
   Change the second integral into the form $\int \frac{f'(x)}{f(x)}\,dx$:
   $= \int 1\,dx + \frac{1}{2}\int \frac{2e^{2x}}{2+e^{2x}}\,dx$
   $= x + \frac{1}{2}\ln|2+e^{2x}| + c$

4. $\int \frac{18x}{9x^2-24x+16}\,dx = \int \frac{18x-24+24}{9x^2-24x+16}\,dx$
   $= \int \frac{18x-24}{9x^2-24x+16}\,dx + \int \frac{24}{9x^2-24x+16}\,dx$
   $= \ln|9x^2-24x+16| + \int \frac{24}{(3x-4)^2}\,dx$
   To solve the second integral, use the substitution $u = 3x-4$:
   $du = 3dx \to \frac{1}{3}du = dx$
   Rewrite the second integral in terms of u:
   $= \ln|9x^2-24x+16| + \int \frac{8}{u^2}\,du$
   $= \ln|9x^2-24x+16| + 8\int u^{-2}\,du$
   $= \ln|9x^2-24x+16| - \frac{8}{u} + c$
   $= \ln|9x^2-24x+16| - \frac{8}{3x-4} + c$

5. $\int \frac{1}{1+\sqrt{e^x}}\,dx = \int \frac{1+\sqrt{e^x}-\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$
   $= \int \frac{1+\sqrt{e^x}}{1+\sqrt{e^x}}\,dx - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$
   $= \int 1\,dx - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$
   $x - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$
   For the remaining integral, use the substitution$u = \sqrt{e^x}$, meaning that $u^2 = e^x$.
   $x = \ln u^2 = 2\ln u$

$dx = \frac{2}{u}\, du$

$x - \int \frac{u}{1+u} \frac{2\, du}{u} = x - 2\int \frac{1}{1+u}\, du$

$x - 2\ln|1+u| + c$

$x - 2\ln|1+\sqrt{e^x}| + c$

6. $7\int \frac{x}{4x^2+20x+25}\, dx$

   We know the denominator differentiates to $8x+20$ so first we will change the numerator to $8x$ by using the Camel Principle multiplicatively:

   $\frac{7}{8}\int \frac{8x}{4x^2+20x+25}\, dx$

   Next, we use it additively to get the 20 we need in the numerator:

   $\frac{7}{8}\int \frac{8x+20-20}{4x^2+20x+25}\, dx = \frac{7}{8}\int \frac{8x+20}{4x^2+20x+25}\, dx - \frac{7}{8}\int \frac{20}{4x^2+20x+25}\, dx$

   $= \frac{7}{8}\ln|4x^2+20x+25 - \frac{7}{8}\int \frac{20}{4x^2+20x+25}\, dx$

   The denominator of the second integral factorises to $(2x+5)^2$, so we can use the substitution $u = 2x+5$

   $du = 2\, dx \to \frac{1}{2}du = dx$

   $= \frac{7}{8}\ln|4x^2+20x+25| - \frac{7}{8}\cdot\frac{1}{2}\int \frac{20}{u^2}\, du$

   $= \frac{7}{8}\ln|4x^2+20x+25| - \frac{7}{16}\int \frac{20}{u^2}\, du$

   $= \frac{7}{8}\ln|4x^2+20x+25| - \frac{140}{16}\int u^{-2}\, du$

   $= \frac{7}{8}\ln|4x^2+20x+25| + \frac{35}{4u} + c$

   $= \frac{7}{8}\ln|4x^2+20x+25| + \frac{35}{8x+20} + c$

7. $\int \sec x\, dx$

   In this case we will use the Camel Principle multiplicatively, multiplying by $\frac{\sec x+\tan x}{\sec x+\tan x}$

   This gives us the integral:

   $\int \frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x}\, dx$

   This is in the format $\frac{f'(x)}{f(x)}$, which integrates to $\ln|f(x)| + c$

   Therefore, our integral is $\ln|\sec x + \tan x| + c$

8. $\int \csc\theta\, d\theta$

   To integrate, first multiply by $\frac{\csc\theta-\cot\theta}{\csc\theta+\cot\theta}$

   This changes the integral to:

   $\int \frac{\csc^2\theta-\csc\theta\cot\theta}{\csc\theta-\cot\theta}$

   This is in the form $\frac{f'(x)}{f(x)}$, therefore the integral is $\ln|\csc\theta - \tan\theta| + c$

9. $\int \frac{1}{1+\tan x}\,dx$

Change the $\tan x$ into $\frac{\sin x}{\cos x}$ and simplify:

$\int \frac{1}{1+\frac{\sin x}{\cos x}}\,dx$

$\int \frac{1}{\frac{\cos x+\sin x}{\cos x}}\,dx$

$\int \frac{\cos x}{\sin x+\cos x}\,dx$

Now we can use the Camel Principle. First, we double the fraction:

$\frac{1}{2}\int \frac{2\cos x}{\sin x+\cos x}\,dx$

Then we add and subtract $\sin x$ from the numerator:

$\frac{1}{2}\int \frac{2\cos x+\sin x-\sin x}{\sin x+\cos x}\,dx$

Separate into two fractions:

$\frac{1}{2}\int \left(\frac{\cos x+\sin x}{\sin x+\cos x}+\frac{\cos x-\sin x}{\sin x+\cos x}\right)dx$

Split into two integrals and simplify:

$\frac{1}{2}\int 1\,dx+\frac{1}{2}\int \frac{\cos x-\sin x}{\sin x+\cos x}\,dx$

The first fraction integrates easily. The second integral is in the form $\int \frac{f'(x)}{f(x)}\,dx$, therefore:

$\frac{x}{2}+\frac{1}{2}\ln|\sin x+\cos x|+c$