Term 2 Week 3

1. Suppose r, s, and t are non-zero real numbers such that the polynomial $x^2 + rx + s$ has s and t as roots, and the polynomial $x^2 + tx + r$ has 5 as a root. Compute s.

2. Suppose a and b are positive integers. Isabella and Vidur both fill up an $a \times b$ table. Isabella fills it up with numbers $1, 2, 3...ab$, putting the numbers $1, 2...b$ in the first row, $b + 1, b + 2...2b$ in the second row, and so on. Vidur fills it up like a multiplication table, putting ij in the cell in row i and column j.

   Examples are shown for a 3 x 4 table below:

1	2	3	4
5	6	7	8
9	10	11	12

Isabella's grid

1	2	3	4
2	4	6	8
3	6	9	12

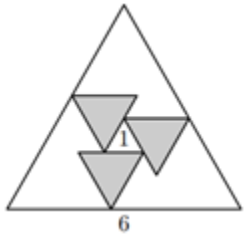
Vidur's grid

   Isabella sums up the numbers in her grid, and Vidur sums up the numbers in his grid. The difference between the sums is 1200. Compute $a + b$.

3. Given $f(x) = x + \frac{1}{2x+\frac{1}{2x+\frac{1}{2x+...}}}$

   Determine the value of $f(99).f'(99)$.

4. Inside an equilateral triangle of side length 6, three congruent equilateral triangles of side length x with sides parallel to the original equilateral triangle are arranged so that each has a vertex on a side of the larger triangle, and a vertex on another one of the smaller triangles, as shown below:



   A smaller equilateral triangle formed between the three congruent equilateral triangles has side length 1. Find the length of x.

5. Compute the sum of all integers n such that $n^2 - 3000$ is a perfect square.