Answers - Trigonometric identities (page ??)

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For each of the following, show that:

1. LHS=$\frac{\sin A+\cos A}{\sin A-\cos A} \times \frac{\sin A+\cos A}{\sin A+\cos A}$

   $\frac{\sin^2 A+2\sin A\cos A+\cos^2 A}{\sin^2 A-\cos^2 A}$

   Using the $\sin^2 A + \cos^2 A = 1$ and the $\cos 2A$ identities:

   $\frac{1+2\sin A\cos A}{1-2\cos^2 A}$=RHS as required

2. LHS=$\frac{\sin 2A}{1+\cos 2A}$

   Using cosine double angle rule:

   $= \frac{\sin 2A}{1+2\cos^2 A-1}$

   $= \frac{2\sin A\cos A}{2\cos^2 A}$

   $= \frac{\sin A}{\cos A}$

   $= \tan A$ as required

3. LHS=$2\sin A\cos A$

   RHS=$\frac{\frac{2\sin A}{\cos A}}{1+\frac{\sin^2 A}{\cos^2 A}}$

   $=\frac{\frac{2\sin A}{\cos A}}{1+\frac{\sin^2 A}{\cos^2 A}} \times \frac{\cos^2 A}{\cos^2 A}$

   $=\frac{2\sin A\cos A}{\cos^2 A+\sin^2 A}$

   $=2\sin A\cos A$ =RHS as required

4. $\frac{\sin 2A}{\sin A} - \frac{\cos 2A}{\cos A} = \sec A$

   LHS=$\frac{2\sin A\cos A}{\sin A} - \frac{2\cos^2 A-1}{\cos A}$

   $=2\cos A - 2\cos A + \frac{1}{\cos A}$

   $=\sec A$ =RHS as required

5. $(\sec A - \tan A)^2 = \frac{1-\sin A}{1+\sin A}$

6. $\tan A = \sqrt{\frac{1-\cos 2A}{1+\cos 2A}}$

7. $\frac{\csc^2 A-1}{\cos^2 A} + \frac{1}{1-\sin^2 A} = \sec^2 A\csc^2 A$

8. $\frac{\cos A}{1+\sin A} = \frac{1-\sin A}{\cos A}$

9. $2\csc 4A + 2\cot 4A = \cot A - \tan A$

10. $\frac{\sin 3A}{\sin 2A-\sin A} = 2\cos A + 1$

11. $\frac{1+\cos A}{1-\cos A} = (\csc A + \cot A)^2$

12. $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$
13. $\cos 3A = 4\cos^3 A - 3\cos A$
14. $\cos 4A = 1 - 8\sin^2 A \cos^2 A$
15. $\sin 5A = 16\sin^5 A - 20\sin^3 A + 5\sin A$
16. $\tan 3A = \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$
17. $\tan 4A = \frac{4\tan A - 4\tan^3 A}{1-6\tan^2 A+\tan^4 A}$
18. $4\sin^3 A\cos 3A + 4\cos^3 A \sin 3A = 3\sin 4A$

Harder problems (including old scholarship questions):

19. $\frac{\csc A - \cot A}{\csc A + \cot A} + \frac{\csc A + \cot A}{\csc A - \cot A} \equiv 2 + 4\cot^2 A$
20. $\frac{1-\sin A}{1-\sec A} - \frac{1+\sin A}{1+\sec A} \equiv 2\cot A(\cos A - \csc A)$
21. $\frac{1+\cos A}{1-\cos A} \equiv (\csc A + \cot A)^2$
22. $\frac{\sin(\pi - B) - \sin A}{\cos A + \cos(\pi - B)} \equiv \frac{\cos A + \cos B}{\sin B + \sin(\pi - A)}$
23. $\frac{\csc A - \sec A}{\csc A + \sec A}(\cot A - \tan A) \equiv \sec A \csc A - 2$
24. $(\sec A - 2\sin A)(\csc A + 2\cos A)\sin A \cos A \equiv (\cos^2 A - \sin^2 A)^2$
25. 2018 Scholarship exam:
    $\frac{\cos\theta}{1+\sin\theta} - \frac{\sin\theta}{1+\cos\theta} = \frac{2(\cos\theta - \sin\theta)}{1+\sin\theta+\cos\theta}$
26. 2017 Scholarship exam:
    $\cos(5\theta) = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$