

Answers - Sum of Roots (page ??)

1. $z^{11} = 1 = \cos 0 + i \sin 0$

$$z = \cos\left(\frac{2\pi k}{11}\right) + i \sin\left(\frac{2\pi k}{11}\right), k = 0, \pm 1, \pm 2, \pm 3, \pm 4$$

Since $z^{11} = 1$ is the same as $z^{11} + z^{10} + \dots - 1 = 0$, we know the sum of the roots is zero.

Also, since $\cos x$ is an even function, we know that $\cos\left(-\frac{2\pi k}{11}\right) = \cos\left(\frac{2\pi k}{11}\right)$.

This means that the sum of the roots is:

$$\begin{aligned}\cos 0 + 2 \cos\left(\frac{2\pi}{11}\right) + 2 \cos\left(\frac{4\pi}{11}\right) + 2 \cos\left(\frac{6\pi}{11}\right) + 2 \cos\left(\frac{8\pi}{11}\right) + 2 \cos\left(\frac{10\pi}{11}\right) &= 0 \\ 1 + 2 \cos\left(\frac{2\pi}{11}\right) + 2 \cos\left(\frac{4\pi}{11}\right) + 2 \cos\left(\frac{6\pi}{11}\right) + 2 \cos\left(\frac{8\pi}{11}\right) + 2 \cos\left(\frac{10\pi}{11}\right) &= 0 \\ 2 \cos\left(\frac{2\pi}{11}\right) + 2 \cos\left(\frac{4\pi}{11}\right) + 2 \cos\left(\frac{6\pi}{11}\right) + 2 \cos\left(\frac{8\pi}{11}\right) + 2 \cos\left(\frac{10\pi}{11}\right) &= -1 \\ \cos\left(\frac{2\pi}{11}\right) + \cos\left(\frac{4\pi}{11}\right) + \cos\left(\frac{6\pi}{11}\right) + \cos\left(\frac{8\pi}{11}\right) + \cos\left(\frac{10\pi}{11}\right) &= -\frac{1}{2}\end{aligned}$$

2. $z^5 - 1 = 0$

$$\alpha^5 - 1 = 0$$

$$(\alpha - 1)(\alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1 = 0)$$

But α is complex, so:

$$\alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1 = 0$$

$$\alpha^4 + \alpha^3 + \alpha^2 + \alpha = -1$$

As required.

3. Sum of the roots is $\sin \theta + \cos \theta$

$$\begin{aligned}
\frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta} &= \frac{\sin \theta}{1 - \frac{\cos \theta}{\sin \theta}} + \frac{\cos \theta}{1 - \frac{\sin \theta}{\cos \theta}} \\
&= \frac{\sin \theta}{\frac{\sin \theta}{\sin \theta} - \frac{\cos \theta}{\sin \theta}} + \frac{\cos \theta}{\frac{\cos \theta}{\cos \theta} - \frac{\sin \theta}{\cos \theta}} \\
&= \frac{\sin^2 \theta}{\sin \theta - \cos \theta} + \frac{\cos^2 \theta}{\cos \theta - \sin \theta} \\
&= \frac{\sin^2 \theta - \cos^2 \theta}{\sin \theta - \cos \theta} \\
&= \frac{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)}{\sin \theta - \cos \theta} \\
&= \sin \theta + \cos \theta
\end{aligned}$$

As required.