

## Answers - Binomial expansion (page ??)

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- $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
- $(2x + y)^4 = (2x)^4 + 4(2x)^3y + 6(2x)^2y^2 + 4(2x)y^3 + y^4$   
 $= 16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$
- $(2x - 3)^5 = (2x)^5 + 5(2x)^4(-3) + 10(2x)^3(-3)^2 + 10(2x)^2(-3)^3 + 5(2x)(-3)^4 + (-3)^5$   
 $= 32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x - 243$
- $(3x + 2y)^4 = (3x)^4 + 4(3x)^3(2y) + 6(3x)^2(2y)^2 + 4(3x)(2y)^3 + (2y)^4$   
 $= 81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$
- $(2x + \frac{1}{x^2})^4 = (2x)^4 + 4(2x)^3(\frac{1}{x^2}) + 6(2x)^2(\frac{1}{x^2})^2 + 4(2x)(\frac{1}{x^2})^3 + (\frac{1}{x^2})^4$   
 $= 16x^4 + 32x + \frac{24}{x^2} + \frac{8}{x^5} + \frac{1}{x^8}$
- We need to find when the powers in a term cancel out and leave a constant.

$$(3x^2)^m (\frac{-1}{3x})^n$$

We can form two equations from this:

$$\frac{x^{2m}}{x^n} = x^0$$

$$2m - n = 0$$

And we know in this question that  $m + n = 12$

Solving, we get  $m = 4, n = 8$ .

This means that if we look in row 12, we look for the column where  $m = 4$  to get the coefficient.

Therefore, our term is  $495(3x)^4(\frac{-1}{3x})^8 = \frac{495}{81} = \frac{55}{9}$

$r \backslash n$	0	1	2	3	4	5	6	7	8	9	10
12	1	12	66	220	495	792	924	792	495	220	66

- We need to find when the powers in a term cancel out to give  $x^2$

Forming two equations from  $(x^2)^m (\frac{1}{x})^n$

$$\frac{x^{2m}}{x^n} = x^2 \rightarrow 2m - n = 2$$

Also,  $m + n = 10$

Solving, we get  $m = 4, n = 6$

From row 10, we see that when  $m = 4$ , the coefficient is 210.

Therefore, our term is  $210(x^2)^4(\frac{1}{x})^6 = 210x^2$

$r \backslash n$	0	1	2	3	4	5	6	7	8	9	10
10	1	10	45	120	210	252	210	120	45	10	1

8. Forming two equations from  $(2x^2)^m(\frac{-3}{x})^n$

$$\frac{x^2m}{x^n} = x^0 \rightarrow 2m - n = 0 // \text{ Also, } m + n = 6$$

Solving, we get  $m = 2, n = 4$

From row 6 we see that when  $m = 2$ , the coefficient is 15.

Therefore our term is  $15(2x^2)^2(\frac{-3}{x})^4 = 15 * 4 * 81 = 4860$

$$\begin{array}{cccccccc} 6 & | & 1 & 6 & 15 & 20 & 15 & 6 & 1 & | \end{array}$$

$$\begin{aligned} 9. \cos^6(\theta) &= \left(\frac{e^{i\theta} + e^{-i\theta}}{2}\right)^6 = \left(\frac{1}{2}\right)^6 (e^{i\theta} + e^{-i\theta})^6 \\ &= \frac{1}{64} (e^{6i\theta} + 6(e^{5i\theta})(e^{-i\theta}) + 15(e^{4i\theta})(e^{-2i\theta}) + 20(e^{3i\theta})(e^{-3i\theta}) + 15(e^{2i\theta})(e^{-4i\theta}) \\ &\quad + 6(e^{i\theta})(e^{-5i\theta}) + e^{-i\theta}) \\ &= \frac{1}{64} (e^{i\theta} + e^{-i\theta} + 6e^{4i\theta} + 6e^{-4i\theta} + 15e^{2i\theta} + 15e^{-2i\theta} + 20) \\ &= \frac{1}{32} \left[ \left(\frac{e^{6i\theta} + e^{-6i\theta}}{2}\right) + 6\left(\frac{e^{4i\theta} + e^{-4i\theta}}{2}\right) + 15\left(\frac{e^{2i\theta} + e^{-2i\theta}}{2}\right) + \frac{20}{2} \right] \\ &= \frac{1}{32} \cos(6\theta) + \frac{3}{16} \cos(4\theta) + \frac{15}{32} \cos(2\theta) + \frac{5}{16} \text{ (As required)} \end{aligned}$$

10.  $(1 + kx)^n$  can be expanded to  $\binom{n}{0}1^n + \binom{n}{1}1^{n-1}(kx) + \binom{n}{2}1^{n-2}(kx)^2 + \dots$

$$1 + nkx + \frac{n(n-1)}{2}k^2x^2 + \dots$$

From this we get the equation  $\frac{n(n-1)}{2} = 120$

$$n^2 - n - 240 = 0$$

$$n = 15, -16$$

Therefore,  $n=15$ . We also know that  $nk = 40$ , therefore  $k = \frac{40}{n} = \frac{40}{15} = \frac{5}{2}$

11.  $(2 - kx)^8$  can be expanded to  $\binom{8}{0}2^8 + \binom{8}{1}2^7(-kx) + \binom{8}{2}2^6(-kx)^2 + \dots$

$$256 - 1024kx + 1792k^2x^2 + \dots$$

Therefore, we know that  $1792k^2 = 1008$

$$k = \frac{3}{4}$$

Using  $-1024k = A$ , we know that  $A = -1024 \times \frac{3}{4} = -768$

12.  $(1 = ax)^n$  can be expanded to  $\binom{n}{0}1^n + \binom{n}{1}1^{n-1}(ax) + \binom{n}{2}1^{n-2}(ax)^2 + \binom{n}{3}1^{n-3}(ax)^3 + \dots$

$$1 + anx + \frac{n(n-1)}{2}a^2x^2 + \frac{n(n-1)(n-2)}{3 \times 2} + \dots$$

From this we know the following:

$$an = -30$$

$$\frac{n^2-n}{2}a^2 = 405$$

$$a^2n^2 - a^2n = 810$$

$$900 + 30n = 810$$

$$n = -3$$

Therefore,  $a = 10$

Finally, the  $x^3$  term has coefficient  $\frac{n(n-1)(n-2)}{3 \times 2} a^3$

Substituting in, we get  $\frac{-3 \times -4 \times -5}{6} \times 1000 = -10000$