Answers - Surface of revolution (page ??)

1. $y = x$

   $y' = 1$

   $A = 2\pi \int_1^2 x\sqrt{1+1^2}\, dx$

   $A = 2\sqrt{2}\pi \int_1^2 x\, dx$

   $A = 2\sqrt{2}\pi \left[\frac{x^2}{2}\right]_1^2$

   $A = 3\pi\sqrt{2}$

2. $y = (x-1)^3$

   $y' = 3(x-1)^2$

   $A = 2\pi \int_1^3 (x-1)^3\sqrt{1+9(x-1)^4}\, dx$

   Use the substitution $u = 1+9(x-1)^4$

   $du = 36(x-1)^3\, dx$

   Recalculate the boundaries:

   $u = 1+9(3-1)^4 = 145$

   $u = 1+9(1-1)^4 = 1$

   So, the integral becomes:

   $A = \frac{\pi}{18}\int_1^{145} u^{\frac{1}{2}}\, du$

   $A = \frac{\pi}{18}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^{145} = 203.04$

3. $y = \sqrt[3]{x}$

   Since it is rotated about the y-**axis** we make x the subject

   $x = y^3$

   $x' = 3y^2$

   $A = 2\pi \int_2^4 y^3\sqrt{1+9y^4}\, dy$

   Use the substitution $u = 1+9y^4$

   $du = 36y^3\, dy$

   $\frac{du}{36} = y^3\, dy$

   Recalculate the boundaries:

   $u = 1+9(2)^4 = 145$

   $u = 1+9(4)^4 = 2305$

   The integral becomes:

$A = \frac{\pi}{18} \int_{145}^{2305} u^{\frac{1}{2}}\, du$

$A = \frac{\pi}{18} \left[\frac{2}{3} u^{\frac{3}{2}} \right]_{145}^{2305} = 12673.18$

4. $y = x^2$ rotated about the y-**axis** between $y = 1$ and $y = 9$

   Since it is rotated about the y-axis, make x the subject.

   $x = y^{\frac{1}{2}}$

   $x' = \frac{1}{2} y^{-\frac{1}{2}}$

   $A = 2\pi \int_1^9 \sqrt{y} \sqrt{1 + \frac{1}{4y}}\, dy$

   $A = 2\pi \int_1^9 \sqrt{y + \frac{1}{4}}\, dy$

   $A = 2\pi \left[\frac{2}{3} (y + \frac{1}{4})^{\frac{3}{2}} \right]_1^9$

   $A = \frac{4\pi}{3} \left[(y + \frac{1}{4})^{\frac{3}{2}} \right]_1^9$

   $A = \frac{4\pi}{3} \left[\left(\frac{37}{4} \right)^{\frac{3}{2}} - \left(\frac{5}{4} \right)^{\frac{3}{2}} \right] = 111.988$

5. Rotated about the y axis so make x the subject.

   $t = 9 - y^2$

   $x = \sqrt{9 - y^2}$

   $x' = \frac{-y}{\sqrt{9 - y^2}}$

   Get the boundaries in terms y:

   $y = \sqrt{9 - 5} = 2$

   $y = \sqrt{9 - 1} = 2\sqrt{2}$

   $A = 2\pi \int_2^{2\sqrt{2}} \sqrt{9 - y^2} \sqrt{1 + \frac{y^2}{9 - y^2}}\, dy$

   $A = 2\pi \int_2^{2\sqrt{2}} \sqrt{(9 - y^2) + y^2}\, dy$

   $A = 2\pi \int_2^{2\sqrt{2}} 3\, dy$

   $A = 2\pi \Big[3y \Big]_2^{2\sqrt{2}}$

   $A = 2\pi [6\sqrt{2} - 6] = 12\pi(\sqrt{2} - 1)$

6. $f(x) = x^3 + \frac{1}{12x}$ from $x = 1$ to $x = 3$ is rotated $360°$ about the x-axis.

   $f'(x) = 3x^2 - \frac{1}{12x^2}$

$$A = 2\pi \int_1^3 (x^3 + \tfrac{1}{12x})\sqrt{1 + (3x^2 - \tfrac{1}{12x^2})^2}\, dx$$

$$A = 2\pi \int_1^3 (x^3 + \tfrac{1}{12x})\sqrt{1 + 9x^4 - \tfrac{1}{2} + \tfrac{1}{144x^4}}\, dx$$

$$A = 2\pi \int_1^3 (x^3 + \tfrac{1}{12x})\sqrt{9x^4 + \tfrac{1}{2} + \tfrac{1}{144x^4}}\, dx$$

$$A = 2\pi \int_1^3 (x^3 + \tfrac{1}{12x})\sqrt{(3x^2 + \tfrac{1}{12x^2})^2}\, dx$$

$$A = 2\pi \int_1^3 (x^3 + \tfrac{1}{12x})(3x^2 + \tfrac{1}{12x^2})\, dx$$

$$A = 2\pi \int_1^3 3x^5 + \tfrac{x}{12} + \tfrac{x}{12} + \tfrac{1}{144x^3}\, dx$$

$$A = 2\pi \int_1^3 3x^5 + \tfrac{x}{3} + \tfrac{1}{144}x^{-3}\, dx$$

$$A = 2\pi \left[\tfrac{x^6}{2} + \tfrac{x^2}{6} - \tfrac{1}{288x^2}\right]_1^3 = 2295.5$$