

Answers - Combinations and permutations (page ??)

1. ${}^{10}C_2 = \frac{10!}{2! \times 8!} = \frac{10 \times 9}{2} = 45$
2. (a) $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
(b) Visualise this with the girls effectively being a sixth member of the group. There are $6!$ ways of arranging them.
Then, within the girls, there are $3!$ ways of arranging them.
This means there are $6! \times 3! = 720 \times 6 = 4320$ possible photos.
3. (a) $6 \times {}^5C_2 \times {}^3C_3 = 6 \times 10 \times 1 = 60$
(b) ${}^6C_2 \times {}^4C_2 \times {}^2C_2 = 15 \times 6 \times 1 = 90$
4. ${}^{20}C_3 \times {}^{30}C_2 = 1140 \times 435 = 495,900$
5. 2 candidates: ${}^8C_2 = 28$
1 candidate: ${}^8C_1 = 8$
0 candidates = 1
Total = 37
6. ${}^{15}C_3 \times {}^9C_1 \times {}^7C_1 = 28,665$
7. Consider the two situations: first, where all 6 people are from the same college. Second, where 4 are from the same college and 2 are from the other one.
6 from same college: ${}^8C_6 = 28$
4 from same college: ${}^8C_4 = 70$
Total is 98
- 8.

$$\frac{p!}{q!(p-q)!} = \frac{p!}{r!(p-r)!}$$
$$\frac{1}{q!(p-q)!} = \frac{1}{r!(p-r)!}$$

There are 2 solutions to consider here. The first gives us the solution $q = r$, which we are told is not a solution.

$$\frac{r!}{(p-q)!} = \frac{q!}{(p-r)!}$$

Here we can equate the numerators and the denominators, giving us $r = q$.

The other way is to cross-multiply different terms:

$$\frac{r!}{q!} = \frac{(p-q)!}{(p-r)!}$$

When we equate the numerators and denominators we get:

$$p - q = r \text{ and } p - r = q$$

Both of which can be rearranged to give the solution $p = q + r$

$$\begin{aligned}
9. \quad & \frac{n!}{r!(n-r)!} = \frac{(n+1)!}{(r-1)!((n+1)-(r-1))!} \\
& \frac{n!}{r!(n-r)!} = \frac{(n+1)!}{(r-1)!(n-r+2)!} \\
& \frac{n!}{r!(n-r)!} = \frac{(n+1)n!}{(r-1)!(n-r+2)(n-r+1)(n-r)!} \\
& \frac{1}{r!} = \frac{n+1}{(r-1)!(n-r+2)(n-r+1)} \\
& \frac{(r-1)!}{r(r-1)!} = \frac{n+1}{(n-r+2)(n-r+1)} \\
& \frac{1}{r} = \frac{n+1}{(n-r+2)(n-r+1)} \\
& (n-r+2)(n-r+1) = r(n+1) \\
& n^2 - rn + n - rn + r^2 - r + 2n - 2r + 2 = rn + r \\
& n^2 - 3rn + 3n + r^2 - 4r + 2 = 0 \\
& n^2 + (3-3r)n + (r^2 - 4r + 2) = 0 \\
& n = \frac{3r - 3 \pm \sqrt{(3-3r)^2 - 4(r^2 - 4r + 2)}}{2} \\
& n = \frac{3r - 3 \pm \sqrt{5r^2 - 2r + 1}}{2}
\end{aligned}$$

Now we try different values for r to see which gives an integer value for n.

$$r = 1; n = 1$$

$$r = 2; n = \frac{3 \pm \sqrt{17}}{2}$$

$$r = 3; n = \frac{6 \pm \sqrt{40}}{2}$$

$$r = 4; n = \frac{9 \pm \sqrt{73}}{2}$$

$$r = 5; n = \frac{12 \pm \sqrt{112}}{2}$$

$$r = 6; n = \frac{15 \pm \sqrt{169}}{2} = \frac{15 \pm 13}{2} = 1, 14$$