# Answers - Binomial expansion (page ??)

1. $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

2. $$\begin{aligned}(2x+y)^4 &= (2x)^4 + 4(2x)^3y + 6(2x)^2y^2 + 4(2x)y^3 + y^4\\ &= 16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4\end{aligned}$$

3. $$\begin{aligned}(2x-3)^5 &= (2x)^5 + 5(2x)^4(-3) + 10(2x)^3(-3)^2 + 10(2x)^2(-3)^3 + 5(2x)(-3)^4 + (-3)^5\\ &= 32x^5 - 240x^4 + 720x^3 - 1080x^2 + 810x - 243\end{aligned}$$

4. $$\begin{aligned}(3x+2y)^4 &= (3x)^4 + 4(3x)^3(2y) + 6(3x)^2(2y)^2 + 4(3x)(2y)^3 + (2y)^4\\ &= 81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4\end{aligned}$$

5. $$\begin{aligned}(2x+\frac{1}{x^2})^4 &= (2x)^4 + 4(2x)^3(\frac{1}{x^2}) + 6(2x)^2(\frac{1}{x^2})^2 + 4(2x)(\frac{1}{x^2})^3 + (\frac{1}{x^2})^4\\ &= 16x^4 + 32x + \frac{24}{x^2} + \frac{8}{x^5} + \frac{1}{x^8}\end{aligned}$$

6. We need to find when the powers in a term cancel out and leave a constant.
   $(3x^2)^m(\frac{-1}{3x})^n$
   We can form two equations from this:
   $\frac{x^{2m}}{x^n} = x^0$
   $2m - n = 0$
   And we know in this question that $m + n = 12$
   Solving, we get $m = 4, n = 8$.
   This means that if we look in row 12, we look for the column where $m = 4$ to get the coefficient.
   Therefore, our term is $495(3x)^4(\frac{-1}{3x})^8 = \frac{495}{81} = \frac{55}{9}$

| n \ r | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 12 | 1 | 12 | 66 | 220 | 495 | 792 | 924 | 792 | 495 | 220 | 66 |

7. We need to find when the powers in a term cancel out to give $x^2$
   Forming two equations from $(x^2)^m(\frac{1}{x})^n$
   $\frac{x^{2m}}{x^n} = x^2 \rightarrow 2m - n = 2$
   Also, $m + n = 10$
   Solving, we get $m = 4, n = 6$
   From row 10, we see that when $m = 4$, the coefficient is 210.
   Therefore, our term is $210(x^2)^4(\frac{1}{x})^6 = 210x^2$

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 1 | 10 | 45 | 120 | 210 | 252 | 210 | 120 | 45 | 10 | 1 |

8. Forming two equations from $(2x^2)^m(\frac{-3}{x})^n$
   $\frac{x^2m}{x^n} = x^0 \rightarrow 2m - n = 0//$ Also, $m + n = 6$

Solving, we get $m = 2, n = 4$
From row 6 we see that when $m = 2$, the coefficient is 15.
Therefore our term is $15(2x^2)^2(\frac{-3}{x})^4 = 15 * 4 * 81 = 4860$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 6 | 1 | 6 | 15 | 20 | 15 | 6 | 1 |

9. $\cos^6(\theta) = (\dfrac{e^{i\theta} + e^{-i\theta}}{2})^6 = (\dfrac{1}{2})^6(e^{i\theta} + e^{-i\theta})^6$

$$= \frac{1}{64}(e^{6i\theta} + 6(e^{5i\theta})(e^{-i\theta}) + 15(e^{4i\theta})(e^{-2i\theta}) + 20(e^{3i\theta})(e^{-3i\theta}) + 15(e^{2i\theta})(e^{-4i\theta})$$
$$+ 6(e^{i\theta})(e^{-5i\theta}) + e^{-i\theta})$$
$$= \frac{1}{64}(e^{i\theta} + e^{-i\theta} + 6e^{4i\theta} + 6e^{-4i\theta} + 15e^{2i\theta} + 15e^{-2i\theta} + 20)$$
$$= \frac{1}{32}[(\frac{e^{6i\theta} + e^{-6i\theta}}{2}) + 6(\frac{e^{4i\theta} + e^{-4i\theta}}{2}) + 15(\frac{e^{2i\theta} + e^{-2i\theta}}{2}) + \frac{20}{2}]$$
$$= \frac{1}{32}\cos(6\theta) + \frac{3}{16}\cos(4\theta) + \frac{15}{32}\cos(2\theta) + \frac{5}{16}\text{(As required)}$$