

Answers - Mixing problems (page ??)

1. $\frac{dS}{dt} = 25 \times 0.03 - 25 \times \frac{S}{5000}$

$$\frac{dS}{dt} = 0.75 - \frac{S}{200}$$

$$\frac{dS}{dt} = \frac{150-S}{200}$$

Separating variables and integrating:

$$\frac{1}{150-S} dS = \frac{1}{200} dt$$

$$\int \frac{1}{150-S} dS = \int \frac{1}{200} dt$$

$$-\ln |150 - S| = \frac{t}{200} + c$$

$$\ln |150 - S| = -\frac{t}{200} + c$$

$$150 - S = Ae^{-\frac{t}{200}}$$

$$S = 150 - Ae^{-\frac{t}{200}}$$

Substituting in the initial value of 20kg to find A:

$$20 = 150 - Ae^0$$

$$A = 130$$

So, the model is:

$$S = 150 - 130e^{-\frac{t}{200}}$$

After half an hour, $t = 30$:

$$S = 150 - 130e^{-\frac{30}{200}} = 38.1\text{kg.}$$

2. $\frac{dA}{dt} = 4 \times 0.5 - 4 \times \frac{A}{60}$

$$\frac{dA}{dt} = 2 - \frac{A}{15}$$

$$\frac{dA}{dt} = \frac{30-A}{15}$$

Separating variables and integrating:

$$\frac{1}{30-A} dA = \frac{1}{15} dt$$

$$\int \frac{1}{30-A} dA = \int \frac{1}{15} dt$$

$$-\ln |30 - A| = \frac{t}{15} + c$$

$$\ln |30 - A| = -\frac{t}{15} + c$$

$$30 - A = Ce^{-\frac{t}{15}}$$

$$A = 30 - Ce^{-\frac{t}{15}}$$

Substituting in the initial value of $(0.15 \times 60 = 9L)$ of alcohol to get C :

$$9 = 30 - Ce^0$$

$$C = 21$$

So, the model is:

$$A = 30 - 21e^{-\frac{t}{15}}$$

After 10 minutes:

$$A = 30 - 21e^{-\frac{10}{15}} = 19.2L$$