

Answers - Partial fractions (page ??)

1. $\frac{x+5}{(x-3)(x+1)} = \frac{A}{x-3} + \frac{B}{x+1}$

$$A(x+1) + B(x-3) = x+5$$

Using critical value method (you don't have to, you could equate coefficients and constant if you want), we substitute in $x = 3$ and $x = -1$:

$$4A = 8 \Rightarrow A = 2$$

$$-4B = 4 \Rightarrow B = -1$$

Partial fraction decomposition is $\frac{2}{x-3} - \frac{1}{x+1}$

2. $\frac{x+26}{x^3+3x-10} = \frac{x+26}{(x+5)(x-2)}$

$$\frac{x+26}{(x+5)(x-2)} = \frac{A}{x+5} + \frac{B}{x-2}$$

$$x+26 = A(x-2) + B(x+5)$$

Using critical value method, we substitute in $x = 2$ and $x = -5$

$$2+26 = 0+7B \Rightarrow B = 4$$

$$-5+26 = -7A \Rightarrow A = 3$$

Giving us: $\frac{3}{x+5} + \frac{4}{x-2}$

3. $\frac{4x-8}{x^2-8x+15} = \frac{4x-8}{(x-3)(x-5)}$

$$\frac{4x-8}{(x-5)(x-3)} = \frac{A}{x-3} + \frac{B}{x-5}$$

$$4x-8 = A(x-5) + B(x-3)$$

Using critical value method we substitute in $x = 5$ and $x = 3$

$$4(3)-8 = -2A \Rightarrow A = -2$$

$$4(5)-8 = 2B \Rightarrow B = 6$$

Giving us $\frac{-2}{x-3} + \frac{6}{x-5}$

4. $\frac{12x-1}{x^2+x-12} = \frac{12x-1}{(x+4)(x-3)}$

$$\frac{12x-1}{(x+4)(x-3)} = \frac{A}{x+4} + \frac{B}{x-3}$$

$$12x-1 = A(x-3) + B(x+4)$$

Using critical values of $x = 3$ and $x = -4$:

$$12(3)-1 = 7B \Rightarrow B = 5$$

$$12(-4)-1 = -7A \Rightarrow A = 7$$

Giving us: $\frac{7}{x+4} + \frac{5}{x-3}$

$$5. \frac{x-5}{(x-2)^2} = \frac{A}{x-2} + \frac{B}{(x-2)^2}$$

$$x - 5 = A(x - 2) + B$$

$$x - 5 = Ax - 2A + B$$

Matching coefficients and constant:

$$x\text{-term : } A = 1$$

$$\text{Constant : } -2A + B = -5 \Rightarrow B = -3$$

$$\text{Giving us: } \frac{1}{x-2} - \frac{3}{(x-2)^2}$$

$$6. \frac{5x+4}{(x-1)(x+2)^2} = \frac{A}{x-1} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

$$5x + 4 = A(x + 2)^2 + B(x - 1)(x + 2) + C(x - 1)$$

$$5x + 4 = Ax^2 + 4Ax + 4A + Bx^2 + Bx - 2B + Cx - C$$

$$5x + 4 = (A + B)x^2 + (4A + B + C)x + 4A - 2B - C$$

Equating coefficients and constant:

$$x^2\text{-term : } A + B = 0$$

$$x\text{-term : } 4A + B + C = 5$$

$$\text{Constant : } 4A - 2B - C = 4$$

Solving simultaneously, $A = 1, B = -1, C = 2$

$$\text{Giving us: } \frac{1}{x-1} - \frac{1}{x+2} + \frac{2}{(x+2)^2}$$

$$7. \frac{2x^2-5x+7}{(x-2)(x-1)^2} = \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$2x^2 - 5x + 7 = A(x - 1)^2 + B(x - 2)(x - 1) + C(x - 2)$$

$$2x^2 - 5x + 7 = Ax^2 - 2Ax + A + Bx^2 - 3Bx + 2B + Cx - 2C$$

$$2x^2 - 5x + 7 = (A + B)x^2 + (-2A - 3B + C)x + A + 2B - 2C$$

Equating coefficients and constant:

$$x^2\text{-term : } A + B = 2$$

$$x\text{-term : } -2A - 3B + C = -5$$

$$\text{Constant : } A + 2B - 2C = 7$$

Solving simultaneously, $A = 5, B = -3, C = -4$

$$\text{Giving us: } \frac{5}{x-2} - \frac{3}{x-1} + \frac{7}{(x-1)^2}$$

$$8. \frac{6-x}{(1-x)(4+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{4+x^2}$$

$$6 - x = A(4 + x^2) + (Bx + C)(1 - x)$$

$$6 - x = 4A + Ax^2 + Bx - Bx^2 + C - Cx$$

$$6 - x = (A - B)x^2 + (B - C)x + 4A + C$$

Equating coefficients and constant:

$$x^2\text{-term : } A - B = 0$$

$$x\text{-term : } B - C = -1$$

$$\text{Constant : } 4A + C = 6$$

Solving simultaneously, $A = 1, B = 1, C = 2$

$$\text{Giving us: } \frac{1}{1-x} + \frac{x+2}{4+x^2}$$

$$9. \frac{5x+2}{(x+1)(x^2-4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-4}$$

$$5x + 2 = A(x^2 - 4) + (Bx + C)(x + 1)$$

$$5x + 2 = Ax^2 - 4A + Bx^2 + Bx + Cx + C$$

$$5x + 2 = (A + B)x^2 + (B + C)x - 4A + C$$

Equating coefficients and constant:

$$x^2\text{-term : } A + B = 0$$

$$x\text{-term : } B + C = 5$$

$$\text{Constant : } -4A + C = 2$$

Solving simultaneously, $A = 1, B = -1, C = 6$

$$\text{Giving us: } \frac{1}{x+1} + \frac{-x+6}{x^2-4}$$