Answers - Endless sums (page ??)

1. Set $y = 2\sqrt{2+y}$ so the expression is $2+y$

   Now we can solve for y:

   $y^2 = 4(2+y) = 8+4y$

   $y^2 - 4y - 8 = 0$

   $y = \frac{4\pm\sqrt{48}}{2} = \frac{4\pm4\sqrt{3}}{2} = 2\pm2\sqrt{3}$

   Since we know the sum is clearly positive, $y = 2+2\sqrt{3}$, meaning the value of the expression is $4+2\sqrt{3}$

2. Set $y = \frac{13}{5\sqrt{3}}\sqrt{4+y}$ so we just need to find the value of y.

   $y^2 = \frac{169}{75}(4+y)$

   $y^2 = \frac{169y}{75} + \frac{676}{75}$

   $75y^2 - 169y - 676 = 0$

   $y = \frac{169\pm481}{150} = \frac{650}{150}, \frac{-312}{150}$

   Since the sum is clearly positive, we know that it value is $\frac{650}{150} = \frac{13}{3}$

3. Start by setting $y = \sqrt{6+y}$ and $z = \sqrt{90+z}$.

   Now we can solve for each and then use these values to solve the original quadratic.

   $y^2 = 6+y$

   $y^2 - y - 6 = 0$

   $y = 3, -2$

   Note: since the series is clearly positive, $y = 3$.

   $z^2 = 90+z$

   $z^2 - z - 90 = 0$

   $z = 10, -9$

   Again, since the series is clearly positive, $z = 10$

   Now we can rewrite the original quadratic as:

   $x^2 - 3x - 10 = 0$

   $x = 5, -2$

4. Start by setting $y = \sqrt{20+y}$ and $z = \sqrt{30+z}$.

   Now we can solve for each and then use these values to solve the original quadratic.

$y^2 = 20 + y$

$y^2 - y - 20 = 0$

$y = 5, -4$

Since the series is clearly positive, $y = 5$

$z^2 = 30 + z$

$z^2 - z - 30 = 0$

$z = 6, -5$

Again, since the series is clearly positive, $z = 6$

Now we can rewrite the original quadratic as:

$x^2 - 5x - 6 = 0$

$x = 6, -1$

5. Every term from the second onwards has a common factor of $\frac{1}{\sqrt{2}}$. Factorising this out, we get:

   $1 + \frac{1}{\sqrt{2}}\left(1 + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{16}} + \frac{1}{\sqrt{64}}\right) = 1 + \frac{1}{\sqrt{2}}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots\right)$

   We know that the infinite sum of $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 2$

   (This is from the formula for the sum to infinity of a geometric sequence with first term 1 and a common ratio of $\frac{1}{2}$: $S_\infty = \frac{1}{1-\frac{1}{2}} = 2$)

   Therefore, the value of the series is $1 + \frac{2}{\sqrt{2}}$

6. Set $y = 1 + \frac{1}{y}$

   $y^2 = y + 1$

   $y^2 - y - 1 = 0$

   $y = \frac{1 \pm \sqrt{5}}{2}$

   Since the expression is clearly positive, the value is $\frac{1+\sqrt{5}}{2}$