

## Answers - Integration by parts - DI method (page ??)

1.  $\int x^2 \sin(2x) dx$

|   | D     | I                       |
|---|-------|-------------------------|
| + | $x^2$ | $\sin(2x)$              |
| - | $2x$  | $-\frac{1}{2} \cos(2x)$ |
| + | $2$   | $-\frac{1}{4} \sin(2x)$ |
| - | $0$   | $\frac{1}{8} \cos(2x)$  |

Stop is reached when we get zero in the D row.

$$\int x^2 \sin(2x) dx = -\frac{x^2}{2} \cos(2x) + \frac{x}{2} \sin(2x) + \frac{1}{4} \cos(2x) + c$$

2.  $\int e^x \cos(x) dx$

|   | D     | I         |
|---|-------|-----------|
| + | $e^x$ | $\cos x$  |
| - | $e^x$ | $\sin x$  |
| + | $e^x$ | $-\cos x$ |

The third row is a “repeat” of the first, so we can stop now. The integral is diagonal products plus the integral of the final row product.

$$\int e^x \cos(x) dx = e^x \sin(x) + e^x \cos(x) - \int e^x \cos(x) dx$$

$$2 \int e^x \cos(x) dx = e^x \sin(x) + e^x \cos(x)$$

$$\int e^x \cos(x) dx = \frac{e^x \sin(x) + e^x \cos(x)}{2} + c$$

3.  $\int (\ln(x))^2 dx$

|   | D                   | I   |
|---|---------------------|-----|
| + | $\ln(x)^2$          | $1$ |
| - | $\frac{2 \ln x}{x}$ | $x$ |

Since the product of the second row can (relatively) easily be integrated, the integral will be:

$$\int (\ln(x))^2 dx = x \ln(x)^2 - \int 2 \ln x dx$$

Using the DI method again for this:

|   | D             | I   |
|---|---------------|-----|
| + | $2 \ln x$     | $1$ |
| - | $\frac{2}{x}$ | $x$ |

The product of the second row can be integrated so we stop, giving us:

$$2 \ln x dx = 2x \ln x - \int 2 dx = 2x \ln x - 2x$$

Therefore, our final integral is:

$$\int (\ln(x))^2 dx = x(\ln(x))^2 - 2x \ln x + 2x + c$$

4.  $\int \sin^3(x) dx$

$$\begin{array}{rcl} & \text{D} & \text{I} \\ + & \sin^2(x) & \sin(x) \\ - & 2\sin(x)\cos(x) & -\cos(x) \end{array}$$

The product of the second row integrates easily so we stop:

$$\int 2\sin(x)\cos^2(x) dx = -\frac{2}{3}\cos^3(x)$$

Therefore, our final integral is:

$$\int \sin^3(x) dx = -\sin^2(x)\cos(x) - \frac{2}{3}\cos^3(x) + c$$

5.  $\int \frac{\ln(x)}{x^2} dx$

$$\begin{array}{rcl} & \text{D} & \text{I} \\ + & \ln x & \frac{1}{x^2} \\ - & \frac{1}{x} & -\frac{1}{x} \end{array}$$

The product of the second row is easy to integrate so we stop:

$$\int \frac{\ln(x)}{x^2} dx = -\frac{\ln(x)}{x} - \int -\frac{1}{x^2} dx$$

$$\int \frac{\ln(x)}{x^2} dx = -\frac{\ln(x)}{x} + \int \frac{1}{x^2} dx$$

$$\int \frac{\ln(x)}{x^2} dx = -\frac{\ln(x)}{x} - \frac{1}{x} + c$$

6.  $\int 4x \cos(2-3x) dx$

$$\begin{array}{rcl} & \text{D} & \text{I} \\ + & 4x & \cos(2-3x) \\ - & 4 & -\frac{1}{3}\sin(2-3x) \\ + & 0 & -\frac{1}{9}\cos(2-3x) \end{array}$$

Stop because we reach zero in the D column, so the integral is:

$$\int 4x \cos(2-3x) dx = -\frac{4x}{3}\sin(2-3x) + \frac{4}{9}\cos(2-3x) + c$$

7.  $\int e^{-x} \cos(x) dx$

$$\begin{array}{rcl} & \text{D} & \text{I} \\ + & e^{-x} & \cos(x) \\ - & -e^{-x} & \sin(x) \\ + & e^{-x} & -\cos(x) \end{array}$$

The third row repeats, so we stop:

$$\int e^{-x} \cos(x) dx = e^{-x} \sin(x) - e^{-x} \cos(x) + \int e^{-x} \times -\cos(x) dx$$

$$\int e^{-x} \cos(x) dx = e^{-x} \sin(x) - e^{-x} \cos(x) - \int e^{-x} \cos(x) dx$$

$$2 \int e^{-x} \cos(x) dx = e^{-x} \sin(x) - e^{-x} \cos(x)$$

$$\int e^{-x} \cos(x) dx = \frac{e^{-x}}{2}(\sin(x) - \cos(x)) + c$$