Answers - The Camel Principle (page ??)

1. $\int \frac{1}{1+e^x}\,dx = \int \frac{1+e^x-e^x}{1+e^x}\,dx$

   $\int \frac{1+e^x}{1+e^x}\,dx - \int \frac{e^x}{1+e^x}\,dx$

   $\int 1\,dx - \int \frac{e^x}{1+e^x}\,dx$

   $= x - \ln|1+e^x| + c$

2. $\int \frac{1}{1+\sqrt{e^x}}\,dx = \int \frac{1+\sqrt{e^x}-\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$

   $= \int \frac{1+\sqrt{e^x}}{1+\sqrt{e^x}}\,dx - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$

   $= \int 1\,dx - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$

   $x - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$

   For the remaining integral, use the substitution$u = \sqrt{e^x}$, meaning that $u^2 = e^x$.

   $x = \ln u^2 = 2\ln u$

   $dx = \frac{2}{u}\,du$

   $x - \int \frac{u}{1+u}\frac{2\,du}{u} = x - 2\int \frac{1}{1+u}\,du$

   $x - 2\ln|1+u| + c$

   $x - 2\ln|1+\sqrt{e^x}| + c$

3. $\int \sec x\,dx$

   In this case we will use the Camel Principle multiplicatively, multiplying by $\frac{\sec x+\tan x}{\sec x+\tan x}$

   This gives us the integral:

   $\int \frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x}\,dx$

   This is in the format $\frac{f'(x)}{f(x)}$, which integrates to $\ln|f(x)| + c$

   Therefore, our integral is $\ln|\sec x + \tan x| + c$

4. $\int \csc\theta\,d\theta$

   To integrate, first multiply by $\frac{\csc\theta-\cot\theta}{\csc\theta+\cot\theta}$

   This changes the integral to:

   $\int \frac{\csc^2\theta-\csc\theta\cot\theta}{\csc\theta-\cot\theta}$

   This is in the form $\frac{f'(x)}{f(x)}$, therefore the integral is $\ln|\csc\theta - \tan\theta| + c$

5. $\int \frac{1}{1+\tan x}\,dx$

   Change the $\tan x$ into $\frac{\sin x}{\cos x}$ and simplify:

   $\int \frac{1}{1+\frac{\sin x}{\cos x}}\,dx$

   $\int \frac{1}{\frac{\cos x+\sin x}{\cos x}}\,dx$

$\int \frac{\cos x}{\sin x+\cos x}\,dx$

Now we can use the Camel Principle. First, we double the fraction:

$\frac{1}{2}\int \frac{2\cos x}{\sin x+\cos x}\,dx$

Then we add and subtract $\sin x$ from the numerator:

$\frac{1}{2}\int \frac{2\cos x+\sin x-\sin x}{\sin x+\cos x}\,dx$

Separate into two fractions:

$\frac{1}{2}\int \left(\frac{\cos x+\sin x}{\sin x+\cos x}+\frac{\cos x-\sin x}{\sin x+\cos x}\right)dx$

Split into two integrals and simplify:

$\frac{1}{2}\int 1\,dx+\frac{1}{2}\int \frac{\cos x-\sin x}{\sin x+\cos x}\,dx$

The first fraction integrates easily. The second integral is in the form $\int \frac{f'(x)}{f(x)}\,dx$, therefore:

$\frac{x}{2}+\frac{1}{2}\ln|\sin x+\cos x|+c$