1 Log problems

You should be familiar with all of the log rules:
$y = \log_b(x) \iff x = b^y$
$\log_b(xy) = \log_b(x) + \log_b(y)$
$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
$\log_b(x^n) = n\log_b(x)$
$\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$
When faced with tricky problems involving logs, we use the above rules to manipulate the equations into something we can solve more easily. A common technique is to use the change of base formula to change a log term into a fraction with a different base. For example:
$\log_8(x) + \log_{16}(x) = 1$
Notice that both terms have bases which are powers of 2, therefore we will change the base to 2 for each term:
$\frac{\log_2(x)}{\log_2(8)} = \frac{\log_2(x)}{3}$
$\frac{\log_2(x)}{\log_2(16)} = \frac{\log_2(x)}{4}$
Giving us an equation of: $\frac{\log_2(x)}{3} + \frac{\log_2(x)}{4} = 1$
We can then easily solve:
$4\log_2(x) + 3\log_2(x) = 12$
$7\log_2(x) = 12$
$\log_2(x) = \frac{12}{7}$
$x = 2^{\frac{12}{7}} = 3.28$
Another technique is to take the log of both sides to help us rearrange the equation into something easier to solve. For example:
$x^{\log_2(x)} = 256x^2$
If we take $\log_2$ of both sides, we get:
$\log_2(x^{\log_2(x)}) = \log_2(256x^2)$
We can now move the power on the LHS out to the front, and also split the RHS into two terms.
$\log_2(x)\log_2(x) = \log_2(256) + \log_2(x^2)$
Simplifying:
$(\log_2(x))^2 = 8 + 2\log_2(x)$
This is a quadratic where the subject is $\log_2(x)$, so if we do a u-substitution where $u = \log_2(x)$ we get:
$u^2 - 2u - 8 = 0$
Solving, we have $u = -2, 4$.
Now we just reverse our substitution to find the value(s) of x:
$\log_2(x) = -2 \rightarrow x = \frac{1}{4}$
$\log_2(x) = 4 \rightarrow x = 16$

Questions

(Answers - page **??**)

1. Solve for x:

   $x^{\log_3(x)} = 81x^3$

2. Solve for x:

   $\log_4(2^x + 48) = x - 1$

3. $\log_x(y) + \log_y(x) = 2$ Find the value of $\frac{x}{y} + \frac{y}{x}$

4. If $\sqrt{\log_a(b)} + \sqrt{\log_b(a)} = 2$, then find the value of $\log_{ab}(a) - \log_{\frac{1}{ab}}(b)$

5. If $2^{3x-5} = 3^{x+3}$ and $x = \log(864^{\log_{10}(y)})$, then find the value of $y^{\log_{10}\frac{8}{3}}$

6. Solve for x:

   $\log_7\big(\log_9(x^2 + \sqrt{x+1} + 8)\big) = 0$

7. If $\log_{16}(x) + \log_8(y) = 11$ and $\log_8(x) + \log_{16}(y) = 10$ then find the value of $\frac{y}{x^2}$

8. Solve for x:

   $\log_{\log_2(x)}(4) = \log_2(\log_4(x))$

9. Solve for x and y:

   $\log_4(x) + \log_9(y) = 2$

   $\log_x(2) + \log_y(3) = 1$

10. If $\log_5(4), \log_5(2^x + \frac{1}{2})$ and $\log_5(2^x - \frac{1}{4})$ are in arithmetic progression, find the value of x and also find the common difference.

11. Solve the system:

    $\log_{10}(x^2 + y^2) = 1 + \log_{10}(13)$

    $\log_{10}(x + y) - \log_{10}(x - y) = 3\log_{10}(2)$

12. Evaluate the expression:

    $\frac{1}{1+\log_a(bc)} + \frac{1}{1+\log_b(ac)} + \frac{1}{1+\log_c(ab)}$