# 1 Binomial expansion

In your formula sheet you will see this on the first page:

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + \binom{n}{n}b^n$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!r!}$$

Some values of $\binom{n}{r}$ are given in the table below.

| n \ r | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | | | | | | | | | | |
| 1 | 1 | 1 | | | | | | | | | |
| 2 | 1 | 2 | 1 | | | | | | | | |
| 3 | 1 | 3 | 3 | 1 | | | | | | | |
| 4 | 1 | 4 | 6 | 4 | 1 | | | | | | |
| 5 | 1 | 5 | 10 | 10 | 5 | 1 | | | | | |
| 6 | 1 | 6 | 15 | 20 | 15 | 6 | 1 | | | | |
| 7 | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | | | |
| 8 | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | | |
| 9 | 1 | 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | 1 | |
| 10 | 1 | 10 | 45 | 120 | 210 | 252 | 210 | 120 | 45 | 10 | 1 |
| 11 | 1 | 11 | 55 | 165 | 330 | 462 | 462 | 330 | 165 | 55 | 11 |
| 12 | 1 | 12 | 66 | 220 | 495 | 792 | 924 | 792 | 495 | 220 | 66 |

This helps us expand out brackets that are raised to a high power. The numbers in the table give the coefficients of the terms when we expand the brackets. For example:

$$(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Notice how the coefficients match the numbers in row 4 in the table.
Also notice that the powers start at 4 for the first term in the brackets and zero for the second term. They then decrease and increase by 1 each term respectively.
In general, the sum of the powers in each term will add to the power we are raising the bracket to (in the example this is 4).
Another example:

$$(2a - 3b)^4 = (2a)^4 + 4(2a)^3(-3b) + 6(2a)^2(-3b)^2 + 4(2a)(-3b)^3 + (-3b)^4$$
$$= 16a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4$$

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# Questions

Expand the following:

1. $(x + y)^3$

2. $(2x + y)^4$

3. $(2x - 3)^5$

4. $(3x + 2y)^4$

5. $(2x + \frac{1}{x^2})^4$

Scholarship questions would tend to look more like this:

6. Find the term independent of $x$ in $(3x^2 - \frac{1}{3x})^{12}$

7. Find the coefficient of the $x^2$ term in $(x^2 + \frac{1}{x})^{10}$

8. Find the term independent of $x$ in $(2x^2 - \frac{3}{x})^6$

9. It can be shown that $\cos(\theta) = \frac{e^{i\theta} + e^{-i\theta}}{2}$ and that $\sin(\theta) = \frac{e^{i\theta} - e^{-i\theta}}{2i}$

   Use these identities, or otherwise, to show that:

   $\cos^6(\theta) = \frac{1}{32}\cos(6\theta) + \frac{3}{16}\cos(4\theta) + \frac{15}{32}\cos(2\theta) + \frac{5}{16}$

10. Given the $k$ is a non zero constant and $n$ is a positive integer, then

    $(1 + kx)^n \equiv 1 + 40x + 120k^2x^2 + \ldots$

    Find the value of $k$ and $n$.

11. Given that $k$ and $A$ are constants with $k > 0$, then

    $(2 - kx)^8 \equiv 256 + Ax + 1008x^2 + \ldots$

    Find the value of $k$ and $A$.

12. $(1 + ax)^n = 1 - 30x + 405x^2 + bx^3 + \ldots$

    Where $a$ and $b$ are constants, and $n$ is a positive integer.

    Determine the value of $n$, $a$ and $b$.