Answers - The Camel Principle (page ??)

1. $\int \frac{1}{1+e^x}\,dx = \int \frac{1+e^x-e^x}{1+e^x}\,dx$

   $\int \frac{1+e^x}{1+e^x}\,dx - \int \frac{e^x}{1+e^x}\,dx$

   $\int 1\,dx - \int \frac{e^x}{1+e^x}\,dx$

   $= x - \ln|1+e^x| + c$

2. $\int \frac{1}{1+\sqrt{e^x}}\,dx = \int \frac{1+\sqrt{e^x}-\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$

   $= \int \frac{1+\sqrt{e^x}}{1+\sqrt{e^x}}\,dx - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$

   $= \int 1\,dx - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$

   $x - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}}\,dx$

   For the remaining integral, use the substitution$u = \sqrt{e^x}$, meaning that $u^2 = e^x$.

   $x = \ln u^2 = 2\ln u$

   $dx = \frac{2}{u}\,du$

   $x - \int \frac{u}{1+u}\frac{2\,du}{u} = x - 2\int \frac{1}{1+u}\,du$

   $x - 2\ln|1+u| + c$

   $x - 2\ln|1+\sqrt{e^x}| + c$

3. $\int \sec x\,dx$
   In this case we will use the Camel Principle multiplicatively, multiplying by $\frac{\sec x+\tan x}{\sec x+\tan x}$

   This gives us the integral:
   $\int \frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x}\,dx$

   This is in the format $\frac{f'(x)}{f(x)}$, which integrates to $\ln|f(x)| + c$
   Therefore, our integral is $\ln|\sec x + \tan x| + c$

4. $\int \csc\theta \, d\theta$

To integrate, first multiply by $\frac{\csc\theta - \cot\theta}{\csc\theta + \cot\theta}$
This changes the integral to:
$\int \frac{\csc^2\theta - \csc\theta\cot\theta}{\csc\theta - \cot\theta}$

This is in the form $\frac{f'(x)}{f(x)}$, therefore the integral is $\ln|\csc\theta - \tan\theta| + c$