

Answers - The Camel Principle (page ??)

$$\begin{aligned}
 1. \int \frac{1}{1+e^x} dx &= \int \frac{1+e^x-e^x}{1+e^x} dx \\
 &= \int \frac{1+e^x}{1+e^x} dx - \int \frac{e^x}{1+e^x} dx \\
 &= \int 1 dx - \int \frac{e^x}{1+e^x} dx \\
 &= x - \ln|1+e^x| + c
 \end{aligned}$$

$$\begin{aligned}
 2. \int \frac{1}{1+\sqrt{e^x}} dx &= \int \frac{1+\sqrt{e^x}-\sqrt{e^x}}{1+\sqrt{e^x}} dx \\
 &= \int \frac{1+\sqrt{e^x}}{1+\sqrt{e^x}} dx - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}} dx \\
 &= \int 1 dx - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}} dx \\
 &= x - \int \frac{\sqrt{e^x}}{1+\sqrt{e^x}} dx
 \end{aligned}$$

For the remaining integral, use the substitution $u = \sqrt{e^x}$, meaning that $u^2 = e^x$.

$$x = \ln u^2 = 2 \ln u$$

$$dx = \frac{2}{u} du$$

$$x - \int \frac{u}{1+u} \frac{2 du}{u} = x - 2 \int \frac{1}{1+u} du$$

$$x - 2 \ln|1+u| + c$$

$$x - 2 \ln|1+\sqrt{e^x}| + c$$

$$3. \int \sec x dx$$

In this case we will use the Camel Principle multiplicatively, multiplying by $\frac{\sec x + \tan x}{\sec x + \tan x}$

This gives us the integral:

$$\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$

This is in the format $\frac{f'(x)}{f(x)}$, which integrates to $\ln|f(x)| + c$

Therefore, our integral is $\ln|\sec x + \tan x| + c$

$$4. \int \csc \theta d\theta$$

To integrate, first multiply by $\frac{\csc \theta - \cot \theta}{\csc \theta + \cot \theta}$

This changes the integral to:

$$\int \frac{\csc^2 \theta - \csc \theta \cot \theta}{\csc \theta - \cot \theta} d\theta$$

This is in the form $\frac{f'(\theta)}{f(\theta)}$, therefore the integral is $\ln|\csc \theta - \cot \theta| + c$