Answers - L'Hôpital's Method (page ??)

1. $\lim\limits_{x\to 1} \dfrac{x^2+2x-3}{x^2-6x+5} = \dfrac{1+2-3}{1-6+5} = \dfrac{0}{0}$ (indeterminate)

   $\lim\limits_{x\to 1} \dfrac{2x+2}{2x-6} = \dfrac{2+2}{2-6} = \dfrac{4}{-4} = -1$

2. $\lim\limits_{x\to -2} \dfrac{x^2-4}{x+2} = \dfrac{4-4}{-2+2} = \dfrac{0}{0}$ (indeterminate)

   $\lim\limits_{x\to -2} \dfrac{(x-2)(x+2)}{x+2} = x-2 = -4$

3. $\lim\limits_{x\to \infty} \dfrac{2x^3-3x^2}{x^4+3x^2} = \dfrac{\frac{2}{x}-\frac{3}{x^2}}{1+\frac{3}{x^2}} = \dfrac{0}{1} = 0$

4. $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = \dfrac{0}{0}$ (indeterminate)

   $\lim\limits_{x\to 0} \dfrac{\cos x}{1} = \dfrac{-1}{1} = -1$

5. $\lim\limits_{x\to 0} \dfrac{\tan x}{\sin x} = \dfrac{0}{0}$ (indeterminate)

   $\lim\limits_{x\to 0} \dfrac{\sec^2 x}{\cos x} = \dfrac{1}{\cos^3 x} = \dfrac{1}{1} = 1$

6. $\lim\limits_{x\to 0} \dfrac{x-\sin x}{1-\cos x} = \dfrac{0-0}{1-1} = \dfrac{0}{0}$ (indeterminate)

   $\lim\limits_{x\to 0} \dfrac{1-\cos x}{1+\sin x} = \dfrac{1-1}{1+0} = 0$

7. $\lim\limits_{x\to 0} \dfrac{3x^2+x^3}{x^2+x^4} = \dfrac{0}{0}$ (indeterminate)

   $\lim\limits_{x\to 0} \dfrac{6x+3x^2}{2x+4x^3} = \dfrac{0}{0}$ (indeterminate)

   $\lim\limits_{x\to 0} \dfrac{6+6x}{2+12x^2} = \dfrac{6}{2} = 3$

8. $\lim_{x\to\infty} \frac{3x^2 + x^3}{x^2 + x^4} = \frac{\frac{3}{x^2} + \frac{1}{x}}{\frac{1}{x^2} + 1} = \frac{0}{1} = 0$

9. $\lim_{x\to\infty} \frac{e^x}{x^2} = \frac{\infty}{\infty}$ (indeterminate)

   $\lim_{x\to\infty} \frac{e^x}{2x} = \frac{\infty}{\infty}$ (indeterminate)

   $\lim_{x\to\infty} \frac{e^x}{2} = \frac{\infty}{2} = \infty$

10. $\lim_{x\to\infty} \frac{\sin\frac{\pi}{x}}{\frac{1}{2x}} = \frac{0}{0}$ (indeterminate)

    $\lim_{x\to\infty} \frac{\cos\frac{\pi}{x} \times -\frac{\pi}{x^2}}{-\frac{1}{2x^2}} = \cos\frac{\pi}{x} \times 2\pi = 1 \times 2\pi = 2\pi$

11. $\lim_{x\to\infty} xe^{-x} = \lim_{x\to\infty} \frac{x}{e^x} = \frac{\infty}{\infty}$ (indeterminate)

    $\lim_{x\to\infty} \frac{1}{e^x} = \frac{1}{\infty} = 0$