1 Binomial expansion

In your formula sheet you will see this on the first page:

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!r!}$$

Some values of $\binom{n}{r}$ are given in the table below.

$n \backslash r$	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66

This helps us expand out brackets that are raised to a high power. The numbers in the table give the coefficients of the terms when we expand the brackets. For example:

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Notice how the coefficients match the numbers in row 4 in the table.
Also notice that the powers start at 4 for the first term in the brackets and zero for the second term. They then decrease and increase by 1 each term respectively.
In general, the sum of the powers in each term will add to the power we are raising the bracket to (in the example this is 4).
Another example:

$$\begin{aligned}(2a-3b)^4 &= (2a)^4 + 4(2a)^3(-3b) + 6(2a)^2(-3b)^2 + 4(2a)(-3b)^3 + (-3b)^4 \\ &= 16a^4 - 96a^3b + 216a^2b^2 - 216ab^3 + 81b^4\end{aligned}$$

Questions

(Answers - page **??**)

Expand the following:

1. $(x+y)^3$
2. $(2x+y)^4$
3. $(2x-3)^5$
4. $(3x+2y)^4$
5. $(2x+\frac{1}{x^2})^4$

Scholarship questions would tend to look more like this:

6. Find the term independent of x in $(3x^2-\frac{1}{3x})^{12}$
7. Find the coefficient of the x^2 term in $(x^2+\frac{1}{x})^{10}$
8. Find the term independent of x in $2x^2-\frac{3}{x})^6$
9. It can be shown that $\cos(\theta)=\frac{e^{i\theta}+e^{-i\theta}}{2}$ and that $\sin(\theta)=\frac{e^{i\theta}-e^{-i\theta}}{2i}$
10. Use these identities, or otherwise, to show that:
    $\cos^6(\theta)=\frac{1}{32}\cos(6\theta)+\frac{3}{16}\cos(4\theta)+\frac{15}{32}\cos(2\theta)+\frac{5}{16}$