# 10.7 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

**Vocabulary: Fill in the blanks.**

1. The origin of the polar coordinate system is called the ________.
2. For the point $(r, \theta)$, $r$ is the ________ ________ from $O$ to $P$ and $\theta$ is the ________ ________, counterclockwise from the polar axis to the line segment $\overline{OP}$.
3. To plot the point $(r, \theta)$, use the ________ coordinate system.
4. The polar coordinates $(r, \theta)$ and the rectangular coordinates $(x, y)$ are related as follows:
   $x =$ ________ $y =$ ________ $\tan\theta =$ ________ $r^2 =$ ________

## Skills and Applications

**Plotting a Point in the Polar Coordinate System** **In Exercises 5–18, plot the point given in polar coordinates and find three additional polar representations of the point, using $-2\pi < \theta < 2\pi$.**

5. $(2, \pi/6)$
6. $(3, 5\pi/4)$
7. $(4, -\pi/3)$
8. $(1, -3\pi/4)$
9. $(2, 3\pi)$
10. $(4, 5\pi/2)$
11. $(-2, 2\pi/3)$
12. $(-3, 11\pi/6)$
13. $(0, 7\pi/6)$
14. $(0, -7\pi/2)$
15. $(\sqrt{2}, 2.36)$
16. $(2\sqrt{2}, 4.71)$
17. $(-3, -1.57)$
18. $(-5, -2.36)$

**Polar-to-Rectangular Conversion** **In Exercises 19–28, a point is given in polar coordinates. Convert the point to rectangular coordinates.**

19. $(0, \pi)$
20. $(0, -\pi)$
21. $(3, \pi/2)$
22. $(3, 3\pi/2)$
23. $(2, 3\pi/4)$
24. $(1, 5\pi/4)$
25. $(-2, 7\pi/6)$
26. $(-3, 5\pi/6)$
27. $(-3, -\pi/3)$
28. $(-2, -4\pi/3)$

**Using a Graphing Utility to Find Rectangular Coordinates** **In Exercises 29–38, use a graphing utility to find the rectangular coordinates of the point given in polar coordinates. Round your results to two decimal places.**

29. $(2, 7\pi/8)$
30. $(3/2, 6\pi/5)$
31. $(1, 5\pi/12)$
32. $(4, 7\pi/9)$
33. $(-2.5, 1.1)$
34. $(-2, 5.76)$
35. $(2.5, -2.9)$
36. $(8.75, -6.5)$
37. $(-3.1, 7.92)$
38. $(-2.04, -5.3)$

**Rectangular-to-Polar Conversion** **In Exercises 39–50, a point is given in rectangular coordinates. Convert the point to polar coordinates. (There are many correct answers.)**

39. $(1, 1)$
40. $(2, 2)$
41. $(-3, -3)$
42. $(-4, -4)$
43. $(3, 0)$
44. $(-6, 0)$
45. $(0, -5)$
46. $(0, 8)$
47. $(-\sqrt{3}, -\sqrt{3})$
48. $(-\sqrt{3}, \sqrt{3})$
49. $(\sqrt{3}, -1)$
50. $(-1, \sqrt{3})$

**Using a Graphing Utility to Find Polar Coordinates** **In Exercises 51–58, use a graphing utility to find one set of polar coordinates of the point given in rectangular coordinates. Round your results to two decimal places.**

51. $(3, -2)$
52. $(6, 3)$
53. $(-5, 2)$
54. $(7, -2)$
55. $(-\sqrt{3}, -4)$
56. $(5, -\sqrt{2})$
57. $\left(\frac{5}{2}, \frac{4}{3}\right)$
58. $\left(-\frac{7}{9}, -\frac{3}{4}\right)$

**Converting a Rectangular Equation to Polar Form** **In Exercises 59–78, convert the rectangular equation to polar form. Assume $a > 0$.**

59. $x^2 + y^2 = 9$
60. $x^2 + y^2 = 16$
61. $y = x$
62. $y = -x$
63. $x = 10$
64. $y = -2$
65. $3x - y + 2 = 0$
66. $3x + 5y - 2 = 0$
67. $xy = 16$
68. $2xy = 1$
69. $x = a$
70. $y = a$
71. $x^2 + y^2 = a^2$
72. $x^2 + y^2 = 9a^2$
73. $x^2 + y^2 - 2ax = 0$
74. $x^2 + y^2 - 2ay = 0$
75. $(x^2 + y^2)^2 = x^2 - y^2$
76. $(x^2 + y^2)^2 = 9(x^2 - y^2)$
77. $y^3 = x^2$
78. $y^2 = x^3$

**Converting a Polar Equation to Rectangular Form** **In Exercises 79–100, convert the polar equation to rectangular form.**

**79.** $r = 5$
**80.** $r = -7$
**81.** $\theta = 2\pi/3$
**82.** $\theta = -5\pi/3$
**83.** $\theta = \pi/2$
**84.** $\theta = 3\pi/2$
**85.** $r = 4 \csc \theta$
**86.** $r = 2 \csc \theta$
**87.** $r = -3 \sec \theta$
**88.** $r = -\sec \theta$
**89.** $r = -2 \cos \theta$
**90.** $r = 4 \sin \theta$
**91.** $r^2 = \cos \theta$
**92.** $r^2 = 2 \sin \theta$
**93.** $r^2 = \sin 2\theta$
**94.** $r^2 = \cos 2\theta$
**95.** $r = 2 \sin 3\theta$
**96.** $r = 3 \cos 2\theta$
**97.** $r = \dfrac{2}{1 + \sin \theta}$
**98.** $r = \dfrac{1}{1 - \cos \theta}$
**99.** $r = \dfrac{6}{2 - 3 \sin \theta}$
**100.** $r = \dfrac{5}{\sin \theta - 4 \cos \theta}$

**Converting a Polar Equation to Rectangular Form** **In Exercises 101–108, describe the graph of the polar equation and find the corresponding rectangular equation.**

**101.** $r = 6$
**102.** $r = 8$
**103.** $\theta = \pi/6$
**104.** $\theta = 3\pi/4$
**105.** $r = 3 \sec \theta$
**106.** $r = 2 \csc \theta$
**107.** $r = 2 \sin \theta$
**108.** $r = -6 \cos \theta$

**109. Ferris Wheel** The center of a Ferris wheel lies at the pole of the polar coordinate system, where the distances are in feet. Passengers enter a car at $(30, -\pi/2)$. It takes 45 seconds for the wheel to complete one clockwise revolution.

(a) Write a polar equation that models the possible positions of a passenger car.

(b) Passengers enter a car. Find and interpret their coordinates after 15 seconds of rotation.

(c) Convert the point in part (b) to rectangular coordinates. Interpret the coordinates.

**110. Ferris Wheel** Repeat Exercise 109 when the distance from a passenger car to the center is 35 feet and it takes 60 seconds to complete one clockwise revolution.

David Lehner/Shutterstock.com

## Exploration

**True or False?** **In Exercises 111 and 112, determine whether the statement is true or false. Justify your answer.**

**111.** If $\theta_1 = \theta_2 + 2\pi n$ for some integer $n$, then $(r, \theta_1)$ and $(r, \theta_2)$ represent the same point in the polar coordinate system.

**112.** If $|r_1| = |r_2|$, then $(r_1, \theta)$ and $(r_2, \theta)$ represent the same point in the polar coordinate system.

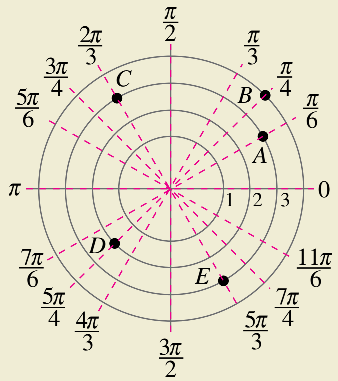
**113. Error Analysis** Describe the error in converting the rectangular coordinates $(1, -\sqrt{3})$ to polar form.

$$\tan \theta = -\sqrt{3}/1 \Rightarrow \theta = \frac{2\pi}{3}$$

$$r = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$$

$$(r, \theta) = \left(2, \frac{2\pi}{3}\right)$$

**114.** **HOW DO YOU SEE IT?** Use the polar coordinate system shown below.




(a) Identify the polar coordinates of points $A$–$E$.

(b) Which points lie on the graph of $r = 3$?

(c) Which points lie on the graph of $\theta = \pi/4$?

**115. Think About It**

(a) Convert the polar equation

$$r = 2(h \cos \theta + k \sin \theta)$$

to rectangular form and verify that it represents a circle.

(b) Use the result of part (a) to convert

$$r = \cos \theta + 3 \sin \theta$$

to rectangular form and find the center and radius of the circle it represents.