Polar Questions

1. A particle moves in a plane so that its position at any time $\theta, 0 \le \theta \le 8$, is given by the polar equation $r(\theta) = 5(1 + \cos\theta)$. When does the particle's distance from the origin change from decreasing to increasing?

   A) $\theta = 0$ only
   B) $\theta = \pi$ only
   C) $\theta = 2\pi$ only
   D) $\theta = 0$ and $\theta = \pi$
   E) $\theta = \pi$ and $\theta = 2\pi$

2. The area of the region enclosed by the polar curve $r = \cos 2\theta$ for $0 \le \theta \le \frac{\pi}{2}$ is

   A) $\frac{\pi}{2}$
   B) π
   C) $\frac{\pi}{8}$
   D) $\frac{\pi}{4}$
   E) 1

3. The area of one leaf of the rose $r = \sin 3\theta$ is

   A) $\frac{\pi}{12}$
   B) $\frac{\pi}{6}$
   C) $\frac{\pi}{4}$
   D) $\frac{\pi}{3}$
   E) $\frac{\pi}{2}$

4. The area outside $r = 1$ and inside $r = 1 + \sin\theta$ is

   A) $2 + \pi$
   B) $2 + \frac{\pi}{2}$
   C) $2 + \frac{\pi}{4}$
   D) $2 - \frac{\pi}{4}$
   E) $2 - \frac{\pi}{2}$

5. The total area of the region enclosed by the polar graph of $r = \cos 3\theta$ is

   A) $\frac{\pi}{12}$
   B) $\frac{\pi}{6}$
   C) $\frac{\pi}{4}$
   D) $\frac{\pi}{3}$
   E) $\frac{\pi+\sqrt{3}}{2}$

6. The area of the region enclosed by the polar curve $r = \sin\theta$ for $0 \le \theta \le \pi$ equals

   A) 1
   B) $\frac{\pi}{2}$
   C) $\frac{\pi}{4}$

D) $\frac{\pi}{8}$
E) π

7. Which of the following gives the area of the region enclosed by the graph of the polar curve $r = 1 + \cos\theta$?

   A) $\int_0^{\pi} \left(1 + \cos^2\theta\right) d\theta$
   B) $\int_0^{\pi} (1 + \cos\theta)^2 d\theta$
   C) $\int_0^{2\pi} (1 + \cos\theta) d\theta$
   D) $\int_0^{2\pi} (1 + \cos\theta)^2 d\theta$
   E) $\frac{1}{2}\int_0^{2\pi} \left(1 + \cos^2\theta\right) d\theta$

8. The area of the region enclosed by the polar curve $r = 2(\cos\theta + \sin\theta)$ is

   A) 1
   B) 2
   C) π
   D) 2π
   E) 4π

9. If the function $r = f(\theta)$ is continuous and nonnegative for $0 \leq \alpha \leq \theta \leq \beta \leq 2\pi$, then the area enclosed by the polar curve $r = f(\theta)$ and the lines $\theta = \alpha$ and $\theta = \beta$ is given by

   A) $\frac{1}{2}\int_\alpha^\beta f\left(\theta^2\right) d\theta$
   B) $\frac{1}{2}\int_\alpha^\beta f(\theta) d\theta$
   C) $\frac{1}{2}\int_\alpha^\beta \theta f\left(\theta^2\right)$
   D) $\frac{1}{2}\int_\alpha^\beta \theta f(\theta) d\theta$
   E) $\frac{1}{2}\int_\alpha^\beta (f(\theta))^2 d\theta$

10. Which of the following integrals gives the total area of the region shared by both polar curves $r = 2\cos\theta$ and $r = 2\sin\theta$

   A) $2\int_0^{\frac{\pi}{4}} \sin^2\theta d\theta$ A) $4\int_0^{\frac{\pi}{4}} \sin^2\theta d\theta$ A) $2\int_0^{\frac{\pi}{2}} \sin^2\theta d\theta$ A) $4\int_0^{\frac{\pi}{4}} \cos^2\theta d\theta$ A) $2\int_0^{\frac{\pi}{4}} \left(\cos^2\theta - \sin^2\theta\right) d\theta$