AOS Math 10, Spring 2024
Derivatives Test (#14)

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May 10, 2024

Student Signature **Class** **Date**

Print Name:

Instructions: Questions 1-14 are multiple choice. For each problem, circle the letter of the best answer. You **must show all work** for credit. Partial credit may be awarded as appropriate. Each question is valued at 3 points.

1. Let $f(x) = \sqrt{x}$. What is the equation of the tangent line to f at the point $(4, 2)$?

   (a) $y = -\frac{1}{2}x + 3$

   (b) $y = \frac{1}{2}x$

   (c) $y = \frac{1}{4}x + 1$

   (d) $y = 2x - 6$

2. What is the derivative of $s(t) = \cos\left(t^2 + 1\right)$?

   (a) $-(t^2 + 1)\sin(t^2 + 1)$

   (b) $-\sin(2t)$

   (c) $-2t\sin(t^2 + 1)$

   (d) $\cos(2t)$

3. If f and h are nonzero differentiable functions, then the derivative of $\dfrac{f}{h}$ is

   (a) $\dfrac{f'}{h'}$

   (b) $\dfrac{f'h + fh'}{h^2}$

   (c) $\dfrac{f'h - fh'}{h^2}$

   (d) $\dfrac{fh' - f'h}{h^2}$

4. The line tangent to the curve $y = \sqrt{16 - x}$ at the point $(0, 4)$ has slope

   (a) 4

   (b) $\dfrac{1}{8}$

   (c) $\dfrac{-1}{8}$

   (d) -4

5. If $y = 6\ln(3x)$ then what is y' ?

(a) $\dfrac{1}{3x}$

(b) $\dfrac{18}{x}$

(c) $\dfrac{2}{x}$

(d) $\dfrac{6}{x}$

6. What is the value of

$$\lim_{\Delta x \to 0} \frac{2(x+\Delta x)^2 - 2x^2}{\Delta x}$$

(a) 4

(b) 2

(c) $4x$

(d) $2x$

7. If $w(t) = \sqrt{t^2 - 1}$ what is the value of $w'(4)$?

(a) $\dfrac{4}{\sqrt{15}}$

(b) $\dfrac{2}{\sqrt{15}}$

(c) $\dfrac{1}{2\sqrt{15}}$

(d) $\dfrac{1}{\sqrt{15}}$

8. At which x value does the graph of $y = 3x^2 - 10x + 15$ have a horizontal tangent line?

(a) $\dfrac{-3}{5}$

(b) $\dfrac{-5}{3}$

(c) $\dfrac{3}{5}$

(d) $\dfrac{5}{3}$

9. If $h(x) = f\left(x^2 + 1\right)$ then which of the following is true?

   (a) $h'(x) = f'\left(x^2 + 1\right)$

   (b) $h'(x) = f'(2x)$

   (c) $h'(x) = 2xf'\left(x^2 + 1\right)$

   (d) $h'(x) = 2xf'(2x)$

10. If $f(x) = \sin(2x + 1)$ and $g(x) = f'(x)$, find $g'(x)$

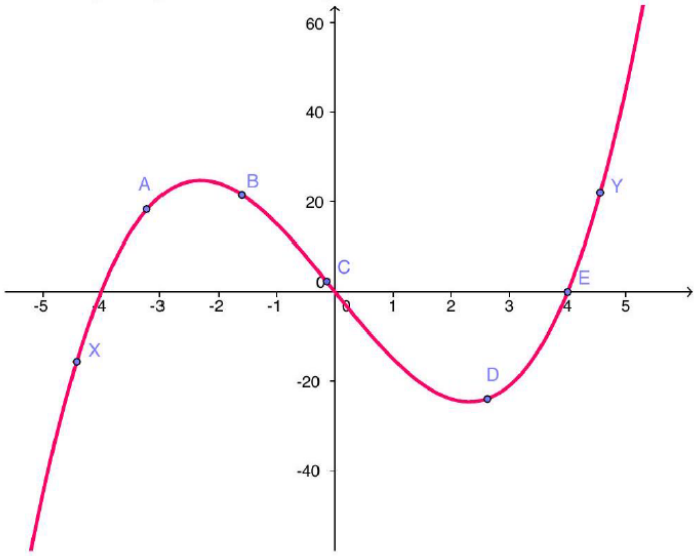
    (a) $g'(x) = -4\sin(2x + 1)$

    (b) $g'(x) = -4x\cos(2x + 1)$

    (c) $g'(x) = 2\sin(2x + 1)$

    (d) $g'(x) = 4\sin(2x + 1)\cos(2x + 1)$

11. The graph of a continuous differentiable function f is shown below.



Using the above graph, select the one true statement below.

(a) $f'(X) < f'(Y) < f'(C)$

(b) $f'(X) < f'(B) < f'(E)$

(c) $f'(C) < f'(D) < f'(Y)$

(d) $f'(A) < f'(B) < f'(C)$

12. Let $f(x) = x^3 - 6x^2 + 10$. At which point(s) on the graph of f is the tangent line parallel to the line $15x - y = 11$?

(a) $(2, -6)$ and $(-2, -22)$

(b) $(2, -6)$ and $(-2, 22)$

(c) $(5, -15)$ and $(-1, 3)$

(d) $(5, -15)$ and $(2, -6)$

13. If $y(x) = \dfrac{\sin(2x)}{x^2}$ find $y'(x)$

(a) $\dfrac{2x\cos(2x) - 2\sin(2x)}{x^3}$

(b) $\dfrac{x^2\sin(2x) + 2\cos(2x)}{x^4}$

(c) $\dfrac{x^2\cos(2x) - \sin(2x)}{x^3}$

(d) $\dfrac{2\cos(2x)}{x}$

14. Calculate $\dfrac{d}{dt}\left(\ln(e^{2t}) - 2t\right)$

(a) 0

(b) $\dfrac{2}{e^{2t}} - 2$

(c) $\dfrac{1}{2t} - 2$

(d) $\dfrac{1}{2e^{2t}} - 2$

Free Response Section: Selected values of f, g, f', g' are given in the table below.

x	0	1	2	3	4
$f(x)$	$\frac{1}{2}$	$\frac{1}{3}$	1	-1	3
$g(x)$	-2	1	$-\frac{1}{2}$	2	$-\frac{1}{3}$
$f'(x)$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{1}{4}$	0	$-\frac{4}{5}$
$g'(x)$	-1	$\frac{2}{3}$	-4	-3	$-\frac{1}{3}$

Using the values in the table, evaluate the following derivatives. **You must show the symbolic derivative as the first part of your answer for credit!**

15. $\frac{d}{dx}\left(f(x)+g(x)\right)$ at $x=4$

16. $\frac{d}{dx}\left(f(x)g(x)\right)$ at $x=1$

17. $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$ at $x=0$

18. $\frac{d}{dx}\left(f(g(x))\right)$ at $x=3$

19. $\frac{d}{dx}\left(g(x+f(x))\right)$ at $x=3$