

AOS Math 10, Spring 2024

Applications of Derivatives Test (#15)

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On my honor, I have not accepted or provided any unauthorized aid on this test, quiz, or assignment.

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May 24, 2024

Student Signature

Class

Date

Print Name:

Instructions: For each problem, circle the letter of the best answer. You **must show all work** for credit. Partial credit may be awarded as appropriate.

1. If $f(x) = \frac{\ln x}{x}$, for all $x > 0$, which of the following is true?
 - (a) f is increasing for all x greater than o .
 - (b) f is decreasing for all x between o and 1 .
 - (c) f is decreasing for all x between 1 and e .
 - (d) f is increasing for all x greater than 1 .
 - (e) f is decreasing for all x greater than e .

2. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$
 - (a) 2
 - (b) -2
 - (c) 1
 - (d) 4
 - (e) 0

3. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b ?
 - (a) 0
 - (b) 1
 - (c) 3
 - (d) -3

4. The function f given by $f(x) = x^3 + 12x - 24$ is
 - (a) decreasing for $x < 0$, increasing for $x > 0$
 - (b) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$
 - (c) decreasing for all x
 - (d) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
 - (e) increasing for all x

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5. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$
- (a) -1
 - (b) $-\frac{\sqrt{5}}{5}$
 - (c) 1
 - (d) $\frac{\sqrt{5}}{5}$
 - (e) 0
6. What is the x -coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?
- (a) -5
 - (b) 5
 - (c) $-\frac{10}{3}$
 - (d) -10
 - (e) 0
7. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$
- (a) -1 and 0 only
 - (b) -1 and 2 only
 - (c) -1, 0, and 2 only
 - (d) 2 only
 - (e) -1 only
8. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?
- (a) It cannot be determined if f has any relative extrema.
 - (b) f has relative maxima at $x = -2$ and at $x = 2$.
 - (c) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
 - (d) f has relative minima at $x = -2$ and at $x = 2$.
 - (e) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.

9. What is the derivative of $y = \csc \sqrt{t}$?

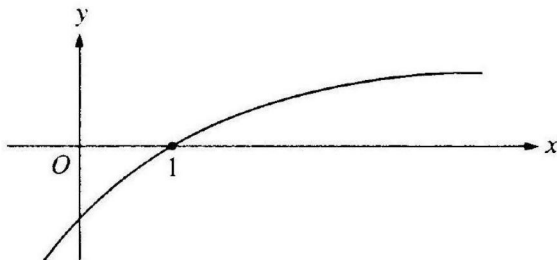
(a) $-\frac{\csc \sqrt{t} \cot \sqrt{t}}{2\sqrt{t}}$

(b) $-\sqrt{t} \cot^2 \sqrt{t}$

(c) $-\cot^2 \sqrt{t}$

(d) $-\csc \sqrt{t} \cot \sqrt{t}$

10. The graph of a twice-differentiable function f is shown in the figure below.



Which of the following is true?

(a) $f''(1) < f(1) < f'(1)$

(b) $f'(1) < f(1) < f''(1)$

(c) $f''(1) < f'(1) < f(1)$

(d) $f(1) < f''(1) < f'(1)$

(e) $f(1) < f'(1) < f''(1)$

Free Response 1

The function

$$f(x) = \frac{x}{x^2 + 4}$$

has first derivative

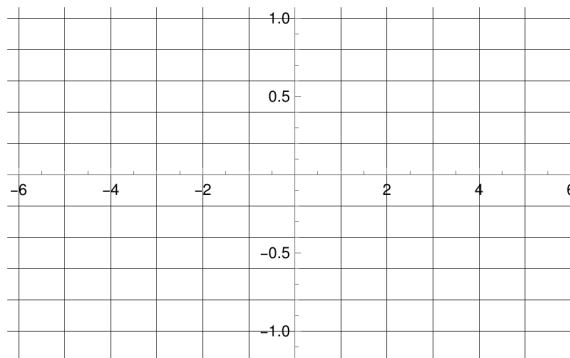
$$f'(x) = \frac{4 - x^2}{(x^2 + 4)^2}$$

and second derivative

$$f''(x) = \frac{2(x^3 - 12x)}{(x^2 + 4)^3}$$

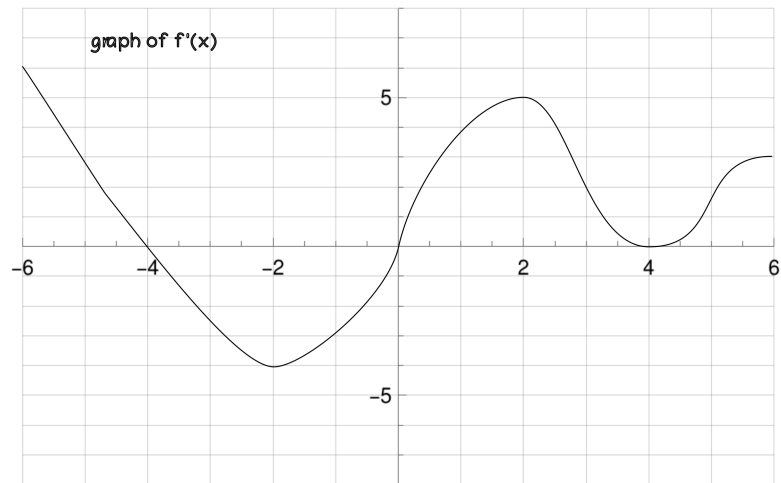
Sketch the graph of $f(x)$ after completing the following questions:

1. State any domain restrictions for $f(x)$
2. Determine any critical points of $f(x)$
3. State intervals on which $f(x)$ is increasing or decreasing
4. State intervals on which $f(x)$ is concave up or concave down
5. Calculate any horizontal asymptotes of $f(x)$



Free Response 2

The graph $f'(x)$ of the derivative of $f(x)$ is shown below. $f'(x)$ has horizontal tangents at $x = -2, 2, 4$ and zeros at $x = -4, 0, 4$. The domains of $f'(x)$ and $f(x)$ are $[-6, 6]$.



1. On which x intervals is the function f decreasing? Justify your answer.
2. At which x value(s) does f have a local minimum? Justify your answer.
3. On which x intervals is the function f concave up? Justify your answer.