AOS Senior AP Calculus BC, Spring 2024
AP Test Review, Exam 1

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Student Signature   **Class**   **Date**

Print Name:

1. Evaluate
$$\lim_{h \to 0} \frac{\sec(3(x+h)) - \sec(3x)}{h}$$

   (a) $3\sec^2(3x)$

   (b) $3\sec(x)\tan(x)$

   (c) $3\tan^2(3x)$

   (d) $9\tan^2(3x)$

   (e) $3\sec(3x)\tan(3x)$

2. Let f be the function given by
$$f(x) = \begin{cases} 2x + 3b & \text{if } x \le 2 \\ 3ax^2 & \text{if } x > 2 \end{cases}$$

   What are all values of a and b for which f is differentiable at $x = 2$?

   (a) $a = \dfrac{-1}{6} \quad b = \dfrac{-2}{3}$

   (b) $a = \dfrac{1}{3} \quad b = \dfrac{-2}{3}$

   (c) $a = \dfrac{-1}{6} \quad b = \dfrac{1}{3}$

   (d) $a = \dfrac{1}{6} \quad b = \dfrac{-1}{3}$

   (e) $a = \dfrac{1}{6} \quad b = \dfrac{-2}{3}$

3. If the function f is continuous for all real numbers and if $f(x) = \dfrac{x^2 - 25}{x - 5}$ when $x \ne 5$, then $f(5) =$

   (a) 5

   (b) 25

   (c) -5

   (d) -10

   (e) 10

4. Evaluate
$$\lim_{x \to e} \frac{\ln x - 1}{x - e}$$

   (a) 1

   (b) e

   (c) $2e$

   (d) $\dfrac{2}{e}$

   (e) $\dfrac{1}{e}$

5. Evaluate

$$\lim_{x\to 0} \frac{1-\cos x}{x^2 + \sin(4x)} =$$

(a) π

(b) 1

(c) -1

(d) $\dfrac{\pi}{2}$

(e) 0

6. If f is the function defined

$$f(x) = \begin{cases} x^2 & \text{for } x < 3 \\ \dfrac{1}{3} & \text{for } x \geq 3 \end{cases}$$

then $\displaystyle\int_{-2}^{4} f(x)dx$ is

(a) $\dfrac{20}{3}$

(b) $\dfrac{28}{3}$

(c) 13

(d) $\dfrac{-28}{3}$

(e) 12

7. Evaluate

$$\int_{0}^{3} \frac{x^2 + 5x + 6}{x + 2}dx =$$

(a) $4 + 2\ln 2$

(b) $\dfrac{15}{2}$

(c) $\dfrac{15}{2} + 2\ln 2$

(d) $\dfrac{17}{2} - 2\ln 2$

(e) $\dfrac{27}{2}$

8. Evaluate

$$\int \frac{\cos\left(\sqrt{x}+1\right)}{\sqrt{x}} dx =$$

(a) $\frac{1}{2}\sin\left(\sqrt{x}+1\right)+C$

(b) $\ln\left|\cos\left(\sqrt{x}+1\right)\right|+C$

(c) $\ln\left|\sin\left(\sqrt{x}+1\right)\right|+C$

(d) $\cos\left(\sqrt{x}+1\right)+C$

(e) $2\sin\left(\sqrt{x}+1\right)+C$

9. Evaluate

$$\int \frac{2x}{x^2+9} dx =$$

(a) $\frac{1}{2}\ln(x^2+9)+C$

(b) $\frac{1}{x^2+9}+C$

(c) $\frac{2}{(x^2+9)}+C$

(d) $\frac{x}{3}\arctan\left(\frac{x}{3}\right)+C$

(e) $\ln(x^2+9)+C$

10. The function g is continuous on the closed interval $[2,10]$. If $\int_9^1 g(x)dx = 25$ and $\int_1^5 \frac{g(x)}{2} dx = -12$, then $\int_5^9 g(x)\,dx =$

(a) 49

(b) 1

(c) 13

(d) -13

(e) -1

11. Using the substitution $u = 2x^2 + 1$, the integral $\displaystyle\int_2^4 2x\left(2x^2 + 1\right)^3 \, dx$ is equal to which of the following?

(a) $\displaystyle 2\int_9^{33} u^3 \, du$

(b) $\displaystyle \frac{1}{2}\int_9^{33} u^3 \, du$

(c) $\displaystyle 2\int_9^{33} u^3 \, du$

(d) $\displaystyle \frac{1}{4}\int_9^{33} u^3 \, du$

(e) $\displaystyle \frac{1}{2}\int_9^{33} u^3 \, du$

12. Evaluate

$$\int \frac{9x + 1}{(2x + 1)(x - 3)} \, dx$$

(a) $\ln|2x + 1| + \ln|x - 3| + C$

(b) $\frac{1}{2}\ln|2x + 1| - 2\ln|x - 3| + C$

(c) $4\ln|2x + 1| - 2\ln|x - 3| + C$

(d) $2\ln|2x + 1| - 4\ln|x - 3| + C$

(e) $\frac{1}{2}\ln|2x + 1| + 4\ln|x - 3| + C$

13. Evaluate

$$\int \frac{1}{x^2 - 16x + 80} \, dx =$$

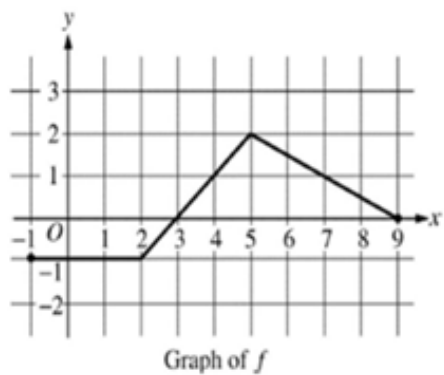(a) $\displaystyle \frac{1}{6}\arctan\left(\frac{x - 8}{6}\right) + C$

(b) $\displaystyle \frac{x}{6}\arctan\left|\frac{x - 8}{6}\right| + C$

(c) $\displaystyle \frac{1}{\ln|x^2 - 16x + 80|} + c$

(d) $2\ln|x - 20| - 4\ln|+4| + C$

(e) $\displaystyle \frac{1}{4}\arctan\left(\frac{x - 8}{4}\right) + C$

14. The graph of a piecewise linear function f is given.

Graph of f

What is the value of $\displaystyle\int_1^7 (4f(x) - 1)\ dx$?

(a) 9.5

(b) 7.5

(c) 9

(d) 10

(e) 8

15. Evaluate

$$\int_1^\infty xe^{-(x^2-1)}\ dx$$

(a) $\dfrac{1}{e}$

(b) $\dfrac{1}{2e}$

(c) 2

(d) divergent

(e) $\dfrac{1}{2}$

16. Integrate

$$\int x^3 e^{2x} \ dx$$

(a) $\dfrac{1}{4}e^{2x}\left(4x^3 - 6x^2 + 6x - 3\right) + C$

(b) $\dfrac{1}{8}e^{2x}\left(4x^3 + 6x^2 - x + 3\right) + C$

(c) $\dfrac{1}{4}e^{2x}\left(4x^3 + 6x^2 + 6x + 3\right) + C$

(d) $\dfrac{1}{8}e^{2x}\left(2x^3 - 3x^2 + 6x - 3\right) + C$

(e) $\dfrac{1}{8}e^{2x}\left(4x^3 - 6x^2 + 6x - 3\right) + C$