Instructions: For each problem, circle the letter of the best answer. You **must show all work** for credit. Partial credit may be awarded as appropriate.

1. If $f(x) = \dfrac{\ln x}{x}$, for all $x > 0$, which of the following is true?

   (a) f is increasing for all x greater than o.

   (b) f is increasing for all x greater than 1 .

   (c) f is decreasing for all x between o and 1 .

   (d) f is decreasing for all x between 1 and e.

   (e) f is decreasing for all x greater than e.

2. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

   (a) 1

   (b) -2

   (c) 4

   (d) 2

   (e) 0

3. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b ?

   (a) 3

   (b) -3

   (c) 0

   (d) 1

4. The function f given by $f(x) = x^3 + 12x - 24$ is

   (a) decreasing for $x < 0$, increasing for $x > 0$

   (b) increasing for all x

   (c) decreasing for all x

   (d) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

   (e) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$

5. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

   (a) 1

   (b) $\frac{\sqrt{5}}{5}$

   (c) 0

   (d) -1

   (e) $-\frac{\sqrt{5}}{5}$

6. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

   (a) -5

   (b) 5

   (c) $-\frac{10}{3}$

   (d) -10

   (e) 0

7. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

   (a) -1 and 0 only

   (b) -1 and 2 only

   (c) -1 only

   (d) 2 only

   (e) $-1, 0$, and 2 only

8. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right) g(x)$, which of the following is true?

   (a) f has relative maxima at $x = -2$ and at $x = 2$.

   (b) f has relative minima at $x = -2$ and at $x = 2$.

   (c) It cannot be determined if f has any relative extrema.

   (d) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.

   (e) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.

9. What is the derivative of $y = \csc\sqrt{t}$?

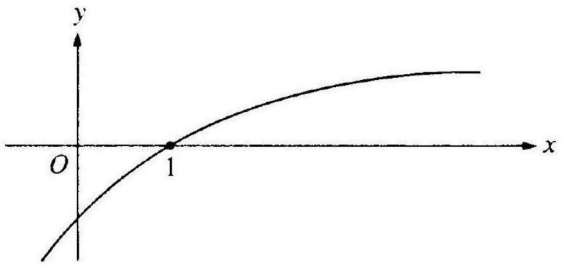
(a) $-\cot^2\sqrt{t}$

(b) $-\dfrac{\csc\sqrt{t}\cot\sqrt{t}}{2\sqrt{t}}$

(c) $-\csc\sqrt{t}\cot\sqrt{t}$

(d) $-\sqrt{t}\cot^2\sqrt{t}$

10. The graph of a twice-differentiable function f is shown in the figure below.



Which of the following is true?

(a) $f''(1) < f'(1) < f(1)$

(b) $f(1) < f'(1) < f''(1)$

(c) $f'(1) < f(1) < f''(1)$

(d) $f''(1) < f(1) < f'(1)$

(e) $f(1) < f''(1) < f'(1)$

Free Response 1

The function

$$f(x) = \frac{x}{x^2 + 4}$$

has first derivative

$$f'(x) = \frac{4 - x^2}{(x^2 + 4)^2}$$

and second derivative

$$f''(x) = \frac{2\left(x^3 - 12x\right)}{(x^2 + 4)^3}$$

Sketch the graph of $f(x)$ after completing the following questions:

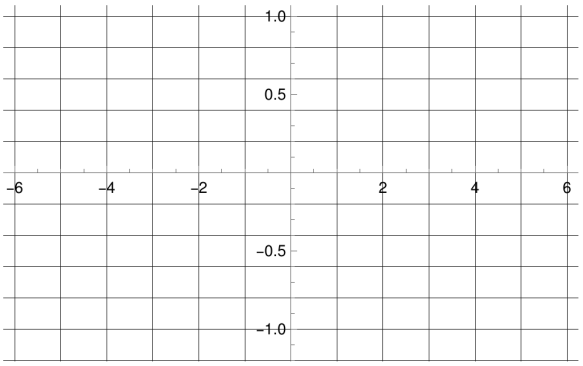
1. State any domain restrictions for $f(x)$

2. Determine any critical points of $f(x)$

3. State intervals on which $f(x)$ is increasing or decreasing

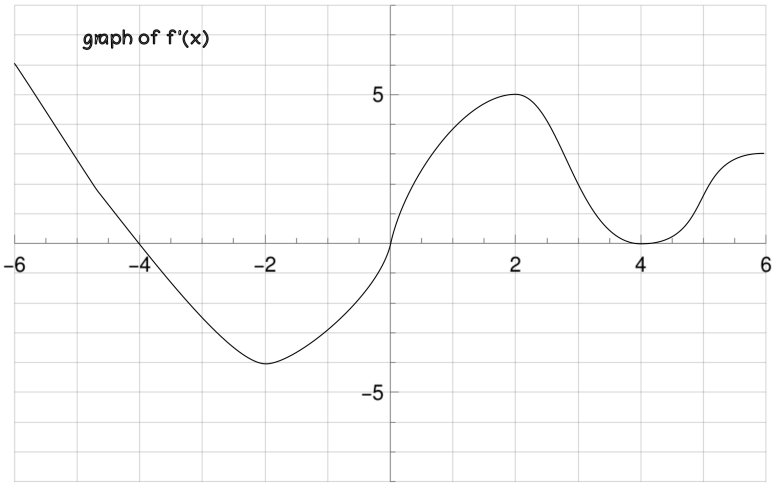
4. State intervals on which $f(x)$ is concave up or concave down

5. Calculate any horizontal asymptotes of $f(x)$



Free Response 2

The graph $f'(x)$ of the derivative of $f(x)$ is shown below. $f'(x)$ has horizontal tangents at $x = -2, 2, 4$ and zeros at $x = -4, 0, 4$. The domains of $f'(x)$ and $f(x)$ are $[-6, 6]$.



1. On which x intervals is the function f decreasing? Justify your answer.

2. At which x value(s) does f have a local minimum? Justify your answer.

3. On which x intervals is the function f concave up? Justify your answer.

KEY

1. A
2. C
3. C
4. B
5. D
6. B
7. C
8. E
9. B
10. B