

AOS Senior AP Calculus BC, Spring 2024

Cumulative, Quarter 3

(Parametric, Polar, Logistic)

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Class

Date

Print Name:

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1. The graph of $r = a + b \sin(\theta)$
 - (a) Has an inner loop whenever $a < b$
 - (b) Never intersects the x -axis
 - (c) Has an inner loop whenever $a > b$
 - (d) Never intersects the y -axis

 2. If a polar graph is defined by $r(\theta)$ and $\frac{dr}{d\theta} > 0$ at a point where $\theta = \alpha$ then
 - (a) The tangent line to the graph at $\theta = \alpha$ has a negative slope
 - (b) The tangent line to the graph $\theta = \alpha$ has a positive slope
 - (c) The graph's radius is increasing at $\theta = \alpha$
 - (d) The graph's radius is decreasing at $\theta = \alpha$

 3. If a particle in the first quadrant is moving towards the x axis then
 - (a) $\frac{d^2x}{dt^2} > 0$
 - (b) $\frac{d^2y}{dt^2} > 0$
 - (c) $\frac{dx}{dt} < 0$
 - (d) $\frac{dy}{dt} < 0$

 4. $x(t) = \cos t$ and $y(t) = \sin t$ are the parametric equations for
 - (a) A parabola
 - (b) A circle
 - (c) A hyperbola
 - (d) A square

 5. If a parametric curve has a point where $x'(a) = 0$ and $y'(a) = 0$ then
 - (a) There is no tangent line at $t = a$
 - (b) There is a vertical tangent line at $t = a$
 - (c) The curve must cross itself
 - (d) There is a horizontal tangent line at $t = a$

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6. A particle moves in a plane from an initial position given by the vector $\vec{r}_0 = \langle x_0, y_0 \rangle$ at time $t = 0$. The particle's velocity at any time t is described by the vector function $\vec{v}(t) = \langle v_x(t), v_y(t) \rangle$. Assuming the velocity function is integrable, which of the following expressions correctly describes the particle's position $\vec{r}(t)$ at any later time t ?

(a) $\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t) dt$

(b) $\vec{r}(t) = \vec{r}_0 + \vec{v}(t)t$

(c) $\vec{r}(t) = \vec{r}_0 + \frac{1}{2}\vec{v}(t)t^2$

(d) $\vec{r}(t) = \vec{r}_0 + \int \vec{v}(t)$

7. The maximum growth rate for a logistic population with carrying capacity L

(a) Can happen more than once during a given solution

(b) Occurs when the population is $L/2$

(c) Always occurs at $t = 0$

(d) Depends on the initial conditions

8. The distance traveled from $t = a$ to $t = b$ of a particle with position vector $\langle x(t), y(t) \rangle$ is given by

(a) $\sqrt{(x(t) - x(0))^2 + (y(t) - y(0))^2}$

(b) $\int_a^b \sqrt{x^2(t) + y^2(t)} dt$

(c) $\int_a^b |x'(t) + y'(t)| dt$

(d) $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$

9. To find the slope of the tangent line to a parametric curve at the point where $t = c$ you should

(a) Evaluate $y'(c)$

(b) Evaluate $y'(c)/x'(c)$

(c) Evaluate $x'(c)$

(d) Evaluate $x'(c)/y'(c)$

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10. A logistic population graph $y = f(t)$ with a max population of L
- (a) Has a decreasing growth rate when $t > 0$
 - (b) Has an increasing growth rate when $t > 0$
 - (c) Has an asymptote at $y = L$
 - (d) Can oscillate for certain initial conditions
11. If $x(t)$ and $y(t)$ are the parametric equations of a curve, the curve will have a horizontal tangent line at $t = c$ if
- (a) $x(c) = 0$ and $x'(c) = 0$
 - (b) $x'(c) = 0$ and $y'(c) \neq 0$
 - (c) $y'(c) = 0$ and $x'(c) \neq 0$
 - (d) $x(c) = 0$ and $y(c) = 0$
12. If n is a positive integer, the graph of $r = \sin(n\theta)$ always
- (a) Completes exactly one period of the graph over $0 \leq \theta < 2\pi$
 - (b) Has one intercept at $(0, 0)$
 - (c) Is a rose with $2n$ petals
 - (d) Is a rose with n petals
13. The area enclosed by a polar curve between $\theta = \alpha$ and $\theta = \beta$ is always
- (a) $\int_{\alpha}^{\beta} r^2(\theta) d\theta$
 - (b) $\int_{\alpha}^{\beta} \frac{1}{2} r^2(\theta) d\theta$
 - (c) Dependent on if the curve intersects itself in the interval $\alpha < \theta < \beta$
 - (d) $\int_{\alpha}^{\beta} r(\theta) d\theta$
14. Which of the following is **not** a polar-rectangular transformation equation?
- (a) $x^2 + y^2 = r^2$
 - (b) $y = r \sin \theta$
 - (c) $x = r \cos \theta$
 - (d) $\tan \theta = \frac{x}{y}$

15. To determine concavity of a parametric curve at the point where $t = c$

(a) Evaluate $\frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{\frac{dx}{dt}}$

(b) Evaluate $x''(c)/y''(c)$

(c) Evaluate $y''(c)/x''(c)$

(d) Evaluate $\frac{d}{dt} \left(\frac{dy}{dt} \right)$

KEY

1. A
2. C
3. D
4. B
5. A
6. A
7. B
8. D
9. B
10. C
11. C
12. B
13. C
14. D
15. A