AOS Math 10, Spring 2024
Applications of Derivatives Test (#15)

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May 24, 2024

Student Signature **Class** **Date**

Print Name:

Instructions: For each problem, circle the letter of the best answer. You **must show all work** for credit. Partial credit may be awarded as appropriate.

1. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

   (a) (*) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$

   (b) $x > 0$

   (c) $-2 < x < 0$ or $x > 2$

   (d) $x > \sqrt{2}$

   (e) $-2 < x < 2$

2. At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

   (a) (*) -1 only

   (b) 0 only

   (c) 1 only

   (d) -1 and 1 only

   (e) $-1, 0$ and 1

3. The graph of $y = \dfrac{-5}{x-2}$ is concave downward for all values of x such that

   (a) (*) $x > 2$

   (b) $x < 0$

   (c) $x < 2$

   (d) $x < 5$

   (e) $x > 0$

4. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

   (a) (*) 4

   (b) 2

   (c) 1

   (d) 0

   (e) -2

5. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b ?

   (a) (*) 0

   (b) -3

   (c) 1

   (d) 3

6. The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

   (a) (*) One

   (b) None

   (c) Two

   (d) Three

   (e) Four

7. If $f(x) = x^2 e^x$, then the graph of f is decreasing for all x such that

   (a) (*) $-2 < x < 0$

   (b) $x < -2$

   (c) $x \succ 2$

   (d) $x < 0$

   (e) $x > 0$

8. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right) g(x)$, which of the following is true?

   (a) (*) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.

   (b) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.

   (c) f has relative minima at $x = -2$ and at $x = 2$.

   (d) f has relative maxima at $x = -2$ and at $x = 2$.

   (e) It cannot be determined if f has any relative extrema.

9. What is the derivative of $y = \sec\sqrt{t}$?

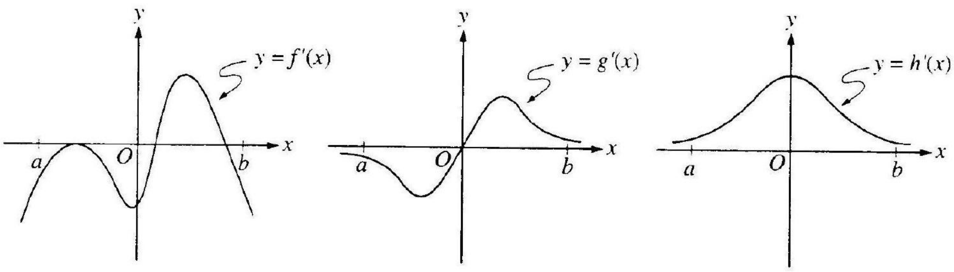
   (a) (*) $\dfrac{\sec\sqrt{t}\tan\sqrt{t}}{2\sqrt{t}}$

   (b) $\sec\sqrt{t}\tan\sqrt{t}$

   (c) $\tan^2\sqrt{t}$

   (d) $\sqrt{t}\tan^2\sqrt{t}$

10. The graphs of the derivatives of the functions f, g, and h are shown below. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?



   (a) (*) f only

   (b) g only

   (c) h only

   (d) f and g only

   (e) f, g, and h

Free Response 1

The function

$$f(x) = \frac{1}{x^2 - 4}$$

has first derivative

$$f'(x) = \frac{-2x}{\left(x^2 - 4\right)^2}$$

and second derivative

$$f''(x) = \frac{6x^2 + 8}{\left(x^2 - 4\right)^3}$$

Sketch the graph of $f(x)$ after completing the following questions:

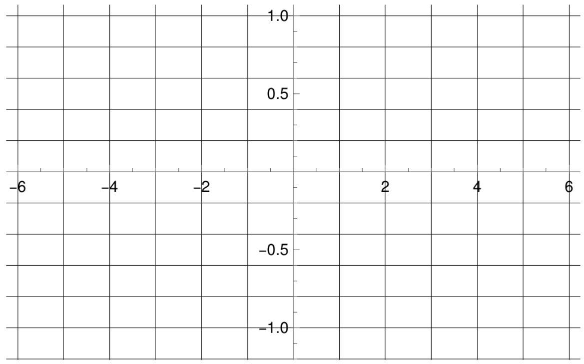
1. State any domain restrictions for $f(x)$

2. Determine any critical points of $f(x)$

3. State intervals on which $f(x)$ is increasing or decreasing

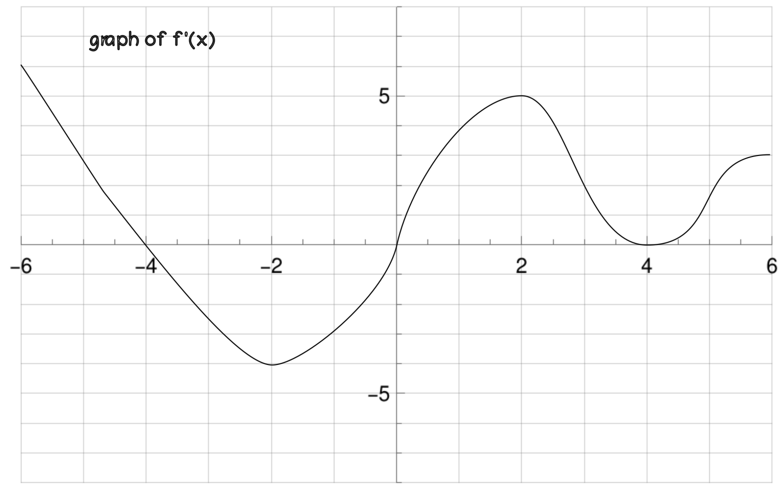
4. State intervals on which $f(x)$ is concave up or concave down

5. Calculate any horizontal asymptotes of $f(x)$



Free Response 2

The graph $f'(x)$ of the derivative of $f(x)$ is shown below. $f'(x)$ has horizontal tangents at $x = -2, 2, 4$ and zeros at $x = -4, 0, 4$. The domains of $f'(x)$ and $f(x)$ are $[-6, 6]$.



1. On which x intervals is the function f increasing? Justify your answer.

2. At which x value(s) does f have a local maximum? Justify your answer.

3. On which x intervals is the function f concave down? Justify your answer.