Day 2 Packet

BC Integration

1.
$$\int_1^e x + \frac{1}{x}\ dx = \left[\frac{x^2}{2} + \ln x\right]_1^e = \frac{e^2-1}{2} + \ln\left|\frac{e}{1}\right| = \frac{e^2-1}{2} + 1 \quad B$$

2.
$$\int_1^\infty xe^{-x^2}\ dx = \frac{-1}{2}\int_1^\infty -2xe^{-x^2}\ dx = \frac{-1}{2}\left[e^{-x^2}\right]_1^\infty = \frac{-1}{2}\left[0 - e^{-1}\right] B$$

3.
$$\begin{aligned}\int x^2 \cos x^3\ dx \\ &= \frac{1}{3}\int 3x^2\cos x^3\ dx \\ &= \frac{1}{3}\int \cos u\ du \\ &= \frac{1}{3}\sin u + C \\ &= B\end{aligned}$$

4.
$$\begin{aligned}\int_{-1}^9 3f(x) + 2\ dx &= 3\int_{-1}^9 f(x) + \int_{-1}^9 2\ dx \\ &= 3[-3.5+6] + 2(10) \\ &= 27.5 \\ &= C\end{aligned}$$

5.
$$\begin{aligned}y(10) &= y(0) + \int_0^{10} y'(t)dt \\ &= 12 + \int_0^{10} e^{t/5}dt \\ &= 12 + \left[5e^{t/5}\right]_0^{10} \\ &= 12 + 5\left(e^2 - 1\right) \\ &= C\end{aligned}$$

6.
$$\begin{aligned}\int_a^b f(x)\ dx &\cong \sum_{k=1}^n f(a + k\Delta x)\Delta x \text{ with } \Delta x = \frac{b-a}{n} = \frac{4}{n} \\ &= \sum e^{(-3+4k/n)}(4/n) \\ &= D\end{aligned}$$

7.
$$\begin{aligned}\int_1^3 f(x+5)\ dx = 7. \quad \text{Let } u = x+5 \\ = \int_6^8 f(u)\ du = 7\end{aligned}$$

$$\text{so} \int_6^8 f(x) + 2\ dx = 7 + 2(2) = 11 \quad D$$

8. The integral of a rate of X is change in X from $t = a$ to $t = b \ldots$ B

9.

$$\int_{-1}^{4} x\left(x^2-3\right)^5\ dx$$
$$u = x^2 - 3$$
$$du = 2x\ dx$$
$$x\ dx = \frac{du}{2}$$
$$= \int_{-2}^{13} u^5\ du/2$$
$$= C$$

10.

u	dv
$+x$	e^{-2x}
-1	$-\frac{1}{2}e^{-2x}$
$+0$	$\frac{1}{4}e^{-2x}$

$$k\int_0^{\infty} xe^{-2x}\ dx =$$
$$k\left(-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right)$$
$$= \left[\frac{-ke^{-2x}}{4}(2x+1)\right]_0^{\infty}$$
$$= \frac{k}{4}$$
$$= 1 \text{ if } k = 4$$
$$= C$$

11.

$$\int_1^e \frac{\ln x}{x^3}\ dx = uv - \int v\ du \qquad \begin{matrix} u = \ln x, dv = x^{-3} \\ du = \frac{1}{x} \quad v = -\frac{1}{2}x^{-2} \end{matrix}$$

$$1 = \frac{-\ln x}{2x^2} - \int \frac{-1}{2x^3}\ dx$$
$$= \left[\frac{-\ln x}{2x^2} - \frac{1}{4x^2}\right]_1^e$$
$$= \left(\frac{-\ln e}{2e^2} - \frac{1}{4e^2}\right) - \left(\frac{-1}{4}\right)$$
$$= \frac{e^2-3}{4e^2} = \text{ A}$$

12.

$$\int f(x)\sin x\ dx = f(x)(-\cos x) - \int 4x^3(-\cos x)\ dx$$
$$\int u\ dv = uv - \int\ du \cdot v$$
$$du = 4x^3$$
$$u = x^4$$

$$E$$

13.
$$\int_{-1}^{1} \frac{1}{x^2}\,dx = \int_{-1}^{0} \frac{1}{x^2}\,dx + \int_{0}^{1} \frac{1}{x^2}\,dx$$

but

$$\begin{aligned}\int_{-1}^{0} \frac{1}{x^2}\,dx &= \left[-\frac{1}{x}\right]_{-1}^{0} \\ &= \lim_{t\to 0}\left[\frac{-1}{x}\right]_{-1}^{t} \\ &= \infty \\ D\end{aligned}$$

14.
$$\begin{aligned}\int_{0}^{1} f'(x)g(x)\,dx = 5 = {} & g(x)f(x)|_0^1 - \int_0^1 f(x)g'(x)\,dx \\ &= (g(1)f(1) - g(0)f(0)) - \int_0^1 f(x)g'(x)\,dx \\ \int_0^1 f(x)g'(x)\,dx &= (3)(4) - (-4)(2) - 5 \\ &= 12 + 8 - 5 = 15 \\ E\end{aligned}$$

15. The first step (not show) is polynomial long division because the fraction is "top heavy" ($x^3 > x$)

$$\begin{aligned}\int_1^3 \frac{3x^3 + 15x^2 + x + 9}{x+5} &= \int_1^3 3x^2 + 1 + \frac{4}{x+5}\,dx \\ &= \left[x^3 + x + 4\ln|x+5|\right]_1^3 \\ &= 26 + 2 + 4\ln\left|\frac{8}{6}\right| \\ &= A\end{aligned}$$

16.
$$\begin{aligned}\int \frac{7x\,dx}{(2x-3)(x+2)} &= \int \frac{A}{2x-3} + \frac{B}{x+2} \\ & A(x+2) + B(2x-3) = 7x \\ & \text{Let } x = -2 \\ & -7B = -14 \text{ so } B = 2 \\ & x = \frac{3}{2} \\ 3.5A &= 10.5 \text{ so } A = 3 \\ \int \frac{7x}{(2x-3)(x+2)} &= \frac{3}{2}|2x-3| + 2\ln|x+2| \\ &= \quad A\end{aligned}$$

17. u-substitution. if $u = \sin x$,

$$\begin{aligned}
\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\, dx &= \int_{x=0}^{x=\pi/2} \frac{du}{1+u^2} \\
&= \int_0^1 \frac{du}{1+u^2} \\
&= [\arctan u]_0^1 \\
&= \frac{\pi}{4} - 0 \\
&B
\end{aligned}$$

18.

$$\begin{aligned}
\int_0^4 f(x)\, dx &= \int_0^2 f(x)\, dx + \int_2^4 f(x)\, dx \\
&= \left(\frac{5+1}{2}\right)(2) + 2(-3) = 0 \\
&= B
\end{aligned}$$

19.

$$\int \frac{1}{1+e^{2x}}$$

$$= \int \frac{e^{-2x}}{e^{-2x}+1}$$

$$u = e^{-2x} + 1$$

$$du = -2e^{-2x}$$

$$\int \frac{e^{-2x}}{e^{-2x}+1}$$

$$\begin{aligned}
&= \frac{-1}{2}\int \frac{du}{u} = -\frac{1}{2}\ln|u| = -\frac{1}{2}\ln\left|1+e^{-2x}\right| \\
&= -\frac{1}{2}\ln\left|e^{-2x}\left(e^{2x}+1\right)\right| \\
&= \frac{-1}{2}\ln\left|e^{-2x}\right| + \frac{-1}{2}\ln\left|e^{2x}+1\right| \\
&= -\frac{1}{2}(-2x) + \frac{-1}{2}\ln\left(1+e^{2x}\right) + C \\
&= D
\end{aligned}$$

20. First complete the square because the denominator has only imaginary roots $b^2 - 4ac < 0$.

$$\int \frac{dx}{x^2 - 10x + 34} = \int \frac{dx}{(x-5)^2 + 9}$$

Let $u = (x-5), a = 3$

$$\int \frac{dx}{(x-5)^2+9} = \int \frac{du}{u^2+a^2} = \frac{1}{3}\arctan\left(\frac{x-5}{3}\right) = C$$

21.

$$\frac{5x+8}{x^2+3x+2} = \frac{2}{x+2} + \frac{3}{x+1} \text{ so}$$
$$\int \frac{5x+8}{x^2+3x+2} = [2\ln|x+2| + 3\ln|x+1|]_0^1$$
$$= 2\ln\left|\frac{3}{2}\right| + 3\ln|2|$$
$$= \ln\frac{9}{4} + \ln 8$$
$$= \ln\frac{9}{4}\cdot 8$$
$$= \ln 18$$
$$= C$$

22.

$$f(2) = 2.4 + \int_{-1}^{2} \sqrt{9-x^2}\, dx = 10.863 = D$$