AOS Senior AP Calculus BC, Spring 2024
Cumulative, Quarter 3
(Parametric, Polar, Logistic)

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Print Name:

1. If a particle in the first quadrant is moving towards the x axis then

   (a) $\frac{d^2y}{dt^2} > 0$

   (b) $\frac{d^2x}{dt^2} > 0$

   (c) $\frac{dy}{dt} < 0$

   (d) $\frac{dx}{dt} < 0$

2. $x(t) = \cos t$ and $y(t) = \sin t$ are the parametric equations for

   (a) A square

   (b) A hyperbola

   (c) A circle

   (d) A parabola

3. To find the slope of the tangent line to a parametric curve at the point where $t = c$ you should

   (a) Evaluate $y'(c)/x'(c)$

   (b) Evaluate $x'(c)/y'(c)$

   (c) Evaluate $x'(c)$

   (d) Evaluate $y'(c)$

4. If a parametric curve has a point where $x'(a) = 0$ and $y'(a) = 0$ then

   (a) There is no tangent line at $t = a$

   (b) The curve must cross itself

   (c) There is a vertical tangent line at $t = a$

   (d) There is a horizontal tangent line at $t = a$

5. The maximum growth rate for a logistic population with carrying capacity L

   (a) Always occurs at $t = 0$

   (b) Depends on the initial conditions

   (c) Can happen more than once during a given solution

   (d) Occurs when the population is $L/2$

6. The area enclosed by a polar curve between $\theta = \alpha$ and $\theta = \beta$ is always

   (a) $\displaystyle\int_{\alpha}^{\beta} r(\theta)\, d\theta$

   (b) $\displaystyle\int_{\alpha}^{\beta} \frac{1}{2} r^2(\theta)\, d\theta$

   (c) $\displaystyle\int_{\alpha}^{\beta} r^2(\theta)\, d\theta$

   (d) Dependent on if the curve intersects itself in the interval $\alpha < \theta < \beta$

7. To determine concavity of a parametric curve at the point where $t = c$

   (a) Evaluate $\displaystyle\frac{d}{dt}\left(\frac{dy}{dt}\right)$

   (b) Evaluate $x''(c)/y''(c)$

   (c) Evaluate $y''(c)/x''(c)$

   (d) Evaluate $\dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

8. The graph of $r = a + b\sin(\theta)$

   (a) Has an inner loop whenever $a > b$

   (b) Has an inner loop whenever $a < b$

   (c) Never intersects the y-axis

   (d) Never intersects the x-axis

9. A logistic population graph $y = f(t)$ with a max population of L

   (a) Has an increasing growth rate when $t > 0$

   (b) Can oscillate for certain initial conditions

   (c) Has a decreasing growth rate when $t > 0$

   (d) Has an asymptote at $y = L$

10. If a polar graph is defined by $r(\theta)$ and $\dfrac{dr}{d\theta} > 0$ at a point where $\theta = \alpha$ then

    (a) The tangent line to the graph $\theta = \alpha$ has a positive slope

    (b) The tangent line to the graph at $\theta = \alpha$ has a negative slope

    (c) The graph's radius is decrasing at $\theta = \alpha$

    (d) The graph's radius is increasing at $\theta = \alpha$

11. If $x(t)$ and $y(t)$ are the parametric equations of a curve, the curve will have a horizontal tangent line at $t = c$ if

    (a) $x(c) = 0$ and $x'(c) = 0$

    (b) $x'(c) = 0$ and $y'(c) \neq 0$

    (c) $y'(c) = 0$ and $x'(c) \neq 0$

    (d) $x(c) = 0$ and $y(c) = 0$

12. A particle moves in a plane from an initial position given by the vector $\vec{r}_0 = \langle x_0, y_0 \rangle$ at time $t = 0$. The particle's velocity at any time t is described by the vector function $\vec{v}(t) = \langle v_x(t), v_y(t) \rangle$. Assuming the velocity function is integrable, which of the following expressions correctly describes the particle's position $\vec{r}(t)$ at any later time t?

    (a) $\vec{r}(t) = \vec{r}_0 + \frac{1}{2}\vec{v}(t)t^2$

    (b) $\vec{r}(t) = \vec{r}_0 + \displaystyle\int_0^t \vec{v}(t)\, dt$

    (c) $\vec{r}(t) = \vec{r}_0 + \vec{v}(t)t$

    (d) $\vec{r}(t) = \vec{r}_0 + \displaystyle\int \vec{v}(t)$

13. The distance traveled from $t = a$ to $t = b$ of a particle with position vector $\langle x(t), y(t) \rangle$ is given by

    (a) $\displaystyle\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

    (b) $\displaystyle\int_a^b |x'(t) + y'(t)|\, dt$

    (c) $\displaystyle\int_a^b \sqrt{x^2(t) + y^2(t)}\, dt$

    (d) $\sqrt{(x(t) - x(0))^2 + (y(t) - y(0))^2}$

14. If n is a positive integer, the graph of $r = \sin(n\theta)$ always

   (a) Completes exactly one period of the graph over $0 \leq \theta < 2\pi$

   (b) Has one intercept at $(0, 0)$

   (c) Is a rose with n petals

   (d) Is a rose with $2n$ petals

15. Which of the following is **not** a polar-rectangular transformation equation?

   (a) $\tan\theta = \dfrac{x}{y}$

   (b) $y = r\sin\theta$

   (c) $x = r\cos\theta$

   (d) $x^2 + y^2 = r^2$

KEY

1. C
2. C
3. A
4. A
5. D
6. D
7. D
8. B
9. D
10. D
11. C
12. B
13. A
14. B
15. A