

**Problem Statement:** Circle  $C_1$  has a radius of 1 and is centered at the origin. Circle  $C_2$  is tangent to circle  $C_1$  at the point  $(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$ , entirely contains circle  $C_1$ , and has twice the area of  $C_1$ . We need to write the equation of  $C_2$  in standard form.

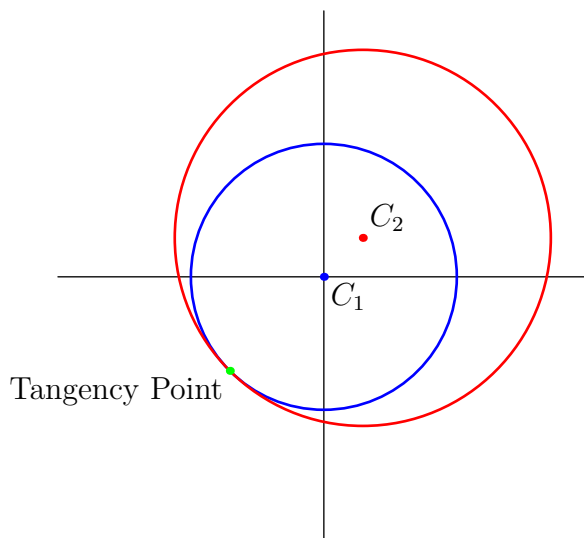


Figure 1: Two tangent circles

**Solution:**

**1. Determine the Radius of  $C_2$ :**

- The area of  $C_2$  is twice that of  $C_1$ . Since the area of a circle is  $\pi r^2$ , and the radius of  $C_1$  ( $r_1$ ) is 1, the radius of  $C_2$  ( $r_2$ ) can be found using  $\pi \times 1^2 \times 2 = \pi r_2^2$ .
- Solving this gives  $r_2 = \sqrt{2}$ .

**2. Find the Center of  $C_2$ :**

- Since  $C_2$  is tangent to  $C_1$  at  $(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$  and contains  $C_1$ , its center must be along the line extending from the origin to the point of tangency, but in the opposite direction.
- The center of  $C_2$  is  $\sqrt{2}$  units away from the origin in this direction. Given the symmetry and the tangency point, the center is at  $(1 - \frac{\sqrt{2}}{2}, 1 - \frac{\sqrt{2}}{2})$ .

**3. Write the Equation of  $C_2$ :**

- The standard form of a circle's equation is  $(x - h)^2 + (y - k)^2 = r^2$ , where  $(h, k)$  is the center and  $r$  is the radius.
- Substituting  $h = 1 - \frac{\sqrt{2}}{2}$ ,  $k = 1 - \frac{\sqrt{2}}{2}$ , and  $r = \sqrt{2}$  gives the equation of  $C_2$  as  $(x - 1 + \frac{\sqrt{2}}{2})^2 + (y - 1 + \frac{\sqrt{2}}{2})^2 = 2$ .

Therefore, the equation of circle  $C_2$  in standard form is:

$$(x - 1 + \frac{\sqrt{2}}{2})^2 + (y - 1 + \frac{\sqrt{2}}{2})^2 = 2$$

This represents the circle with twice the area of  $C_1$ , tangent to  $C_1$  at  $(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$ , and enclosing  $C_1$  completely.