AOS Math 10, Spring 2024
Cumulative, Quarter 3
(Parametric, Polar, Vectors, Complex)

_____/ *40*

ACADEMIES OF LOUDOUN HONOR CODE

Honesty and integrity are the foundations of good academic work. Whether you are working on a problem set, lab report, project, presentation, or paper, do not engage in plagiarism, unauthorized collaboration, cheating, or facilitating academic dishonesty. Our expectation is for our students to be successful while being trustworthy. The honor code is not intended to be punitive, but rather a guide for all students and faculty to follow. For these reasons, the Academies of Loudoun will uphold the following Honor Code:

On my honor, I have not accepted or provided any unauthorized aid on this test, quiz, or assignment.

As an Academies of Loudoun student, you agreed to uphold the Academies Honor Code. Please write the Honor Code Pledge below and sign this document.

_____

_____

_____

_____

Student Signature **Class** **Date**

Print Name:

1. Eliminate the parameter t: $x = t^3 - 2$ and $y = 1 - t^2$

   (a) $y = 1 - \sqrt[3]{(x+2)^2}$

   (b) $y = 1 + \sqrt[3]{(2x+2)^2}$

   (c) $y = 1 + \sqrt[3]{(x+2)^3}$

   (d) $y = 1 - \sqrt[3]{(2x+2)^2}$

2. Eliminate the parameter t: $x = 4\sin(t) + 1$ and $y = 3\cos(t) - 2$

   (a) $\dfrac{(x-1)^2}{4} + \dfrac{(y+2)^2}{3} = 1$

   (b) $\dfrac{(x-1)^2}{3} + \dfrac{(y+2)^2}{4} = 1$

   (c) $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

   (d) $\dfrac{(x-1)^2}{16} + \dfrac{(y+2)^2}{9} = 1$

3. Convert the polar coordinate $(6, -2\pi/3)$ to rectangular coordinates

   (a) $(-3, -3)$

   (b) $(3\sqrt{3}, -3\sqrt{3})$

   (c) $(-3, -3\sqrt{3})$

   (d) $(3, 3\sqrt{3})$

4. Convert this equation to polar coordinates: $x^2 - y^2 = 16$

   (a) $r^2 = \dfrac{4}{\cos^2\theta + \sin^2\theta}$

   (b) $r^2 = \dfrac{16}{\cos^2\theta - \sin^2\theta}$

   (c) $r^2 = \dfrac{16}{\cos^2\theta + \sin^2\theta}$

   (d) $r^2 = \dfrac{4}{\cos\theta - \sin\theta}$

5. Convert this equation to rectangular coordinates: $r = 3\sec\theta$

   (a) $x = 1$

   (b) $x = 3$

   (c) $y = 3$

   (d) $y = 1$

6. Which of the following is the graph of $r = \cos(3\theta)$

   (a) A 3 leaf rose with no x intercept

   (b) A 6 leaf rose with no x intercept

   (c) A 3 leaf rose with x-intercepts $(0,0), (1,0)$

   (d) A 6 leaf rose with x-intercepts $(0,0), (\pm 1, 0)$

7. A baseball pitcher throws a baseball with an initial speed of 138 feet per second at an angle of 20° to the horizontal. The ball leaves the pitcher's hand at a height of 4 feet above the ground. Write the equations of motion, v_x and v_y for velocity and s_x, s_y for position.

   (a) $v_x(t) = 129.7$, $v_y(t) = 47.2 - 16t$, $s_x(t) = 129.7t$, $s_y(t) = 47.2t - 16t^2 + 4$

   (b) $v_x(t) = 129.7$, $v_y(t) = 47.2 + 32t$, $s_x(t) = 129.7t$, $s_y(t) = 47.2t - 16t^2$

   (c) $v_x(t) = 129.7$, $v_y(t) = 47.2 - 32t$, $s_x(t) = 129.7t$, $s_y(t) = 47.2t - 16t^2 + 4$

   (d) $v_x(t) = 129.7$, $v_y(t) = 47.2 - 32t$, $s_x(t) = 129.7t$, $s_y(t) = 47.2t - 16t^2$

8. Let $u = \langle -3, 5\rangle$ and $\vec{v} = \langle 1, 4\rangle$ and $\vec{w} = \langle 6, -3\rangle$ find $\vec{u} + 2\vec{v} - \vec{w}$

   (a) $\langle -8, 16\rangle$

   (b) $\langle -7, 16\rangle$

   (c) $\langle -7, 14\rangle$

   (d) $\langle -6, 14\rangle$

9. Given $\vec{u} = \langle 3\sqrt{3}, -5\rangle$, find $\|\vec{u}\|$

   (a) $2\sqrt{17}$

   (b) $3\sqrt{17}$

   (c) $2\sqrt{13}$

   (d) $3\sqrt{13}$

10. Given $\vec{u} = \langle -10, 9 \rangle$, find a unit vector in the direction of $\vec{u}$

(a) $\langle -\frac{10}{\sqrt{181}}, -\frac{9}{\sqrt{181}} \rangle$

(b) $\langle -\frac{10}{\sqrt{19}}, -\frac{9}{\sqrt{19}} \rangle$

(c) $\langle -\frac{10}{\sqrt{181}}, \frac{9}{\sqrt{181}} \rangle$

(d) $\langle -\frac{10}{\sqrt{19}}, \frac{9}{\sqrt{19}} \rangle$

11. Which vector is perpendicular to $\langle \frac{2}{3}, -\frac{17}{2} \rangle$

(a) $\langle -9, \frac{18}{3} \rangle$

(b) $\langle -9, \frac{12}{17} \rangle$

(c) $\langle -9, -\frac{17}{18} \rangle$

(d) $\langle -9, -17 \rangle$

12. Which vector is parallel to $\langle \frac{2}{3}, -\frac{17}{2} \rangle$

(a) $\langle 25, -2 \rangle$

(b) $\langle 4, -51 \rangle$

(c) $\langle 2, -25 \rangle$

(d) $\langle -51, 4 \rangle$

13. What is the radian angle between $\langle 5, 1 \rangle$ and $\langle 2, -3 \rangle$

(a) 1.480

(b) 1.080

(c) 1.180

(d) 1.580

14. If vector $\vec{x}$ has magnitude 9 and makes an angle of 3.4 radians with the positive x axis, find the components of x and write as $a\hat{i} + b\hat{j}$.

    (a) $-2.30\hat{i} - 8.70\hat{j}$

    (b) $-8.70\hat{i} - 2.30\hat{j}$

    (c) $-9.20\hat{i} + 3.20\hat{j}$

    (d) $8.70\hat{i} + 2.30\hat{j}$

15. Write the complex number $-3 + 9i$ in polar form.

    (a) $9.49e^{1.33i}$

    (b) $9.59e^{1.79i}$

    (c) $9.29e^{1.81i}$

    (d) $9.49e^{1.89i}$

16. Divide $10 - 9i$ by $2 - 4i$, and express your answer in the form $a + bi$.

    (a) $-\frac{4}{5} - \frac{29}{10}i$

    (b) $-\frac{4}{5} + \frac{29}{10}i$

    (c) $\frac{14}{5} - \frac{11}{10}i$

    (d) $\frac{14}{5} + \frac{11}{10}i$

17. Simplify the product $\sqrt{7}e^{-i\pi/3} \cdot 3e^{i\pi/5}$.

    (a) $3\sqrt{7}e^{2\pi i/15}$

    (b) $3\sqrt{7}e^{\pi i/15}$

    (c) $3\sqrt{7}e^{-2\pi i/15}$

    (d) $3\sqrt{7}e^{-\pi i/15}$

18. Solve the equation $z^2 - 2z + 5 = 0$ for z and express your answers in rectangular form.

    (a) $1 \pm \sqrt{5}i$

    (b) $1 \pm 3i$

    (c) $1 \pm 2i$

    (d) $1 \pm 2i^2$

19. If $z = 2 + i$ is one root of a quadratic equation $x^2 + bx + c$ with real coefficients, what is bc?

(a) -10

(b) 8

(c) 4

(d) -20

20. Factor $z^2 + 9$ into a product of two binomials.

(a) $(z + 3\sqrt{i})(z - 3\sqrt{i})$

(b) $(z + 3i)(z + 3i)$

(c) $(z + 3i)(z - 3i)$

(d) $(z + 3)(z - 3)$

21. (Bonus): By multiplying two complex numbers, prove the addition identities for sin and cos.

KEY

1. A
2. D
3. C
4. B
5. B
6. C
7. C
8. B
9. C
10. C
11. B
12. B
13. C
14. B
15. D
16. D
17. C
18. C
19. D
20. C