Vector Pre-Test

Part A: Basic Operations and Concepts

1. **Vector Addition:** Given $\vec{a} = \langle 3, 4 \rangle$ and $\vec{b} = \langle 2, -1 \rangle$, find $\vec{a} + \vec{b}$.
2. **Vector Subtraction:** Given $\vec{a} = \langle -1, 5 \rangle$ and $\vec{b} = \langle 4, 3 \rangle$, find $\vec{a} - \vec{b}$.
3. **Scalar Multiplication:** If $\vec{a} = \langle 2, -3 \rangle$, find $5\vec{a}$.
4. **Magnitude of a Vector:** Calculate the magnitude of $\vec{a} = \langle 6, -8 \rangle$.
5. **Direction of a Vector:** Determine the argument of $\vec{b} = \langle 0, 5 \rangle$.
6. **Unit Vector:** Find a unit vector in the direction of $\vec{a} = \langle 4, 4 \rangle$.

Part B: Advanced Operations

7. **Dot Product:** Calculate the dot product of $\vec{a} = \langle 1, 2, 3 \rangle$ and $\vec{b} = \langle -2, 0, 4 \rangle$.
8. **Cross Product:** Find the cross product of $\vec{a} = \langle 1, 0, -1 \rangle$ and $\vec{b} = \langle 2, -1, 3 \rangle$.
9. **Vector in $\mathbf{a}\hat{\imath} + \mathbf{b}\hat{\jmath}$ Notation:** Express $\vec{c} = \langle 3, -4 \rangle$ in $\mathbf{a}\hat{\imath} + \mathbf{b}\hat{\jmath}$ notation.
10. **Resolving a Vector into Components:** Resolve $\vec{a} = 5\hat{\imath} - 3\hat{\jmath}$ into its component form.

Part C: Drawing and Illustration

11. **Drawing Vectors:** Draw the vector $\vec{a} = \langle 4, 3 \rangle$ from the origin in a Cartesian plane.
12. **Triangle Law of Addition:** Illustrate the triangle law of addition for $\vec{a} = \langle 2, 3 \rangle$ and $\vec{b} = \langle -1, 1 \rangle$.
13. **Parallelogram Law of Addition:** Show the parallelogram law of addition for $\vec{a} = \langle 3, 2 \rangle$ and $\vec{b} = \langle -2, 4 \rangle$.

Part D: 3D Vectors

14. **3D Vector Addition:** Given $\vec{a} = \langle 1, -2, 1 \rangle$ and $\vec{b} = \langle 3, 0, -1 \rangle$, find $\vec{a} + \vec{b}$.
15. **Magnitude of a 3D Vector:** Find the magnitude of $\vec{a} = \langle 2, -2, 1 \rangle$.
16. **Unit Vector in 3D:** Find a unit vector in the direction of $\vec{a} = \langle -1, 2, 2 \rangle$.

Part E: Word Problems

17. **Navigation:** A plane flies 100 km due east ($\hat{\imath}$) and then 200 km due north ($\hat{\jmath}$). Represent this path as a vector and find its magnitude.
18. **Physics:** A force of 10 N is applied in the direction $\langle 3, 4 \rangle$. Find the components of this force in the east ($\hat{\imath}$) and north ($\hat{\jmath}$) directions.
19. **Engineering:** A beam is subjected to two forces: $F_1 = 50\hat{\imath} - 30\hat{\jmath} + 20\hat{k}$ N and $F_2 = -20\hat{\imath} + 40\hat{\jmath} - 10\hat{k}$ N. Find the resultant force on the beam.

Part F: Bonus Question

20. **Challenging Problem:** Given vectors $\vec{a} = \langle 4, -3, 2 \rangle$ and $\vec{b} = \langle -2, 1, 5 \rangle$, find a vector $\vec{c}$ that is perpendicular to both $\vec{a}$ and $\vec{b}$ and has a magnitude of 10 units.

Remember to show all your work for full credit, especially for the drawing and word problem sections. Good luck!

Vector Arithmetic Summary

Basic Concepts

- **Vectors** are mathematical entities with both magnitude (length) and direction, represented in 2D or 3D space.
- **Notation:** Vectors are often denoted as $\vec{a}$ or in component form as $\langle x, y \rangle$ in 2D and $\langle x, y, z \rangle$ in 3D, or using unit vector notation as $a\hat{i} + b\hat{j}$ (in 2D) and $a\hat{i} + b\hat{j} + c\hat{k}$ (in 3D).

Vector Operations

- **Addition:** Vector addition is performed component-wise. Given two vectors $\vec{a} = \langle a_x, a_y \rangle$ and $\vec{b} = \langle b_x, b_y \rangle$, their sum is $\vec{a} + \vec{b} = \langle a_x + b_x, a_y + b_y \rangle$.
- **Subtraction:** Similar to addition, vector subtraction is component-wise: $\vec{a} - \vec{b} = \langle a_x - b_x, a_y - b_y \rangle$.
- **Scalar Multiplication:** Multiplying a vector by a scalar changes its magnitude without affecting its direction: $k\vec{a} = \langle ka_x, ka_y \rangle$.
- **Magnitude (or Length):** The magnitude of a vector $\vec{a} = \langle a_x, a_y \rangle$ is given by $\|\vec{a}\| = \sqrt{a_x^2 + a_y^2}$. In 3D, for $\vec{a} = \langle a_x, a_y, a_z \rangle$, it is $\sqrt{a_x^2 + a_y^2 + a_z^2}$.
- **Direction (or Argument):** The direction of a vector can be found using trigonometry, typically as an angle measured from the positive x-axis.

Advanced Concepts

- **Dot Product:** The dot product of $\vec{a}$ and $\vec{b}$ is a scalar: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$ (in 2D) or $a_x b_x + a_y b_y + a_z b_z$ (in 3D), which can indicate the angle between the vectors.
- **Cross Product:** Available only in 3D, the cross product of $\vec{a}$ and $\vec{b}$ results in a vector $\vec{c}$ that is perpendicular to both $\vec{a}$ and $\vec{b}$, with magnitude equal to the area of the parallelogram formed by $\vec{a}$ and $\vec{b}$.

Unit Vectors and Decomposition

- **Unit Vectors:** A unit vector has a magnitude of 1 and points in the direction of the given vector. It's calculated as $\hat{a} = \frac{\vec{a}}{\|\vec{a}\|}$.
- **Decomposition:** A vector can be decomposed into its components along the axes, which is useful for analyzing its direction and magnitude in terms of basic vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$.

Applications

- **Triangle and Parallelogram Laws:** These laws illustrate vector addition graphically. The triangle law forms a triangle from the tail of the first vector to the head of the second, while the parallelogram law forms a parallelogram to find the resultant vector.
- **Real-World Problems:** Vectors find applications in various fields, including physics (for force and velocity), engineering (for stress and displacement), and navigation (for displacement and direction).

Understanding these fundamentals of vector arithmetic is essential for solving problems related to vector addition, subtraction, scalar multiplication, magnitudes, and more, in both two and three dimensions.