

AOS Math 10, Spring 2024

Applications of Derivatives Test (#15)

_____/____



ACADEMIES OF LOUDOUN HONOR CODE



Honesty and integrity are the foundations of good academic work. Whether you are working on a problem set, lab report, project, presentation, or paper, do not engage in plagiarism, unauthorized collaboration, cheating, or facilitating academic dishonesty. Our expectation is for our students to be successful while being trustworthy. The honor code is not intended to be punitive, but rather a guide for all students and faculty to follow. For these reasons, the Academies of Loudoun will uphold the following Honor Code:

On my honor, I have not accepted or provided any unauthorized aid on this test, quiz, or assignment.

As an Academies of Loudoun student, you agreed to uphold the Academies Honor Code. Please write the Honor Code Pledge below and sign this document.

May 24, 2024

Student Signature

Class

Date

Print Name:

KEY

1. B
2. E
3. C
4. C
5. D
6. A
7. C
8. E
9. B
10. D

Instructions: For each problem, circle the letter of the best answer. You **must show all work** for credit. Partial credit may be awarded as appropriate.

1. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

- (a) $x > \sqrt{2}$
- (b) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$
- (c) $-2 < x < 2$
- (d) $-2 < x < 0$ or $x > 2$
- (e) $x > 0$

2. At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

- (a) $-1, 0$ and 1
- (b) 0 only
- (c) -1 and 1 only
- (d) 1 only
- (e) -1 only

3. The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that

- (a) $x < 5$
- (b) $x < 2$
- (c) $x > 2$
- (d) $x < 0$
- (e) $x > 0$

4. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

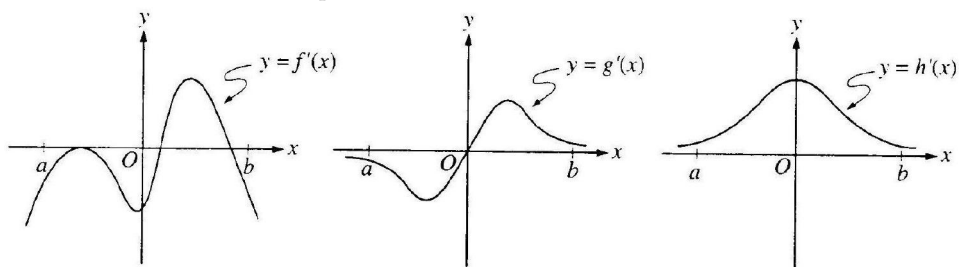
- (a) 1
- (b) 0
- (c) 4
- (d) -2
- (e) 2

-
5. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b ?
- (a) -3
 - (b) 3
 - (c) 1
 - (d) 0
6. The derivative of f is $x^4(x - 2)(x + 3)$. At how many points will the graph of f have a relative maximum?
- (a) One
 - (b) Three
 - (c) None
 - (d) Four
 - (e) Two
7. If $f(x) = x^2e^x$, then the graph of f is decreasing for all x such that
- (a) $x < -2$
 - (b) $x < 0$
 - (c) $-2 < x < 0$
 - (d) $x > 0$
 - (e) $x > 2$
8. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?
- (a) f has relative maxima at $x = -2$ and at $x = 2$.
 - (b) It cannot be determined if f has any relative extrema.
 - (c) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
 - (d) f has relative minima at $x = -2$ and at $x = 2$.
 - (e) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.

9. What is the derivative of $y = \sec \sqrt{t}$?

- (a) $\tan^2 \sqrt{t}$
- (b) $\frac{\sec \sqrt{t} \tan \sqrt{t}}{2\sqrt{t}}$
- (c) $\sqrt{t} \tan^2 \sqrt{t}$
- (d) $\sec \sqrt{t} \tan \sqrt{t}$

10. The graphs of the derivatives of the functions f, g , and h are shown below. Which of the functions f, g , or h have a relative maximum on the open interval $a < x < b$?



- (a) g only
- (b) f and g only
- (c) h only
- (d) f only
- (e) f, g , and h

Free Response 1

The function

$$f(x) = \frac{1}{x^2 - 4}$$

has first derivative

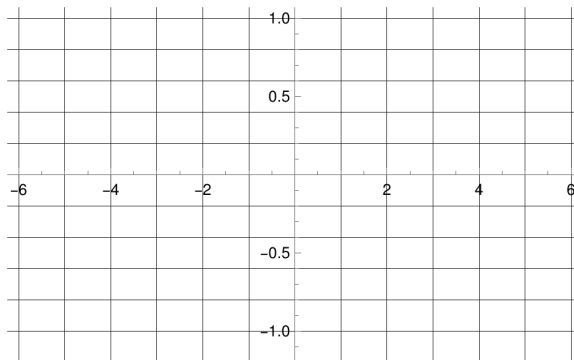
$$f'(x) = \frac{-2x}{(x^2 - 4)^2}$$

and second derivative

$$f''(x) = \frac{6x^2 + 8}{(x^2 - 4)^3}$$

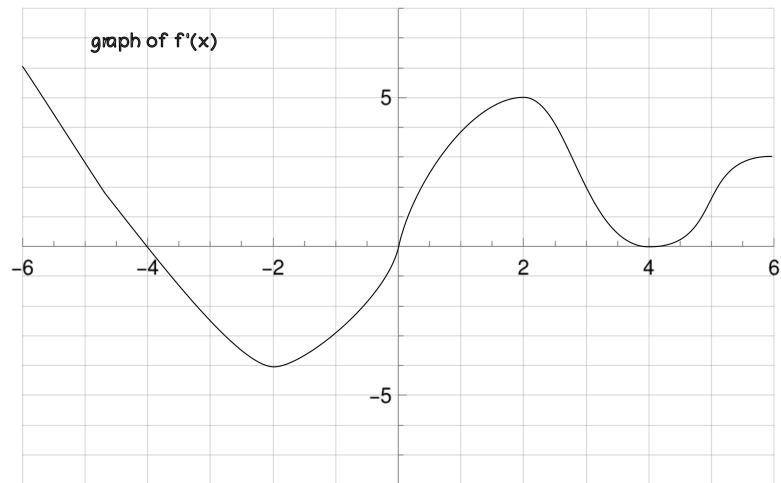
Sketch the graph of $f(x)$ after completing the following questions:

1. State any domain restrictions for $f(x)$
2. Determine any critical points of $f(x)$
3. State intervals on which $f(x)$ is increasing or decreasing
4. State intervals on which $f(x)$ is concave up or concave down
5. Calculate any horizontal asymptotes of $f(x)$



Free Response 2

The graph $f'(x)$ of the derivative of $f(x)$ is shown below. $f'(x)$ has horizontal tangents at $x = -2, 2, 4$ and zeros at $x = -4, 0, 4$. The domains of $f'(x)$ and $f(x)$ are $[-6, 6]$.



1. On which x intervals is the function f increasing? Justify your answer.
2. At which x value(s) does f have a local maximum? Justify your answer.
3. On which x intervals is the function f concave down? Justify your answer.