Solution Using a vertical major axis, as shown in Figure 10.61, choose an equation of the form $r = ep/(1 + e\sin\,\theta)$. The vertices of the ellipse occur when $\theta = \pi/2$ and $\theta = 3\pi/2$, and the length of the major axis is the sum of the r-values of the vertices. That is,

$$2a = \frac{0.967p}{1 + 0.967} + \frac{0.967p}{1 - 0.967} \approx 29.79p \approx 35.88.$$

So, $p \approx 1.204$ and $ep \approx (0.967)(1.204) \approx 1.164$. Substituting this value for ep in the equation, you have

$$r = \frac{1.164}{1 + 0.967\sin\,\theta}$$

where r is measured in astronomical units. To find the closest point to the sun (a focus), substitute $\theta = \pi/2$ into this equation to obtain

$$r = \frac{1.164}{1 + 0.967\sin\,(\pi/2)}$$
$$\approx 0.59 \text{ astronomical unit } \approx 55{,}000{,}000 \text{ miles.}$$

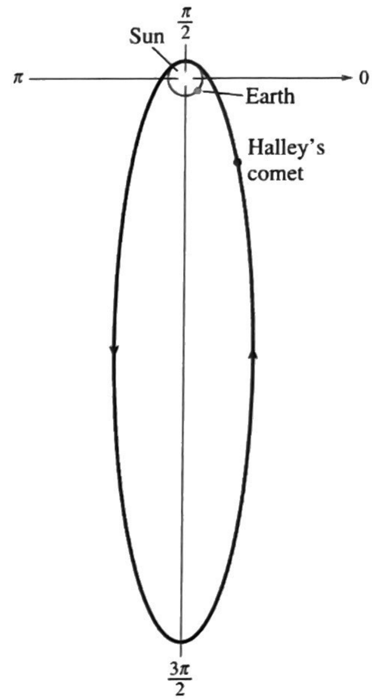
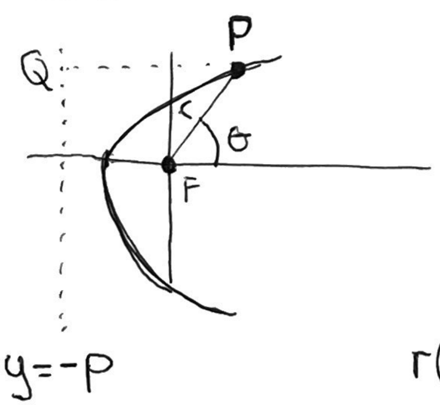


FIGURE 10.61

CHECKPOINT

Encke's comet has an elliptical orbit with an eccentricity of $e \approx 0.847$. The length of the major axis of the orbit is approximately 4.420 astronomical units. Find a polar equation for the orbit. How close does Encke's comet come to the sun?

Derivation



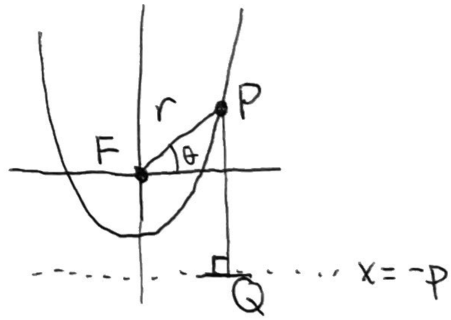
$$\begin{cases} PF = r \\ PQ = p + r\cos\theta \\ PF = e \cdot PQ \end{cases}$$

$$r = ep + er\cos\theta$$

$$r(1 - e\cos\theta) = ep$$

$$r = \frac{ep}{1 - e\cos\theta}$$

Any conic, vertical directrix



$$\begin{cases} PF = r \\ PQ = p + r\sin\theta \\ PF = e \cdot PQ \end{cases}$$

$$r = ep + er\sin\theta$$

$$r = \frac{ep}{1 - e\sin\theta}$$

Any conic, horizontal directrix