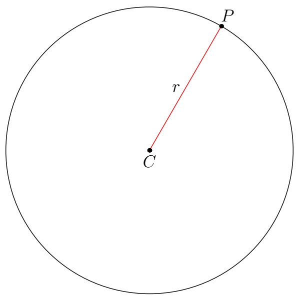
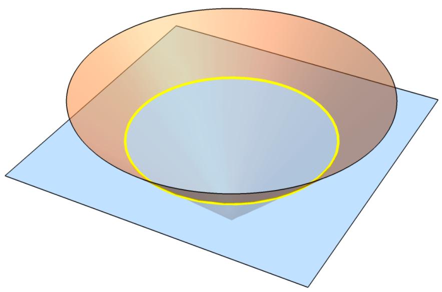
The Circle

The circle is one of the most basic shapes, known to humans for tens of thousands of years according to the archaeological record. It defines everybody's favorite constant, π, and is the basis of analytic trigonometry.

Geometric Definition: The circle is the locus of points all equidistant from a given *center*. The distance from any such point to the center is called the *radius*.





Parent Equation: The parent equation $x^2 + y^2 = 1$ is the equation for the unit circle, a circle centered at the origin with radius 1.

Standard Form Equation: $(x - h)^2 + (y - k)^2 = r^2$ is a circle with radius r centered at (h, k)

General Form Equation: $Ax^2 + Cy^2 + Dx + Ey + F = 0$. This is the general form for any (non-rotated) conic.

Geometry Review: Given a line segment joining (x_1, y_1) and (x_2, y_2)

- the length of the segment is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- The midpoint of the segment is $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

Completing the Square: You can always rewrite $x^2 + bx + c$ as

- $x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$
- which is $(x + \frac{b}{2})^2 + (c - \left(\frac{b}{2}\right)^2)$

Problems

Unless otherwise specified, any circle equations should be given in standard form

Fundamental Concepts

1. Find the radius of the circle $x^2 + y^2 = 100$.
2. Find the radius of the circle $x^2 + y^2 = 72$.
3. Find the radius and center of the circle $(x - 8)^2 + (y + 4)^2 = 20$.

4. Translate the circle given here by 5 units up and 3 units to the left: $(x+8)^2 + (y+2)^2 = 20$
5. Translate the circle given here so that its center is half as close to the origin: $(x+\sqrt{7})^2 + (y+\frac{\sqrt{5}}{2})^2 = \pi$
6. Write the equation of a circle with the same center but half the area of the circle $(x-1)^2+(y-5)^2 = 84$
7. Find the radius and center of the circle $(3x-4)^2 = 100 - (3y+1)^2$
8. Write the equation of a circle with radius 5 and center $(3, -4)$. Write both standard and general form.
9. Write the equation of a circle with radius $\frac{\sqrt{17}}{3}$ and center $(\frac{1}{3}, \frac{-2}{3})$. Eliminate all denominators. Write both standard and general form.
10. A diameter of a certain circle joins the points $(-12, 7)$ and $(12, 14)$. Find the equation of the circle.
11. Write the circle $(x+8)^2 + (y-\sqrt{2})^2 = 15\sqrt{2}$ in general form.
12. Complete the square to write the following in standard form: $x^2 + 10x + y^2 - 20y - 50 = 0$
13. Complete the square to write the following in standard form: $4x^2 + 8x + 4y^2 - 20y = 100$
14. Write a "completing the square" circle problem and trade with a friend.

Deeper Understanding

15. An equilateral triangle has its base as the line segment joining the points $(-4, -2)$ and $(4, -2)$. Find the third point of the triangle. Circumscribe a circle around this triangle and find the equation of the circle. (Hint: the center of the circle is the average of the three triangle vertices).
16. Given the circle $x^2 + y^2 = 72$, find four points on the circle that form the vertices of a square.
17. In the general form equation of a circle $Ax^2 + Cy^2 + Dx + Ey + F = 0$, can A be greater than C? Can it be less than C? Why or why not?
18. Write an inequality which constrains F in terms of the other coefficients in the general form equation of a circle. (Hint: find the center and radius of the general equation first by completing the square.)
19. Find the intersection points of the circle $x^2 + y^2 = 80$ with the line $y = 2x + 1$. Round your answer to 3 places.
20. A regular hexagon is inscribed in a unit circle. What is its perimeter?
21. The circle $x^2 + y^2 = 25$ contains the points $(3, 4)$ and $(3, -4)$. The tangent line to the circle at the point $(3, 4)$ is perpendicular to the line segment from the origin to $(3, 4)$. Write the equation of this tangent line. Also write the equation of the tangent line through $(3, -4)$.
22. Circle C_1 has radius 1 and is centered at the origin. Circle C_2 is tangent to circle C_1 at the point $(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$, entirely contains circle C_1 and has twice the area of C_1. Write the equation of C_2 in standard form.