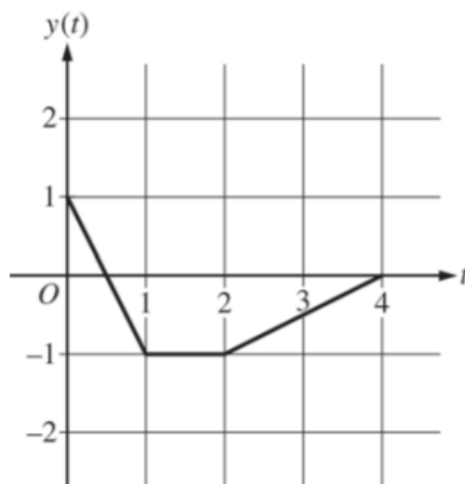# AP Calculus BC – Parametric Equations AP Test Practice

FRQ.1 (calculator)

At time t, the position of a particle moving in the xy-plane is given by the parametric functions $(x(t), y(t))$, where $\dfrac{dx}{dt} = t^2 + \sin(3t^2)$. The graph of y, consisting of three line segments, is shown in the figure above.
At $t = 0$, the particle is at position $(5, 1)$.

(a) Find the position of the particle at $t = 3$.

(b) Find the slope of the line tangent to the path of the particle at $t = 3$.

(c) Find the speed of the particle at $t = 3$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

FRQ.2 (calculator)

At time $t \geq 0$, a particle moving along a curve in the xy-plane has position $(x(t), y(t))$ with velocity vector $v(t) = \left(\cos(t^2), e^{0.5t}\right)$. At $t = 1$, the particle is at the point $(3, 5)$.

(a) Find the x-coordinate of the position of the particle at time $t = 2$.

(b) For $0 < t < 1$, there is a point on the curve at which the line tangent to the curve has a slope of 2. At what time is the object at that point?

(c) Find the time at which the speed of the particle is 3.

(d) Find the total distance traveled by the particle from time $t = 0$ to time $t = 1$.

FRQ.3 (calculator)

For $t \geq 0$, a particle is moving along a curve so that its position at time t is $(x(t), y(t))$. At time $t = 2$, the particle is at position $(1, 5)$. It is known that $\dfrac{dx}{dt} = \dfrac{\sqrt{t+2}}{e^t}$ and $\dfrac{dy}{dt} = \sin^2 t$.

(a) Is the horizontal movement of the particle to the left or to the right at time $t = 2$? Explain your answer. Find the slope of the path of the particle at time $t = 2$.

(b) Find the x-coordinate of the particle's position at time $t = 4$.

(c) Find the speed of the particle at time $t = 4$. Find the acceleration vector of the particle at time $t = 4$.

(d) Find the distance traveled by the particle from time $t = 2$ to $t = 4$.

MC.1

1. At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle t^2, 5t \rangle$. What is the acceleration vector of the particle at time $t = 3$?

(A) $\left\langle 9, \dfrac{45}{2} \right\rangle$ (B) $\langle 6, 5 \rangle$ (C) $\langle 2, 0 \rangle$ (D) $\sqrt{306}$ (E) $\sqrt{61}$

MC.2

Which of the following gives the length of the path described by the parametric equations $x = \sin(t^3)$ and $y = e^{5t}$ from $t = 0$ to $t = \pi$?

(A) $\displaystyle\int_0^\pi \sqrt{\sin^2(t^3) + e^{10t}}\, dt$

(B) $\displaystyle\int_0^\pi \sqrt{\cos^2(t^3) + e^{10t}}\, dt$

(C) $\displaystyle\int_0^\pi \sqrt{9t^4 \cos^2(t^3) + 25e^{10t}}\, dt$

(D) $\displaystyle\int_0^\pi \sqrt{3t^2 \cos(t^3) + 5e^{5t}}\, dt$

(E) $\displaystyle\int_0^\pi \sqrt{\cos^2(3t^2) + e^{10t}}\, dt$

MC.3

28. In the xy-plane, a particle moves along the parabola $y = x^2 - x$ with a constant speed of $2\sqrt{10}$ units per second. If $\dfrac{dx}{dt} > 0$, what is the value of $\dfrac{dy}{dt}$ when the particle is at the point $(2, 2)$?

(A) $\dfrac{2}{3}$ (B) $\dfrac{2\sqrt{10}}{3}$ (C) 3   (D) 6   (E) $6\sqrt{10}$

MC.4

The position of a particle moving in the xy-plane is given by the vector $\langle 4t^3, y(2t) \rangle$, where y is a twice-differentiable function of t. At time $t = \dfrac{1}{2}$, what is the acceleration vector of the particle?

(A) $\langle 3,\, 2y''(1) \rangle$

(B) $\langle 6,\, 4y''(1) \rangle$

(C) $\langle 12,\, 2y''(1) \rangle$

(D) $\langle 12,\, 4y''(1) \rangle$

MC.5

If $x(t) = t^2 + 4$ and $y(t) = t^4 + 3$, for $t > 0$, then in terms of t, $\dfrac{d^2 y}{dx^2} =$

(A) $\dfrac{1}{2}$ (B) 2   (C) $4t$ (D) $6t^2$ (E) $12t^2$

MC.6

A particle moves in the xy-plane with position given by $(x(t), y(t)) = \left(5 - 2t, t^2 - 3\right)$ at time t. In whic direction is the particle moving as it passes through the point $(3, -2)$?

(A) Up and to the left

(B) Down and to the left

(C) Up and to the right

(D) Down and to the right

(E) Straight up

MC.7

A particle moves in the xy-plane so that its position for $t \geq 0$ is given by the parametric equations $x = \ln(t + 1)$ and $y = kt^2$, where k is a positive constant. The line tangent to the particle's path at the point where $t = 3$ has slope 8. What is the value of k?

(A) $\dfrac{1}{192}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{4}{3}$ (D) $\dfrac{16}{3}$

MC.9 (calculator)

The velocity vector of a particle moving in the xy-plane has components given by $\dfrac{dx}{dt} = \sin\left(t^2\right)$ and $\dfrac{dy}{dt} = e^{\cos t}$. At time $t = 4$, the position of the particle is $(2, 1)$. What is the y-coordinate of the position vector at time $t = 3$?

(A) 0.410   (B) 0.590   (C) 0.851   (D) 1.410

MC.10 (calculator)

The position of an object moving along a path in the xy-plane is given by the parametric equations $x(t) = 5 \sin(\pi t)$ and $y(t) = (2t - 1)^2$. The speed of the particle at time $t = 0$ is

(A) 3.422

(B) 11.708

(C) 15.580

(D) 16.209