2.4 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. A ________ number has the form $a + bi$, where $a \neq 0$, $b = 0$.
2. An ________ number has the form $a + bi$, where $a \neq 0$, $b \neq 0$.
3. A ________ ________ number has the form $a + bi$, where $a = 0$, $b \neq 0$.
4. The imaginary unit i is defined as $i =$ ________, where $i^2 =$ ________.
5. When a is a positive real number, the ________ ________ root of $-a$ is defined as $\sqrt{-a} = \sqrt{a}i$.
6. The numbers $a + bi$ and $a - bi$ are called ________ ________, and their product is a real number $a^2 + b^2$.

Skills and Applications

Equality of Complex Numbers **In Exercises 7–10, find real numbers a and b such that the equation is true.**

7. $a + bi = 9 + 8i$
8. $a + bi = 10 - 5i$
9. $(a - 2) + (b + 1)i = 6 + 5i$
10. $(a + 2) + (b - 3)i = 4 + 7i$

Writing a Complex Number in Standard Form **In Exercises 11–22, write the complex number in standard form.**

11. $2 + \sqrt{-25}$
12. $4 + \sqrt{-49}$
13. $1 - \sqrt{-12}$
14. $2 - \sqrt{-18}$
15. $\sqrt{-40}$
16. $\sqrt{-27}$
17. 23
18. 50
19. $-6i + i^2$
20. $-2i^2 + 4i$
21. $\sqrt{-0.04}$
22. $\sqrt{-0.0025}$

Adding or Subtracting Complex Numbers **In Exercises 23–30, perform the operation and write the result in standard form.**

23. $(5 + i) + (2 + 3i)$
24. $(13 - 2i) + (-5 + 6i)$
25. $(9 - i) - (8 - i)$
26. $(3 + 2i) - (6 + 13i)$
27. $(-2 + \sqrt{-8}) + (5 - \sqrt{-50})$
28. $(8 + \sqrt{-18}) - (4 + 3\sqrt{2}i)$
29. $13i - (14 - 7i)$
30. $25 + (-10 + 11i) + 15i$

Multiplying Complex Numbers **In Exercises 31–38, perform the operation and write the result in standard form.**

31. $(1 + i)(3 - 2i)$
32. $(7 - 2i)(3 - 5i)$
33. $12i(1 - 9i)$
34. $-8i(9 + 4i)$
35. $(\sqrt{2} + 3i)(\sqrt{2} - 3i)$
36. $(4 + \sqrt{7}i)(4 - \sqrt{7}i)$
37. $(6 + 7i)^2$
38. $(5 - 4i)^2$

Multiplying Conjugates **In Exercises 39–46, write the complex conjugate of the complex number. Then multiply the number by its complex conjugate.**

39. $9 + 2i$
40. $8 - 10i$
41. $-1 - \sqrt{5}i$
42. $-3 + \sqrt{2}i$
43. $\sqrt{-20}$
44. $\sqrt{-15}$
45. $\sqrt{6}$
46. $1 + \sqrt{8}$

A Quotient of Complex Numbers in Standard Form **In Exercises 47–54, write the quotient in standard form.**

47. $\dfrac{2}{4 - 5i}$
48. $\dfrac{13}{1 - i}$
49. $\dfrac{5 + i}{5 - i}$
50. $\dfrac{6 - 7i}{1 - 2i}$
51. $\dfrac{9 - 4i}{i}$
52. $\dfrac{8 + 16i}{2i}$
53. $\dfrac{3i}{(4 - 5i)^2}$
54. $\dfrac{5i}{(2 + 3i)^2}$

Performing Operations with Complex Numbers **In Exercises 55–58, perform the operation and write the result in standard form.**

55. $\dfrac{2}{1 + i} - \dfrac{3}{1 - i}$
56. $\dfrac{2i}{2 + i} + \dfrac{5}{2 - i}$
57. $\dfrac{i}{3 - 2i} + \dfrac{2i}{3 + 8i}$
58. $\dfrac{1 + i}{i} - \dfrac{3}{4 - i}$

Writing a Complex Number in Standard Form **In Exercises 59–66, write the complex number in standard form.**

59. $\sqrt{-6}\sqrt{-2}$
60. $\sqrt{-5}\sqrt{-10}$
61. $(\sqrt{-15})^2$
62. $(\sqrt{-75})^2$
63. $\sqrt{-8} + \sqrt{-50}$
64. $\sqrt{-45} - \sqrt{-5}$
65. $(3 + \sqrt{-5})(7 - \sqrt{-10})$
66. $(2 - \sqrt{-6})^2$

Complex Solutions of a Quadratic Equation In Exercises 67–76, use the Quadratic Formula to solve the quadratic equation.

67. $x^2 - 2x + 2 = 0$
68. $x^2 + 6x + 10 = 0$
69. $4x^2 + 16x + 17 = 0$
70. $9x^2 - 6x + 37 = 0$
71. $4x^2 + 16x + 21 = 0$
72. $16t^2 - 4t + 3 = 0$
73. $\frac{3}{2}x^2 - 6x + 9 = 0$
74. $\frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0$
75. $1.4x^2 - 2x + 10 = 0$
76. $4.5x^2 - 3x + 12 = 0$

Simplifying a Complex Number In Exercises 77–86, simplify the complex number and write it in standard form.

77. $-6i^3 + i^2$
78. $4i^2 - 2i^3$
79. $-14i^5$
80. $(-i)^3$
81. $(\sqrt{-72})^3$
82. $(\sqrt{-2})^6$
83. $\dfrac{1}{i^3}$
84. $\dfrac{1}{(2i)^3}$
85. $(3i)^4$
86. $(-i)^6$

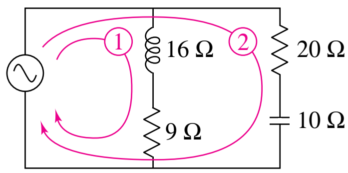
87. **Impedance of a Circuit**
The opposition to current in an electrical circuit is called its impedance. The impedance z in a parallel circuit with two pathways satisfies the equation

$$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2}$$

where z_1 is the impedance (in ohms) of pathway 1 and z_2 is the impedance (in ohms) of pathway 2.

(a) The impedance of each pathway in a parallel circuit is found by adding the impedances of all components in the pathway. Use the table to find z_1 and z_2

	Resistor	Inductor	Capacitor
Symbol	$a\ \Omega$	$b\ \Omega$	$c\ \Omega$
Impedance	a	bi	$-ci$



(b) Find the impedance z.

88. **Cube of a Complex Number** Cube each complex number.
(a) $-1 + \sqrt{3}i$ (b) $-1 - \sqrt{3}i$

Exploration

True or False? In Exercises 89–92, determine whether the statement is true or false. Justify your answer.

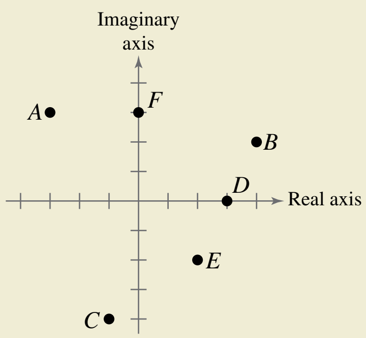
89. The sum of two complex numbers is always a real number.
90. There is no complex number that is equal to its complex conjugate.
91. $-i\sqrt{6}$ is a solution of $x^4 - x^2 + 14 = 56$.
92. $i^{44} + i^{150} - i^{74} - i^{109} + i^{61} = -1$

93. **Pattern Recognition** Find the missing values.

$i^1 = i$ $\quad i^2 = -1$ $\quad i^3 = -i$ $\quad i^4 = 1$
$i^5 = \square$ $\quad i^6 = \square$ $\quad i^7 = \square$ $\quad i^8 = \square$
$i^9 = \square$ $\quad i^{10} = \square$ $\quad i^{11} = \square$ $\quad i^{12} = \square$

What pattern do you see? Write a brief description of how you would find i raised to any positive integer power.

94. **HOW DO YOU SEE IT?** The coordinate system shown below is called the complex plane. In the complex plane, the point (a, b) corresponds to the complex number $a + bi$.



Match each complex number with its corresponding point.
(i) 3 (ii) $3i$ (iii) $4 + 2i$
(iv) $2 - 2i$ (v) $-3 + 3i$ (vi) $-1 - 4i$

95. **Error Analysis** Describe the error.

$$\sqrt{-6}\sqrt{-6} = \sqrt{(-6)(-6)} = \sqrt{36} = 6 \quad ✗$$

96. **Proof** Prove that the complex conjugate of the product of two complex numbers $a_1 + b_1 i$ and $a_2 + b_2 i$ is the product of their complex conjugates.

97. **Proof** Prove that the complex conjugate of the sum of two complex numbers $a_1 + b_1 i$ and $a_2 + b_2 i$ is the sum of their complex conjugates.

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Solving a Polynomial Equation In Exercises 29–32, find all real solutions of the polynomial equation.

29. $-5x^3 + 11x^2 - 4x - 2 = 0$
30. $8x^3 + 10x^2 - 15x - 6 = 0$
31. $x^4 + 6x^3 + 3x^2 - 16x + 6 = 0$
32. $x^4 + 8x^3 + 14x^2 - 17x - 42 = 0$

Using the Rational Zero Test In Exercises 33–36, (a) list the possible rational zeros of f, (b) sketch the graph of f so that some of the possible zeros in part (a) can be disregarded, and then (c) determine all real zeros of f.

33. $f(x) = x^3 + x^2 - 4x - 4$
34. $f(x) = -3x^3 + 20x^2 - 36x + 16$
35. $f(x) = -4x^3 + 15x^2 - 8x - 3$
36. $f(x) = 4x^3 - 12x^2 - x + 15$

Using the Rational Zero Test In Exercises 37–40, (a) list the possible rational zeros of f, (b) use a graphing utility to graph f so that some of the possible zeros in part (a) can be disregarded, and then (c) determine all real zeros of f.

37. $f(x) = -2x^4 + 13x^3 - 21x^2 + 2x + 8$
38. $f(x) = 4x^4 - 17x^2 + 4$
39. $f(x) = 32x^3 - 52x^2 + 17x + 3$
40. $f(x) = 4x^3 + 7x^2 - 11x - 18$

Finding a Polynomial Function with Given Zeros In Exercises 41–46, find a polynomial function with real coefficients that has the given zeros. (There are many correct answers.)

41. $1, 5i$
42. $4, -3i$
43. $2, 2, 1 + i$
44. $-1, 5, 3 - 2i$
45. $\frac{2}{3}, -1, 3 + \sqrt{2}i$
46. $-\frac{5}{2}, -5, 1 + \sqrt{3}i$

Finding a Polynomial Function with Given Zeros In Exercises 47–50, find the polynomial function f with real coefficients that has the given degree, zeros, and solution point.

	Degree	Zeros	Solution Point
47.	4	$-2, 1, i$	$f(0) = -4$
48.	4	$-1, 2, \sqrt{2}i$	$f(1) = 12$
49.	3	$-3, 1 + \sqrt{3}i$	$f(-2) = 12$
50.	3	$-2, 1 - \sqrt{2}i$	$f(-1) = -12$

Factoring a Polynomial In Exercises 51–54, write the polynomial (a) as the product of factors that are irreducible over the *rationals*, (b) as the product of linear and quadratic factors that are irreducible over the *reals*, and (c) in completely factored form.

51. $f(x) = x^4 + 2x^2 - 8$
52. $f(x) = x^4 + 6x^2 - 27$
53. $f(x) = x^4 - 2x^3 - 3x^2 + 12x - 18$
(*Hint:* One factor is $x^2 - 6$.)
54. $f(x) = x^4 - 3x^3 - x^2 - 12x - 20$
(*Hint:* One factor is $x^2 + 4$.)

Finding the Zeros of a Polynomial Function In Exercises 55–60, use the given zero to find all the zeros of the function.

	Function	Zero
55.	$f(x) = x^3 - x^2 + 4x - 4$	$2i$
56.	$f(x) = 2x^3 + 3x^2 + 18x + 27$	$3i$
57.	$g(x) = x^3 - 8x^2 + 25x - 26$	$3 + 2i$
58.	$g(x) = x^3 + 9x^2 + 25x + 17$	$-4 + i$
59.	$h(x) = x^4 - 6x^3 + 14x^2 - 18x + 9$	$1 - \sqrt{2}i$
60.	$h(x) = x^4 + x^3 - 3x^2 - 13x + 14$	$-2 + \sqrt{3}i$

Finding the Zeros of a Polynomial Function In Exercises 61–72, write the polynomial as the product of linear factors and list all the zeros of the function.

61. $f(x) = x^2 + 36$
62. $f(x) = x^2 + 49$
63. $h(x) = x^2 - 2x + 17$
64. $g(x) = x^2 + 10x + 17$
65. $f(x) = x^4 - 16$
66. $f(y) = y^4 - 256$
67. $f(z) = z^2 - 2z + 2$
68. $h(x) = x^3 - 3x^2 + 4x - 2$
69. $g(x) = x^3 - 3x^2 + x + 5$
70. $f(x) = x^3 - x^2 + x + 39$
71. $g(x) = x^4 - 4x^3 + 8x^2 - 16x + 16$
72. $h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

Finding the Zeros of a Polynomial Function In Exercises 73–78, find all the zeros of the function. When there is an extended list of possible rational zeros, use a graphing utility to graph the function in order to disregard any of the possible rational zeros that are obviously not zeros of the function.

73. $f(x) = x^3 + 24x^2 + 214x + 740$
74. $f(s) = 2s^3 - 5s^2 + 12s - 5$
75. $f(x) = 16x^3 - 20x^2 - 4x + 15$
76. $f(x) = 9x^3 - 15x^2 + 11x - 5$
77. $f(x) = 2x^4 + 5x^3 + 4x^2 + 5x + 2$
78. $g(x) = x^5 - 8x^4 + 28x^3 - 56x^2 + 64x - 32$

6.6 Exercises

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Vocabulary: Fill in the blanks.

1. The ________ ________ of the complex number $z = a + bi$ is $z = r(\cos\theta + i\sin\theta)$, where r is the ________ of z and θ is an ________ of z.
2. ________ Theorem states that if $z = r(\cos\theta + i\sin\theta)$ is a complex number and n is a positive integer, then $z^n = r^n(\cos n\theta + i\sin n\theta)$.
3. The complex number $u = a + bi$ is an ________ ________ of the complex number z when $z = u^n = (a + bi)^n$.
4. Successive nth roots of a complex number have arguments that differ by ________.

Skills and Applications

Trigonometric Form of a Complex Number In Exercises 5–24, plot the complex number. Then write the trigonometric form of the complex number.

5. $1 + i$
6. $5 - 5i$
7. $1 - \sqrt{3}i$
8. $4 - 4\sqrt{3}i$
9. $-2(1 + \sqrt{3}i)$
10. $\frac{5}{2}(\sqrt{3} - i)$
11. $-5i$
12. $12i$
13. 2
14. 4
15. $-7 + 4i$
16. $3 - i$
17. $2\sqrt{2} - i$
18. $-3 - i$
19. $5 + 2i$
20. $8 + 3i$
21. $3 + \sqrt{3}i$
22. $3\sqrt{2} - 7i$
23. $-8 - 5\sqrt{3}i$
24. $-9 - 2\sqrt{10}i$

Writing a Complex Number in Standard Form In Exercises 25–32, write the standard form of the complex number. Then plot the complex number.

25. $2(\cos 60° + i\sin 60°)$
26. $5(\cos 135° + i\sin 135°)$
27. $\sqrt{48}[\cos(-30°) + i\sin(-30°)]$
28. $\sqrt{8}(\cos 225° + i\sin 225°)$
29. $\frac{9}{4}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$
30. $6\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$
31. $5[\cos(198° 45') + i\sin(198° 45')]$
32. $9.75[\cos(280° 30') + i\sin(280° 30')]$

Writing a Complex Number in Standard Form In Exercises 33–36, use a graphing utility to write the complex number in standard form.

33. $5\left(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9}\right)$
34. $10\left(\cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}\right)$
35. $2(\cos 155° + i\sin 155°)$
36. $9(\cos 58° + i\sin 58°)$

Multiplying Complex Numbers In Exercises 37–40, find the product. Leave the result in trigonometric form.

37. $\left[2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]\left[6\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)\right]$
38. $\left[\frac{3}{4}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]\left[4\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)\right]$
39. $\left[\frac{5}{3}(\cos 120° + i\sin 120°)\right]\left[\frac{2}{3}(\cos 30° + i\sin 30°)\right]$
40. $\left[\frac{1}{2}(\cos 100° + i\sin 100°)\right]\left[\frac{4}{5}(\cos 300° + i\sin 300°)\right]$

Dividing Complex Numbers In Exercises 41–44, find the quotient. Leave the result in trigonometric form.

41. $\dfrac{3(\cos 50° + i\sin 50°)}{9(\cos 20° + i\sin 20°)}$
42. $\dfrac{\cos 120° + i\sin 120°}{2(\cos 40° + i\sin 40°)}$
43. $\dfrac{\cos\pi + i\sin\pi}{\cos(\pi/3) + i\sin(\pi/3)}$
44. $\dfrac{5(\cos 4.3 + i\sin 4.3)}{4(\cos 2.1 + i\sin 2.1)}$

Multiplying or Dividing Complex Numbers In Exercises 45–50, (a) write the trigonometric forms of the complex numbers, (b) perform the operation using the trigonometric forms, and (c) perform the operation using the standard forms, and check your result with that of part (b).

45. $(2 + 2i)(1 - i)$
46. $(\sqrt{3} + i)(1 + i)$
47. $-2i(1 + i)$
48. $3i(1 - \sqrt{2}i)$
49. $\dfrac{3 + 4i}{1 - \sqrt{3}i}$
50. $\dfrac{1 + \sqrt{3}i}{6 - 3i}$

Multiplying in the Complex Plane In Exercises 51 and 52, find the product in the complex plane.

51. $\left[2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)\right]\left[\frac{1}{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]$
52. $\left[2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]\left[3\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right]$

Finding a Power of a Complex Number In Exercises 53–68, use DeMoivre's Theorem to find the power of the complex number. Write the result in standard form.

53. $[5(\cos 20° + i \sin 20°)]^3$
54. $[3(\cos 60° + i \sin 60°)]^4$
55. $\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)^{12}$
56. $\left[2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)\right]^8$
57. $[5(\cos 3.2 + i \sin 3.2)]^4$
58. $(\cos 0 + i \sin 0)^{20}$
59. $[3(\cos 15° + i \sin 15°)]^4$
60. $\left[2\left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}\right)\right]^6$
61. $(1 + i)^5$
62. $(2 + 2i)^6$
63. $(-1 + i)^6$
64. $(3 - 2i)^8$
65. $2(\sqrt{3} + i)^{10}$
66. $4(1 - \sqrt{3}i)^3$
67. $(3 - 2i)^5$
68. $(\sqrt{5} - 4i)^3$

Graphing Powers of a Complex Number In Exercises 69 and 70, represent the powers z, z^2, z^3, and z^4 graphically. Describe the pattern.

69. $z = \frac{\sqrt{2}}{2}(1 + i)$
70. $z = \frac{1}{2}(1 + \sqrt{3}i)$

Finding the *n*th Roots of a Complex Number In Exercises 71–86, (a) use the formula on page 450 to find the roots of the complex number, (b) write each of the roots in standard form, and (c) represent each of the roots graphically.

71. Square roots of $5(\cos 120° + i \sin 120°)$
72. Square roots of $16(\cos 60° + i \sin 60°)$
73. Cube roots of $8\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$
74. Fifth roots of $32\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$
75. Cube roots of $-\frac{125}{2}(1 + \sqrt{3}i)$
76. Cube roots of $-4\sqrt{2}(-1 + i)$
77. Square roots of $-25i$
78. Fourth roots of $625i$
79. Fourth roots of 16
80. Fourth roots of i
81. Fifth roots of 1
82. Cube roots of 1000
83. Cube roots of -125
84. Fourth roots of -4
85. Fifth roots of $4(1 - i)$
86. Sixth roots of $64i$

Solving an Equation In Exercises 87–94, use the formula on page 450 to find all solutions of the equation and represent the solutions graphically.

87. $x^4 + i = 0$
88. $x^3 + 1 = 0$
89. $x^5 + 243 = 0$
90. $x^3 - 27 = 0$
91. $x^4 + 16i = 0$
92. $x^6 + 64i = 0$
93. $x^3 - (1 - i) = 0$
94. $x^4 + (1 + i) = 0$

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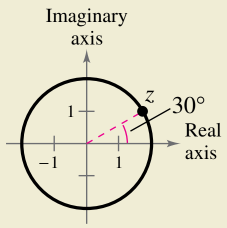
95. Ohm's Law Ohm's law for alternating current circuits is $E = IZ$, where E is the voltage in volts, I is the current in amperes, and Z is the impedance in ohms. Each variable is a complex number.

(a) Write E in trigonometric form when $I = 6(\cos 41° + i \sin 41°)$ amperes and $Z = 4[\cos(-11°) + i \sin(-11°)]$ ohms.

(b) Write the voltage from part (a) in standard form.

(c) A voltmeter measures the magnitude of the voltage in a circuit. What would be the reading on a voltmeter for the circuit described in part (a)?

96. HOW DO YOU SEE IT? The figure shows one of the fourth roots of a complex number z.



(a) How many roots are not shown?

(b) Describe the other roots.

Exploration

True or False? In Exercises 97 and 98, determine whether the statement is true or false. Justify your answer.

97. Geometrically, the nth roots of any complex number z are all equally spaced around the unit circle.

98. The product of two complex numbers is zero only when the modulus of one (or both) of the complex numbers is zero.

99. Quotient of Two Complex Numbers Given two complex numbers $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$, $z_2 \neq 0$, show that

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)].$$

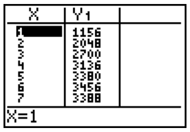
100. Negative of a Complex Number Show that the negative of $z = r(\cos \theta + i \sin \theta)$ is $-z = r[\cos(\theta + \pi) + i \sin(\theta + \pi)]$.

101. Complex Conjugates Show that

$$\bar{z} = r[\cos(-\theta) + i \sin(-\theta)]$$

is the complex conjugate of $z = r(\cos \theta + i \sin \theta)$. Then find (a) $z\bar{z}$ and (b) $z/\bar{z}$, $\bar{z} \neq 0$.

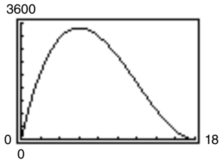
93. (a) $V(x) = x(36 - 2x)^2$ (b) Domain: $0 < x < 18$

(c)


X	Y1
1	1156
2	2048
3	2700
4	3136
5	3380
6	3456
7	3388

X=1

6 in. × 24 in. × 24 in.

(d)


$x = 6$; The results are the same.

95. (a) Relative maximum: (4.44, 1512.60)
Relative minimum: (11.97, 189.37)
(b) Increasing: (3, 4.44), (11.97, 16)
Decreasing: (4.44, 11.97)
(c) Answers will vary.

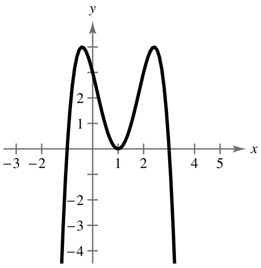
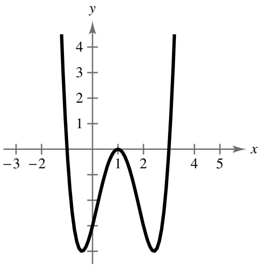
97. $x \approx 200$

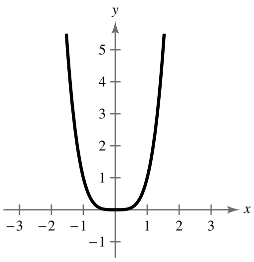
99. True. A polynomial function falls to the right only when the leading coefficient is negative.

101. False. The graph falls to the left and to the right or the graph rises to the left and to the right.

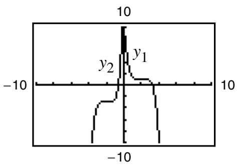
103. Answers will vary. *Sample answers:*

$a_4 < 0$ $a_4 > 0$

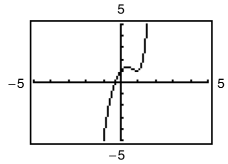



105.


(a) Upward shift of two units; Even
(b) Left shift of two units; Neither
(c) Reflection in the y-axis; Even
(d) Reflection in the x-axis; Even
(e) Horizontal stretch; Even (f) Vertical shrink; Even
(g) $g(x) = x^3, x \ge 0$; Neither (h) $g(x) = x^{16}$; Even

107.


(a) y_1 is decreasing, y_2 is increasing.
(b) Yes; a; If $a > 0$, then the graph is increasing, and if $a < 0$, then the graph is decreasing.

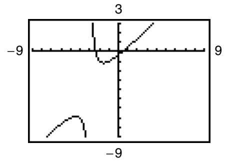
(c)


No; f is not strictly increasing or strictly decreasing, so f cannot be written in the form $f(x) = a(x - h)^5 + k$.

Section 2.3 *(page 142)*

1. $f(x)$: dividend; $d(x)$: divisor;
$q(x)$: quotient; $r(x)$: remainder

3. improper **5.** Factor **7.** Answers will vary.

9. (a) and (b)


(c) Answers will vary.

11. $2x + 4,\ x \ne -3$ **13.** $x^2 - 3x + 1,\ x \ne -\frac{5}{4}$

15. $x^3 + 3x^2 - 1,\ x \ne -2$ **17.** $6 - \dfrac{1}{x + 1}$

19. $x - \dfrac{x + 9}{x^2 + 1}$ **21.** $2x - 8 + \dfrac{x - 1}{x^2 + 1}$

23. $x + 3 + \dfrac{6x^2 - 8x + 3}{(x - 1)^3}$ **25.** $2x^2 - 2x + 6,\ x \ne 4$

27. $6x^2 + 25x + 74 + \dfrac{248}{x - 3}$ **29.** $4x^2 - 9,\ x \ne -2$

31. $-x^2 + 10x - 25,\ x \ne -10$ **33.** $x^2 + x + 4 + \dfrac{21}{x - 4}$

35. $10x^3 + 10x^2 + 60x + 360 + \dfrac{1360}{x - 6}$

37. $x^2 - 8x + 64,\ x \ne -8$

39. $-3x^3 - 6x^2 - 12x - 24 - \dfrac{48}{x - 2}$

41. $-x^3 - 6x^2 - 36x - 36 - \dfrac{216}{x - 6}$

43. $4x^2 + 14x - 30,\ x \ne -\frac{1}{2}$

45. $f(x) = (x - 3)(x^2 + 2x - 4) - 5,\ f(3) = -5$

47. $f(x) = \left(x + \frac{2}{3}\right)(15x^3 - 6x + 4) + \frac{34}{3},\ f\left(-\frac{2}{3}\right) = \frac{34}{3}$

49. $f(x) = \left(x - 1 + \sqrt{3}\right)\left[-4x^2 + \left(2 + 4\sqrt{3}\right)x + \left(2 + 2\sqrt{3}\right)\right]$,
$f\left(1 - \sqrt{3}\right) = 0$

51. (a) -2 (b) 1 (c) 36 (d) 5

53. (a) -35 (b) $-\frac{5}{8}$ (c) -10 (d) -211

55. $(x + 3)(x + 2)(x + 1)$; Solutions: $-3, -2, -1$

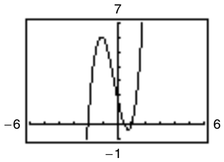
57. $(2x - 1)(x - 5)(x - 2)$; Solutions: $\frac{1}{2}, 5, 2$

59. $\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)(x + 2)$; Solutions: $-\sqrt{3}, \sqrt{3}, -2$

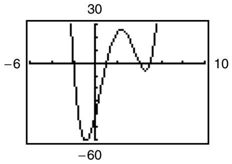
61. $(x - 1)\left(x - 1 - \sqrt{3}\right)\left(x - 1 + \sqrt{3}\right)$;
Solutions: $1, 1 + \sqrt{3}, 1 - \sqrt{3}$

CHAPTER 2

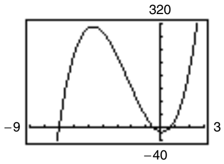
63. (a) Answers will vary. (b) $2x - 1$
(c) $f(x) = (2x - 1)(x + 2)(x - 1)$ (d) $\frac{1}{2}, -2, 1$
(e)



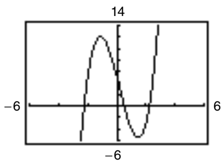
65. (a) Answers will vary. (b) $(x - 4)(x - 1)$
(c) $f(x) = (x - 5)(x + 2)(x - 4)(x - 1)$ (d) $5, -2, 4, 1$
(e)



67. (a) Answers will vary. (b) $x + 7$
(c) $f(x) = (x + 7)(2x + 1)(3x - 2)$ (d) $-7, -\frac{1}{2}, \frac{2}{3}$
(e)



69. (a) Answers will vary. (b) $x - \sqrt{5}$
(c) $f(x) = (x - \sqrt{5})(x + \sqrt{5})(2x - 1)$ (d) $\pm\sqrt{5}, \frac{1}{2}$
(e)



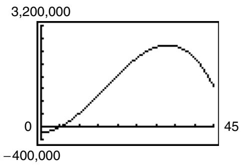
71. (a) $2, \pm 2.236$ (b) 2
(c) $f(x) = (x - 2)(x - \sqrt{5})(x + \sqrt{5})$

73. (a) $-2, 0.268, 3.732$ (b) -2
(c) $h(t) = (t + 2)[t - (2 + \sqrt{3})][t - (2 - \sqrt{3})]$

75. (a) $0, 3, 4, \pm 1.414$ (b) 0
(c) $h(x) = x(x - 4)(x - 3)(x + \sqrt{2})(x - \sqrt{2})$

77. $x^2 - 7x - 8,\ x \neq -8$ **79.** $x^2 + 3x,\ x \neq -2, -1$

81. (a)



(b) \$250,366
(c) Answers will vary.

83. False. $-\frac{4}{7}$ is a zero of f.

85. True. The degree of the numerator is greater than the degree of the denominator.

87. $x^{2n} + 6x^n + 9,\ \ x^n \neq -3$ **89.** $k = -1$, not 1.

91. $c = -210$ **93.** $k = 7$

Section 2.4 *(page 150)*

1. real **3.** pure imaginary **5.** principal square

7. $a = 9, b = 8$ **9.** $a = 8, b = 4$ **11.** $2 + 5i$

13. $1 - 2\sqrt{3}i$ **15.** $2\sqrt{10}i$ **17.** 23 **19.** $-1 - 6i$

21. $0.2i$ **23.** $7 + 4i$ **25.** 1 **27.** $3 - 3\sqrt{2}i$

29. $-14 + 20i$ **31.** $5 + i$ **33.** $108 + 12i$ **35.** 11

37. $-13 + 84i$ **39.** $9 - 2i, 85$ **41.** $-1 + \sqrt{5}i, 6$

43. $-2\sqrt{5}i, 20$ **45.** $\sqrt{6}, 6$ **47.** $\frac{8}{41} + \frac{10}{41}i$

49. $\frac{12}{13} + \frac{5}{13}i$ **51.** $-4 - 9i$ **53.** $-\frac{120}{1681} - \frac{27}{1681}i$

55. $-\frac{1}{2} - \frac{5}{2}i$ **57.** $\frac{62}{949} + \frac{297}{949}i$ **59.** $-2\sqrt{3}$ **61.** -15

63. $7\sqrt{2}i$ **65.** $(21 + 5\sqrt{2}) + (7\sqrt{5} - 3\sqrt{10})i$

67. $1 \pm i$ **69.** $-2 \pm \dfrac{1}{2}i$ **71.** $-2 \pm \dfrac{\sqrt{5}}{2}i$

73. $2 \pm \sqrt{2}i$ **75.** $\dfrac{5}{7} \pm \dfrac{5\sqrt{13}}{7}i$ **77.** $-1 + 6i$

79. $-14i$ **81.** $-432\sqrt{2}i$ **83.** i **85.** 81

87. (a) $z_1 = 9 + 16i, z_2 = 20 - 10i$
(b) $z = \dfrac{11{,}240}{877} + \dfrac{4630}{877}i$

89. False. *Sample answer:* $(1 + i) + (3 + i) = 4 + 2i$

91. True.

$$\begin{aligned} x^4 - x^2 + 14 &= 56 \\ (-i\sqrt{6})^4 - (-i\sqrt{6})^2 + 14 &\stackrel{?}{=} 56 \\ 36 + 6 + 14 &\stackrel{?}{=} 56 \\ 56 &= 56 \end{aligned}$$

93. $i, -1, -i, 1, i, -1, -i, 1$; The pattern repeats the first four results. Divide the exponent by 4.
When the remainder is 1, the result is i.
When the remainder is 2, the result is -1.
When the remainder is 3, the result is $-i$.
When the remainder is 0, the result is 1.

95. $\sqrt{-6}\sqrt{-6} = \sqrt{6}i\sqrt{6}i = 6i^2 = -6$ **97.** Proof

Section 2.5 *(page 162)*

1. Fundamental Theorem of Algebra **3.** Rational Zero

5. linear; quadratic; quadratic **7.** Descartes's Rule of Signs

9. 3 **11.** 5 **13.** 2 **15.** $\pm 1, \pm 2$

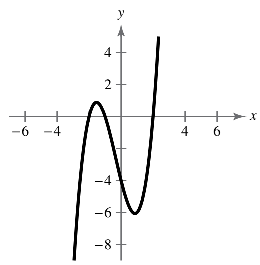
17. $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm\frac{9}{2}, \pm\frac{15}{2}, \pm\frac{45}{2}$

19. $-2, -1, 3$ **21.** No rational zeros **23.** $-6, -1$

25. $-1, \dfrac{1}{2}$ **27.** $-2, 3, \pm\dfrac{2}{3}$ **29.** $1, \dfrac{3}{5} \pm \dfrac{\sqrt{19}}{5}$

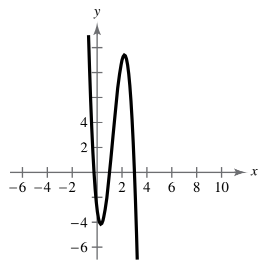
31. $-3, 1, -2 \pm \sqrt{6}$

33. (a) $\pm 1, \pm 2, \pm 4$
(b)



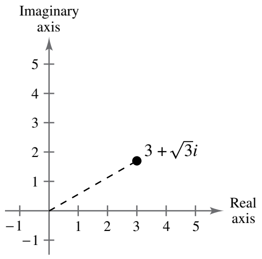
(c) $-2, -1, 2$

35. (a) $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$
(b)



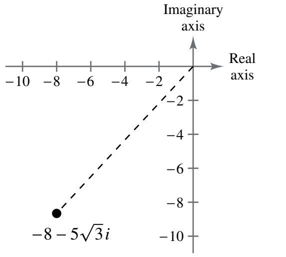
(c) $-\frac{1}{4}, 1, 3$

21.



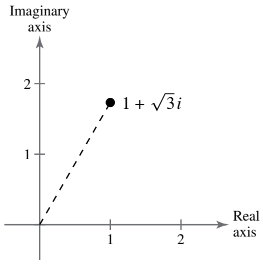
$2\sqrt{3}\left(\cos\dfrac{\pi}{6}+i\sin\dfrac{\pi}{6}\right)$

23.

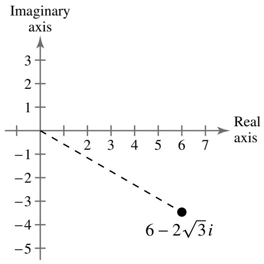


$\sqrt{139}(\cos 3.97 + i\sin 3.97)$

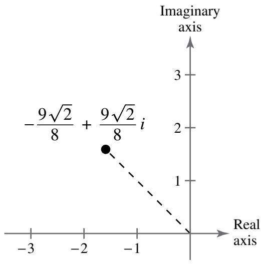
25. $1+\sqrt{3}i$



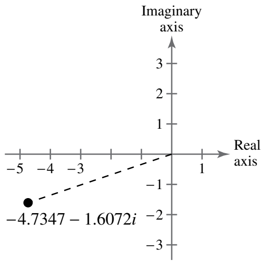
27. $6-2\sqrt{3}i$



29. $-\dfrac{9\sqrt{2}}{8}+\dfrac{9\sqrt{2}}{8}i$



31. $-4.7347-1.6072i$



33. $4.6985+1.7101i$ **35.** $-1.8126+0.8452i$

37. $12\left(\cos\dfrac{\pi}{3}+i\sin\dfrac{\pi}{3}\right)$ **39.** $\dfrac{10}{9}(\cos 150^\circ + i\sin 150^\circ)$

41. $\dfrac{1}{3}(\cos 30^\circ + i\sin 30^\circ)$ **43.** $\cos\dfrac{2\pi}{3}+i\sin\dfrac{2\pi}{3}$

45. (a) $\left[2\sqrt{2}\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)\right]\left[\sqrt{2}\left(\cos\dfrac{7\pi}{4}+i\sin\dfrac{7\pi}{4}\right)\right]$
(b) $4(\cos 0 + i\sin 0) = 4$ (c) 4

47. (a) $\left[2\left(\cos\dfrac{3\pi}{2}+i\sin\dfrac{3\pi}{2}\right)\right]\left[\sqrt{2}\left(\cos\dfrac{\pi}{4}+i\sin\dfrac{\pi}{4}\right)\right]$
(b) $2\sqrt{2}\left(\cos\dfrac{7\pi}{4}+i\sin\dfrac{7\pi}{4}\right)=2-2i$
(c) $-2i-2i^2=-2i+2=2-2i$

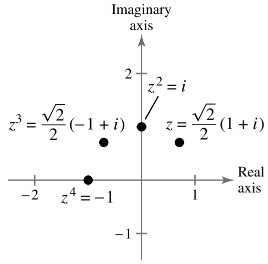
49. (a) $[5(\cos 0.93 + i\sin 0.93)] \div \left[2\left(\cos\dfrac{5\pi}{3}+i\sin\dfrac{5\pi}{3}\right)\right]$
(b) $\frac{5}{2}(\cos 1.97 + i\sin 1.97) \approx -0.982+2.299i$
(c) About $-0.982+2.299i$

51. -1 **53.** $\dfrac{125}{2}+\dfrac{125\sqrt{3}}{2}i$ **55.** -1

57. $608.0+144.7i$ **59.** $\dfrac{81}{2}+\dfrac{81\sqrt{3}}{2}i$ **61.** $-4-4i$

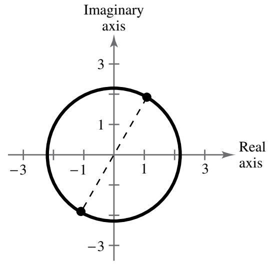
63. $8i$ **65.** $1024-1024\sqrt{3}i$ **67.** $-597-122i$

69.

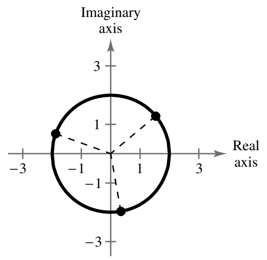


The absolute value of each is 1, and the consecutive powers of z are each 45° apart.

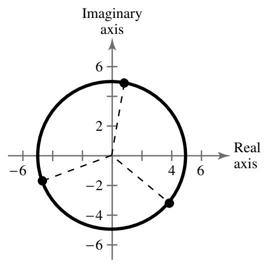
71. (a) $\sqrt{5}(\cos 60^\circ + i\sin 60^\circ)$
$\sqrt{5}(\cos 240^\circ + i\sin 240^\circ)$
(b) $\dfrac{\sqrt{5}}{2}+\dfrac{\sqrt{15}}{2}i, -\dfrac{\sqrt{5}}{2}-\dfrac{\sqrt{15}}{2}i$
(c)



73. (a) $2\left(\cos\dfrac{2\pi}{9}+i\sin\dfrac{2\pi}{9}\right)$
$2\left(\cos\dfrac{8\pi}{9}+i\sin\dfrac{8\pi}{9}\right)$
$2\left(\cos\dfrac{14\pi}{9}+i\sin\dfrac{14\pi}{9}\right)$
(b) $1.5321+1.2856i$, $-1.8794+0.6840i$, $0.3473-1.9696i$
(c)

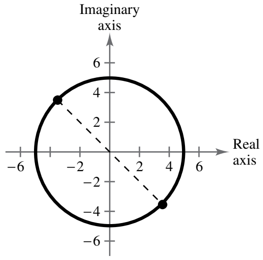


75. (a) $5\left(\cos\dfrac{4\pi}{9}+i\sin\dfrac{4\pi}{9}\right)$
$5\left(\cos\dfrac{10\pi}{9}+i\sin\dfrac{10\pi}{9}\right)$
$5\left(\cos\dfrac{16\pi}{9}+i\sin\dfrac{16\pi}{9}\right)$
(b) $0.8682+4.9240i$, $-4.6985-1.7101i$, $3.8302-3.2139i$
(c)



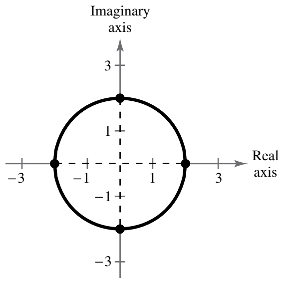
77. (a) $5\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)$
$5\left(\cos\dfrac{7\pi}{4}+i\sin\dfrac{7\pi}{4}\right)$
(b) $-\dfrac{5\sqrt{2}}{2}+\dfrac{5\sqrt{2}}{2}i$, $\dfrac{5\sqrt{2}}{2}-\dfrac{5\sqrt{2}}{2}i$

(c)



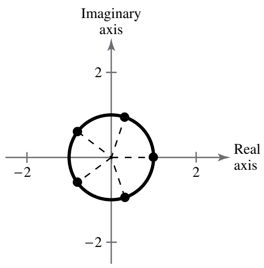
79. (a) $2(\cos 0 + i \sin 0)$
$2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$
$2(\cos \pi + i \sin \pi)$
$2\left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2}\right)$
(b) $2, 2i, -2, -2i$

(c)



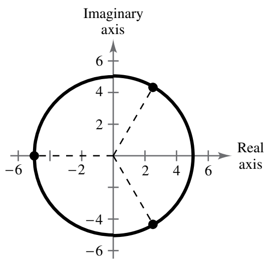
81. (a) $\cos 0 + i \sin 0$
$\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5}$
$\cos \frac{4\pi}{5} + i \sin \frac{4\pi}{5}$
$\cos \frac{6\pi}{5} + i \sin \frac{6\pi}{5}$
$\cos \frac{8\pi}{5} + i \sin \frac{8\pi}{5}$
(b) $1, 0.3090 + 0.9511i,$
$-0.8090 + 0.5878i,$
$-0.8090 - 0.5878i,$
$0.3090 - 0.9511i$

(c)



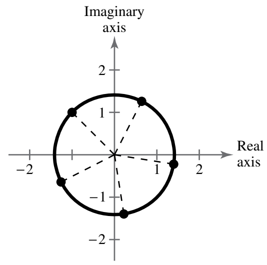
83. (a) $5\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)$
$5(\cos \pi + i \sin \pi)$
$5\left(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3}\right)$
(b) $\frac{5}{2} + \frac{5\sqrt{3}}{2}i, -5,$
$\frac{5}{2} - \frac{5\sqrt{3}}{2}i$

(c)

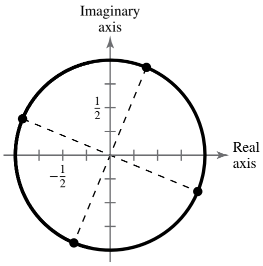


85. (a) $\sqrt{2}\left(\cos \frac{7\pi}{20} + i \sin \frac{7\pi}{20}\right)$
$\sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)$
$\sqrt{2}\left(\cos \frac{23\pi}{20} + i \sin \frac{23\pi}{20}\right)$
$\sqrt{2}\left(\cos \frac{31\pi}{20} + i \sin \frac{31\pi}{20}\right)$
$\sqrt{2}\left(\cos \frac{39\pi}{20} + i \sin \frac{39\pi}{20}\right)$
(b) $0.6420 + 1.2601i,$
$-1 + i,$
$-1.2601 - 0.6420i,$
$0.2212 - 1.3968i,$
$1.3968 - 0.2212i$

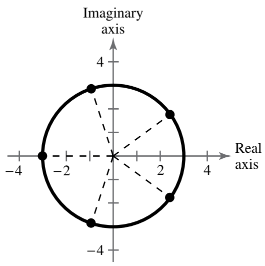
(c)



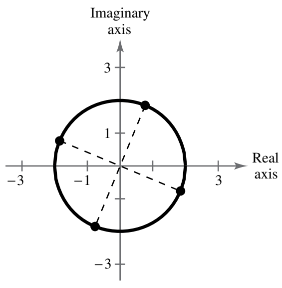
87. $\cos \frac{3\pi}{8} + i \sin \frac{3\pi}{8}$
$\cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8}$
$\cos \frac{11\pi}{8} + i \sin \frac{11\pi}{8}$
$\cos \frac{15\pi}{8} + i \sin \frac{15\pi}{8}$



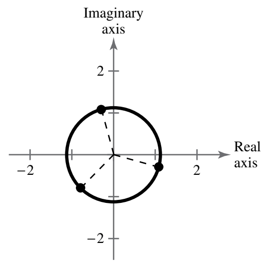
89. $3\left(\cos \frac{\pi}{5} + i \sin \frac{\pi}{5}\right)$
$3\left(\cos \frac{3\pi}{5} + i \sin \frac{3\pi}{5}\right)$
$3(\cos \pi + i \sin \pi)$
$3\left(\cos \frac{7\pi}{5} + i \sin \frac{7\pi}{5}\right)$
$3\left(\cos \frac{9\pi}{5} + i \sin \frac{9\pi}{5}\right)$



91. $2\left(\cos \frac{3\pi}{8} + i \sin \frac{3\pi}{8}\right)$
$2\left(\cos \frac{7\pi}{8} + i \sin \frac{7\pi}{8}\right)$
$2\left(\cos \frac{11\pi}{8} + i \sin \frac{11\pi}{8}\right)$
$2\left(\cos \frac{15\pi}{8} + i \sin \frac{15\pi}{8}\right)$



93. $\sqrt[6]{2}\left(\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12}\right)$
$\sqrt[6]{2}\left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}\right)$
$\sqrt[6]{2}\left(\cos \frac{23\pi}{12} + i \sin \frac{23\pi}{12}\right)$



95. (a) $E = 24(\cos 30° + i \sin 30°)$ volts
(b) $E = 12\sqrt{3} + 12i$ volts (c) $|E| = 24$ volts

97. False. They are equally spaced around the circle centered at the origin with radius $\sqrt[n]{r}$.

99. Answers will vary.

101. Answers will vary; (a) r^2 (b) $\cos 2\theta + i \sin 2\theta$

Review Exercises *(page 456)*

1. $C = 72°$, $b \approx 12.21$, $c \approx 12.36$

3. $A = 26°$, $a \approx 24.89$, $c \approx 56.23$

5. $C = 66°$, $a \approx 2.53$, $b \approx 9.11$

7. $B = 108°$, $a \approx 11.76$, $c \approx 21.49$

9. $A \approx 20.41°$, $C \approx 9.59°$, $a \approx 20.92$

11. $B \approx 39.48°$, $C \approx 65.52°$, $c \approx 48.24$ **13.** 19.1

15. 47.2 **17.** About 31.1 m

19. $A \approx 16.99°$, $B \approx 26.00°$, $C \approx 137.01°$

21. $A \approx 29.92°$, $B \approx 86.18°$, $C \approx 63.90°$

23. $A = 36°$, $C = 36°$, $b \approx 17.80$

25. $A \approx 45.76°$, $B \approx 91.24°$, $c \approx 21.42$

27. No; $A \approx 77.52°$, $B \approx 38.48°$, $a \approx 14.12$

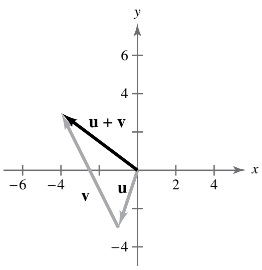
29. Yes; $A \approx 28.62°$, $B \approx 33.56°$, $C \approx 117.82°$

31. About 4.3 ft, about 12.6 ft **33.** 7.64 **35.** 8.36

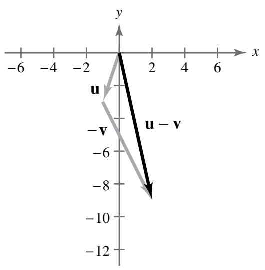
37. Equivalent; **u** and **v** have the same magnitude and direction.

39. $\langle 7, -7 \rangle$; $7\sqrt{2}$

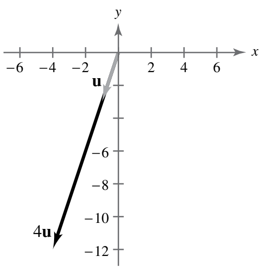
41. (a) $\langle -4, 3 \rangle$



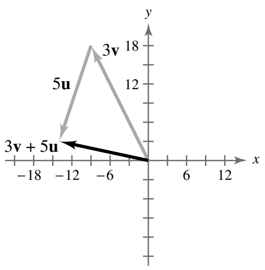
(b) $\langle 2, -9 \rangle$



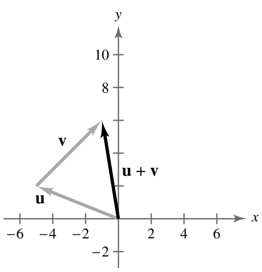
(c) $\langle -4, -12 \rangle$



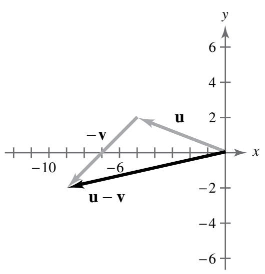
(d) $\langle -14, 3 \rangle$



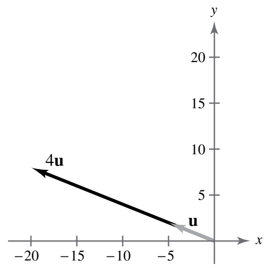
43. (a) $\langle -1, 6 \rangle$



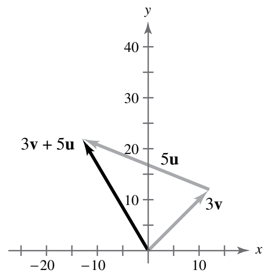
(b) $\langle -9, -2 \rangle$



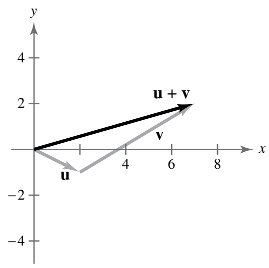
(c) $\langle -20, 8 \rangle$



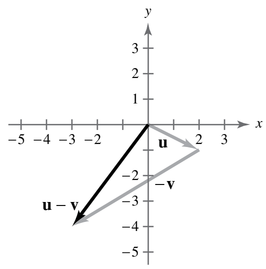
(d) $\langle -13, 22 \rangle$



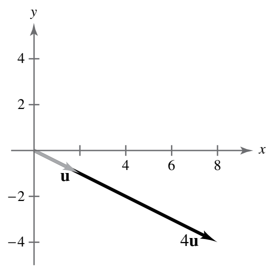
45. (a) $7\mathbf{i} + 2\mathbf{j}$



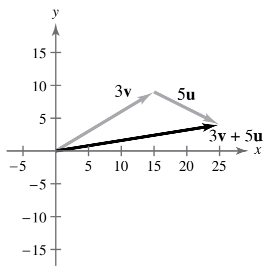
(b) $-3\mathbf{i} - 4\mathbf{j}$



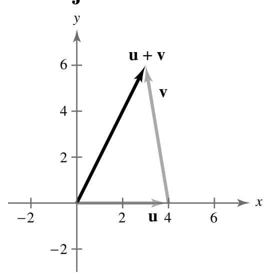
(c) $8\mathbf{i} - 4\mathbf{j}$



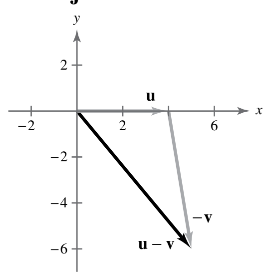
(d) $25\mathbf{i} + 4\mathbf{j}$



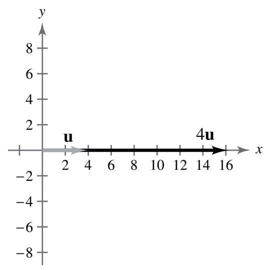
47. (a) $3\mathbf{i} + 6\mathbf{j}$



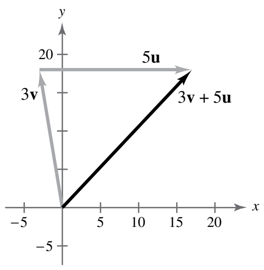
(b) $5\mathbf{i} - 6\mathbf{j}$



(c) $16\mathbf{i}$



(d) $17\mathbf{i} + 18\mathbf{j}$



49. $-\mathbf{i} + 5\mathbf{j}$ **51.** $6\mathbf{i} + 4\mathbf{j}$