

AOS Math 10, Spring 2024

Applications of Derivatives Test (#15)

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ACADEMIES OF LOUDOUN HONOR CODE



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May 24, 2024

Student Signature

Class

Date

Print Name:

Instructions: For each problem, circle the letter of the best answer. You **must show all work** for credit. Partial credit may be awarded as appropriate.

1. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

- (a) $x > \sqrt{2}$
- (b) $x > 0$
- (c) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$
- (d) $-2 < x < 2$
- (e) $-2 < x < 0$ or $x > 2$

2. At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

- (a) -1 and 1 only
- (b) 1 only
- (c) 0 only
- (d) -1, 0 and 1
- (e) -1 only

3. The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that

- (a) $x > 0$
- (b) $x < 0$
- (c) $x > 2$
- (d) $x < 5$
- (e) $x < 2$

4. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

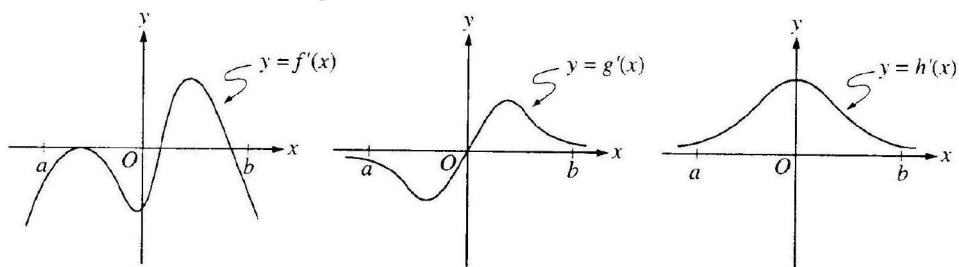
- (a) -2
- (b) 0
- (c) 4
- (d) 2
- (e) 1

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5. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b ?
- (a) -3
 - (b) 1
 - (c) 3
 - (d) 0
6. The derivative of f is $x^4(x - 2)(x + 3)$. At how many points will the graph of f have a relative maximum?
- (a) None
 - (b) Two
 - (c) Four
 - (d) One
 - (e) Three
7. If $f(x) = x^2e^x$, then the graph of f is decreasing for all x such that
- (a) $-2 < x < 0$
 - (b) $x < -2$
 - (c) $x < 0$
 - (d) $x > 0$
 - (e) $x > 2$
8. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?
- (a) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
 - (b) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
 - (c) It cannot be determined if f has any relative extrema.
 - (d) f has relative maxima at $x = -2$ and at $x = 2$.
 - (e) f has relative minima at $x = -2$ and at $x = 2$.

9. What is the derivative of $y = \sec \sqrt{t}$?

- (a) $\sec \sqrt{t} \tan \sqrt{t}$
- (b) $\tan^2 \sqrt{t}$
- (c) $\frac{\sec \sqrt{t} \tan \sqrt{t}}{2\sqrt{t}}$
- (d) $\sqrt{t} \tan^2 \sqrt{t}$

10. The graphs of the derivatives of the functions f, g , and h are shown below. Which of the functions f, g , or h have a relative maximum on the open interval $a < x < b$?



- (a) h only
- (b) f only
- (c) f, g , and h
- (d) f and g only
- (e) g only

Free Response 1

The function

$$f(x) = \frac{1}{x^2 - 4}$$

has first derivative

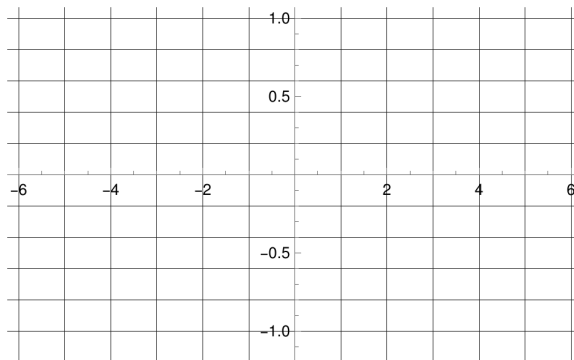
$$f'(x) = \frac{-2x}{(x^2 - 4)^2}$$

and second derivative

$$f''(x) = \frac{6x^2 + 8}{(x^2 - 4)^3}$$

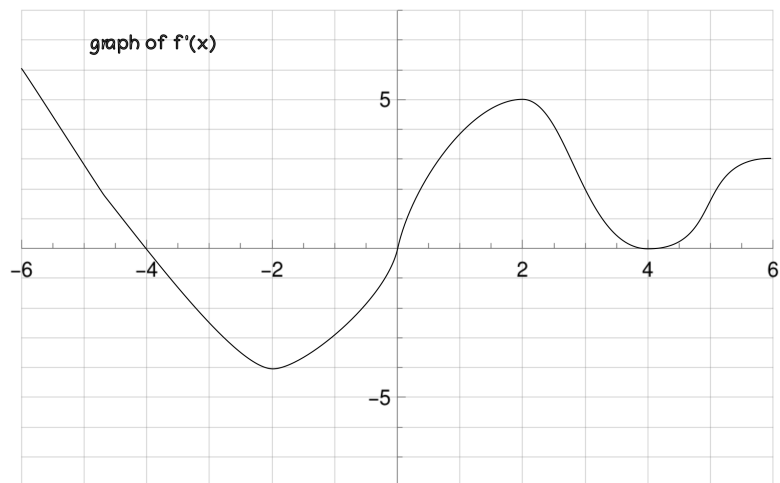
Sketch the graph of $f(x)$ after completing the following questions:

1. State any domain restrictions for $f(x)$
2. Determine any critical points of $f(x)$
3. State intervals on which $f(x)$ is increasing or decreasing
4. State intervals on which $f(x)$ is concave up or concave down
5. Calculate any horizontal asymptotes of $f(x)$



Free Response 2

The graph $f'(x)$ of the derivative of $f(x)$ is shown below. $f'(x)$ has horizontal tangents at $x = -2, 2, 4$ and zeros at $x = -4, 0, 4$. The domains of $f'(x)$ and $f(x)$ are $[-6, 6]$.



1. On which x intervals is the function f increasing? Justify your answer.
2. At which x value(s) does f have a local maximum? Justify your answer.
3. On which x intervals is the function f concave down? Justify your answer.

KEY

1. C
2. E
3. C
4. C
5. D
6. D
7. A
8. B
9. C
10. B