AOS Math 10, Spring 2024
Derivatives Test (#14)

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May 10, 2024

Student Signature **Class** **Date**

Print Name:

Instructions: For each problem, circle the letter of the best answer. You **must show all work** for credit. Partial credit may be awarded as appropriate.

1. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

   (a) $x > 0$

   (b) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$

   (c) $-2 < x < 0$ or $x > 2$

   (d) $x > \sqrt{2}$

   (e) $-2 < x < 2$

2. If $f(x) = x + \dfrac{1}{x}$, then the set of values for which f increases is

   (a) $(-\infty, -1] \cup [1, \infty)$

   (b) $[-1, 1]$

   (c) $(-\infty, \infty)$

   (d) $(0, \infty)$

   (e) $(-\infty, 0) \cup (0, \infty)$

3. At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

   (a) -1 only

   (b) o only

   (c) 1 only

   (d) -1 and 1 only

   (e) $-1, 0$ and 1

4. The graph of $y = \dfrac{-5}{x - 2}$ is concave downward for all values of x such that

   (a) $x < 0$

   (b) $x < 2$

   (c) $x < 5$

   (d) $x > 0$

   (e) $x > 2$

5. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b ?

   (a) -3

   (b) 0

   (c) 1

   (d) 3

6. The function f given by $f(x) = x^3 + 12x - 24$ is

   (a) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$

   (b) decreasing for $x < 0$, increasing for $x > 0$

   (c) increasing for all x

   (d) decreasing for all x

   (e) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

7. The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

   (a) None

   (b) One

   (c) Two

   (d) Three

   (e) Four

8. If $f(x) = x^2e^x$, then the graph of f is decreasing for all x such that

   (a) $x < -2$

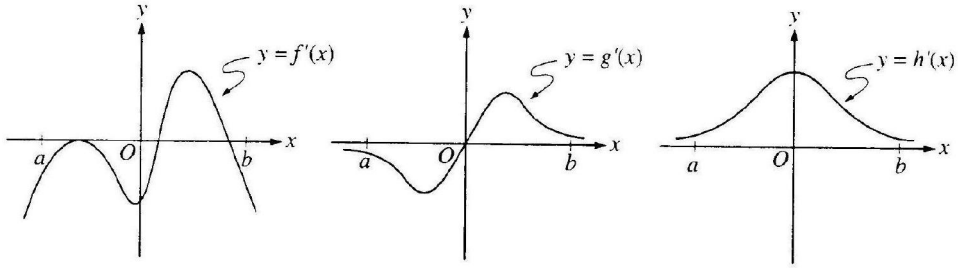
   (b) $-2 < x < 0$

   (c) $x \succ 2$

   (d) $x < 0$

   (e) $x > 0$

9. The graphs of the derivatives of the functions f, g, and h are shown below. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?



(a) f only

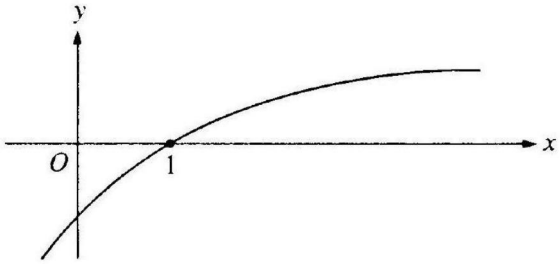
(b) g only

(c) h only

(d) f and g only

(e) f, g, and h

10. The graph of a twice-differentiable function f is shown in the figure below.



Which of the following is true?

(a) $f(1) < f'(1) < f''(1)$

(b) $f(1) < f''(1) < f'(1)$

(c) $f'(1) < f(1) < f''(1)$

(d) $f''(1) < f(1) < f'(1)$

(e) $f''(1) < f'(1) < f(1)$

Free Response

The function

$$f(x) = \frac{1}{x^2 - 4}$$

has first derivative

$$f'(x) = \frac{-2x}{(x^2 - 4)^2}$$

and second derivative

$$f''(x) = \frac{6x^2 + 8}{(x^2 - 4)^3}$$

Sketch the graph of $f(x)$ after completing the following questions:

1. State any domain restrictions for $f(x)$

2. Determine any critical points of $f(x)$

3. State intervals on which $f(x)$ is increasing or decreasing

4. State intervals on which $f(x)$ is concave up or concave down

5. Calculate any horizontal asymptotes of $f(x)$