AOS Math 10, Spring 2024
Cumulative, Quarter 3
(Parametric, Polar, Vectors, Complex)

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Student Signature **Class** **Date**

Print Name:

1. Eliminate the parameter t: $x = t^3 - 2$ and $2y = 1 - t^2$

   (a) $y = 1 + \sqrt[3]{(x+2)^2}$

   (b) $y = 1 - \sqrt[3]{(x+2)^2}$

   (c) $y = 1 + \sqrt{(x+2)^3}$

   (d) $y = 1 - \sqrt[3]{(x+2)^2}$

2. Eliminate the parameter t: $x = 4\sin(t) + 1$ and $y = 3\cos(t) - 2$

   (a) $\dfrac{(x-1)^2}{3} + \dfrac{(y+1)^2}{4} = 1$

   (b) $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

   (c) $\dfrac{(x-1)^2}{16} + \dfrac{(y+1)^2}{9} = 1$

   (d) $\dfrac{(x-1)^2}{4} + \dfrac{(y+1)^2}{3} = 1$

3. Convert the polar coordinate $(6, -2\pi/3)$ to rectangular coordinates

   (a) $(3, 3\sqrt{3})$

   (b) $(-3, -3)$

   (c) $(3, 3\sqrt{3})$

   (d) $(-3, -3\sqrt{3})$

   (e) $(3\sqrt{3}, -3\sqrt{3})$

4. Convert this equation to polar coordinates: $x^2 - y^2 = 16$

   (a) $r^2 = \dfrac{16}{\cos^2\theta + \sin^2\theta}$

   (b) $r^2 = \dfrac{4}{\cos^2\theta + \sin^2\theta}$

   (c) $r^2 = \dfrac{4}{\cos\theta - \sin\theta}$

   (d) $r^2 = \dfrac{16}{\cos^2\theta - \sin^2\theta}$

5. Convert this equation to rectangular coordinates: $r = 3\sec\theta$

   (a) $x = 1$

   (b) $x = 3$

   (c) $y = 1$

   (d) $y = 3$

6. Which of the following is the graph of $r = \cos(3\theta$.

   (a) A 6 leaf rose with x-intercepts $(0,0), (\pm 1, 0)$

   (b) A 6 leaf rose with no x intercept

   (c) A 3 leaf rose with no x intercept

   (d) A 3 leaf rose with x-intercepts $(0,0), (1,0)$

7. A baseball pitcher throws a baseball with an initial speed of 138 feet per second at an angle of 20° to the horizontal. The ball leaves the pitcher's hand at a height of 4 feet above the ground. Write the equations of motion, v_x and v_y for velocity and s_x, s_y for position.

   (a) $v_x(t) = 129.7,\ v_y(t) = 47.2 - 32t,\ s_x(t) = 129.7t,\ s_y(t) = 47.2t - 16t^2 + 4$

   (b) $v_x(t) = 129.7,\ v_y(t) = 47.2 - 32t,\ s_x(t) = 129.7t,\ s_y(t) = 47.2t - 16t^2$

   (c) $v_x(t) = 129.7,\ v_y(t) = 47.2 + 32t,\ s_x(t) = 129.7t,\ s_y(t) = 47.2t - 16t^2$

   (d) $v_x(t) = 129.7,\ v_y(t) = 47.2 - 16t,\ s_x(t) = 129.7t,\ s_y(t) = 47.2t - 16t^2 + 4$

8. Let $u = \langle -3, 5\rangle$ and $\vec{v} = \langle 1, 4\rangle$ and $\vec{w} = \langle 6, -3\rangle$ find $\vec{u} + 2\vec{v} - \vec{w}$

   (a) $\langle -7, 16\rangle$

   (b) $\langle -6, 14\rangle$

   (c) $\langle -7, 14\rangle$

   (d) $\langle -8, 16\rangle$

9. Given $\vec{u} = \langle 3\sqrt{3}, -5\rangle$, find $\|\vec{u}\|$

   (a) $3\sqrt{13}$

   (b) $2\sqrt{17}$

   (c) $2\sqrt{13}$

   (d) $3\sqrt{17}$

10. Given $\vec{u} = \langle -10, 9 \rangle$, find a unit vector in the direction of $\vec{u}$

(a) $\langle -\frac{10}{\sqrt{19}}, \frac{9}{\sqrt{19}} \rangle$

(b) $\langle -\frac{10}{\sqrt{181}}, \frac{9}{\sqrt{181}} \rangle$

(c) $\langle -\frac{10}{\sqrt{181}}, -\frac{9}{\sqrt{181}} \rangle$

(d) $\langle -\frac{10}{\sqrt{19}}, -\frac{9}{\sqrt{19}} \rangle$

11. Which vector is perpendicular to $\langle \frac{2}{3}, -\frac{17}{2} \rangle$

(a) $\langle -9, -17 \rangle$

(b) $\langle -9, -\frac{17}{18} \rangle$

(c) $\langle -9, -\frac{6}{17} \rangle$

(d) $\langle 9, \frac{18}{3} \rangle$

12. Which vector is parallel to $\langle \frac{2}{3}, -\frac{17}{2} \rangle$

(a) $\langle -51, 4 \rangle$

(b) $\langle 4, -51 \rangle$

(c) $\langle 25, -2 \rangle$

(d) $\langle 2, -25 \rangle$

13. What is the radian angle between $\langle 5, 1 \rangle$ and $\langle 2, -3 \rangle$

(a) 1.580

(b) 1.480

(c) 1.180

(d) 1.080

14. If vector $\vec{x}$ has magnitude 9 and makes an angle of 3.4 radians with the positive x axis, find the components of x and write as $a\hat{\imath} + b\hat{\jmath}$.

   (a) $8.70\hat{\imath} + 2.30\hat{\jmath}$

   (b) $-9.20\hat{\imath} + 3.20\hat{\jmath}$

   (c) $-8.70\hat{\imath} - 2.30\hat{\jmath}$

   (d) $-2.30\hat{\imath} - 8.70\hat{\jmath}$

15. Write the complex number $-3 + 9i$ in polar form.

   (a) $9.49e^{1.89i}$

   (b) $9.29e^{1.81i}$

   (c) $9.49e^{1.33i}$

   (d) $9.59e^{1.79i}$

16. Divide $10 - 9i$ by $2 - 4i$, and express your answer in the form $a + bi$.

   (a) $-\frac{4}{5} - \frac{29}{10}i$

   (b) $\frac{14}{5} + \frac{11}{10}i$

   (c) $\frac{14}{5} - \frac{11}{10}i$

   (d) $-\frac{4}{5} + \frac{29}{10}i$

17. Simplify the product $\sqrt{7}e^{-i\pi/3} \cdot 3e^{i\pi/5}$.

   (a) $3\sqrt{7}e^{2\pi i/15}$

   (b) $3\sqrt{7}e^{-2\pi i/15}$

   (c) $3\sqrt{7}e^{\pi i/15}$

   (d) $3\sqrt{7}e^{-\pi i/15}$

18. Solve the equation $z^2 - 2z + 5 = 0$ for z and express your answers in rectangular form.

   (a) $1 \pm 3i$

   (b) $1 \pm 2i$

   (c) $1 \pm \sqrt{5}i$

   (d) $1 \pm 2i^2$

19. If $z = 2 + i$ is one root of a quadratic equation $x^2 + bx + c$ with real coefficients, what is bc?

(a) -10

(b) -20

(c) 4

(d) 8

20. If $z^4 = 16e^{2\pi i/5}$, find all values of z in polar form.

(a) $z = 2e^{\pi i/10}, 2e^{3\pi i/10}, 2e^{5\pi i/10}, 2e^{7\pi i/10}$

(b) $z = 2, 2e^{\pi i/2}, 2e^{\pi i}, 2e^{3\pi i/2}$

(c) $z = 2e^{\pi i/10}, 2e^{3\pi i/5}, 2e^{11\pi i/10}, 2e^{8\pi i/5}$

(d) $z = 2e^{2\pi i/5}, 2e^{4\pi i/5}, 2e^{6\pi i/5}, 2e^{8\pi i/5}$

21. Factor $z^2 + 9$ into a product of two binomials.

(a) $(z + 3i)(z - 3i)$

(b) $(z + 3)(z - 3)$

(c) $(z + 3i)(z + 3i)$

(d) $(z + 3\sqrt{i})(z - 3\sqrt{i})$

22. (Bonus): By multiplying two complex numbers, prove the addition identities for sin and cos.

KEY

1. D
2. C
3. D
4. D
5. B
6. D
7. A
8. A
9. C
10. B
11. C
12. B
13. C
14. C
15. A
16. B
17. B
18. B
19. B
20. C
21. A