BC Review, Mostly Series

1. Write the general and first four terms of the Maclaurin series for $\sin x, \cos x, e^x, \ln(1+x)$

2. Let f be a function having derivatives of all orders for all real numbers. The third-degree Taylor polynomial for f about $x = 2$ is given by $P_3(x) = 7 - 9(x-2)^2 - 3(x-2)^3$.

3. Suppose the fourth derivative of f satisfies the inequality $\left| f^{(4)}(x) \right| \leq 6$ for all x on the closed interval $[0, 2]$. Use the Lagrange error bound to justify why $f(0)$ is negative.

4. Find the Lagrange error in estimating $e \approx 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24}$

5. Find the Lagrange error in estimating $\cos(0.3) \approx 1 - \frac{(0.3)^2}{2!} + \frac{(0.3)^4}{4!}$

6. Write the first four terms of the Maclaurin series for $f(x) = x^2 \sin(3x) - 4$

7. Write the first four terms of the Maclaurin series for $f(x) = 2x + 3x \ln(x^2)$

8. Evaluate: $\displaystyle\sum_{n=3}^{\infty} \frac{3^{n-2}}{4^{n+2}}$

9. Evaluate: $\displaystyle\sum_{n=1}^{\infty} \frac{3^{2n-2}}{4^{n+2}}$

10. Evaluate: $\displaystyle\sum_{n=0}^{\infty} x^n$ (hint: it's a geometric series)

11. Evaluate: $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^n$ (hint: it's a geometric series)

12. Take derivatives of both sides of 9. Then do the same to 10.

13. Evaluate: $\displaystyle\int \frac{2x+3}{x^2 - 7x + 10} \, dx$

14. What's the formula for total distance traveled? Total displacement? In 1D and in 2D?

15. What is the arc length of the parabola $y = 2x^2 - 5$ from $x = 1$ to $x = 4$? (Calculator can be used to integrate)

16. Evaluate $\displaystyle\int_0^{\infty} e^{-x} \, dx$

17. Evaluate $\displaystyle\int_0^1 \frac{e^x}{1 - e^x} \, dx$