Complex Pre-Test Key

1. $8 + 2i$
2. $1 + 4i$
3. $4 - 8i$
4. $-3 + 3i$
5. $6 + 12i$
6. $-2 + 6i$
7. -6
8. -20
9. $(1 + 2i)(3 + 4i) = 3 + 8i^2 + 4i + 6i = -5 + 10i$
10. $(-2 + 3i)(1 - 5i) = -2 - 15\left(i^2\right) + 10i + 3i = 13 + 13i$
11. $(4 + 3i)(4 - 3i) = 4^2 - (3i)^2 = 16 + 9 = 25$
12. $(2 + 5i)(2 - 5i) = 4 + 25 = 29$
13. $\frac{4+2i}{2-3i} \cdot \frac{2+3i}{2+3i} = \frac{(8-6)+(12+4)i}{4+9} = \frac{-2}{13} + \frac{16}{13}i$
14. $\frac{-1+4i}{3+i} \cdot \frac{3-i}{3-i} = \frac{(-3+4)+(12i+i)}{9+1} = \frac{1}{10} + \frac{13}{10}i$
15. $\sqrt{3^2 + 4^2 = 5}$
16. $\sqrt{4+2} = \sqrt{6}$
17. $\arctan(\sqrt{3}/1) = \pi/3$
18. $\arctan(-1/1) = \pi/4 + \pi = 3\pi/4$
19. $x = (-1 \pm \sqrt{1-4})/2 = (-1 \pm \sqrt{-3})/2 = -\frac{1}{2} + \frac{i\sqrt{3}}{2}, -\frac{1}{2} - \frac{i\sqrt{3}}{2}$
20. $x^3 + 1 = (x + 1)\left(x^2 - x + 1\right) \Rightarrow x = 1$ or $x = \frac{1 \pm \sqrt{1-4}}{2}$

$$\Rightarrow x = 1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$$

19. $\left(2e^{i\pi/2}\right)\left(3e^{i\pi/3}\right) = 6e^{i(7\pi/12)}$
20. $8e^{i2\pi/3}$
21. $8e^{i3\pi/4}$
22. $81e^{4i\pi/3}$
23. Since θ and $\theta + 2\pi$ have the same sin and cos, any imaginary $e^{i\theta}$ is equal to $e^{(2k\pi+\theta)i}$ for integers k. Since $-1 = e^{\pi i}$, write $-8 = 8(-1) = 8(e^{\pi i + 2k\pi})$ and raise to the $1/3$ power to take cube roots.

$$\left(8e^{i\pi+2k\pi}\right)^{1/3} = 2e^{i(\pi/3+2k\pi/3)} \quad k = 0, 1, 2$$
$$= 2e^{i\pi/3}, 2e^{i\pi}, 2e^{i5\pi/3}$$

24. $(2+i)^{1/4}$ Convert to polar form. $\ldots r = \sqrt{2^2+1^2} = \sqrt{5} \quad \theta = \tan^{-1}(1/2)$ = oops this was supposed to be a reference angle. Let's pretend $\theta = \pi/6$.

$$(2+i)^{1/4} = \left(\sqrt{5}e^{i\pi/6}\right)^{1/4} = 5^{1/8}\left(e^{i\pi/6+2k\pi}\right)^{1/4}; \quad k = 0, 1, 2, 3$$
$$= 5^{1/8}e^{\pi i/24}, 5^{1/8}e^{13\pi i/24}, 5^{1/8}e^{25\pi i/24}, 5^{1/8}e^{37\pi i/24}$$

25. Write -4 as a power of e. Use the fact that $e^{\pi i} = -1$

$$\begin{aligned}
&\ln(-4) = x \\
&-4 = e^x \\
&-4 = re^{i\theta} \\
&\text{so } r = 4, e^{i\theta} = -1, \theta = \pi \\
&-4 = 4e^{i\pi} = e^{\ln 4}e^{i\pi} = e^{i\pi+\ln 4} = e^x \\
&x = \ln 4 + \pi i
\end{aligned}$$

26. $\ln(-16) = \ln(16) + \pi i$
27. $i^{23} = i^{23 \bmod 4} = i^3 = -i$
28. $i^{45} = i^{+5 \bmod 4} = i^1 = i$
29. $(-1)^i = e^{i\ln(-1)} = e^{i(\pi i)} = e^{-\pi}$
30. $i^i = e^{i\ln i} = e^{i(\pi/2)} = e^{-\pi/2}$