Directions:

Solve the following problems using the available space for scratchwork. Indicate your answers on the front page. Do not spend too much time on any one problem.

Note: Let ln(x) denote the natural logarithm of x with base e.

01. B	07. B	13. A	19. B	25. A	31. C
02. A	08. C	14. E	20. D	26. B	32. B
03. A	09. B	15. E	21. A	27. C	33. E
04. D	10. A	16. E	22. D	28. C	34. E
05. D	11. A	17. B	23. C	29. C	35. B
06. A	12. A	18. C	24. B	30. C	36. C

01

The graph in the xy-plane represented by $x = \cos t$ and $y = 1 - \cos 2t$ for $-\infty < t < \infty$ is

A) Part of a hyperbola

B) Part of a parabola

C) An ellipse

D) A semicircle

E) A straight line

$y = 1 - (\cos^2 t - \sin^2 t)$

$y = 1 - \cos^2 t + (1 - \cos^2 t)$

$y = 2 - 2\cos^2 t$

$y = 2 - 2x^2$

parabola

ANSWER

B

02

If $x = t - t^2$ and $y = \sqrt{2t+5}$, then $\frac{dy}{dx}$ at $t = 2$ is

A) $-\frac{1}{9}$

B) -1

C) $\frac{3}{2}$

D) -9

E) $-\frac{1}{18}$

$x' = 1 - 2t$

$x'(2) = 1 - 4$

$= -3$

$y' = \frac{2}{2\sqrt{2t+5}}$

$y' = \frac{1}{\sqrt{2t+5}}$

$y'(2) = \frac{1}{\sqrt{9}}$

$= \frac{1}{3}$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1/3}{-3} = -\frac{1}{9}$

ANSWER

A

03

Two particles, Alpha and Beta, race from the y-axis to the vertical line $x = 6\pi$. Alpha's position is given by the parametric equations $x_\alpha = 3t - 3\sin t$ and $y_\alpha = 3 - 3\cos t$, while Beta's position is given by $x_\beta = 3t - 4\sin t$ and $y_\beta = 3 - 4\cos t$ for $t \geq 0$. Which statement best describes the race and its outcome?

$x_\alpha = 3t - 3\sin t = 6\pi$

$t = 2\pi$

$x_\beta = 3t - 4\sin t = 6\pi$

$t = 5.007$

A) Alpha moves slower and loses

B) Alpha takes a shorter path and wins ×

Beta wins

C) Beta starts out in the wrong direction and loses ×

D) Beta moves faster but loses ×

E) Alpha and Beta tie ×

ANSWER

A

04

A curve in the xy-plane is defined parametrically by the equations $x = t^2 + t$ and $y = t^2 - t$. For what value of t is the tangent line to the curve horizontal?

$\frac{dy}{dx} = \frac{2t-1}{2t+1} = 0$ $\quad$ $2t-1=0$

$2t=1$

$t=1/2$

A) $t = -1$

B) $t = -\frac{1}{2}$

C) $t = 0$

D) $t = \frac{1}{2}$

E) $t = 1$

ANSWER

D

05

A curve is given parametrically by the equations $x = 3 - 4\sin t$ and $y = 4 + 3\cos t$ for $0 \le t \le 2\pi$. What are all points (x, y) at which the curve has a vertical tangent?

t	x	y	
0	3	7	→ (3,7)
π	3	1	(3,1)
2π	3	7	

$\frac{dy}{dx} = \frac{-4\cos t}{-3\sin t}$

$-3\sin t = 0$

$\sin t = 0$

$t = 0, \pi, 2\pi$

A) $(-1,4)$

B) $(3,7)$

C) $(-1,4)$ and $(3,7)$

D) $(3,7)$ and $(3,1)$

E) $(4,-1)$ and $(4,7)$

ANSWER

D

06

If $x = t^2 + 2t$ and $y = 3\ln(t+1)$, then $\frac{dy}{dx}$ at $t = \frac{1}{2}$ is

A) $\frac{2}{3}$

$\frac{dy}{dx} = \frac{\frac{3}{t+1}}{2t+2} = \frac{\frac{3}{3/2}}{3} = \frac{1}{3/2} = 2/3$

B) $\frac{4}{5}$

C) $\frac{3}{2}$

D) 3

E) $\frac{1}{2}$

ANSWER

A

07

The movement of a particle in the plane is $x(t) = \sin t$ and $y(t) = \cos^2 t$. If t is in the interval $(0, \pi)$, when is it stationary?

A) 0

$\frac{dy}{dx} = \frac{-2\sin t\cos t}{\cos t}$

B) $\frac{\pi}{2}$

check $-2\sin t\cos t = 0$

C) $\frac{\pi}{4}$

$t = 0, \pi/2, \pi$

D) $\frac{3\pi}{4}$

NOT IN INTERVAL

E) π

ANSWER

B

08

If $x = t^3 - 3t$ and $y = (t^2+1)^2$, then $\frac{dy}{dx}$ at $t = 2$ is

A) 40

B) 9

C) $\frac{40}{9}$

D) $\frac{9}{40}$

E) 0

$x' = 3t^2 - 3$ $\quad y' = 2(t^2+1)\cdot(2t)$

$x'(2) = 9$ $\quad y'(2) = 2(5)(4)$

$= 40$

$\frac{dy}{dx} = \frac{40}{9}$

ANSWER

C

09

The equation of a line tangent to the curve $x(t) = t^2$ and $y(t) = t^3 - 1$ at the point $(4,7)$ is

$4 = t^2$ $\quad 7 = t^3 - 1$

$\pm 2 = t$ $\quad 8 = t^3$

$2 = t$

$t = 2$

$x' = 2t$ $\quad y' = 3t^2$

$x' = 4$ $\quad y' = 12$

A) $x - 3y = -5$

B) $3x - y = 5$

C) $4x - 7y = 0$

D) $4x + 7y = 12$

E) $x^2 + y^3 = 1$

$\frac{dy}{dx} = \frac{12}{4} = 3$

$y - 7 = 3(x-4)$

$y - 7 = 3x - 12$

$y - 3x = -5$

$3x - y = 5$

ANSWER

B

10

Which of the following is an equation of the line tangent to the curve with parametric equations $x = 3e^{-t}$ and $y = 6e^{t}$ at the point where $t = 0$?

A) $2x + y - 12 = 0$

B) $-2x + y - 12 = 0$

C) $x - 2y + 9 = 0$

D) $2x - y = 0$

E) $x + 2y - 15 = 0$

$x' = -3e^{-t}$ $\quad y' = 6e^{t}$ $\quad x(0) = 3$ $\quad y(0) = 6$

$x'(0) = -3$ $\quad y'(0) = 6$ $\quad (3, 6)$

$\frac{dy}{dx} = \frac{6}{-3} = -2$

$y - 6 = -2(x - 3)$

$y - 6 = -2x + 6$

$2x + y - 12 = 0$

ANSWER: A

11

A curve in the plane is defined parametrically by the equations $x = 2t + 3$ and $y = t^2 + 2t$. An equation of the line tangent to the curve at $t = 1$ is

A) $y = 2x - 7$

B) $y = x - 2$

C) $y = 2x$

D) $y = 2x - 1$

E) $y = \frac{1}{2}x + \frac{1}{2}$

$x' = 2$

$x'(1) = 2$

$y' = 2t + 2$

$y'(1) = 4$

$\frac{dy}{dx} = \frac{4}{2} = 2$

$x(1) = 5$

$y(1) = 3$

$(5, 3)$

$y - 3 = 2(x - 5)$

$y - 3 = 2x - 10$

$y = 2x - 7$

ANSWER: A

12

If $x = 2t^2$ and $y = t^3$, then $\frac{d^2y}{dx^2}$ at $t = 3$ is

* MAKE SURE TO DIVIDE SECOND DERIVATIVE BY AN ADDITIONAL dx/dt

A) $\frac{1}{16}$

$x'(t) = 4t$ $\quad y'(t) = 3t^2$

$x'(3) = 12$ $\quad y'(3) = 27$

B) $\frac{9}{2}$

C) $\frac{3}{4}$

D) $\frac{1}{4}$

$\frac{dy}{dx} = \frac{3t^2}{4t} = \frac{3t}{4}$

E) $\frac{9}{4}$

$\frac{d^2y}{dx^2} = \frac{\frac{3(4) - 0(3t)}{(4)^2}}{4t} = \frac{12}{4^3(3)} = \frac{12}{192} = \frac{1}{16}$

ANSWER

A

13

If $x = \sin t$ and $y = cos^2 t$, then $\frac{d^2y}{dx^2}$ at $t = \pi$ is

A) -2

$\frac{dy}{dx} = \frac{-2\cos t \sin t}{\cos t} = -2\sin t$

B) $-\frac{1}{4}$

C) 0

$\frac{d^2y}{dx^2} = \frac{-2\cos t}{\cos t} = -2$

D) $\frac{1}{4}$

E) 2

ANSWER

A

14

A particle moves in the xy-plane so that at any time $t, t > 0$. Its coordinates are $x = e^t \sin t$ and $y = e^t \cos t$. At $t = \pi$, its velocity vector is

A) $\langle e^{\pi}, -e^{\pi} \rangle$

B) $\langle 0, -e^{\pi} \rangle$

C) $\langle -e^{\pi}, e^{\pi} \rangle$

D) $\langle e^{\pi}, e^{\pi} \rangle$

E) $\langle -e^{\pi}, -e^{\pi} \rangle$

$S(t) = \langle e^t \sin t, e^t \cos t \rangle$

$v(t) = \langle e^t \sin t + e^t \cos t, e^t \cos t - e^t \sin t \rangle$

$v(\pi) = \langle -e^{\pi}, -e^{\pi} \rangle$

ANSWER

E

15

$\int 3 - 4\cos t \, dt$

$3t - 4\sin t + C$

$0 - 0 + C = 0$

$C = 0$

$\int 4\sin t \, dt$

$-4\cos t + C$

$-4(1) + C = -1$

$C = 3$

The velocity vector of a particle moving in the xy-plane is $\langle 3 - 4\cos t, 4\sin t \rangle$ for all $t \geq 0$. When $t = 0$, the particle is at the point $(0, -1)$. Which statement best describes the motion of the particle?

A) The particle moves around a circle

B) The particle moves along a sine graph

C) The particle moves to the left for all t

D) The particle moves to the right with a regular up and down motion

E) The particle moves generally to the right with an up and down motion, but periodically loops to the left

$\langle 3t - 4\sin t, 3 - 4\cos t \rangle$

Graph

ANSWER

E

16

The velocity vector of a particle moving in the coordinate plane is $\langle 4t, -2t \rangle$ for $t \geq 0$. The path of the particle lies on

A) A hyperbola

B) An ellipse

C) A line

D) A parabola

E) A ray

$x' = 4t$ $y' = -2t$

$\frac{dy}{dx} = \frac{-2t}{4t} = -\frac{1}{2}$

CONSTANT SLOPE → LINEAR

Since $t \geq 0$, endpoint exists

ANSWER

C

17

The position of a particle in the xy-plane is given by $x = 4t^2$ and $y = \sqrt{t}$. At $t = 4$, the acceleration vector is

A) $\langle 8, -\frac{1}{64} \rangle$

B) $\langle 8, -\frac{1}{32} \rangle$

C) $\langle 8, \frac{1}{32} \rangle$

D) $\langle 32, -\frac{1}{32} \rangle$

E) $\langle 32, \frac{1}{4} \rangle$

$s = \langle 4t^2, t^{1/2} \rangle$

$v = \langle 8t, \frac{1}{2}t^{-1/2} \rangle$

$a = \langle 8, -\frac{1}{4}t^{-3/2} \rangle$

$-\frac{1}{4(4)^{3/2}} = \frac{-1}{4(8)} = -\frac{1}{32}$

$= \langle 8, -\frac{1}{32} \rangle$

ANSWER

B

18

The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = 9\cos t$ and $y(t) = 4\sin t$ for $t \geq 0$, then at $t = 3$, the acceleration vector is

A) $\langle -8.910, 0.564 \rangle$

B) $\langle -1.270, -3.960 \rangle$

C) $\langle 8.910, -0.564 \rangle$

D) $\langle 8.910, 0.564 \rangle$

E) $\langle -0.564, 8.910 \rangle$

$s = \langle 9\cos t, 4\sin t \rangle$

$v = \langle -9\sin t, 4\cos t \rangle$

$a = \langle -9\cos t, -4\sin t \rangle$

$a = \langle -9\cos 3, -4\sin 3 \rangle$

$= \langle 8.910, -.564 \rangle$

ANSWER

C

19

A particle moves in the xy-plane so that at any time t its coordinates are $x = t^2$ and $y = 4 - t^3$. At $t = 1$, its acceleration vector is

A) $\langle 2, -3 \rangle$

B) $\langle 2, -6 \rangle$

C) $\langle 1, -6 \rangle$

D) $\langle 2, 6 \rangle$

E) $\langle 1, -2 \rangle$

$s = \langle t^2, 4 - t^3 \rangle$

$v = \langle 2t, -3t^2 \rangle$

$a = \langle 2, -6t \rangle$

$a = \langle 2, -6 \rangle$

ANSWER

B

20

The particle moves in the xy-plane so that, at any time t, its coordinates are $x = \frac{t^3-2t^2}{4}$ and $y = t^2 - t$. At $t = 2$, its acceleration vector is

A) $\langle 0, 0 \rangle$

B) $\langle 1, 2 \rangle$

C) $\langle 2, 0 \rangle$

D) $\langle 2, 2 \rangle$

E) $\langle -2, -2 \rangle$

$S = \langle \frac{1}{4}t^3 - \frac{1}{2}t^2, t^2 - t \rangle$

$v = \langle \frac{3}{4}t^2 - t, 2t - 1 \rangle$

$a = \langle \frac{3}{2}t - 1, 2 \rangle$

$a = \langle 3 - 1, 2 \rangle$

$a = \langle 2, 2 \rangle$

ANSWER

D

21

A particle moves in the xy-plane so that, at any time t, its coordinates are $x = t^3 + t$ and $y = t^5 - 2t^2$. At $t = 2$, its acceleration vector is

A) $\langle 12, 156 \rangle$

B) $\langle 12, 164 \rangle$

C) $\langle 6, 156 \rangle$

D) $\langle 6, 164 \rangle$

E) $\langle 13, 72 \rangle$

$S = \langle t^3 + t, t^5 - 2t^2 \rangle$

$v = \langle 3t^2 + 1, 5t^4 - 4t \rangle$

$a = \langle 6t, 20t^3 - 4 \rangle$

$a = \langle 12, 20(8) - 4 \rangle$

$a = \langle 12, 156 \rangle$

ANSWER

A

22

A particle moves in the xy-plane so that at any time t, its coordinates are $x = \alpha\cos(\beta t)$ and $y = \alpha\sin(\beta t)$, where α and β are constants. The y-component of the acceleration of the particle at any time t is

A) $-\beta^2 y$

B) $-\beta^2 x$

C) $-\alpha\beta\sin(\beta t)$

D) $-\alpha\beta^2\sin(\beta t)$

E) $-\alpha\beta^2\cos(\beta t)$

$v_y(t) = \beta\alpha\cos(\beta t)$

$a_y(t) = -\alpha\beta^2\sin(\beta t)$

ANSWER

D

23

A particle moves in the plane according to $x = t\cos t$ and $x = t\sin t$. Which of the following vectors is orthogonal (perpendicular) to the acceleration vector at $t = \pi$.

A) $\langle 1, \pi \rangle$

B) $\langle 2, -\pi \rangle$

C) $\langle 2, \pi \rangle$

D) $\langle \pi, 2 \rangle$

E) $\langle \pi, 1 \rangle$

$v = \langle \cos t - t\sin t,\ \sin t + t\cos t \rangle$

$a = \langle -\sin t - \sin t - t\cos t,\ \cos t + \cos t - t\sin t \rangle$

$a = \langle 0 - 0 - \pi(-1),\ (-1) + (-1) - \pi(0) \rangle$

$a = \langle \pi, -2 \rangle$

$a \to -2/\pi$

perp $\to \pi/2$

$\langle 2, \pi \rangle$

ANSWER

C

24

A particle moves in the xy-plane so that its velocity vector at time t is $v(t) = \langle t^2, \sin \pi t\rangle$ and the position vector at time $t = 0$ is $\langle 1, 0\rangle$. What is the position vector of the particle when $t = 3$?

A) $\langle 9, \frac{1}{\pi}\rangle$

B) $\langle 10, \frac{2}{\pi}\rangle$

C) $\langle 6, -2\pi\rangle$

D) $\langle 10, 2\pi\rangle$

E) $\langle 10, 2\rangle$

$V_x(3) - V_x(0) = \int_0^3 t^2 dt$

$V_x(3) - 1 = \frac{1}{3}t^3\Big|_0^3$

$V_x(3) = 1 + 9$

$V_x(3) = 10$

$V_y(3) - V_y(0) = \int_0^3 \sin(\pi t)dt$

$V_y(3) - 0 = -\frac{1}{\pi}\cos(\pi t)\Big|_0^3$

$V_y(3) = -\frac{1}{\pi}[-1 - 1]$

$V_y(3) = \frac{2}{\pi}$

$\langle 10, 2/\pi\rangle$

ANSWER: B

25

The velocity vector of a particle moving in the xy-[lane is given by $v = \langle 2\sin t, 3\cos t\rangle$ for $t \geq 0$. At $t = 0$, the particle is at the point $(1, 1)$. What is the position vector at $t = 2$?

A) $\langle 3.832, 3.728\rangle$

B) $\langle 1.832, -1.728\rangle$

C) $\langle 1.819, -1.248\rangle$

D) $\langle 1.735, -0.532\rangle$

E) $\langle 0, 3\rangle$

$V_x(2) - V_x(0) = \int_0^2 2\sin t\, dt$

$V_x(2) - 1 = -2\cos t\Big|_0^2$

$V_x(2) = 1 - 2[\cos 2 - 1]$

$V_x(2) = 3 - 2\cos 2$

$V_x(2) = 3.832$

$V_y(2) - V_y(0) = \int_0^2 3\cos t\, dt$

$V_y(2) - 1 = 3\sin t\Big|_0^2$

$V_y(2) = 1 + 3[\sin 2 - 0]$

$V_y(2) = 1 + 3\sin 2$

$V_y(2) = 3.728$

ANSWER: A

26

A particle moves in a plane so that its position at any time θ, $0 \le \theta \le 8$, is given by the polar equation $r(\theta) = 5(1 + \cos\theta)$. When does the particle's distance from the origin change from decreasing to increasing?

$r(\theta) = 5(1 + \cos\theta)$ $\quad$ $r'(\theta) = -5\sin\theta = 0$

CARDIOID $\quad$ $\theta = 0, \pi, 2\pi$

$-$ $\quad$ $+$ $\quad$ $-$

0 $\quad$ π $\quad$ 2π $\quad$ 8

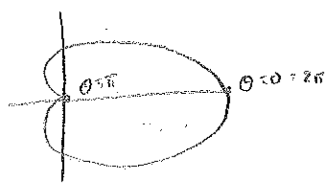
A) $\theta = 0$ only

B) $\theta = \pi$ only

C) $\theta = 2\pi$ only

D) $\theta = 0$ and $\theta = \pi$

E) $\theta = \pi$ and $\theta = 2\pi$



ANSWER

B

27

The area of the region enclosed by the polar curve $r = \cos 2\theta$ for $0 \le \theta \le \frac{\pi}{2}$ is

↑
4 Leaf Rose

$\cos 2(0) = \cos(0) = 1$

$\cos 2(\pi/2) = \cos\pi = -1$

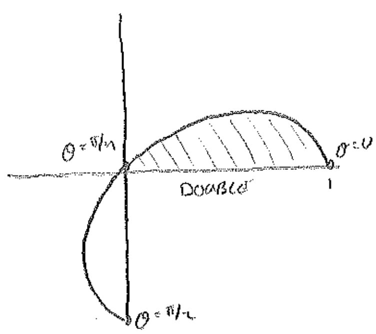
A) $\frac{\pi}{2}$

B) π

C) $\frac{\pi}{8}$

D) $\frac{\pi}{4}$

E) 1



$2\left[\frac{1}{2}\int_0^{\pi/4}(\cos 2\theta)^2\,d\theta\right]$

$\int_0^{\pi/4}(\cos 2\theta)^2\,d\theta$ $\quad$ $u = 2\theta$, $du = 2\,d\theta$ $\quad$ $\pi/4 \to \pi/2$, $0 \to 0$

$\frac{1}{2}\int_0^{\pi/2}\cos^2 u\,du$

$\frac{1}{2}\int_0^{\pi/2}\frac{1}{2} + \frac{1}{2}\cos 2u\,du$

$\frac{1}{4}u + \frac{1}{8}\sin 2u\Big|_0^{\pi/2}$

$\left[\frac{1}{4}\left(\frac{\pi}{2}\right) + \frac{1}{8}\sin(\pi)\right] - [0 + 0]$

$\frac{\pi}{8} + 0$

ANSWER

C

28

The area of one leaf of the rose $r = \sin 3\theta$ is

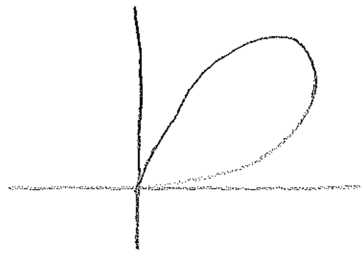
A) $\frac{\pi}{12}$

B) $\frac{\pi}{6}$

C) $\frac{\pi}{4}$

D) $\frac{\pi}{3}$

E) $\frac{\pi}{2}$



$r = \sin 3\theta = 0$

$3\theta = 0 \quad 3\theta = \pi$

$\theta = 0 \quad \theta = \pi/3$

$\frac{1}{2}\int_0^{\pi/3} (\sin 3\theta)^2 d\theta \quad u = 3\theta \quad du = 3d\theta \quad \pi/3 \to \pi \quad 0 \to 0$

$\frac{1}{2}\int_0^{\pi} \sin^2 u \, du$

$\frac{1}{2}\int_0^{\pi} \frac{1}{2} - \frac{1}{2}\cos 2u \, du$

$\frac{1}{4}u - \frac{1}{8}\sin 2u \Big|_0^{\pi}$

$\left[\frac{1}{4}\pi - \frac{1}{8}\sin(2\pi)\right] - [0 - 0]$

$\frac{\pi}{4} - 0 - 0$

$\pi/4$

ANSWER

C

29

The area outside $r = 1$ and inside $r = 1 + \sin\theta$ is

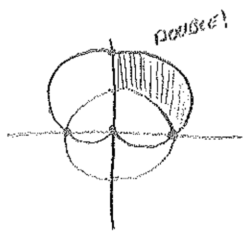
A) $2 + \pi$

B) $2 + \frac{\pi}{2}$

C) $2 + \frac{\pi}{4}$

D) $2 - \frac{\pi}{4}$

E) $2 - \frac{\pi}{2}$



$2\left[\frac{1}{2}\int_0^{\pi/2} (1+\sin\theta)^2 - (1)^2\right] d\theta$

$\int_0^{\pi/2} 1 + 2\sin\theta + \sin^2\theta - 1 \, d\theta$

$\int_0^{\pi/2} 2\sin\theta + \sin^2\theta \, d\theta$

$-2\cos\theta \Big|_0^{\pi/2} + \int_0^{\pi/2} \frac{1}{2} - \frac{1}{2}\cos 2\theta \, d\theta$

$-2[0 - 1] + \left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]\Big|_0^{\pi/2}$

$2 + \left[\left(\frac{\pi}{4} - 0\right) + (0 - 0)\right]$

$2 + \pi/4$

ANSWER

C

30

The total area of the region enclosed by the polar graph of $r = \cos 3\theta$ is

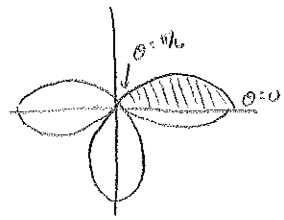
A) $\frac{\pi}{12}$

B) $\frac{\pi}{6}$

C) $\frac{\pi}{4}$

D) $\frac{\pi}{3}$

E) $\frac{\pi+\sqrt{3}}{2}$



$\cos(3\theta) = 0$

$3\theta = \pi/2$

$\theta = \pi/6$

$6\left[\frac{1}{2}\int_0^{\pi/6} (\cos 3\theta)^2 \, d\theta\right]$

$3\int_0^{\pi/6} (\cos 3\theta)^2 \, d\theta$ $\quad u = 3\theta \quad du = 3d\theta \quad 0 \to 0 \quad \pi/6 \to \pi/2$

$\int_0^{\pi/2} \cos^2 u \, du$

$\int_0^{\pi/2} \frac{1}{2} + \frac{1}{2}\cos 2u \, du$

$\frac{1}{2}u + \frac{1}{4}\sin 2u \Big|_0^{\pi/2}$

$\left[\frac{\pi}{4} + \frac{1}{4}\overset{0}{\cancel{\sin \pi}}\right] - [0 + 0]$

$\pi/4$

ANSWER: C

31

The area of the region enclosed by the polar curve $r = \sin\theta$ for $0 \le \theta \le \pi$ equals

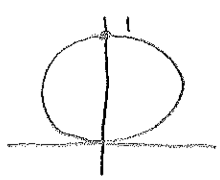
A) 1

B) $\frac{\pi}{2}$

C) $\frac{\pi}{4}$

D) $\frac{\pi}{8}$

E) π



$A = \pi r^2$

$A = \pi\left(\frac{1}{2}\right)^2$

$A = \frac{\pi}{4}$

ANSWER: C

32

Which of the following gives the area of the region enclosed by the graph of the polar curve $r = 1 + \cos\theta$?

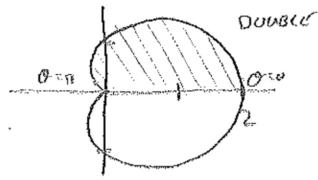
A) $\int_0^{\pi}(1+\cos^2\theta)\,d\theta$

B) $\int_0^{\pi}(1+\cos\theta)^2\,d\theta$

C) $\int_0^{2\pi}(1+\cos\theta)\,d\theta$

D) $\int_0^{2\pi}(1+\cos\theta)^2\,d\theta$

E) $\frac{1}{2}\int_0^{2\pi}(1+\cos^2\theta)\,d\theta$



$2\left[\frac{1}{2}\int_0^{\pi}(1+\cos\theta)^2\,d\theta\right]$

$\int_0^{\pi}(1+\cos\theta)^2\,d\theta$

ANSWER

B

33

The area of the region enclosed by the polar curve $r = 2(\cos\theta + \sin\theta)$ is

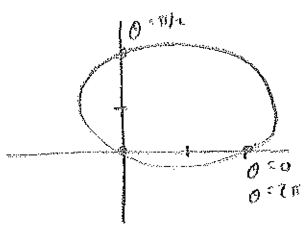
A) 1

B) 2

C) π

D) 2π

E) 4π



$2(\cos\theta + \sin\theta) = 2$

$\cos\theta + \sin\theta = 1$

when $\theta = 0, \pi/2, 2\pi$

$\frac{1}{2}\int_0^{2\pi}[2(\cos\theta+\sin\theta)]^2\,d\theta$

$\frac{1}{2}\int_0^{2\pi}4[\cos^2\theta+2\cos\theta\sin\theta+\sin^2\theta]\,d\theta$

$2\int_0^{2\pi}[1+2\sin\theta\cos\theta]\,d\theta$ $\quad u=\sin\theta$, $du=\cos\theta\,d\theta$

$2\theta+4\int_0^{2\pi}u\,du$

$2\theta+2\sin^2\theta\Big|_0^{2\pi}$

$(4\pi+0)-(0)$

4π

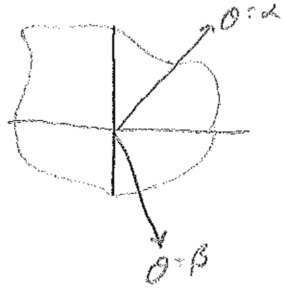
ANSWER

E

34

key!

If the function $r = f(\theta)$ is continuous and nonnegative for $0 \leq \alpha \leq \theta \leq \beta \leq 2\pi$, then the area enclosed by the polar curve $r = f(\theta)$ and the lines $\theta = \alpha$ and $\theta = \beta$ is given by



$A = \frac{1}{2}\int_\alpha^\beta [f(\theta)]^2 d\theta$

A) $\frac{1}{2}\int_\alpha^\beta f(\theta^2)\, d\theta$

B) $\frac{1}{2}\int_\alpha^\beta f(\theta)\, d\theta$

C) $\frac{1}{2}\int_\alpha^\beta \theta f(\theta^2)\, d\theta$

D) $\frac{1}{2}\int_\alpha^\beta \theta f(\theta)\, d\theta$

E) $\frac{1}{2}\int_\alpha^\beta (f(\theta))^2\, d\theta$

ANSWER

E

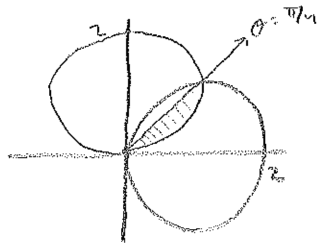
35

Which of the following integrals gives the total area of the region shared by both polar curves $r = 2\cos\theta$ and $r = 2\sin\theta$?

$2\cos\theta = 2\sin\theta$

$\cos\theta = \sin\theta$

$\theta = \pi/4$



A) $2\int_0^{\frac{\pi}{4}} \sin^2\theta\, d\theta$

B) $4\int_0^{\frac{\pi}{4}} \sin^2\theta\, d\theta$

C) $2\int_0^{\frac{\pi}{2}} \sin^2\theta\, d\theta$

D) $4\int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$

E) $2\int_0^{\frac{\pi}{4}} (\cos^2\theta - \sin^2\theta)\, d\theta$

$2\left[\frac{1}{2}\int_0^{\pi/4} (2\sin\theta)^2 d\theta\right]$

$\int_0^{\pi/4} 4\sin^2\theta\, d\theta$

OR

$\int_{\pi/4}^{\pi/2} 4\cos^2\theta\, d\theta$

ANSWER

B

36

The area enclosed by the polar curve $r = 6\cos\theta + 8\sin\theta$ from $\theta = 0$ to $\theta = \pi$ is

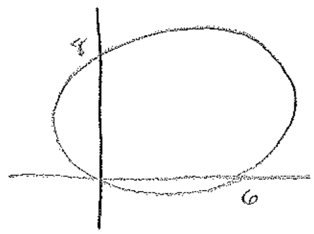
A) 28.274

B) 50.265

C) 78.540

D) 113.097

E) 201.062



$\frac{1}{2}\int_0^{\pi} (6\cos\theta + 8\sin\theta)^2 d\theta$

78.540

ANSWER

C