

# AOS Math 10, Spring 2024

## Applications of Derivatives Test (#15)

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**May 24, 2024**

**Student Signature**

**Class**

**Date**

**Print Name:**

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**Instructions:** For each problem, circle the letter of the best answer. You **must show all work** for credit. Partial credit may be awarded as appropriate.

1. If  $f(x) = \frac{\ln x}{x}$ , for all  $x > 0$ , which of the following is true?
  - (a)  $f$  is increasing for all  $x$  greater than  $e$ .
  - (b)  $f$  is increasing for all  $x$  greater than 1 .
  - (c)  $f$  is decreasing for all  $x$  between  $e$  and 1 .
  - (d)  $f$  is decreasing for all  $x$  between 1 and  $e$ .
  - (e)  $f$  is decreasing for all  $x$  greater than  $e$ .
  
2. The absolute maximum value of  $f(x) = x^3 - 3x^2 + 12$  on the closed interval  $[-2, 4]$  occurs at  $x =$ 
  - (a) 1
  - (b) -2
  - (c) 4
  - (d) 2
  - (e) 0
  
3. If the graph of  $y = x^3 + ax^2 + bx - 4$  has a point of inflection at  $(1, -6)$ , what is the value of  $b$  ?
  - (a) 3
  - (b) -3
  - (c) 0
  - (d) 1
  
4. The function  $f$  given by  $f(x) = x^3 + 12x - 24$  is
  - (a) decreasing for  $x < 0$ , increasing for  $x > 0$
  - (b) increasing for all  $x$
  - (c) decreasing for all  $x$
  - (d) decreasing for  $x < -2$ , increasing for  $-2 < x < 2$ , decreasing for  $x > 2$
  - (e) increasing for  $x < -2$ , decreasing for  $-2 < x < 2$ , increasing for  $x > 2$

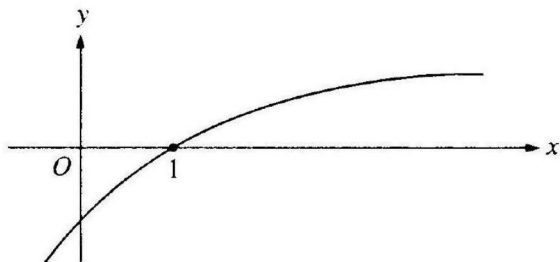
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5. The function  $f$  given by  $f(x) = 3x^5 - 4x^3 - 3x$  has a relative maximum at  $x =$
- (a) 1
  - (b)  $\frac{\sqrt{5}}{5}$
  - (c) 0
  - (d) -1
  - (e)  $-\frac{\sqrt{5}}{5}$
6. What is the  $x$ -coordinate of the point of inflection on the graph of  $y = \frac{1}{3}x^3 + 5x^2 + 24$  ?
- (a) -5
  - (b) 5
  - (c)  $-\frac{10}{3}$
  - (d) -10
  - (e) 0
7. If  $f''(x) = x(x+1)(x-2)^2$ , then the graph of  $f$  has inflection points when  $x =$
- (a) -1 and 0 only
  - (b) -1 and 2 only
  - (c) -1 only
  - (d) 2 only
  - (e) -1, 0, and 2 only
8. If  $g$  is a differentiable function such that  $g(x) < 0$  for all real numbers  $x$  and if  $f'(x) = (x^2 - 4)g(x)$ , which of the following is true?
- (a)  $f$  has relative maxima at  $x = -2$  and at  $x = 2$ .
  - (b)  $f$  has relative minima at  $x = -2$  and at  $x = 2$ .
  - (c) It cannot be determined if  $f$  has any relative extrema.
  - (d)  $f$  has a relative minimum at  $x = -2$  and a relative maximum at  $x = 2$ .
  - (e)  $f$  has a relative maximum at  $x = -2$  and a relative minimum at  $x = 2$ .

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9. What is the derivative of  $y = \csc \sqrt{t}$  ?

- (a)  $-\cot^2 \sqrt{t}$
- (b)  $-\frac{\csc \sqrt{t} \cot \sqrt{t}}{2\sqrt{t}}$
- (c)  $-\csc \sqrt{t} \cot \sqrt{t}$
- (d)  $-\sqrt{t} \cot^2 \sqrt{t}$

10. The graph of a twice-differentiable function  $f$  is shown in the figure below.



Which of the following is true?

- (a)  $f''(1) < f'(1) < f(1)$
- (b)  $f(1) < f'(1) < f''(1)$
- (c)  $f'(1) < f(1) < f''(1)$
- (d)  $f''(1) < f(1) < f'(1)$
- (e)  $f(1) < f''(1) < f'(1)$

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## Free Response 1

The function

$$f(x) = \frac{x}{x^2 + 4}$$

has first derivative

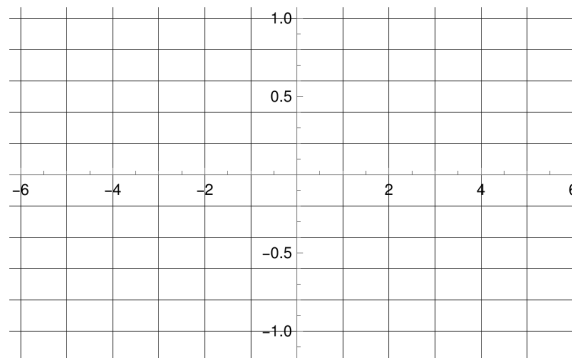
$$f'(x) = \frac{4 - x^2}{(x^2 + 4)^2}$$

and second derivative

$$f''(x) = \frac{2(x^3 - 12x)}{(x^2 + 4)^3}$$

Sketch the graph of  $f(x)$  after completing the following questions:

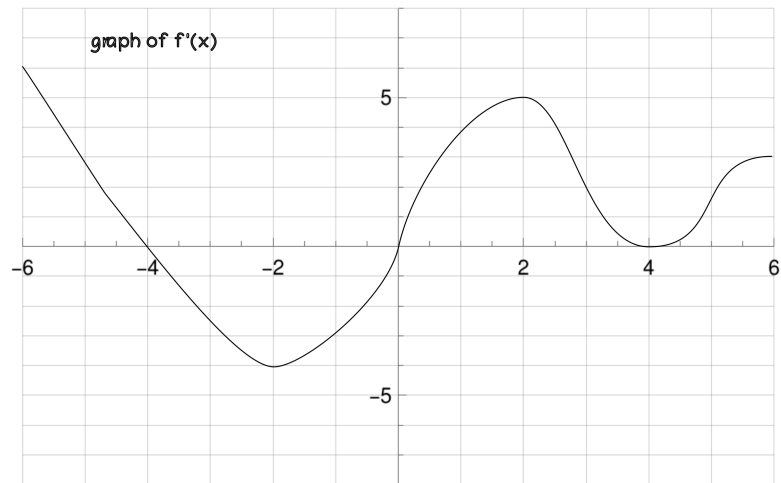
1. State any domain restrictions for  $f(x)$
2. Determine any critical points of  $f(x)$
3. State intervals on which  $f(x)$  is increasing or decreasing
4. State intervals on which  $f(x)$  is concave up or concave down
5. Calculate any horizontal asymptotes of  $f(x)$



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## Free Response 2

The graph  $f'(x)$  of the derivative of  $f(x)$  is shown below.  $f'(x)$  has horizontal tangents at  $x = -2, 2, 4$  and zeros at  $x = -4, 0, 4$ . The domains of  $f'(x)$  and  $f(x)$  are  $[-6, 6]$ .



1. On which  $x$  intervals is the function  $f$  decreasing? Justify your answer.
2. At which  $x$  value(s) does  $f$  have a local minimum? Justify your answer.
3. On which  $x$  intervals is the function  $f$  concave up? Justify your answer.

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**KEY**

1. A
2. C
3. C
4. B
5. D
6. B
7. C
8. E
9. B
10. B