

AOS Senior AP Calculus BC, Spring 2024

Cumulative, Quarter 3

(Parametric, Polar, Logistic)

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Class

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1. Eliminate the parameter t : $x = t^3 - 2$ and $2y = 1 - t^2$

(a) $y = 1 + \sqrt[3]{(x+2)^2}$

(b) $y = 1 + \sqrt{(x+2)^3}$

(c) $y = 1 - \sqrt[3]{(x+2)^2}$

(d) $y = 1 - \sqrt{(x+2)^2}$

2. Eliminate the parameter t : $x = 4\sin(t) + 1$ and $y = 3\cos(t) - 2$

(a) $\frac{(x-1)^2}{4} + \frac{(y+1)^2}{3} = 1$

(b) $\frac{(x-1)^2}{3} + \frac{(y+1)^2}{4} = 1$

(c) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(d) $\frac{(x-1)^2}{16} + \frac{(y+1)^2}{9} = 1$

3. Convert the polar coordinate $(6, -2\pi/3)$ to rectangular coordinates

(a) $(3, 3\sqrt{3})$

(b) $(3\sqrt{3}, -3\sqrt{3})$

(c) $(-3, -3)$

(d) $(3, 3\sqrt{3})$

(e) $(-3, -3\sqrt{3})$

4. Convert this equation to polar coordinates: $x^2 - y^2 = 16$

(a) $r^2 = \frac{4}{\cos^2 \theta + \sin^2 \theta}$

(b) $r^2 = \frac{16}{\cos^2 \theta + \sin^2 \theta}$

(c) $r^2 = \frac{4}{\cos \theta - \sin \theta}$

(d) $r^2 = \frac{16}{\cos^2 \theta - \sin^2 \theta}$

5. Convert this equation to rectangular coordinates: $r = 3 \sec \theta$

(a) $x = 1$

(b) $y = 3$

(c) $y = 1$

(d) $x = 3$

6. Which of the following is the graph of $r = \cos(3\theta)$.

(a) A 6 leaf rose with x -intercepts $(0, 0), (\pm 1, 0)$

(b) A 3 leaf rose with no x intercept

(c) A 6 leaf rose with no x intercept

(d) A 3 leaf rose with x -intercepts $(0, 0), (1, 0)$

7. A baseball pitcher throws a baseball with an initial speed of 138 feet per second at an angle of 20° to the horizontal. The ball leaves the pitcher's hand at a height of 4 feet above the ground. Write the equations of motion, v_x and v_y for velocity and s_x, s_y for position.

(a) $v_x(t) = 129.7, v_y(t) = 47.2 - 16t, s_x(t) = 129.7t, s_y(t) = 47.2t - 16t^2 + 4$

(b) $v_x(t) = 129.7, v_y(t) = 47.2 + 32t, s_x(t) = 129.7t, s_y(t) = 47.2t - 16t^2$

(c) $v_x(t) = 129.7, v_y(t) = 47.2 - 32t, s_x(t) = 129.7t, s_y(t) = 47.2t - 16t^2$

(d) $v_x(t) = 129.7, v_y(t) = 47.2 - 32t, s_x(t) = 129.7t, s_y(t) = 47.2t - 16t^2 + 4$

8. Let $u = \langle -3, 5 \rangle$ and $\vec{v} = \langle 1, 4 \rangle$ and $\vec{w} = \langle 6, -3 \rangle$ find $\vec{u} + 2\vec{v} - \vec{w}$

(a) $\langle -8, 16 \rangle$

(b) $\langle -6, 14 \rangle$

(c) $\langle -7, 14 \rangle$

(d) $\langle -7, 16 \rangle$

9. Given $\vec{u} = \langle 3\sqrt{3}, -5 \rangle$, find $\|\vec{u}\|$

(a) $3\sqrt{13}$

(b) $2\sqrt{17}$

(c) $3\sqrt{17}$

(d) $2\sqrt{13}$

10. Given $\vec{u} = \langle -10, 9 \rangle$, find a unit vector in the direction of \vec{u}

(a) $\langle -\frac{10}{\sqrt{19}}, \frac{9}{\sqrt{19}} \rangle$

(b) $\langle -\frac{10}{\sqrt{181}}, -\frac{9}{\sqrt{181}} \rangle$

(c) $\langle -\frac{10}{\sqrt{19}}, -\frac{9}{\sqrt{19}} \rangle$

(d) $\langle -\frac{10}{\sqrt{181}}, \frac{9}{\sqrt{181}} \rangle$

11. Which vector is perpendicular to $\langle \frac{2}{3}, -\frac{17}{2} \rangle$

(a) $\langle 9, \frac{18}{3} \rangle$

(b) $\langle -9, -\frac{17}{18} \rangle$

(c) $\langle -9, -17 \rangle$

(d) $\langle -9, -\frac{6}{17} \rangle$

12. Which vector is parallel to $\langle \frac{2}{3}, -\frac{17}{2} \rangle$

(a) $\langle 2, -25 \rangle$

(b) $\langle -51, 4 \rangle$

(c) $\langle 25, -2 \rangle$

(d) $\langle 4, -51 \rangle$

13. What is the radian angle between $\langle 5, 1 \rangle$ and $\langle 2, -3 \rangle$

(a) 1.080

(b) 1.480

(c) 1.580

(d) 1.180

14. If vector \vec{x} has magnitude 9 and makes an angle of 3.4 radians with the positive x axis, find the components of x and write as $a\hat{i} + b\hat{j}$.

- (a) $8.70\hat{i} + 2.30\hat{j}$
- (b) $-2.30\hat{i} - 8.70\hat{j}$
- (c) $-9.20\hat{i} + 3.20\hat{j}$
- (d) $-8.70\hat{i} - 2.30\hat{j}$

15. Write the complex number $-3 + 9i$ in polar form.

- (a) $9.59e^{1.79i}$
- (b) $9.29e^{1.81i}$
- (c) $9.49e^{1.33i}$
- (d) $9.49e^{1.89i}$

16. Divide $10 - 9i$ by $2 - 4i$, and express your answer in the form $a + bi$.

- (a) $-\frac{4}{5} + \frac{29}{10}i$
- (b) $-\frac{4}{5} - \frac{29}{10}i$
- (c) $\frac{14}{5} - \frac{11}{10}i$
- (d) $\frac{14}{5} + \frac{11}{10}i$

17. Simplify the product $\sqrt{7}e^{-i\pi/3} \cdot 3e^{i\pi/5}$.

- (a) $3\sqrt{7}e^{2\pi i/15}$
- (b) $3\sqrt{7}e^{-\pi i/15}$
- (c) $3\sqrt{7}e^{\pi i/15}$
- (d) $3\sqrt{7}e^{-2\pi i/15}$

18. Solve the equation $z^2 - 2z + 5 = 0$ for z and express your answers in rectangular form.

- (a) $1 \pm 3i$
- (b) $1 \pm \sqrt{5}i$
- (c) $1 \pm 2i^2$
- (d) $1 \pm 2i$

19. If $z = 2 + i$ is one root of a quadratic equation $x^2 + bx + c$ with real coefficients, what is bc ?

- (a) -10
- (b) 4
- (c) 8
- (d) -20

20. If $z^4 = 16e^{2\pi i/5}$, find all values of z in polar form.

- (a) $z = 2, 2e^{\pi i/2}, 2e^{\pi i}, 2e^{3\pi i/2}$
- (b) $z = 2e^{\pi i/10}, 2e^{3\pi i/10}, 2e^{5\pi i/10}, 2e^{7\pi i/10}$
- (c) $z = 2e^{2\pi i/5}, 2e^{4\pi i/5}, 2e^{6\pi i/5}, 2e^{8\pi i/5}$
- (d) $z = 2e^{\pi i/10}, 2e^{3\pi i/5}, 2e^{11\pi i/10}, 2e^{8\pi i/5}$

21. Factor $z^2 + 9$ into a product of two binomials.

- (a) $(z + 3)(z - 3)$
- (b) $(z + 3i)(z + 3i)$
- (c) $(z + 3\sqrt{i})(z - 3\sqrt{i})$
- (d) $(z + 3i)(z - 3i)$

22. (Bonus): By multiplying two complex numbers, prove the addition identities for \sin and \cos .