Parametrics Test

1. At time $t \geq 0$, a particle moving in the xy-plane has velocity vector given by $v(t) = \langle t^2, 5t \rangle$.
    1. What is the acceleration vector of the particle at time $t = 3$?
    2. What is the speed of the particle at time $t = 2$?
    3. If the initial position at time $t = 0$ is $(1, 2)$, find the position of the particle at time $t = 4$
    4. What distance is covered by the particle from time $t = 0$ to time $t = 4$?
2. Write an integral expression for the length of the path described by $x(t) = \sin(t^3)$ and $y(t) = e^{3t}$ from $t = 0$ to $t = \pi$
3. The position of a particle moving in the xy-plane is given by the vector $\langle 4t^3, y(2t) \rangle$ At time $t = \frac{1}{2}$, what is the acceleation vector of the particle?
4. If $x(t) = t^2 + 4$ and $y(t) = t^4 + 3$ for any $t > 0$ then what is $\frac{d^2y}{dx^2}$ in terms of t?
5. A particle moves in the xy-plane so that its position for $t \geq 0$ is given by $x(t) = \ln(t + 1)$ and $y = kt^2$, for some positive constant k. The tangent line to the particle's path at the point where $t = 3$ has slope 8. What is the value of k?
6. A particle moves on a plane curve such that at any time t > 0, its x-coordinate is $t - t^2 + t^3$ while its y-coordinate is $(2 - t^2)^2$. Find the magnitude of the particle's acceleration at $t = 1$.
7. The position of an object moving in the xy-plane with position function $f(t) = \langle 1 + \sin t, t + \cos t \rangle$, for $t \geq 0$. What is the maximum speed attained by the object?
8. A particle moves along the path $f(t) = \langle 2t^{3/2}, 3t - 1 \rangle$, for $t \geq 0$. What is the average speed (average rate of change) of the particle from time $t = 0$ to time $t = 3$?
9. If $x(t) = 3t^2 - 4$ and $y(t) = e^{2t} - 1$, write y as a function of x when $t \geq 2$.
10. Using the fifth-degree Taylor polynomial for $\sin x$, what is the error in approximating $\sin(1)$ by $P_5(1)$? That is, give an upper bound on $|P_5(1) - \sin(1)|$.