AOS Math 10, Spring 2024
Derivatives Test (#14)

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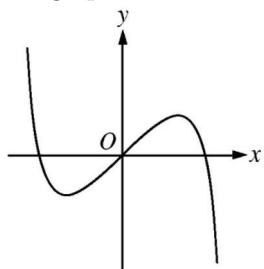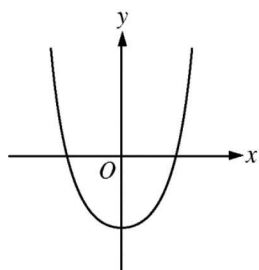|		
_____		**May 10, 2024**
Student Signature	**Class**	**Date**

Print Name:

Instructions: For each problem, circle the letter of the best answer. You **must show all work** for credit. Partial credit may be awarded as appropriate.
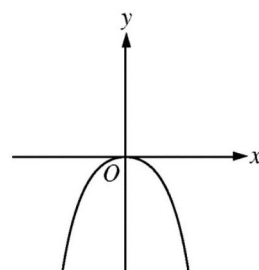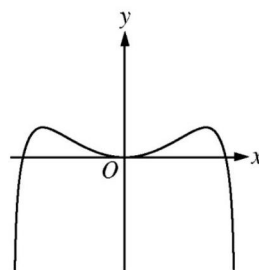
1. The graph of the function f is shown below.

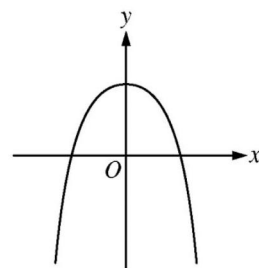Which of the following could be the graph of f', the derivative of f ?

(a)

(b)

(c)
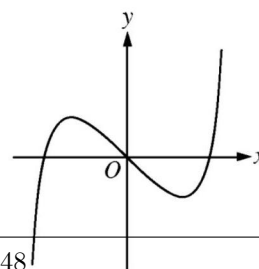

(d)

2. Let h be the function defined by $h(x) = \int_{\pi/4}^{x} \sin^2 t\, dt$. Which of the following is an equation for the line tangent to the graph of h at the point where $x = \frac{\pi}{4}$?

   (a) $y = \frac{1}{2}$

   (b) $y = \sqrt{2}x$

   (c) $y = x - \frac{\pi}{4}$

   (d) $y = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

   (e) $y = \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

3. The number of people who have entered a museum on a certain day is modeled by a function $f(t)$, where t is measured in hours since the museum opened that day. The number of people who have left the museum since it opened that same day is modeled by a function $g(t)$. If $f'(t) = 380\,(1.02^t)$ and $g'(t) = 240 + 240\sin\left(\frac{\pi(t-4)}{12}\right)$, at what time t, for $1 \leq t \leq 11$, is the number of people in the museum at a maximum?

   (a) 1

   (b) 7.888

   (c) 9.446

   (d) 10.974

   (e) 11

4. A particle moves along a straight line with velocity given by $v(t) = 5 + e^{t/3}$ for time $t \geq 0$. What is the acceleration of the particle at time $t = 4$?

   (a) 0.422

   (b) 0.698

   (c) 1.265

   (d) 8.794

   (e) 28.381

5. The function f is defined on the open interval $0.4 < x < 2.4$ and has first derivative f' given by $f'(x) = \sin\left(x^2\right)$. Which of the following statements are true?
   I. f has a relative maximum on the interval $0.4 < x < 2.4$.
   II. f has a relative minimum on the interval $0.4 < x < 2.4$.
   III. The graph of f has two points of inflection on the interval $0.4 < x < 2.4$.

   (a) I only

   (b) II only

   (c) III only

   (d) I and III only

   (e) II and III only

6. The first derivative of the function g is given by $g'(x) = \cos(\pi x^2)$ for $-0.5 < x < 1.5$. On which of the following intervals is g decreasing?

   (a) $-0.5 < x < 0$

   (b) $0 < x < 1$

   (c) $0.707 < x < 1.225$

   (d) $1.225 < x < 1.414$

   (e) $1.414 < x < 1.5$

7. The height above the ground of a passenger on a Ferris wheel t minutes after the ride begins is modeled by the differentiable function H, where $H(t)$ is measured in meters. Which of the following is an interpretation of the statement $H'(7.5) = 15.708$?

   (a) The Ferris wheel is turning at a rate of 15.708 meters per minute when the passenger is 7.5 meters above the ground.

   (b) The Ferris wheel is turning at a rate of 15.708 meters per minute 7.5 minutes after the ride begins.

   (c) The passenger's height above the ground is increasing by 15.708 meters per minute when the passenger is 7.5 meters above the ground.

   (d) The passenger's height above the ground is increasing by 15.708 meters per minute 7.5 minutes after the ride begins.

   (e) The passenger is 15.708 meters above the ground 7.5 minutes after the ride begins.

8. Let f be a twice-differentiable function on the open interval (a, b). If $f'(x) > 0$ on (a, b) and $f''(x) < 0$ on (a, b), which of the following could be the graph of f ?

(a)

(b)

(c)

(d)

(e)
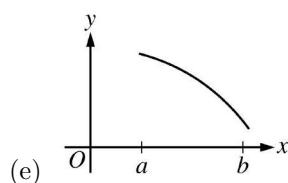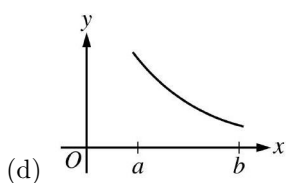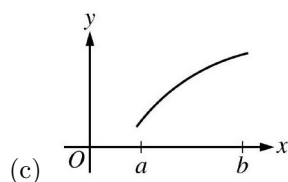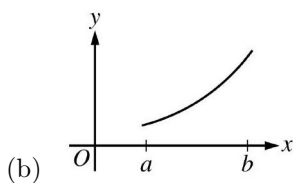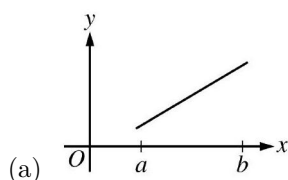
9. In the xy-plane, the graph of the twice-differentiable function $y = f(x)$ is concave up on the open interval $(0, 2)$ and is tangent to the line $y = 3x - 2$ at $x = 1$. Which of the following statements must be true about the derivative of f ?

(a) $f'(x) \leq 3$ on the interval $(0.9, 1)$.

(b) $f'(x) \geq 3$ on the interval $(0.9, 1)$.

(c) $f'(x) < 0$ on the interval $(0.9, 1.1)$.

(d) $f'(x) > 0$ on the interval $(0.9, 1.1)$.

(e) $f'(x)$ is constant on the interval $(0.9, 1.1)$.

10. Let f be the function given by $f(x) = 3 - 2x$. If g is a function with derivative given by $g'(x) = f(x)f'(x)(x-3)$, on what intervals is g increasing?

(a) $\left(-\infty, \frac{3}{2}\right]$ and $[3, \infty)$

(b) $\left(-\infty, \frac{3}{2}\right]$ only

(c) $\left[\frac{3}{2}, 3\right]$ only

(d) $\left[\frac{3}{2}, \infty\right)$

(e) $[3, \infty)$ only

11. A curve C is defined by the parametric equations $x(t) = 3 + t^2$ and $y(t) = t^3 + 5t$. Which of the following is an equation of the line tangent to the graph of C at the point where $t = 1$?

(a) $y = \frac{1}{4}x + 5$

(b) $y = 4x - 10$

(c) $y = 4x + 6$

(d) $y = 8x - 26$

(e) $y = 8x + 6$

12. Let f be the function given by $f(x) = x^3 - 2x^2 + 5x - 16$. For what value of x in the closed interval $[0, 5]$ does the instantaneous rate of change of f equal the average rate of change of f over that interval?

(a) 0

(b) $\frac{5}{3}$

(c) $\frac{5}{2}$

(d) 3

(e) 5

13. The position of a particle moving along the x-axis is given by a twice-differentiable function $x(t)$. If $x(2) < 0$, $x'(2) < 0$, and $x''(2) < 0$, which of the following statements must be true about the particle at time $t = 2$?

(a) The particle is moving toward the origin at a decreasing speed.

(b) The particle is moving toward the origin at an increasing speed.

(c) The particle is moving away from the origin at a decreasing speed.

(d) The particle is moving away from the origin at an increasing speed.

(e) The particle is moving away from the origin at a constant speed.

14. If $0 \leq b \leq 2$, for what value of b is $\int_0^b \cos\left(e^x\right) dx$ a minimum?

    (a) 0

    (b) 0.452

    (c) 1.145

    (d) 1.550

    (e) 2

15. A cup has the shape of a right circular cone. The height of the cup is 12 cm, and the radius of the opening is 3 cm. Water is poured into the cup at a constant rate of 2 cm^3/sec. What is the rate at which the water level is rising when the depth of the water in the cup is 5 cm ? (The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h$.)

    (a) $\frac{32}{25\pi}$cm/sec

    (b) $\frac{96}{125\pi}$cm/sec

    (c) $\frac{2}{3\pi}$cm/sec

    (d) $\frac{2}{9\pi}$cm/sec

    (e) $\frac{1}{200\pi}$cm/sec

16. A particle moves along the x-axis so that at time $t \geq 0$ the position of the particle is given by $x(t) = 0.5t^4 - 1.5t^3 - 2t^2 + 6t - 1$. What is the velocity of the particle at the first instance the particle is at the origin?

    (a) -4.071

    (b) -2.048

    (c) 0

    (d) 5.153

    (e) 6

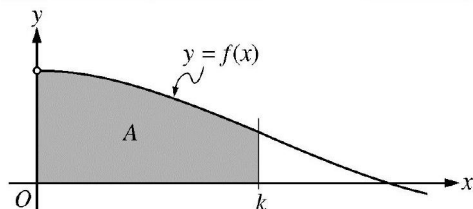17. The graph of f', the derivative of a function f, is shown below.

Which of the following could be the graph of f ?

I.

II.

III.
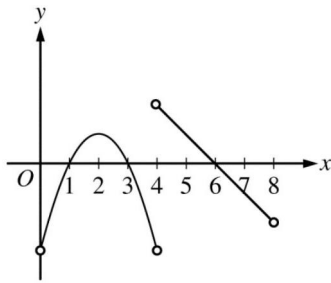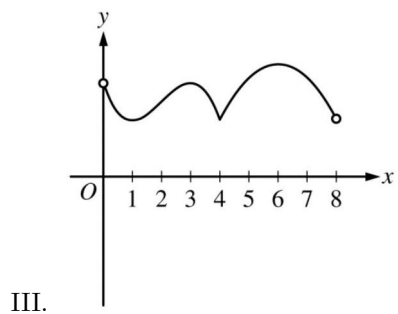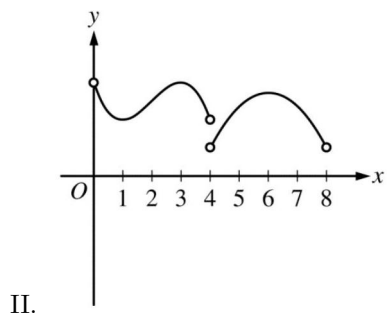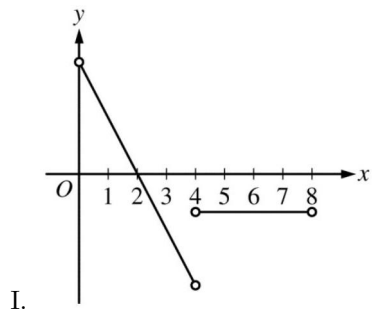
(a) I only

(b) II only

(c) III only

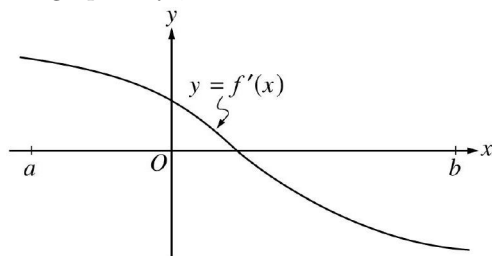(d) I and II only

(e) II and III only

18. The figure below

shows the region A, which is bounded by the x - and y-axes, the graph of $f(x) = \frac{\sin x}{x}$ for $x > 0$, and the vertical line $x = k$. If k increases at a rate of $\frac{\pi}{4}$ units per second, how fast is the area of region A increasing when $k = \frac{\pi}{6}$?

   (a) 0

   (b) $\frac{3}{4}$

   (c) $\frac{3}{\pi}$

   (d) $\frac{\sqrt{3}}{2}$

   (e) $2\sqrt{3}$

19. The number of gallons of water in a storage tank at time t, in minutes, is modeled by $w(t) = 25 - t^2$ for $0 \leq t \leq 5$. At what rate, in gallons per minute, is the amount of water in the tank changing at time $t = 3$ minutes?

   (a) 66

   (b) 16

   (c) -3

   (d) -6

20. Let f be the function defined by $f(x) = -3 + 6x^2 - 2x^3$. What is the largest open interval on which the graph of f is both concave up and increasing?

   (a) $(0, 1)$

   (b) $(1, 2)$

   (c) $(0, 2)$

   (d) $(2, \infty)$

21. A particle moves along the x-axis so that at time $t > 0$ its position is given by $x(t) = 12e^{-t} \sin t$. What is the first time t at which the velocity of the particle is zero?

   (a) $\frac{\pi}{4}$

   (b) $\frac{\pi}{2}$

   (c) $\frac{3\pi}{4}$

   (d) π

22. The graph of f', the derivative of the function f, is shown in the figure below.


Which of the following statements must be true?

> I. f is continuous on the open interval (a, b).
> II. f is decreasing on the open interval (a, b).
> III. The graph of f is concave down on the open interval (a, b).

(a) I only

(b) I and II only

(c) I and III only

(d) II and III only

23. An isosceles right triangle with legs of length s has area $A = \frac{1}{2}s^2$. At the instant when $s = \sqrt{32}$ centimeters, the area of the triangle is increasing at a rate of 12 square centimeters per second. At what rate is the length of the hypotenuse of the triangle increasing, in centimeters per second, at that instant?

(a) $\frac{3}{4}$

(b) 3

(c) $\sqrt{32}$

(d) 48

24. The graph of a differentiable function f is shown in the figure below
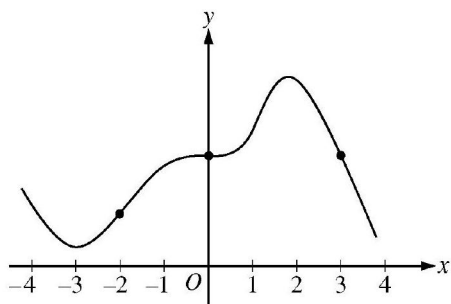
Which of the following is true?

(a) $f'(-2) < f'(0) < f'(3)$

(b) $f'(-2) < f'(3) < f'(0)$

(c) $f'(3) < f'(-2) < f'(0)$

(d) $f'(3) < f'(0) < f'(-2)$

25. A file is downloaded to a computer at a rate modeled by the differentiable function $f(t)$, where t is the time in seconds since the start of the download and $f(t)$ is measured in megabits per second. Which of the following is the best interpretation of $f'(5) = 2.8$?

(a) At time $t = 5$ seconds, the rate at which the file is downloaded to the computer is 2.8 megabits per second.

(b) At time $t = 5$ seconds, the rate at which the file is downloaded to the computer is increasing at a rate of 2.8 megabits per second per second.

(c) Over the time interval $0 \leq t \leq 5$ seconds, 2.8 megabits of the file are downloaded to the computer.

(d) Over the time interval $0 \leq t \leq 5$ seconds, the average rate at which the file is downloaded to the computer is 2.8 megabits per second.