



NATIONAL MATH + SCIENCE INITIATIVE

AP Calculus

Theorems (IVT, EVT, and MVT)

Student Handout

2016-2017 EDITION

Use the following link or scan the QR code to complete the evaluation for the Study Session https://www.surveymonkey.com/r/S_SSS



Theorems (IVT, EVT, and MVT)

Students should be able to apply and have a geometric understanding of the following:

- Intermediate Value Theorem
- Mean Value Theorem for derivatives
- Extreme Value Theorem

Multiple Choice

1. (calculator not allowed)

If f is continuous for $a \leq x \leq b$ and differentiable for $a < x < b$, which of the following could be false?

- (A) $f'(c) = \frac{f(b) - f(a)}{b - a}$ for some c such that $a < c < b$.
- (B) $f'(c) = 0$ for some c such that $a < c < b$.
- (C) f has a minimum value on $a \leq x \leq b$.
- (D) f has a maximum value on $a \leq x \leq b$.
- (E) $\int_a^b f(x) dx$ exists.

2. (calculator not allowed)

The function f is defined on the closed interval $[2, 4]$ and $f(2) = f(3) = f(4)$. On the open interval $(2, 4)$, f is continuous and strictly decreasing. Which of the following statements is true?

- (A) f attains neither a minimum value nor a maximum value on the closed interval $[2, 4]$.
- (B) f attains a minimum value but does not attain a maximum value on the closed interval $[2, 4]$.
- (C) f attains a maximum value but does not attain a minimum value on the closed interval $[2, 4]$.
- (D) f attains both a minimum value and a maximum value on the closed interval $[2, 4]$.

3. (calculator not allowed)

Let f be a function with first derivative defined by $f'(x) = \frac{2x^2 - 5}{x^2}$ for $x > 0$. It is known that $f(1) = 7$ and $f(5) = 11$. What value of x in the open interval $(1, 5)$ satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1, 5]$?

- (A) 1 (B) $\sqrt{\frac{5}{2}}$ (C) $\sqrt[3]{10}$ (D) $\sqrt{5}$

4. (calculator not allowed)

x	0	1	2
$f(x)$	1	k	2

The function f is continuous on the closed interval $[0, 2]$ and has values that are given in the table above. The equation $f(x) = \frac{1}{2}$ must have at least two solutions in the interval $[0, 2]$ if $k =$

- (A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 3

5. (calculator not allowed)

Let g be a continuous function on the closed interval $[0, 1]$. Let $g(0) = 1$ and $g(1) = 0$. Which of the following is NOT necessarily true?

- (A) There exists a number h in $[0, 1]$ such that $g(h) \geq g(x)$ for all x in $[0, 1]$.
 (B) For all a and b in $[0, 1]$, if $a = b$, then $g(a) = g(b)$.
 (C) There exists a number h in $[0, 1]$ such that $g(h) = \frac{1}{2}$.
 (D) There exists a number h in $[0, 1]$ such that $g(h) = \frac{3}{2}$.
 (E) For all h in the open interval $(0, 1)$, $\lim_{x \rightarrow h} g(x) = g(h)$.

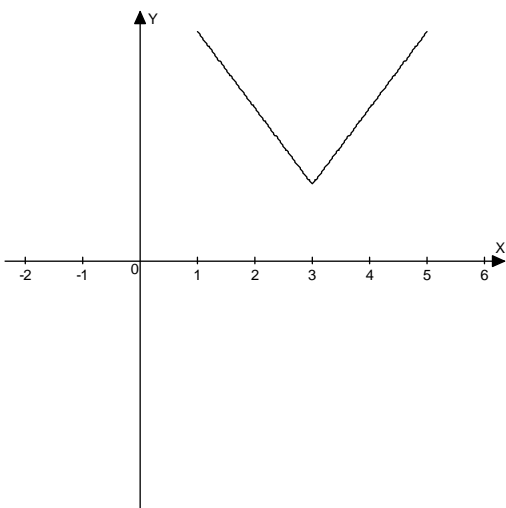
6. (calculator not allowed)

If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c ?

- (A) $\frac{2\pi}{3}$
- (B) $\frac{3\pi}{4}$
- (C) $\frac{5\pi}{6}$
- (D) π
- (E) $\frac{3\pi}{2}$

7. (calculator not allowed)

Which of the following theorems may be applied to the graph below,
 $y = |x - 3| + b$, $b > 0$, over the interval $[2, 4]$?



- I. Mean Value Theorem
- II. Intermediate Value Theorem
- III. Extreme Value Theorem

- (A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III

8. (calculator not allowed)

The function f is defined by $f(x) = 4x^2 - 5x + 1$. The application of the Mean Value Theorem to f on the interval $0 < x < 2$ guarantees the existence of a value c , where $0 < c < 2$ such that $f'(c) =$

- (A) 1 (B) 3 (C) 7 (D) 8

9. (calculator not allowed)

A function of f is continuous on the closed interval $[2, 5]$ with $f(2) = 17$ and $f(5) = 17$. Which of the following additional conditions guarantees that there is a number c in the open interval $(2, 5)$ such that $f'(c) = 0$?

- (A) No additional conditions are necessary
 (B) f has a relative extremum on the open interval $(2, 5)$.
 (C) f is differentiable on the open interval $(2, 5)$.
 (D) $\int_2^5 f(x) dx$ exists

10. (calculator not allowed)

x	0	2	4	8
$f(x)$	3	4	9	13
$f'(x)$	0	1	1	2

The table above gives values of a differentiable function f and its derivatives at selected values of x . If h is the function given by $h(x) = f(2x)$, which of the following statements must be true?

- (I) h is increasing on $2 < x < 4$.
 (II) There exists c , where $0 < c < 4$, such that $h(c) = 12$.
 (III) There exists c , where $0 < c < 2$, such that $h'(c) = 3$.
- (A) II only
 (B) I and III only
 (C) II and III only
 (D) I, II, and III

11. (calculator allowed)

The function f is continuous for $-2 \leq x \leq 1$ and differentiable for $-2 < x < 1$. If $f(-2) = -5$ and $f(1) = 4$, which of the following statements could be false?

- (A) There exists c , where $-2 < c < 1$, such that $f(c) = 0$.
- (B) There exists c , where $-2 < c < 1$, such that $f'(c) = 0$.
- (C) There exists c , where $-2 < c < 1$, such that $f(c) = 3$.
- (D) There exists c , where $-2 < c < 1$, such that $f'(c) = 3$.
- (E) There exists c , where $-2 \leq c \leq 1$ such that $f(c) \geq f(x)$ for all x on the closed interval $-2 \leq x \leq 1$.

12. (calculator not allowed)

x	0	2	5	9	11
$g(x)$	1	2.8	1.7	1	3.4

The table above shows selected values of a continuous function g . For $0 \leq x \leq 11$, what is the fewest possible number of times $g(x) = 2$?

- (A) One
- (B) Two
- (C) Three
- (D) Four

Free Response

13. (calculator not allowed)

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.

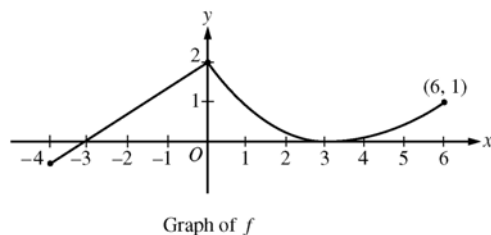
14. (calculator not allowed)

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

Train A runs back and forth on an east-west section of railroad track. Train A's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

(b) Do the data in the table support the conclusion that train A's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.

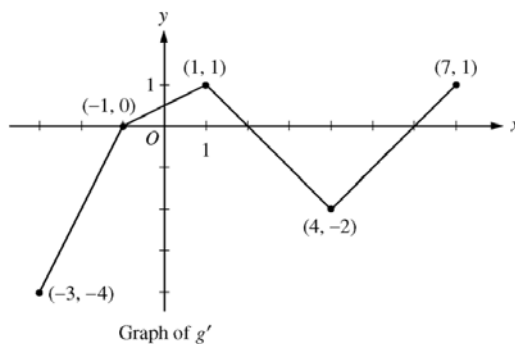
15. (calculator allowed)



A continuous function f is defined on the closed interval $-4 \leq x \leq 6$. The graph of f consists of a line segment and a curve that is tangent to the x -axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

- (c) Is there a value of a , $-4 \leq a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c , $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

16. (calculator not allowed)



Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g' , the derivative of g , is shown above for $-3 \leq x \leq 7$.

- (d) Find the average rate of change of $g'(x)$ on the interval $-3 \leq x \leq 7$. Does the Mean Value Theorem applied on the interval $-3 \leq x \leq 7$ guarantee a value of c , for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

17. (calculator allowed)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x . The function h is given by $h(x) = f(g(x)) - 6$.

(a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

(b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

18. (calculator not allowed)

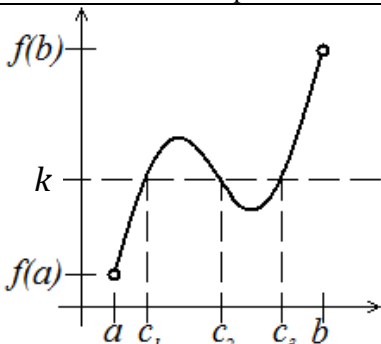
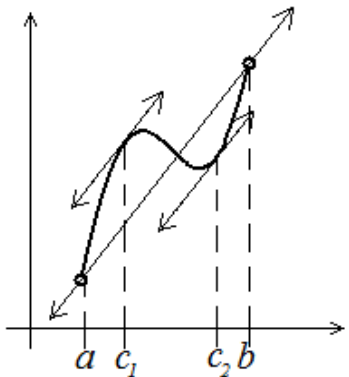
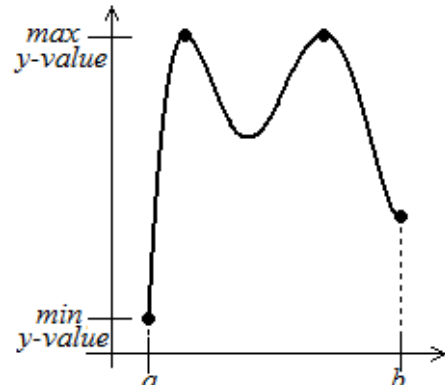
Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

(a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

(b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

(d) Let $h(x) = f(x) - x$. Explain why there must be a value r for $2 < r < 5$ such that $h(r) = 0$.

Reference Page

Name	Formal Statement	Restatement	Graph	Notes
IVT	<p>If $f(x)$ is continuous on a closed interval $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there exists at least one value c in $[a, b]$ such that $f(c)=k$.</p>	<p>On a continuous function, you will hit every y-value between two given y-values at least once.</p>		<p>When writing a justification using the IVT, you must state the function is continuous even if this information is provided in the question.</p>
MVT	<p>If $f(x)$ is continuous on the closed interval $[a, b]$ and differentiable on (a, b), then there must exist at least one value c in (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$</p>	<p>If conditions are met (very important!) there is at least one point where the slope of the tangent line equals the slope of the secant line.</p>		<p>When writing a justification using the MVT, you must state the function is differentiable (continuity is implied by differentiability) even if this information is provided in the question.</p> <p>(Questions may ask students to justify why the MVT cannot be applied often using piecewise functions that are not differentiable over an open interval.)</p>
EVT	<p>A continuous function $f(x)$ on a closed interval $[a, b]$ attains both an absolute maximum $f(c) \geq f(x)$ for all x in the interval and an absolute minimum $f(c) \leq f(x)$ for all x in the interval</p>	<p>Every continuous function on a closed interval has a highest y-value and a lowest y-value.</p>		<p>When writing a justification using the EVT, you must state the function is continuous on a closed interval even if this information is provided in the question.</p>