Derivatives Questions Draft

1. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.
   (A) $x > 0$
   (B) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$
   (C) $-2 < x < 0$ or $x > 2$
   (D) $x > \sqrt{2}$
   (E) $-2 < x < 2$

2. If $f(x) = x + \frac{1}{x}$, then the set of values for which f increases is
   (A) $(-\infty, -1] \cup [1, \infty)$
   (B) $[-1,1]$
   (C) $(-\infty, \infty)$
   (D) $(0, \infty)$
   (E) $(-\infty, 0) \cup (0, \infty)$

3. If $f(x) = \frac{\ln x}{x}$, for all $x > 0$, which of the following is true?

   (A) f is increasing for all x greater than 0 .
   (B) f is increasing for all x greater than 1 .
   (C) f is decreasing for all x between 0 and 1 .
   (D) f is decreasing for all x between 1 and e.
   (E) f is decreasing for all x greater than e.

4. At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?
   (A) -1 only
   (B) 0 only
   (C) 1 only
   (D) -1 and 1 only
   (E) $-1,0$ and 1

5. The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that
   (A) $x < 0$
   (B) $x < 2$
   (C) $x < 5$
   (D) $x > 0$
   (E) $x > 2$

6. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2,4]$ occurs at $x =$
   (A) 4
   (B) 2
   (C) 1
   (D) 0
   (E) -2

7. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b ?
   (A) -3
   (B) 0
   (C) 1
   (D) 3

8. The function f given by $f(x) = x^3 + 12x - 24$ is

   (A) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$
   (B) decreasing for $x < 0$, increasing for $x > 0$
   (C) increasing for all x
   (D) decreasing for all x
   (E) decreasing for $x < -2$, increasing for $-2 < x < 2$, decreasing for $x > 2$

9. The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?
   (A) None
   (B) One
   (C) Two
   (D) Three
   (E) Four

10. If $f(x) = x^2e^x$, then the graph of f is decreasing for all x such that
   (A) $x < -2$
   (B) $-2 < x < 0$
   (C) $x \succ 2$
   (D) $x < 0$
   (E) $x > 0$

11. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

   (A) $x < 0$

   (B) $x > 0$
   (C) $x < -2$ or $x \succ \frac{2}{3}$
   (D) $x < \frac{2}{3}$ or $x > 2$
   (E) $\frac{2}{3} < x < 2$

12. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at x =
   (A) -1
   (B) $-\frac{\sqrt{5}}{5}$
   (C) 0
   (D) $\frac{\sqrt{5}}{5}$
   (E) 1

13. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?
   (A) 5
   (B) 0
   (C) $-\frac{10}{3}$
   (D) -5
   (E) -10

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?
   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5

15. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when x =
(A) -1 only
(B) 2 only
(C) -1 and 0 only
(D) -1 and 2 only
(E) $-1, 0$, and 2 only

16. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(A) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$
(B) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
(C) $(0, \infty)$
(D) $(-\infty, 0)$
(E) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

17. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = (x^2 - 4)g(x)$, which of the following is true?

(A) f has a relative maximum at $x = -2$ and a relative minimum at $x = 2$.
(B) f has a relative minimum at $x = -2$ and a relative maximum at $x = 2$.
(C) f has relative minima at $x = -2$ and at $x = 2$.
(D) f has relative maxima at $x = -2$ and at $x = 2$.
(E) It cannot be determined if f has any relative extrema.

18. If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflection for the graph of f ?
(A) -1
(B) 0
(C) 1
(D) 0 and 1
(E) $-1, 0$, and 1

19. What is the derivative of $y = \sec\sqrt{t}$?

(A) $\sec\sqrt{t}\tan\sqrt{t}$

(B) $\tan^2\sqrt{t}$

(C) $\frac{\sec\sqrt{t}\tan\sqrt{t}}{2\sqrt{t}}$

(D) $\sqrt{t}\tan^2\sqrt{t}$

20. At which x coordinate is the tangent line to $y = 3x^2 - 8x + 12$ parallel to the line $4x + 2y = 7$?

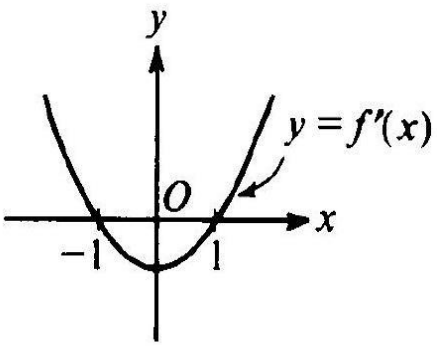
(A) 0

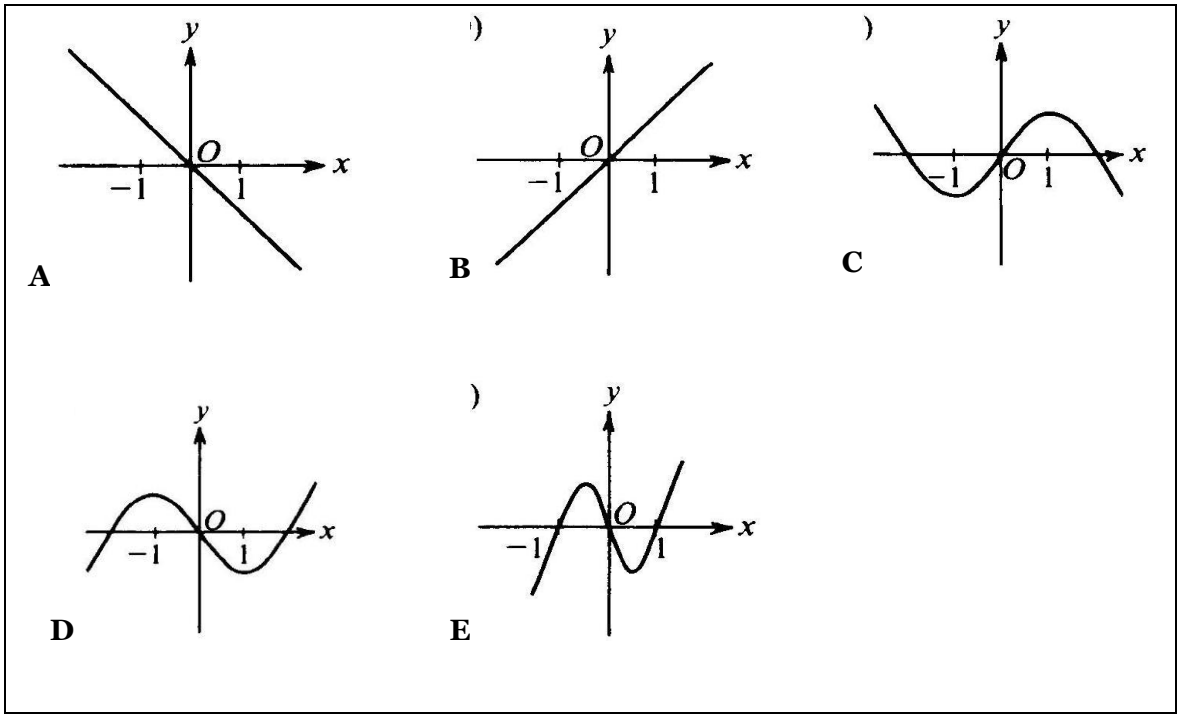
(B) 1

(C) 2

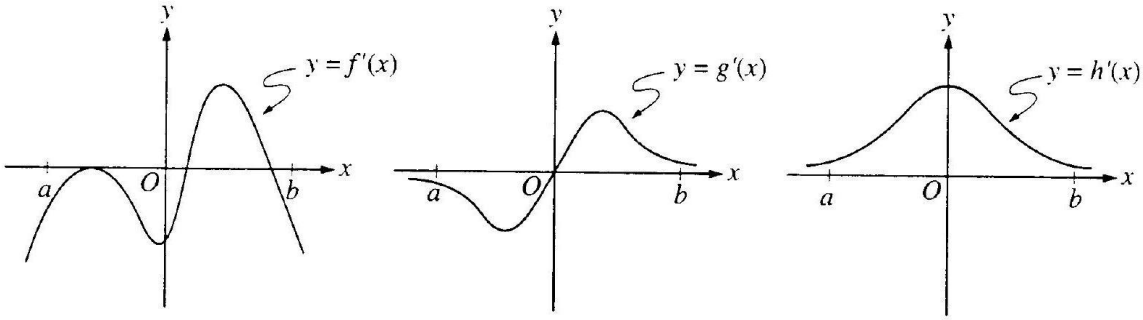
(D) -2

21. The graph of the derivative of f is shown in the figure to the right. Which of the following could be the graph of f ?



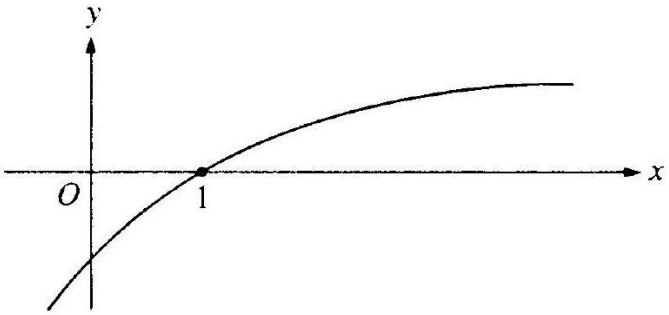


22. The graphs of the derivatives of the functions f, g, and h are shown below. Which of the functions f, g, or h have a relative maximum on the open interval $a < x < b$?



(A) f only
(B) g only
(C) h only
(D) f and g only
(E) f, g, and h

23. The graph of a twice-differentiable function f is shown in the figure below.



Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$
(B) $f(1) < f''(1) < f'(1)$
(C) $f'(1) < f(1) < f''(1)$
(D) $f''(1) < f(1) < f'(1)$
(E) $f''(1) < f'(1) < f(1)$

Free Response

The function

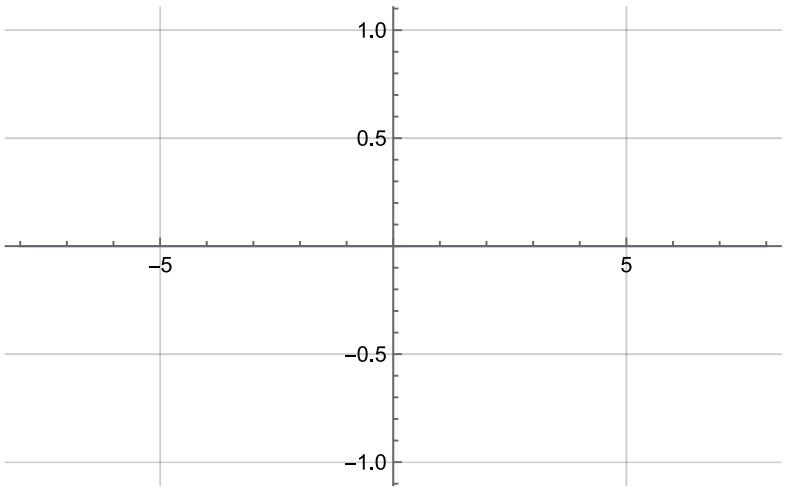
$$f\ \ (x) = \frac{1}{x^2 - 4}$$

has first derivative

$$f'(x) = \frac{-2x}{(x^2 - 4)^2}$$

and second derivative

$$f''(x) = \frac{6x^2 + 8}{(x^2 - 4)^3}$$



Sketch the graph of $f(x)$ after completing the following questions:

1. State any domain restrictions for $f(x)$

2. Determine any critical points of $f(x)$

3. State intervals on which $f(x)$ is increasing or decreasing

4. State intervals on which $f(x)$ is concave up or concave down

5. Calculate any horizontal asymptotes of $f(x)$