AOS Math 10, Spring 2024
Derivatives Test (#14)

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May 10, 2024

Student Signature **Class** **Date**

Print Name:

Instructions: For each problem, circle the letter of the best answer. You **must show all work** for credit. Partial credit may be awarded as appropriate.

1. Given the function defined by $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph of f is concave up.

   (a) $x > 0$

   (b) $-\sqrt{2} < x < 0$ or $x > \sqrt{2}$

   (c) $-2 < x < 0$ or $x > 2$

   (d) $x > \sqrt{2}$

   (e) $-2 < x < 2$

2. If $f(x) = x + \dfrac{1}{x}$, then the set of values for which f increases is

   (a) $(-\infty, -1] \cup [1, \infty)$

   (b) $[-1, 1]$

   (c) $(-\infty, \infty)$

   (d) $(0, \infty)$

   (e) $(-\infty, 0) \cup (0, \infty)$

3. If f(x) $= \dfrac{\ln x}{\mathrm{x}}$, for all x > 0, which of the following is true?

   (a) f is increasing for all x greater than o.

   (b) f is increasing for all x greater than 1 .

   (c) f is decreasing for all x between o and 1 .

   (d) f is decreasing for all x between 1 and e.

   (e) f is decreasing for all x greater than e.

4. At what values of x does $f(x) = 3x^5 - 5x^3 + 15$ have a relative maximum?

   (a) -1 only

   (b) o only

   (c) 1 only

   (d) -1 and 1 only

   (e) $-1, 0$ and 1

5. The graph of $y = \dfrac{-5}{x-2}$ is concave downward for all values of x such that

   (a) $x < 0$

   (b) $x < 2$

   (c) $x < 5$

   (d) $x > 0$

   (e) $x > 2$

6. The absolute maximum value of $f(x) = x^3 - 3x^2 + 12$ on the closed interval $[-2, 4]$ occurs at $x =$

   (a) 4

   (b) 2

   (c) 1

   (d) 0

   (e) -2

7. If the graph of $y = x^3 + ax^2 + bx - 4$ has a point of inflection at $(1, -6)$, what is the value of b ?

   (a) -3

   (b) 0

   (c) 1

   (d) 3

8. The function f given by $f(x) = x^3 + 12x - 24$ is

   (a) increasing for $x < -2$, decreasing for $-2 < x < 2$, increasing for $x > 2$

   (b) decreasing for x < 0, increasing for x > 0

   (c) increasing for all x

   (d) decreasing for all x

   (e) decreasing for x < -2, increasing for $-2 <$ x < 2, decreasing for x > 2

9. The derivative of f is $x^4(x-2)(x+3)$. At how many points will the graph of f have a relative maximum?

   (a) None

   (b) One

   (c) Two

   (d) Three

   (e) Four

10. If $f(x) = x^2e^x$, then the graph of f is decreasing for all x such that

   (a) $x < -2$

   (b) $-2 < x < 0$

   (c) $x \succ 2$

   (d) $x < 0$

   (e) $x > 0$

11. The graph of $y = 3x^4 - 16x^3 + 24x^2 + 48$ is concave down for

   (a) $x < 0$

   (b) $x > 0$

   (c) $x < -2$ or $x > \dfrac{2}{3}$

   (d) $x < \dfrac{2}{3}$ or $x > 2$

   (e) $\dfrac{2}{3} < x < 2$

12. The function f given by $f(x) = 3x^5 - 4x^3 - 3x$ has a relative maximum at $x =$

   (a) -1

   (b) $-\dfrac{\sqrt{5}}{5}$

   (c) 0

   (d) $\dfrac{\sqrt{5}}{5}$

   (e) 1

13. What is the x-coordinate of the point of inflection on the graph of $y = \frac{1}{3}x^3 + 5x^2 + 24$?

(a) 5

(b) 0

(c) $-\frac{10}{3}$

(d) -5

(e) -10

14. A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero?

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

15. If $f''(x) = x(x+1)(x-2)^2$, then the graph of f has inflection points when $x =$

(a) -1 only

(b) 2 only

(c) -1 and o only

(d) -1 and 2 only

(e) $-1, 0$, and 2 only

16. The function f is given by $f(x) = x^4 + x^2 - 2$. On which of the following intervals is f increasing?

(a) $\left(-\frac{1}{\sqrt{2}}, \infty\right)$

(b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(c) $(0, \infty)$

(d) $(-\infty, 0)$

(e) $\left(-\infty, -\frac{1}{\sqrt{2}}\right)$

17. If g is a differentiable function such that $g(x) < 0$ for all real numbers x and if $f'(x) = \left(x^2 - 4\right) g(x)$, which of the following is true?

    (a) f has a relative maximum at x = −2 and a relative minimum at x = 2.

    (b) f has a relative minimum at x = −2 and a relative maximum at x = 2.

    (c) f has relative minima at x = −2 and at x = 2.

    (d) f has relative maxima at x = −2 and at x = 2.

    (e) It cannot be determined if f has any relative extrema.

18. If f is the function defined by $f(x) = 3x^5 - 5x^4$, what are all the x-coordinates of points of inflection for the graph of f ?

    (a) -1

    (b) 0

    (c) 1

    (d) 0 and 1

    (e) $-1, 0$, and 1

19. What is the derivative of $y = \sec \sqrt{t}$?

    (a) $\sec \sqrt{t} \tan \sqrt{t}$

    (b) $\tan^2 \sqrt{t}$

    (c) $\dfrac{\sec \sqrt{t} \tan \sqrt{t}}{2\sqrt{t}}$

    (d) $\sqrt{t} \tan^2 \sqrt{t}$

20. At which x coordinate is the tangent line to $y = 3x^2 - 8x + 12$ parallel to the line $4x + 2y = 7$?

    (a) 0

    (b) 1

    (c) 2

    (d) -2

21. The graph of the derivative of f is shown in the figure to the right. Which of the following could be the graph of f ?

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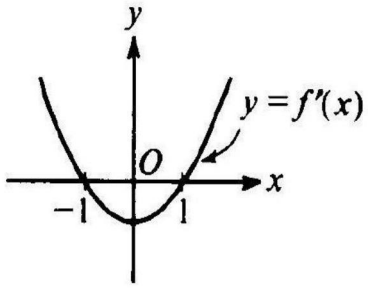
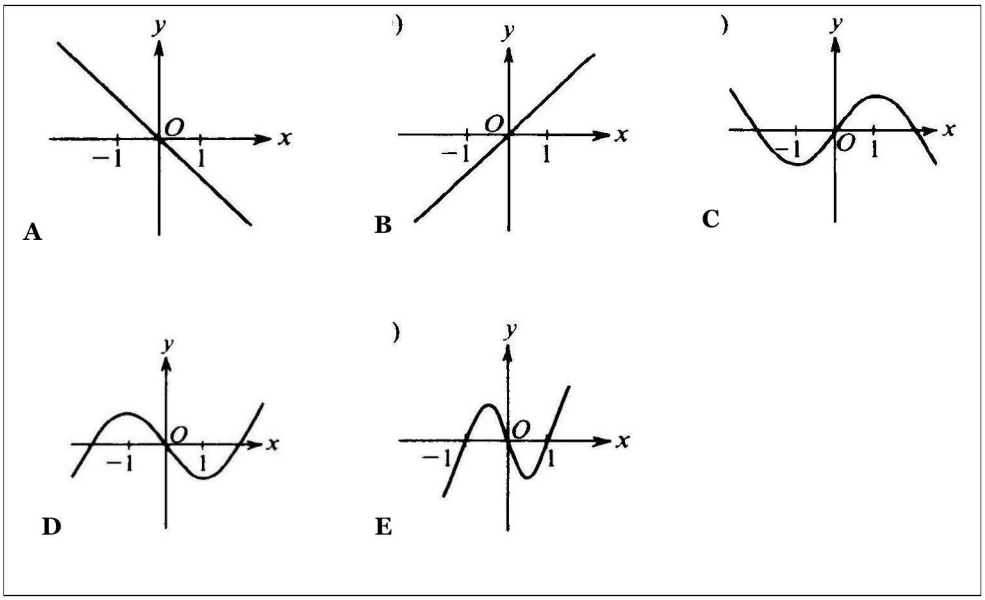




(a) A

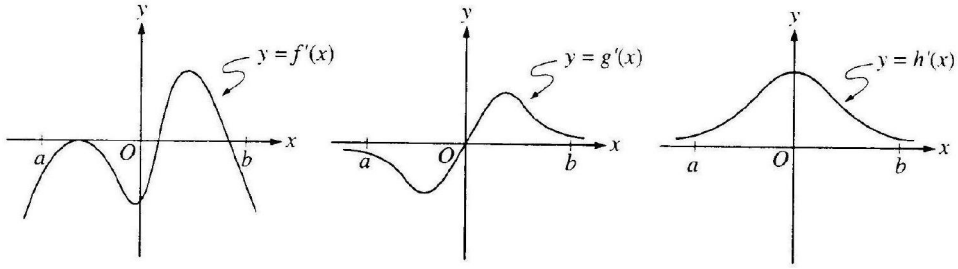
(b) B

(c) C

(d) D

(e) E

22. The graphs of the derivatives of the functions $f, g,$ and h are shown below. Which of the functions $f, g,$ or h have a relative maximum on the open interval $a < x < b$?



(a) f only

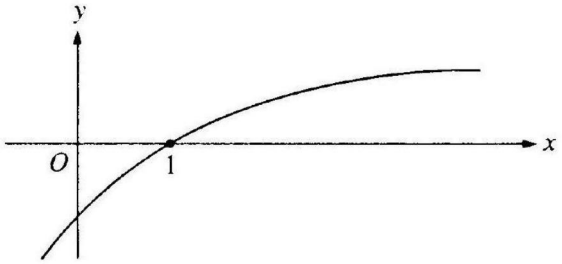
(b) g only

(c) h only

(d) f and g only

(e) $f, g,$ and h

23. The graph of a twice-differentiable function f is shown in the figure below.



Which of the following is true?

(a) $f(1) < f'(1) < f''(1)$

(b) $f(1) < f''(1) < f'(1)$

(c) $f'(1) < f(1) < f''(1)$

(d) $f''(1) < f(1) < f'(1)$

(e) $f''(1) < f'(1) < f(1)$

Free Response

The function

$$f(x) = \frac{1}{x^2 - 4}$$

has first derivative

$$f'(x) = \frac{-2x}{(x^2 - 4)^2}$$

and second derivative

$$f''(x) = \frac{6x^2 + 8}{(x^2 - 4)^3}$$

Sketch the graph of $f(x)$ after completing the following questions:

1. State any domain restrictions for $f(x)$ 2. Determine any critical points of $f(x)$ 3. State intervals on which $f(x)$ is increasing or decreasing 4. State intervals on which $f(x)$ is concave up or concave down 5. Calculate any horizontal asymptotes of $f(x)$