AOS Senior AP Calculus BC, Spring 2024
Cumulative, Quarter 3
(Parametric, Polar, Logistic)

_______/ 30

ACADEMIES OF LOUDOUN HONOR CODE

Honesty and integrity are the foundations of good academic work. Whether you are working on a problem set, lab report, project, presentation, or paper, do not engage in plagiarism, unauthorized collaboration, cheating, or facilitating academic dishonesty. Our expectation is for our students to be successful while being trustworthy. The honor code is not intended to be punitive, but rather a guide for all students and faculty to follow. For these reasons, the Academies of Loudoun will uphold the following Honor Code:

On my honor, I have not accepted or provided any unauthorized aid on this test, quiz, or assignment.

As an Academies of Loudoun student, you agreed to uphold the Academies Honor Code. Please write the Honor Code Pledge below and sign this document.

____

____

____

____

Student Signature **Class** **Date**

Print Name:

1. To determine concavity of a parametric curve at the point where $t = c$

    (a) Evaluate $\dfrac{d}{dt}\left(\dfrac{dy}{dt}\right)$

    (b) Evaluate $\dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

    (c) Evaluate $y''(c)/x''(c)$

    (d) Evaluate $x''(c)/y''(c)$

2. Which of the following is **not** a polar-rectangular transformation equation?

    (a) $y = r\sin\theta$

    (b) $\tan\theta = \dfrac{x}{y}$

    (c) $x^2 + y^2 = r^2$

    (d) $x = r\cos\theta$

3. If a polar graph is defined by $r(\theta)$ and $\dfrac{dr}{d\theta} > 0$ at a point where $\theta = \alpha$ then

    (a) The graph's radius is decrasing at $\theta = \alpha$

    (b) The graph's radius is increasing at $\theta = \alpha$

    (c) The tangent line to the graph at $\theta = \alpha$ has a negative slope

    (d) The tangent line to the graph $\theta = \alpha$ has a positive slope

4. The graph of $r = a + b\sin(\theta)$

    (a) Has an inner loop whenever $a > b$

    (b) Never intersects the x-axis

    (c) Never intersects the y-axis

    (d) Has an inner loop whenever $a < b$

5. $x(t) = \cos t$ and $y(t) = \sin t$ are the parametric equations for

    (a) A hyperbola

    (b) A square

    (c) A parabola

    (d) A circle

6. A particle moves in a plane from an initial position given by the vector $\vec{r}_0 = \langle x_0, y_0 \rangle$ at time $t = 0$. The particle's velocity at any time t is described by the vector function $\vec{v}(t) = \langle v_x(t), v_y(t) \rangle$. Assuming the velocity function is integrable, which of the following expressions correctly describes the particle's position $\vec{r}(t)$ at any later time t?

   (a) $\vec{r}(t) = \vec{r}_0 + \vec{v}(t)t$

   (b) $\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t)\,dt$

   (c) $\vec{r}(t) = \vec{r}_0 + \int \vec{v}(t)$

   (d) $\vec{r}(t) = \vec{r}_0 + \frac{1}{2}\vec{v}(t)t^2$

7. If a particle in the first quadrant is moving towards the x axis then

   (a) $\frac{d^2x}{dt^2} > 0$

   (b) $\frac{dx}{dt} < 0$

   (c) $\frac{dy}{dt} < 0$

   (d) $\frac{d^2y}{dt^2} > 0$

8. The distance traveled from $t = a$ to $t = b$ of a particle with position vector $\langle x(t), y(t) \rangle$ is given by

   (a) $\sqrt{(x(t) - x(0))^2 + (y(t) - y(0))^2}$

   (b) $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

   (c) $\int_a^b \sqrt{x^2(t) + y^2(t)}\,dt$

   (d) $\int_a^b |x'(t) + y'(t)|\,dt$

9. A logistic population graph $y = f(t)$ with a max population of L

   (a) Has an asymptote at $y = L$

   (b) Has an increasing growth rate when $t > 0$

   (c) Can oscillate for certain initial conditions

   (d) Has a decreasing growth rate when $t > 0$

10. If $x(t)$ and $y(t)$ are the parametric equations of a curve, the curve will have a horizontal tangent line at $t = c$ if

   (a) $x(c) = 0$ and $y(c) = 0$

   (b) $x(c) = 0$ and $x'(c) = 0$

   (c) $x'(c) = 0$ and $y'(c) \neq 0$

   (d) $y'(c) = 0$ and $x'(c) \neq 0$

11. The area enclosed by a polar curve between $\theta = \alpha$ and $\theta = \beta$ is always

   (a) $\displaystyle\int_\alpha^\beta r^2(\theta)\, d\theta$

   (b) $\displaystyle\int_\alpha^\beta \frac{1}{2} r^2(\theta)\, d\theta$

   (c) $\displaystyle\int_\alpha^\beta r(\theta)\, d\theta$

   (d) Dependent on if the curve intersects itself in the interval $\alpha < \theta < \beta$

12. The maximum growth rate for a logistic population with carrying capacity L

   (a) Occurs when the population is $L/2$

   (b) Depends on the initial conditions

   (c) Always occurs at $t = 0$

   (d) Can happen more than once during a given solution

13. To find the slope of the tangent line to a parametric curve at the point where $t = c$ you should

   (a) Evaluate $x'(c)/y'(c)$

   (b) Evaluate $y'(c)/x'(c)$

   (c) Evaluate $x'(c)$

   (d) Evaluate $y'(c)$

14. If a parametric curve has a point where $x'(a) = 0$ and $y'(a) = 0$ then

   (a) There is no tangent line at $t = a$

   (b) There is a vertical tangent line at $t = a$

   (c) The curve must cross itself

   (d) There is a horizontal tangent line at $t = a$

15. If n is a positive integer, the graph of $r = \sin(n\theta)$ always

(a) Has one intercept at $(0, 0)$

(b) Is a rose with $2n$ petals

(c) Is a rose with n petals

(d) Completes exactly one period of the graph over $0 \le \theta < 2\pi$

KEY

1. B
2. B
3. B
4. D
5. D
6. B
7. C
8. B
9. A
10. D
11. D
12. A
13. B
14. A
15. A