1.

$$\int_1^e x + \frac{1}{x}\ dx = \left[\frac{x^2}{2} + \ln x\right]_1^e = \frac{e^2-1}{2} + \ln\left|\frac{e}{1}\right| = \frac{e^2-1}{2} + 1 \quad B$$

2.

$$\int_1^\infty xe^{-x^2}\ dx = \frac{-1}{2}\int_1^\infty -2xe^{-x^2}\ dx = \frac{-1}{2}\left[e^{-x^2}\right]_1^\infty = \frac{-1}{2}\left[0 - e^{-1}\right] B$$

3.

$$\begin{aligned}\int x^2 \cos x^3\ dx & \\ &= \frac{1}{3}\int 3x^2\cos x^3\ dx \\ &= \frac{1}{3}\int \cos u\ du \\ &= \frac{1}{3}\sin u + C \\ &= B\end{aligned}$$

4.

$$\begin{aligned}\int_{-1}^9 3f(x) + 2\ dx &= 3\int_{-1}^9 f(x) + \int_{-1}^9 2\ dx \\ &= 3[-3.5 + 6] + 2(10) \\ &= 27.5 \\ &= C\end{aligned}$$

5.

$$\begin{aligned}y(10) &= y(0) + \int_0^{10} y'(t)dt \\ &= 12 + \int_0^{10} e^{t/5}dt \\ &= 12 + \left[5e^{t/5}\right]_0^{10} \\ &= 12 + 5\left(e^2 - 1\right)\end{aligned}$$

6.

$$\begin{aligned}\int_a^b f(x)\ dx &\cong \sum_{k=1}^n f(a + k\Delta x)\Delta x \text{ with } \Delta x = \frac{b-a}{n} = \frac{4}{n} \\ &= \sum e^{(-3+4k/n)}(4/n) \\ &= D\end{aligned}$$

7.

$$\int_1^3 f(x+5)\ dx = 7. \quad \text{Let } u = x+5$$

$$= \int_6^8 f(u)\ du = 7$$

$$\text{so} \int_6^8 f(x) + 2\ dx = 7 + 2(2) = 11 \quad D$$

8. B

9.

$$\int_{-1}^4 x\left(x^2-3\right)^5\ dx$$

$$u = x^2 - 3$$

$$du = 2x\ dx$$

$$x\ dx = \frac{du}{2}$$

$$= \int_{-2}^{13} u^5\ du/2$$

$$= C$$

10

u	dv
$+x$	e^{-2x}
-1	$-\frac{1}{2}e^{-2x}$
$+0$	$\frac{1}{4}e^{-2x}$

$$k\int_0^\infty xe^{-2x}\ dx =$$

$$k\left(-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right)$$

$$= \left[\frac{-ke^{-2x}}{4}(2x+1)\right]_0^\infty$$

$$= \frac{k}{4}$$

$$= 1 \text{ if } k = 4$$

11.

$$\int_1^e \frac{\ln x}{x^3}\, dx = uv - \int v\, du \qquad \begin{matrix} u = \ln x, dv = x^{-3} \\ du = \frac{1}{x} \quad v = -\frac{1}{2}x^{-2} \end{matrix}$$

$$\begin{aligned} &= \frac{-\ln x}{2x^2} - \int \frac{-1}{2x^3}\, dx \\ &= \left[\frac{-\ln x}{2x^2} - \frac{1}{4x^2}\right]_1^e \\ &= \left(\frac{-\ln e}{2e^2} - \frac{1}{4e^2}\right) - \left(\frac{-1}{4}\right) \\ &= \frac{e^2 - 3}{4e^2} = \text{ A} \end{aligned}$$

12.

$$\int f(x)\sin x\, dx = f(x)(-\cos x) - \int 4x^3(-\cos x)\, dx$$

$$\int u\, dv = uv - \int\, du \cdot v$$

$$du = 4x^3$$

$$u = x^4$$

$$E$$

13.

$$\int_{-1}^1 \frac{1}{x^2}\, dx = \int_{-1}^0 \frac{1}{x^2}\, dx + \int_0^1 \frac{1}{x^2}\, dx$$

but

$$\int_{-1}^0 \frac{1}{x^2}\, dx = \left[-\frac{1}{x}\right]_{-1}^0$$

$$= \lim_{t\to 0}\left[\frac{-1}{x}\right]_{-1}^t$$

$$= \infty$$

$$D$$

14

$$
\begin{aligned}
\int_0^1 f'(x)g(x)\ dx = 5 = &\ g(x)f(x)|_0^1 - \int_0^1 f(x)g'(x)\ dx \\
&= (g(1)f(1) - g(0)f(0)) - \int_0^1 f(x)g'(x)\ dx \\
\int_0^1 f(x)g'(x)\ dx &= (3)(4) - (-4)(2) - 5 \\
&= 12 + 8 - 5 = 15 \\
&X
\end{aligned}
$$

15.

$$
\begin{aligned}
\int_1^3 \frac{3x^3 + 15x^2 + x + 9}{x+5} &= \int_1^3 3x^2 + 1 + \frac{4}{x+5}\ dx \\
&= \left[x^3 + x + 4\ln|x+5|\right]_1^3 \\
&= 26 + 2 + 4\ln\left|\frac{8}{6}\right| \\
&= A
\end{aligned}
$$

16.

$$
\begin{aligned}
\int \frac{7x\ dx}{(2x-3)(x+2)} &= \int \frac{A}{2x-3} + \frac{B}{x+2} \\
&A(x+2) + B(2x-3) = 7x \\
&\text{Let } x = -2 \\
&-7B = -14 \text{ so } B = 2 \\
&x = \frac{3}{2} \\
&3.5A = 10.5 \text{ so } A = 3 \\
\int \frac{7x}{(2x-3)(x+2)} &= \frac{3}{2}|2x-3| + 2\ln|x+2| \\
&= \quad A
\end{aligned}
$$

17.

$$
\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\ dx = \int_{x=0}^{x=\pi/2} \frac{du}{1+u^2}
$$

if $u = \sin x$

$$\int_{x=0}^{x=\pi/2} \frac{du}{1+u^2}$$

$$\begin{aligned}
&= \int_0^1 \frac{du}{1+u^2} \\
&= [\arctan u]_0^1 \\
&= \frac{\pi}{4} - 0 \\
&X
\end{aligned}$$

18.

$$\begin{aligned}
\int_0^4 f(x)\ dx &= \int_0^2 f(x)\ dx + \int_2^4 f(x)\ dx \\
&= \left(\frac{5+1}{2}\right)(2) + 2(-3) = 0 \\
&= B
\end{aligned}$$

19.

$$\int \frac{1}{1+e^{2x}}$$

$$\begin{aligned}
&= \int \frac{e^{-2x}}{e^{-2x}+1} \\
u &= e^{-2x}+1 \\
du &= -2e^{-2x}
\end{aligned}$$

$$\int \frac{e^{-2x}}{e^{-2x}+1}$$

$$\begin{aligned}
&= \frac{-1}{2}\int \frac{du}{u} = -\frac{1}{2}\ln|u| = -\frac{1}{2}\ln\left|1+e^{-2x}\right| \\
&= -\frac{1}{2}\ln\left|e^{-2x}\left(e^{2x}+1\right)\right| \\
&= \frac{-1}{2}\ln\left|e^{-2x}\right| + \frac{-1}{2}\ln\left|e^{2x}+1\right| \\
&= -\frac{1}{2}(-2x) + \frac{-1}{2}\ln\left(1+e^{2x}\right) + C \\
&= D
\end{aligned}$$

20.

$$\int \frac{dx}{x^2-10x+34} = \int \frac{dx}{(x-5)^2+9}$$

Let $u = (x-5), a = 3$

$$\int \frac{dx}{(x-5)^2+9} = \int \frac{du}{u^2+a^2} = \frac{1}{3}\arctan\left(\frac{x-5}{3}\right)$$

21.

$$\begin{aligned}
&\frac{5x+8}{x^2+3x+2} = \frac{2}{x+2} + \frac{3}{x+1} \text{ so} \\
&\int \frac{5x+8}{x^2+3x+2} = [2\ln|x+2| + 3\ln|x+1|]_0^1 \\
&= 2\ln\left|\frac{3}{2}\right| + 3\ln|2| \\
&= \ln\frac{9}{4} + \ln 8 \\
&= \ln\frac{9}{4}\cdot 8 \\
&= \ln 18 \\
&= C
\end{aligned}$$

1.

$$f(2) = 2.4 + \int_{-1}^{2} \sqrt{9-x^2}\, dx = 10.863$$