

AOS Math 10, Spring 2024

Derivatives Test (#14)

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May 10, 2024

Student Signature

Class

Date

Print Name:

Instructions: Questions 1-14 are multiple choice. For each problem, circle the letter of the best answer. You **must show all work** for credit. Partial credit may be awarded as appropriate. Each question is valued at 3 points.

1. Let $f(x) = \sqrt{x}$. What is the equation of the tangent line to f at the point $(4, 2)$?

(a) $y = -\frac{1}{2}x + 3$

(b) $y = \frac{1}{4}x + 1$

(c) $y = \frac{1}{2}x$

(d) $y = 2x - 6$

2. What is the derivative of $s(t) = \cos(t^2 + 1)$?

(a) $\cos(2t)$

(b) $-(t^2 + 1)\sin(t^2 + 1)$

(c) $-2t\sin(t^2 + 1)$

(d) $-\sin(2t)$

3. If f and h are nonzero differentiable functions, then the derivative of $\frac{f}{h}$ is

(a) $\frac{fh' - f'h}{h^2}$

(b) $\frac{f'h - fh'}{h^2}$

(c) $\frac{f'h + fh'}{h^2}$

(d) $\frac{f'}{h'}$

4. The line tangent to the curve $y = \sqrt{16 - x}$ at the point $(0, 4)$ has slope

(a) $\frac{1}{8}$

(b) -4

(c) $-\frac{1}{8}$

(d) 4

5. If $y = 6 \ln(3x)$ then what is y' ?

- (a) $\frac{6}{x}$
- (b) $\frac{18}{x}$
- (c) $\frac{2}{x}$
- (d) $\frac{1}{3x}$

6. What is the value of

$$\lim_{\Delta x \rightarrow 0} \frac{2(x + \Delta x)^2 - 2x^2}{\Delta x}$$

- (a) $2x$
- (b) 4
- (c) 2
- (d) $4x$

7. If $w(t) = \sqrt{t^2 - 1}$ what is the value of $w'(4)$?

- (a) $\frac{1}{2\sqrt{15}}$
- (b) $\frac{2}{\sqrt{15}}$
- (c) $\frac{1}{\sqrt{15}}$
- (d) $\frac{4}{\sqrt{15}}$

8. At which x value does the graph of $y = 3x^2 - 10x + 15$ have a horizontal tangent line?

- (a) $\frac{-5}{3}$
- (b) $\frac{3}{5}$
- (c) $\frac{5}{3}$
- (d) $\frac{-3}{5}$

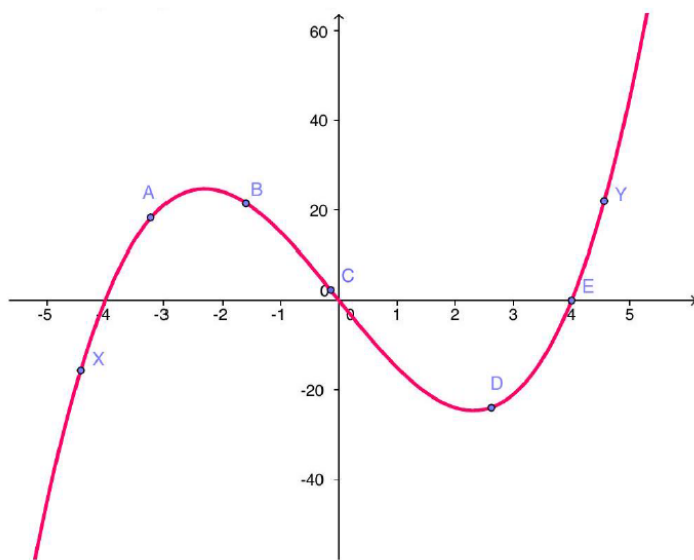
9. If $h(x) = f(x^2 + 1)$ then which of the following is true?

- (a) $h'(x) = f'(x^2 + 1)$
- (b) $h'(x) = 2xf'(x^2 + 1)$
- (c) $h'(x) = f'(2x)$
- (d) $h'(x) = 2xf'(2x)$

10. If $f(x) = \sin(2x + 1)$ and $g(x) = f'(x)$, find $g'(x)$

- (a) $g'(x) = -4x \cos(2x + 1)$
- (b) $g'(x) = 2 \sin(2x + 1)$
- (c) $g'(x) = 4 \sin(2x + 1) \cos(2x + 1)$
- (d) $g'(x) = -4 \sin(2x + 1)$

11. The graph of a continuous differentiable function f is shown below.



Using the above graph, select the one true statement below.

- (a) $f'(C) < f'(D) < f'(Y)$
- (b) $f'(A) < f'(B) < f'(C)$
- (c) $f'(X) < f'(B) < f'(E)$
- (d) $f'(X) < f'(Y) < f'(C)$

12. Let $f(x) = x^3 - 6x^2 + 10$. At which point(s) on the graph of f is the tangent line parallel to the line $15x - y = 11$?

- (a) $(2, -6)$ and $(-2, -22)$
- (b) $(5, -15)$ and $(2, -6)$
- (c) $(2, -6)$ and $(-2, 22)$
- (d) $(5, -15)$ and $(-1, 3)$

13. If $y(x) = \frac{\sin(2x)}{x^2}$ find $y'(x)$

- (a) $\frac{x^2 \cos(2x) - \sin(2x)}{x^3}$
- (b) $\frac{x^2 \sin(2x) + 2 \cos(2x)}{x^4}$
- (c) $\frac{2x \cos(2x) - 2 \sin(2x)}{x^3}$
- (d) $\frac{2 \cos(2x)}{x}$

14. Calculate $\frac{d}{dt} (\ln(e^{2t}) - 2t)$

- (a) 0
- (b) $\frac{1}{2t} - 2$
- (c) $\frac{1}{2e^{2t}} - 2$
- (d) $\frac{2}{e^{2t}} - 2$

Free Response Section: Selected values of f, g, f', g' are given in the table below.

x	0	1	2	3	4
$f(x)$	$\frac{1}{2}$	$\frac{1}{3}$	1	-1	3
$g(x)$	-2	1	$-\frac{1}{2}$	2	$-\frac{1}{3}$
$f'(x)$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{1}{4}$	0	$-\frac{4}{5}$
$g'(x)$	-1	$\frac{2}{3}$	-4	-3	$-\frac{1}{3}$

Using the values in the table, evaluate the following derivatives. **You must show the symbolic derivative as the first part of your answer for credit!**

15. $\frac{d}{dx} (f(x) + g(x))$ at $x = 4$

16. $\frac{d}{dx} (f(x)g(x))$ at $x = 1$

17. $\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right)$ at $x = 0$

18. $\frac{d}{dx} (f(g(x)))$ at $x = 3$

19. $\frac{d}{dx} (g(x + f(x)))$ at $x = 3$