AOS AP CALCULUS BC 2024
Major Summative #10
Polar Calculus

_____/36.5

ACADEMIES OF LOUDOUN HONOR CODE

Honesty and integrity are the foundations of good academic work. Whether you are working on a problem set, lab report, project, presentation, or paper, do not engage in plagiarism, unauthorized collaboration, cheating, or facilitating academic dishonesty. Our expectation is for our students to be successful while being trustworthy. The honor code is not intended to be punitive, but rather a guide for all students and faculty to follow. For these reasons, the Academies of Loudoun will uphold the following Honor Code:

On my honor, I have not accepted or provided any unauthorized aid on this test, quiz, or assignment.

As an Academies of Loudoun student, you agreed to uphold the Academies Honor Code. Please write the Honor Code Pledge below and sign this document.

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Student Signature: _____

Class: _____ ***Date:*** _____

Print Name_____

Multiple Choice.

Correct work must be shown for full credit. Choose the letter for the best answer.(4 pts each)

1. What is the slope of the line tangent to the polar curve $r = 3\theta$ at the point where $\theta = \frac{\pi}{2}$?

   A) 0
   B) 3
   C) -3
   D) $-\frac{\pi}{2}$
   E) $-\frac{2}{\pi}$

2. The polar curve $r = 1 + sin(\theta)cos(\theta)$ has horizontal tangents at which of the following angles θ? (Circle **all** answers that apply)

   A) 0.435
   B) -2.706
   C) 1.135
   D) -2.007
   E) 1.345

3. A particle moves in a plane so that its position at any time $\theta, 0 \leq \theta \leq 8$, is given by the polar equation $r(\theta) = 5(1 + cos\theta)$. When does the particle's distance from the origin change from decreasing to increasing?

   A) $\theta = 0$ only
   B) $\theta = \pi$ only
   C) $\theta = 2\pi$ only
   D) $\theta = 0$ and $\theta = \pi$
   E) $\theta = \pi$ and $\theta = 2\pi$

4. The area of the region enclosed by the polar curve $r = cos2\theta$ for $0 \leq \theta \leq \frac{\pi}{2}$ is

A) 1

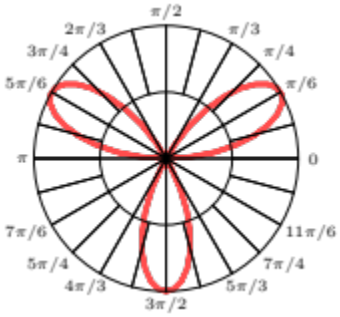
B) $\frac{\pi}{2}$

C) π

D) $\frac{\pi}{8}$

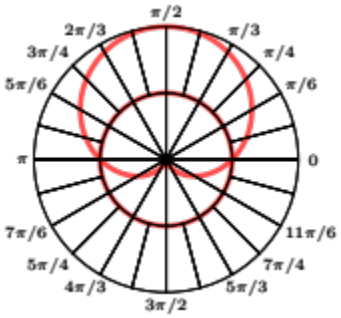
E) $\frac{\pi}{4}$

5. The area of one leaf of the rose $r = sin3\theta$ is



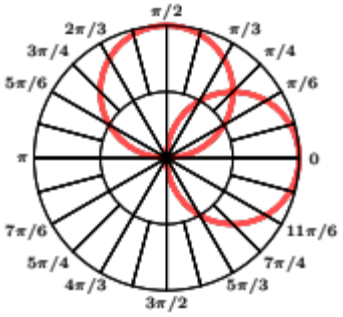
A) 1.5708
B) 0.2618
C) 0.5236
D) 0.7854
E) 1.0472

6. The area outside $r = 1$ and inside $r = 1 + \sin\theta$ is



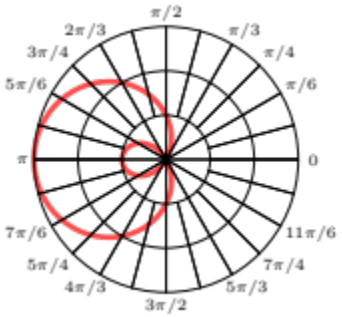
A) 0.4292
B) 3.5708
C) 2.7854
D) 1.2146
E) 5.1416

7. Which of the following approximates the total area of the region common to the interior of both polar curves $r = 2\cos\theta$ and $r = 2\sin\theta$?



A) 0.28540
B) 0.57079
C) 0.78432
D) 1.57082
E) 3.14159

8. The area of the inner loop of the curve $r = 1 - 2cos(\theta)$ is



A. $\frac{1}{2}\int_{\frac{-\pi}{3}}^{\frac{\pi}{3}}\left(1-2cos(\theta)\right)^2 d\theta$

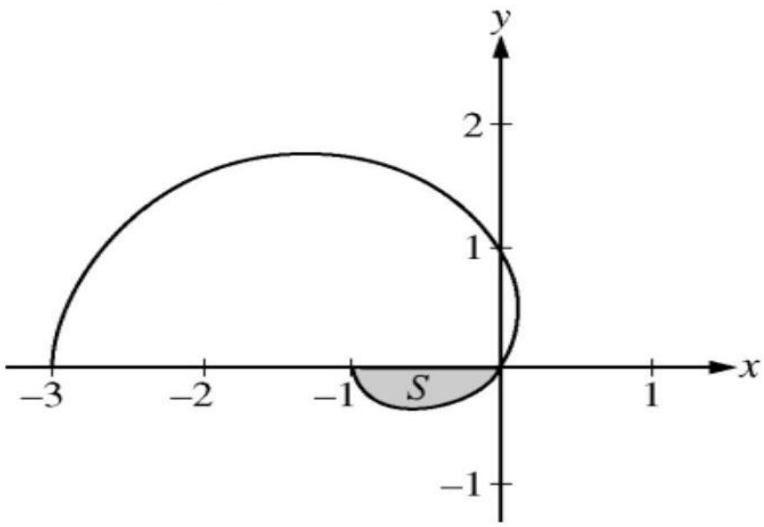
B. $\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{5\pi}{3}}\left(1-2cos(\theta)\right)^2 d\theta$

C. $\frac{1}{2}\int_{0}^{\frac{\pi}{3}}\left(1-2cos(\theta)\right)^2 d\theta$

D. $\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{-\pi}{3}}\left(1-2cos(\theta)\right)^2 d\theta$

E. $2\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\left(1-2cos(\theta)\right)^2 d\theta$

Free Response Question. (NON CALCULATOR)



The graph of the polar curve $r = 1 - 2cos\theta$ for $0 \leq \theta \leq \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x-axis.

(a) Write an integral expression for the area of S.

(b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

(c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.