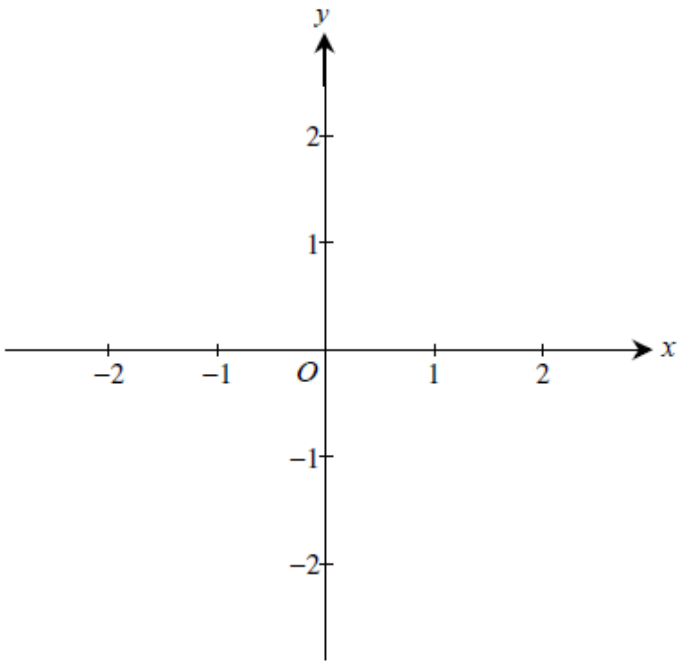
AP CALCULUS FREE RESPONSE QUESTIONS – POLAR FUNCTIONS

Please do not write on this packet. Show all work on separate paper.

1993 #4 – no calculator

Consider the polar curve $r = 2\sin(3\theta)$ for $0 \le \theta \le \pi$.

(a) In the xy-plane provided below, sketch the curve.



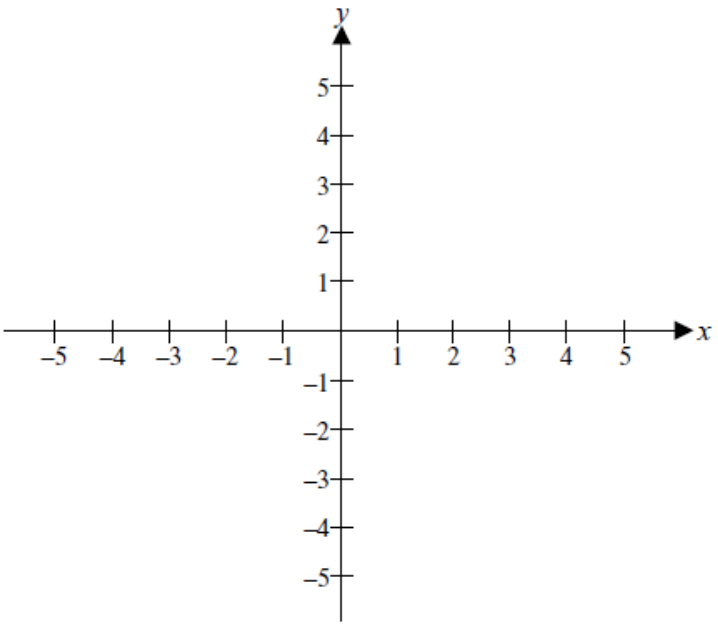
(b) Find the area of the region inside the curve.

(c) Find the slope of the curve at the point where $\theta = \frac{\pi}{4}$.

1990 #4 – no calculator

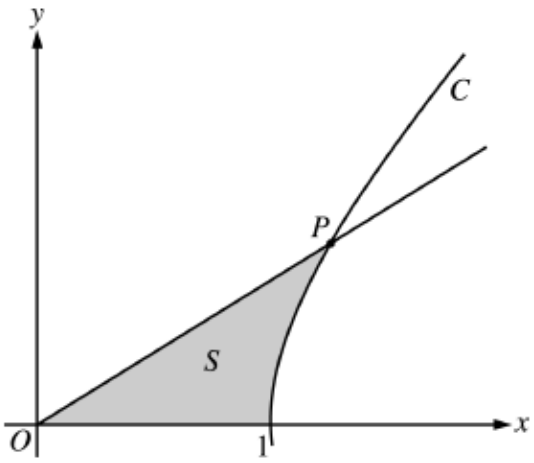
Let R be the region inside the graph of the polar curve $r = 2$ and outside the graph of the polar curve $r = 2(1 - \sin\theta)$.

(a) Sketch the two polar curves in the xy-plane provided below and shade the region R.



(b) Find the area of R.

2003 #3 – calculator



3. The figure above shows the graphs of the line $x = \frac{5}{3}y$ and the curve C given by $x = \sqrt{1 + y^2}$. Let S be the shaded region bounded by the two graphs and the x-axis. The line and the curve intersect at point P.

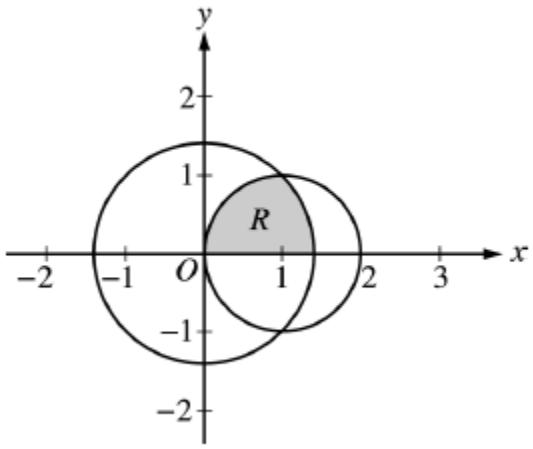
   (a) Find the coordinates of point P and the value of $\frac{dx}{dy}$ for curve C at point P.

   (b) Set up and evaluate an integral expression with respect to y that gives the area of S.

   (c) Curve C is a part of the curve $x^2 - y^2 = 1$. Show that $x^2 - y^2 = 1$ can be written as the polar equation $r^2 = \frac{1}{\cos^2\theta - \sin^2\theta}$.

   (d) Use the polar equation given in part (c) to set up an integral expression with respect to the polar angle θ that represents the area of S.

2003B #2 – calculator



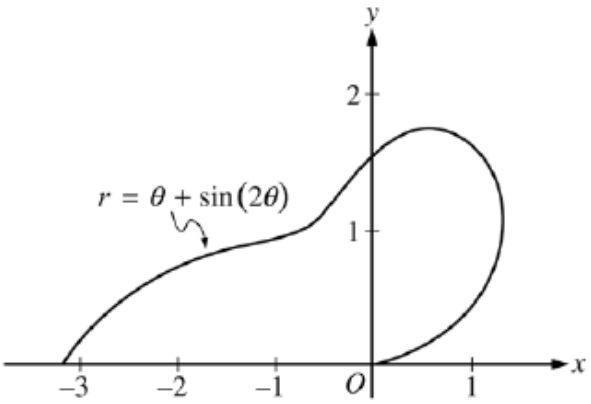
2. The figure above shows the graphs of the circles $x^2 + y^2 = 2$ and $(x-1)^2 + y^2 = 1$. The graphs intersect at the points $(1, 1)$ and $(1, -1)$. Let R be the shaded region in the first quadrant bounded by the two circles and the x-axis.

   (a) Set up an expression involving one or more integrals with respect to x that represents the area of R.

   (b) Set up an expression involving one or more integrals with respect to y that represents the area of R.

   (c) The polar equations of the circles are $r = \sqrt{2}$ and $r = 2\cos\theta$, respectively. Set up an expression involving one or more integrals with respect to the polar angle θ that represents the area of R.

2005 #2 – calculator



2. The curve above is drawn in the xy-plane and is described by the equation in polar coordinates $r = \theta + \sin(2\theta)$ for $0 \le \theta \le \pi$, where r is measured in meters and θ is measured in radians. The derivative of r with respect to θ is given by $\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$.

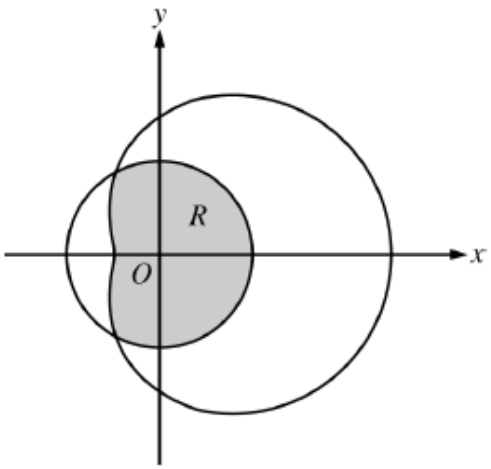
   (a) Find the area bounded by the curve and the x-axis.

   (b) Find the angle θ that corresponds to the point on the curve with x-coordinate -2.

   (c) For $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$, $\frac{dr}{d\theta}$ is negative. What does this fact say about r ? What does this fact say about the curve?

   (d) Find the value of θ in the interval $0 \le \theta \le \frac{\pi}{2}$ that corresponds to the point on the curve in the first quadrant with greatest distance from the origin. Justify your answer.

2007 #3 – calculator



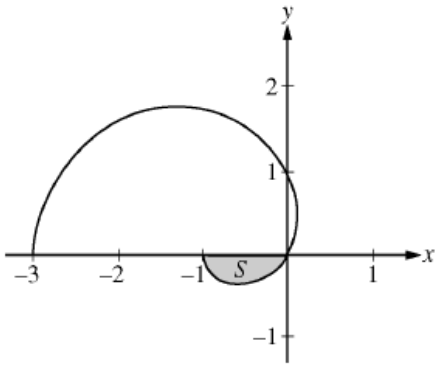
3. The graphs of the polar curves $r = 2$ and $r = 3 + 2\cos\theta$ are shown in the figure above. The curves intersect when $\theta = \frac{2\pi}{3}$ and $\theta = \frac{4\pi}{3}$.

   (a) Let R be the region that is inside the graph of $r = 2$ and also inside the graph of $r = 3 + 2\cos\theta$, as shaded in the figure above. Find the area of R.

   (b) A particle moving with nonzero velocity along the polar curve given by $r = 3 + 2\cos\theta$ has position $(x(t), y(t))$ at time t, with $\theta = 0$ when $t = 0$. This particle moves along the curve so that $\frac{dr}{dt} = \frac{dr}{d\theta}$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

   (c) For the particle described in part (b), $\frac{dy}{dt} = \frac{dy}{d\theta}$. Find the value of $\frac{dy}{dt}$ at $\theta = \frac{\pi}{3}$ and interpret your answer in terms of the motion of the particle.

2009B #4 – no calculator



4. The graph of the polar curve $r = 1 - 2\cos\theta$ for $0 \le \theta \le \pi$ is shown above. Let S be the shaded region in the third quadrant bounded by the curve and the x-axis.

   (a) Write an integral expression for the area of S.

   (b) Write expressions for $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ in terms of θ.

   (c) Write an equation in terms of x and y for the line tangent to the graph of the polar curve at the point where $\theta = \frac{\pi}{2}$. Show the computations that lead to your answer.