# The Circle

The circle is one of the most basic shapes, known to humans for tens of thousands of years according to the archaeological record. It defines everybody’s favorite constant, , and is the basis of analytic trigonometry.

**Geometric Definition**: The circle is the locus of points all equidistant from a given *center*. The distance from any such point to the center is called the *radius*.

![Schematic of a Circle](data:application/pdf;base64,)

Schematic of a Circle

**Parent Equation**: The parent equation is the equation for the unit circle, a circle centered at the origin with radius 1.

**Standard Form Equation**: is a circle with radius centered at

**General Form Equation**: . This is the general form for any (non-rotated) conic.

**Geometry Review**: Given a line segment joining and

* the length of the segment is
* The midpoint of the segment is

**Completing the Square**: You can always rewrite as

* which is

## Problems

Unless otherwise specified, any circle equations should be given in standard form

#### Fundamental Concepts

1. Find the radius of the circle .
2. Find the radius of the circle .
3. Find the radius and center of the circle .
4. Translate the circle given here by 5 units up and 3 units to the left:
5. Translate the circle given here so that its center is half as close to the origin:
6. Find the radius and center of the circle
7. Find the radius and center of the circle
8. Write the equation of a circle with radius 5 and center . Write both standard and general form.
9. Write the equation of a circle with radius and center . Eliminate all denominators. Write both standard and general form.
10. A diameter of a certain circle joins the points and . Find the equation of the circle.
11. Write the circle in general form.
12. Complete the square to write the following in standard form:
13. Complete the square to write the following in standard form:
14. Write a “completing the square” circle problem and trade with a friend.

#### Deeper Understanding

1. An equilateral triangle has its base as the line segment joining the points and . Find the third point of the triangle. Circumscribe a circle around this triangle and find the equation of the circle. (Hint: the center of the circle is the average of the three triangle vertices).
2. Given the circle , find four points on the circle that form the vertices of a square.
3. In the general form equation of a circle , can be greater than ? Can it be less than ? Why or why not?
4. Write an inequality which constrains in terms of the other coefficients in the general form equation of a circle. (Hint: find the center and radius of the general equation first by completing the square.)
5. Find the intersection points of the circle with the line . Round your answer to 3 places.
6. A regular hexagon is inscribed in a unit circle. What is its perimeter?
7. The circle contains the points and . The tangent line to the circle at the point is perpendicular to the line segment from the origin to . Write the equation of this tangent line. Also write the equation of the tangent line through .
8. Circle has radius 1 and is centered at the origin. Circle is tangent to circle at the point , entirely contains circle and has twice the area of . Write the equation of in standard form.