# Sequences and Series

## Useful formulae

In the preceding sums, is the first term and is the last term and is the number of terms.

## Practice with arithmetic and geometric sequences

Write a formula for the th term of each sequence below.

For each of the sequences 1-6 find the term

For each the sequences 1-6 find the sum

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For the convergent geometric sequences determine

For the convergent geometric sequences determine the first index for which

For sequence 1, solve the equation

For sequence 2, solve the equation

If a geometric sequence has and find the common ratio and the term

If an arithmetic sequence has and , write a formula for and the sum of the first 12 terms, starting with

## Take it to the limit

Evaluate the following infinite limits of sequences.

1. where is the th Fibonacci number (this is most likely a calculator or computer question)

## Sums of powers

The sums and in general fascinated mathematicians for centuries. We will derive some of these formulas in this section.

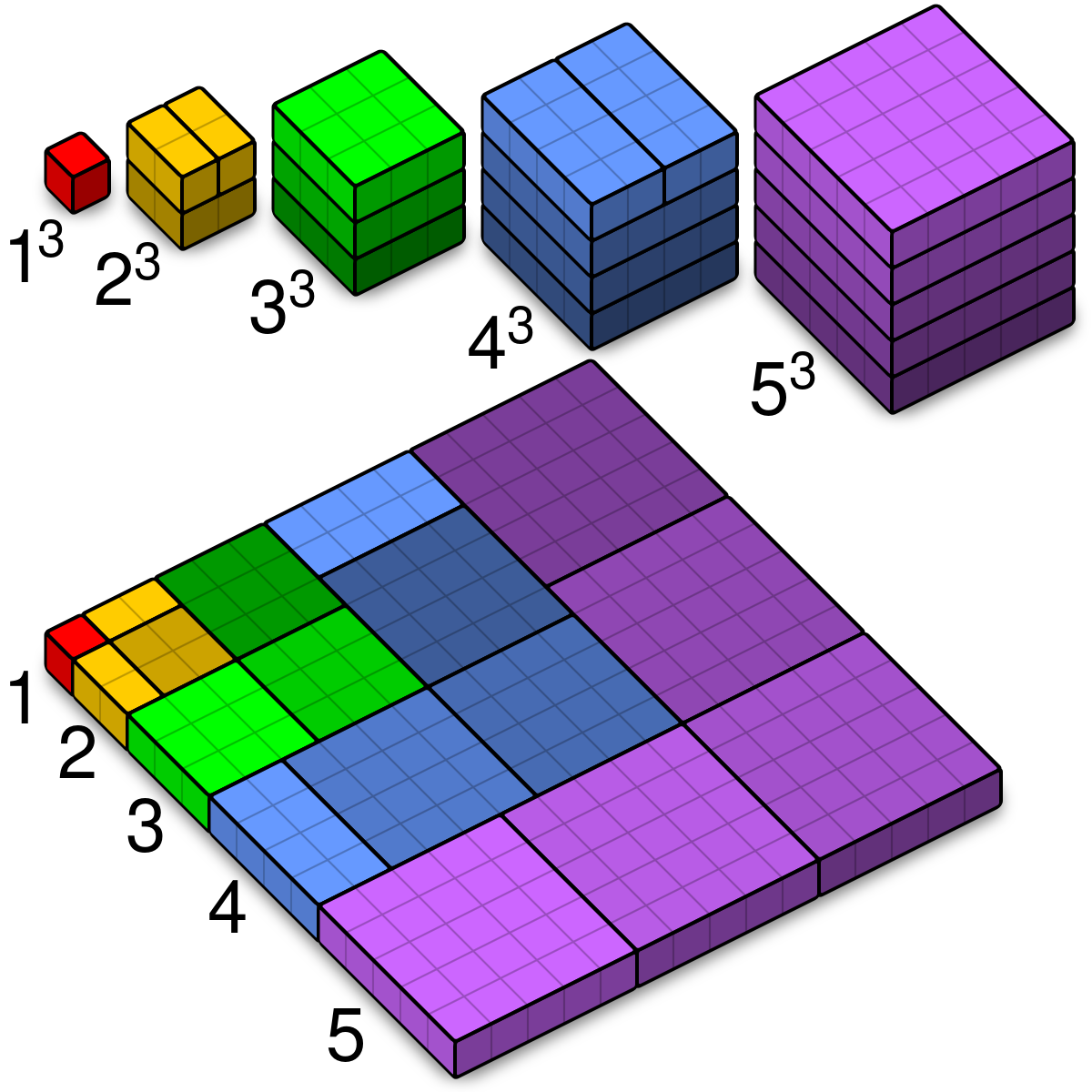
First, the following formulas you should be able to verify on your own

The next is not obvious

But by inferring the pattern in the last 3 formula you should be able to complete

(These formulas involving “falling and rising factorials” are easily established using a branch of mathematics called *discrete calculus*)

Now your job is to find the equations for and by manipulating the above formulas.

Though messy, this procedure can be continued to find for any . These lead to the famous *Bernoulli Numbers*.

## Sums of Cubes

We share a “proof without words” relating the formulas for and , a.k.a Nichomachus’ Theorem. Can you see the connection to the formula? Or can you figure out the formula from the picture?

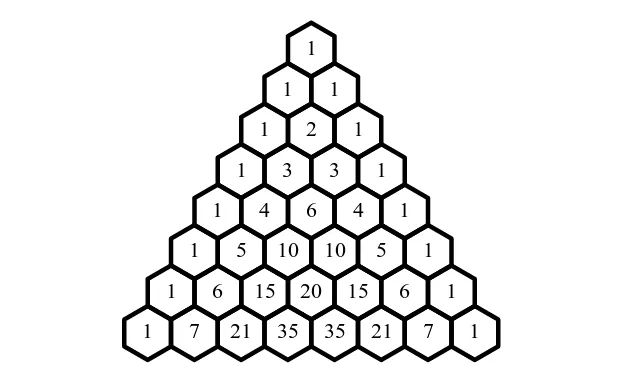
## Shape Numbers

The sequence of the sum of the first integers is are called the *triangular numbers.* This is because they can be shown to count the number of dots in increasingly large equilateral triangles. The sequence of sums of the triangular numbers is called the *tetrahedral numbers*. Can you find a formula for them?

There are also the more familiar *square numbers* and. What interesting formulas can you find with them?

And, of course, other shapes. Interesting ones to consider are the *hexagonal numbers* (and do they relate to the triangular numbers?) and the *pentagonal numbers* which have a nice closed form formula.

## Pascal’s Triangle

The familiar Pascal’s triangle is probably most associated with binomial expansions but it encodes a host of other interesting patterns. Look in the triangle below for the triangular and tetrahedral numbers? Can you explain why they are here? Can you find a formula that describes them?

It is well known that the sums of the rows of Pascal’s Triangle yield powers of two (check it out: ). Somewhat less well known is that a similar type of sum of Pascal’s Triangle yields the Fibonacci numbers: . Can you find it?