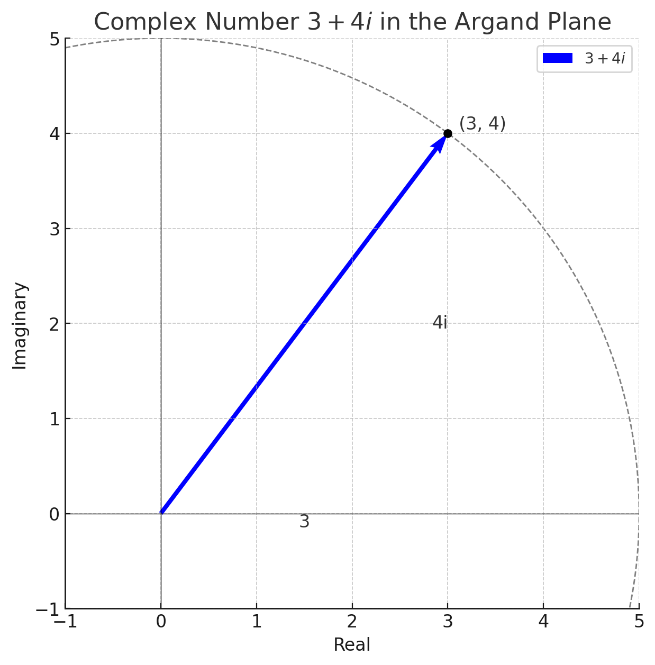
# Intro TO Complex Numbers

## Dangerously brief, curiously suspect, history of numbers

Leopold Kronecker, 19th century German mathematician, is credited with saying (in German)[[1]](#footnote-1) “Whole numbers were made by God, all else is the work of man.” If you think about it, whole numbers are great for counting but, one day, somebody had to solve an equation like and this required the invention of (discovery of?) rational numbers. These were great for a while until someone had to solve .[[2]](#footnote-2). So a new number, ◈, was born (they didn’t really use the symbol ◈ but we have to call it something.) So . The first algebraic number, (now called ) was soon found to be like and and it was discovered how to add and multiply these things together. And everyone was happy. Until some wise guy tried to solve . And then there was trouble.

## Complex Numbers

Just like a symbol (in our questionable history above) was created to indicate , a symbol was introduced to mean . That symbol is, of course . is the imaginary unit. Multiples of are imaginary numbers. And when imaginary and real numbers are added togther, a complex number is born. A complex number is a number that can be expressed in the form , where and are real numbers, and is the imaginary unit with the property that . The real part of the complex number is , and the imaginary part is . Complex numbers extend the concept of one-dimensional number lines to a two-dimensional complex plane, and a whole new world of mathematical possibilities arises.

**Example** The complex number has a real part of 3 and an imaginary part of 4.

A complex number

In the complex plane, also known as the Argand plane, complex numbers are represented as points or vectors. The horizontal axis represents the real part, and the vertical axis represents the imaginary part.

**Example** The complex number is represented as a point (3, 4) in the complex plane, or as a vector from the origin (0, 0) to the point (3, 4).

## Basic Operations

### Addition and Subtraction

Adding or subtracting complex numbers involves combining their real parts and their imaginary parts separately.

**Example - Addition** Given and , .

**Example - Subtraction** Given and , .

### Magnitude (Modulus)

The magnitude (or modulus) of a complex number is the distance from the origin to the point in the complex plane, calculated as .

**Example** For , Magnitude .

### Direction (Argument)

The direction (or argument) of a complex number is the angle formed with the positive real axis. It can be found using the arctan function. Be aware that this is the same process as converting rectangular coordinates to polar coordinates. You will need to be aware of the correct quadrant for your angle.

**Example** Find the argument of the complex number . **Solution** Argument .

**Example** Find the argument of the complex number . **Solution** Argument . But the original point is in Quadrant II and this angle is Quadrant IV. To fix, we add radians: .

### Multiplication

Multiplication can be performed by treating complex number as binomials and using the fact that . Here’s a detailed example. Consider two complex numbers: - -

To multiply these complex numbers, we multiply the binomials using the distributive property:

We can combine like terms and use to simplify:

So, the product .

### Division

To divide one complex number by another, you essentially perform multiplication by the reciprocal of the divisor, just as with real numbers. The key to simplifying such division is to eliminate the imaginary part from the denominator, which is achieved by multiplying both the numerator and the denominator by the conjugate of the denominator.

The **conjugate** of a complex number is . Multiplying a complex number by its conjugate results in a real number, specifically , since . This is the same term you have seen applied to radicals: the conjugate of is because the product of these two numbers is rational

Given two complex numbers, and , to find , follow these steps:

1. **Find the conjugate** of the denominator , which is .
2. **Multiply** both the numerator and the denominator by this conjugate.
3. **Simplify** the resulting expression to get the quotient in standard form.

**Example** Divide by . **Solution:**

1. **Conjugate of** : The conjugate of is .
2. **Multiply**: Multiply both and by :
3. **Simplify**:

* Finally, divide each part by 13:

**Example** Divide by . **Solution:**

1. **Conjugate of** : The conjugate is .
2. **Multiply**:
3. **Simplify**:

* Simplifying further:

## Checkpoint

1. **Addition**: Find .
2. **Subtraction**: Calculate .
3. **Multiplication**: Multiply by .
4. **Division**: Divide by .
5. **Conjugate**: Find the conjugate of .
6. **Magnitude**: Calculate the magnitude of .
7. **Argument**: Find the argument of in radians.
8. **Convert**: Find a complex number with magnitue 10 and argument

1. “Die ganzen Zahlen hat der liebe Gott gemacht, alles andere ist Menschenwerk” [↑](#footnote-ref-1)
2. We believe they were finding the diagonal of a unit square [↑](#footnote-ref-2)