Here’s a new vector math test tailored to your specifications, utilizing both bracket notation and unit vector notation . The test progresses through addition and subtraction, includes combined operations, moves on to magnitudes and unit vectors, tests understanding of vector direction, requires constructing vectors from given lengths and directions, assesses the concept of perpendicularity, and concludes with finding specific vectors relative to another.

### Vector Math Test

#### Adding and Subtracting Vectors

1. **Addition in Bracket Notation:** Given and , find .
2. **Subtraction in Notation:** If and , find .
3. **Addition in 3D:** Given and , find .
4. **Subtraction in 3D Notation:** If and , find .

#### Combined Operations

1. **Combining Operations with 3 Terms:** Given , , and , find .
2. **Combining Operations with 4 Terms in 3D:** For , , , and , calculate .

#### Magnitude of Vectors

1. **Magnitude in 2D:** Find the magnitude of .
2. **Magnitude in 3D:** Calculate the magnitude of .

#### Unit Vectors

1. **Unit Vector in 2D:** Find a unit vector in the direction of .
2. **Unit Vector in 3D:** Determine a unit vector in the direction of .

#### Vector Direction

1. **Direction of a Vector:** Given , find the direction of in radians.

#### Constructing Vectors from Length and Direction

1. **Vector from Length and Direction in 2D:** Construct a vector with a length of 5 units and a direction of 45 degrees from the positive x-axis.
2. **Rotating vectors**: Given vector $ =

#### Perpendicularity

1. **Are These Vectors Perpendicular?** Given and , determine if and are perpendicular.
2. **Are These Vectors Perpendicular in 3D?** For and , determine if they are perpendicular.

#### Specific Vectors Relative to Another

1. **Vector Perpendicular to with Length 5:** Given , find a vector that is perpendicular to and has a magnitude of 5.
2. **Vector Parallel to with Length 5:** Given , find a vector that is parallel to and has a magnitude of 5.

### Vector Word Problems

#### Problem 18: Adding Forces in Engineering

A crane is lifting a heavy load with two cables. The first cable applies a force of 400 N directly upwards, represented by N. The second cable applies a force at an angle of 30 degrees to the horizontal, with a magnitude of 300 N.

**a)** Represent the second force, , in component form using and .

**b)** Calculate the total force exerted by the two cables on the load in component form.

**c)** Determine the magnitude and direction of the total force.

#### Problem 19: Plane Flying in the Wind

A plane aims to fly directly north at an airspeed of 250 km/h. However, there’s a wind blowing from the west at 60 km/h.

**a)** Represent the plane’s velocity as a vector, , and the wind’s velocity as a vector, , in component form.

**b)** Calculate the plane’s actual velocity relative to the ground, , taking into account the wind’s effect.

**c)** Determine the plane’s actual speed and direction of travel relative to the ground.

#### Problem 20: Boat Going Down a River

A boat wants to cross a river that is 500 meters wide. The river’s current flows southward at a speed of 3 km/h, and the boat can travel at a speed of 8 km/h relative to the water.

**a)** If the boat aims to reach the exact opposite point on the other bank, at what angle (to the west of north) must it head?

**b)** Represent the boat’s intended velocity in the water, , and the river’s current, , in component form.

**c)** Calculate how long it will take for the boat to cross the river and the actual distance downstream it will have landed from its starting point.

For these problems, students are encouraged to use vector addition to find resultant vectors, decompose vectors into their components for calculation, and apply trigonometry to find magnitudes and directions. These practical scenarios not only test their understanding of vector arithmetic but also illustrate the real-world applicability of vectors in physics and engineering.