# Applications of Vectors

First we include 2 incredibly important vector applications, the dot product and the cross product. Following this are a number of problems.

### Dot Product (Scalar Product)

* **Definition**: The dot product between two vectors and is a scalar quantity that is the result of multiplying the magnitudes of the two vectors by the cosine of the angle () between them. It is denoted as .
* **Formula**: where and are the magnitudes of vectors and , respectively.
* **Component-wise Calculation**: For vectors in 2D Cartesian coordinates, and , the dot product is . In 3D Cartesian coordinates, and , the dot product is . In any dimension, the dot product is simply the sum of the component-wise products of the 2 vectors.
* **Properties**:
  + Commutative:
  + Distributive (over ):
  + Scalar Result: The result is a scalar (not a vector).
  + is not defined since the dot product is a scalar.
* **Applications**: Dot product is used to find the angle between two vectors, project one vector onto another, and in various physics calculations like work. One handy application is that and are perpendicular if and only if their dot product is 0. Another is that . In word, the dot product of a vector with itself is its squared magnitude.

### Cross Product (Vector Product)

* **Definition**: The cross product between two 3D vectors and is a vector that is perpendicular to the plane containing and . It is denoted as . It is only defined in three dimensions.
* **Formula**: , where is the angle between and , and is a unit vector perpendicular to the plane containing and .
* **Component-wise Calculation**: For vectors in Cartesian coordinates, and , the cross product is .
* **Properties**:
  + Commutative: ? (TBD)
  + Distributive:
  + Vector Result: The result is a vector.
* **Applications**: The main application is to find a vector perpendicular to 2 given vectors. Cross product can also be used to find the area of a parallelogram formed by two vectors, and in physics to compute torque, and the relationship between electric and magnetic fields.

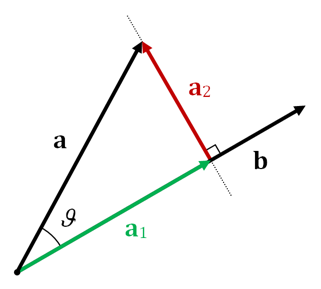
### Practice Problems

1. Given vectors and , find the dot product .
2. Given vectors and , find the angle between and .
3. Given vectors and , determine if and are perpendicular.
4. (\*) Given vector , find a vector that is in the xy-plane and makes a angle with .
5. Let be any 2D vector in the plane and let prove that the angle between and is .
6. (\*) Use the result of the previous problem to prove the addition law of cosines.
7. Rewrite as a matrix times a vector.
8. Given vectors and , find the cross product .
9. The cross product can be defined as a matrix determinant. Look up the formula and write it down.
10. Given vectors and , determine if and are parallel by using the cross product.
11. Given vectors and , find the area of the parallelogram spanned by and .
12. Given vectors and , find the area of the triangle formed by , , and the origin.
13. Use vectors to find the area of the triangle with vertices at and (6,-4)$
14. Pick two vectors and and describe the relationship between and .
15. What is ?
16. What is ?
17. What is ?
18. (\*) Write as a Linear Combination of , , . Given the three 3D vectors , , and defined as follows:

* And given vector , write as a linear combination of vectors , , and . In other words, find scalars , , and such that: . (hint:) Use matrices (and your calculator is helpful!).

1. Explain geometrically when a problem like 18 can’t be solved.
2. Explain algebraically when a problem like 18 can’t be solved.
3. Find a vector parallel to the line and a vector perpendicular to the line
4. (\*\*) To find the distance from a point to a line defined by the equation , you can use the following formula:

Prove this formula using dot products. (It is helpful to write a line as the set of points for any real constant .)

1. The projection of onto is shown in the diagram on the right as the vector . Derive a formula for .
2. A boat wishes sail due east across a north-south flowing river. The river current is moving 20 knots at an angle of 10 degrees east of north while and the wind is blowing 15 knots towards an angle of 25 degrees north of west. Which bearing and what engine speed should the boat maintain to achieve its heading while traveling at a speed of 18 knots?
3. A boat is being pulled by three tugboats. Tugboat A is pulling with a force of 100 Newtons directly north. Tugboat B is pulling with a force of 50 Newtons at an angle of 30° to the east of north. Tugboat C is pulling with a force of 75 Newtons at an angle of 45° to the west of north. Calculate the magnitude and direction of the resultant force acting on the boat.