Ejercicios de optimización de funciones

Encontrar un máximo o un mínimo local

7.3 Locate the minimum of the function

$f(x) = 3 + 6x + 5x^2 + 3x^3 + 4x^4$

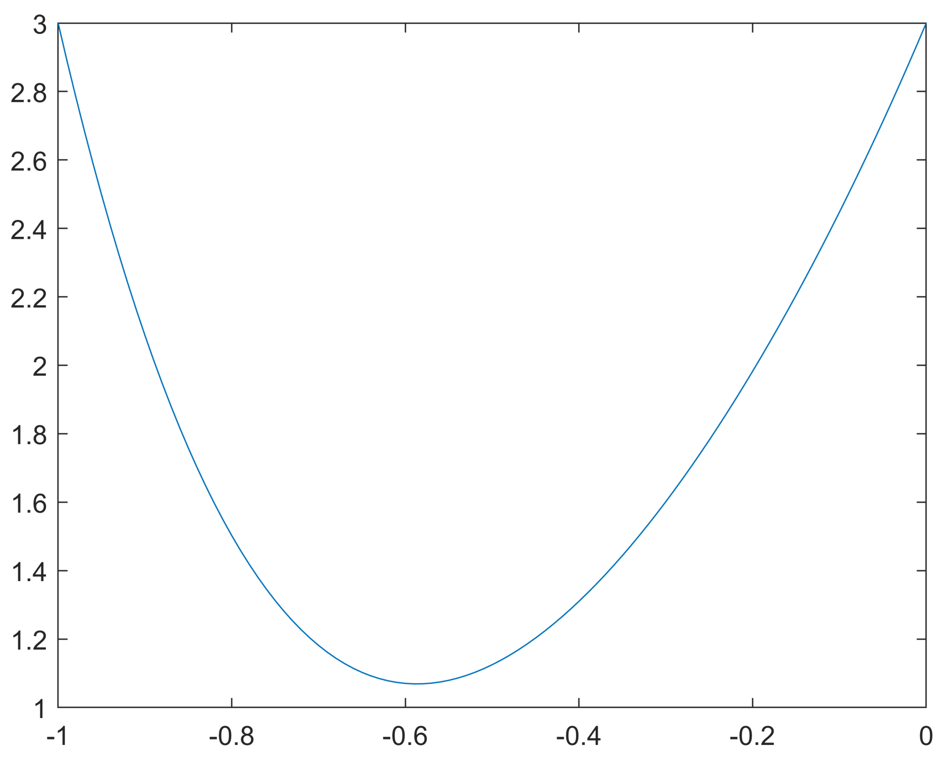
```
f = @(x) 3 + 6*x + 5*x.^2 + 3*x.^3 + 4*x.^4
```

```
f = function_handle with value:
    @(x)3+6*x+5*x.^2+3*x.^3+4*x.^4
```

```
min = nROpt(f, -1, 1)
```

```
min = -0.5867
```

```
x = -1:0.01:0;
plot(x, f(x))
```



7.2 Determine the maximum and the corresponding value of x for the function

$f(x) = -x^2 + 8x - 12$

```
f = @(x) -x.^2 + 8*x -12
```

```
f = function_handle with value:
    @(x)-x.^2+8*x-12
```

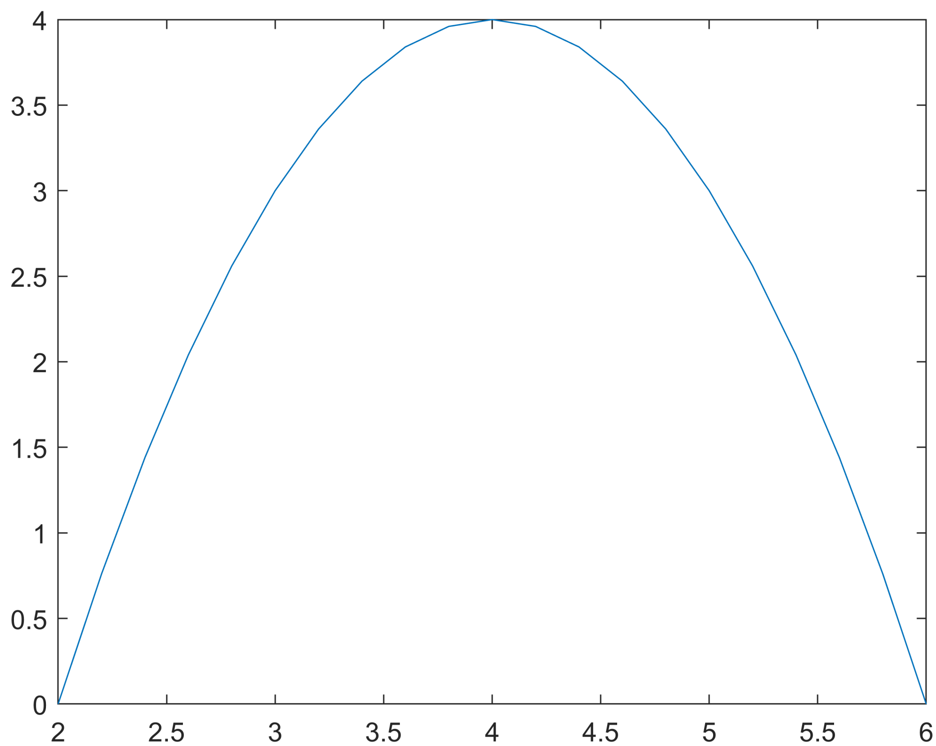
```
max = nROpt(@(x) -f(x), -1, 1)
```

```
max = 4
```

```
fmax = f(max)
```

```
fmax = 4
```

```
x = 2:0.2:6;
plot(x, f(x))
```



7.4 Given

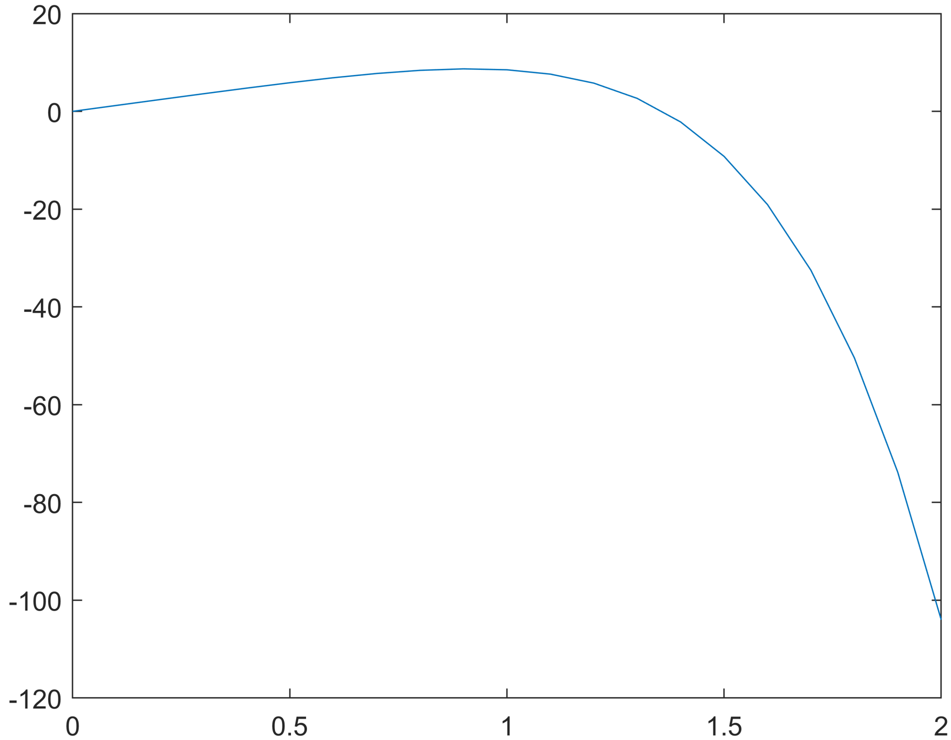
$$f(x) = -1.5x^6 - 2x^4 + 12x$$

(a) Plot the function.

```
f = @(x) -1.5*x.^6 - 2*x.^4 + 12*x
```

```
f = function_handle with value:
    @(x)-1.5*x.^6-2*x.^4+12*x
```

```
x = 0:0.1:2;
plot(x, f(x))
```



(b) Use analytical methods to prove that the function is concave for all values of *x*.

Si f es dos veces diferenciable, entonces f es estrictamente cóncava si y solo si f'' no es positiva:

Función f a derivar: $f(x) = -1.5x^6 - 2x^4 + 12x$

```
df = diff(sym(f)) % primera derivada
```

df = $-9\,x^5 - 8\,x^3 + 12$

```
ddf = diff(diff(sym(f))) % segunda derivada
```

ddf = $-45\,x^4 - 24\,x^2$

Como $f'' < 0 \ \ \forall\, x \Rightarrow f$ es estrictamente cóncava.

(c) Find the maximum *f*(*x*) and the corresponding value of *x*.

```
max = goldenSS(@(x) -f(x), -10, 10)
```

```
max = 0.9169
```

```
fmax = f(max)
```

```
fmax = 8.6979
```

Código de las funciones

Golden section search

```
function [x, i] = goldenSS(f, a, b)
    REL_TOL = sqrt(eps);
    MAX_ITER = 53;
    x = (a + b) / 2;
    g = (1 + sqrt(5)) / 2 - 1;
    dist = g * (b - a);

    x1 = a + dist;
    fx1 = f(x1);
    x2 = b - dist;
    fx2 = f(x2);

    i = 0;
    flag = true;
    while flag
        dist = g * dist;
        if fx1 < fx2
            a = x2;
            x2 = x1;
            fx2 = fx1;
            x1 = a + dist;
            fx1 = f(x1);
        else
            b = x1;
            x1 = x2;
            fx1 = fx2;
            x2 = b - dist;
            fx2 = f(x2);
        end
        x = (a + b) / 2;
        i = i + 1;
        flag = i < MAX_ITER && abs((b - a) / x) > REL_TOL;
    end
end
```

Newton optimization

```
function [x, i, m] = nROpt(f, a, b)
    REL_TOL = sqrt(eps);
    MAX_ITER = 53;
    x = (a + b) / 2;
    df = diff(sym(f));
    ddf = matlabFunction(diff(df));
    df = matlabFunction(df);

    if nargin(ddf) == 0
        ddf = @(x) ddf();
    end

    i = 0;
    flag = true;
```

```
    while flag
        xp = x;
        x = xp - df(xp) / ddf(xp);
        i = i + 1;
        flag = i < MAX_ITER && abs((x - xp) / x) > REL_TOL;
    end
    m = ddf(x) > 0;
end
```