Números complejos

N_1 Naturales (1,2) contar 50,000 aC (marcas en huesos)

N_0 Naturales (0,1,2)

Z Enteros (-2,-1-0,1,2,3)

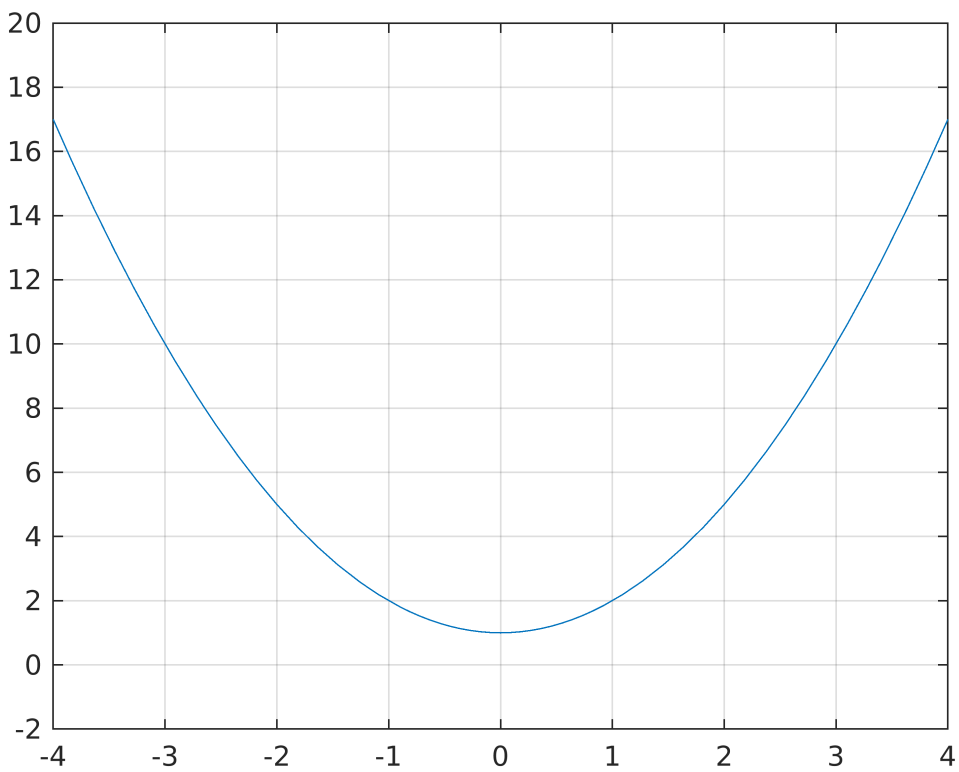
Racionales (3/2)

R Reales (pi, e)

C Complejos ($x^2 + 1 = 0;\ x = \sqrt{-1} = \pm i$)

Números multidimensionales: vectores y matrices

```
y = @(x) x.^2 + 1;
fplot(y,[-4,4]);
grid on
axis([-4,4,-2,20]);
```




```
p= [1,0,1];
raices = roots(p)
```

```
raices = 2×1 complex
   0.0000 + 1.0000i
   0.0000 - 1.0000i
```

The fundamental theorem of algebra, combined with the factor theorem (a polynomial f(x) has a factor (x-k) if and only if f(k)=0), states that a polynomial p of degree n has n roots in the complex plane, if they are counted with their multiplicities.

```
p = [1, -2, -1, 2];  % x^3 -2*x^2 -x + 2 = 0
x = roots(p)
```

```
x = 3×1
   -1.0000
    2.0000
    1.0000
```

```
p = [1,2,-7,4];    % (x-1)^2*(x+4)=(x^2-2*x+1)(x+4)= x^3+2*x^2-7*x+4
x = roots(p)
```

```
x = 3×1 complex
  -4.0000 + 0.0000i
   1.0000 + 0.0000i
   1.0000 - 0.0000i
```

The complex conjugate root theorem states that if the coefficients of a polynomial are real, then the non-real roots appear in pairs of the type $a \pm i\, b$

```
p = [1, -2, 5]       % x^2 - 2*x + 5 = 0
```

```
p = 1×3
     1    -2     5
```

```
x = roots(p)
```

```
x = 2×1 complex
   1.0000 + 2.0000i
   1.0000 - 2.0000i
```

Funciones

Construcción

```
z = 3 + 2i
```

```
z = 3.0000 + 2.0000i
```

```
z = 3 + 2j
```

```
z = 3.0000 + 2.0000i
```

```
z = 3 + j*2
```

```
z = 3.0000 + 2.0000i
```

```
z = 3 + 2*j
```

```
z = 3.0000 + 2.0000i
```

```
z = complex(3,2)
```

```
z = 3.0000 + 2.0000i
```

Descomposición

```
parteReal = real(z)
```

```
parteReal = 3
```

```
parteImaginaria = imag(z)
```

```
parteImaginaria = 2
```

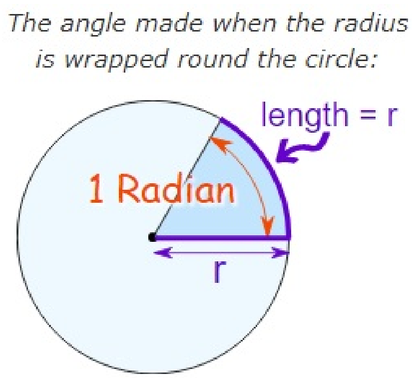
```

r = abs(z)           % complex magnitud
```

```
r = 3.6056
```

```
theta = angle(z)     % phase angle [-pi,pi] radians
```

```
theta = 0.5880
```



https://www.mathsisfun.com/geometry/radians.html

2*pi radianes = 360 grados

Conjugados

```
zC = z'              % (complejo) conjugado
```

```
zC = 3.0000 - 2.0000i
```

```
zC = conj(z)
```

```
zC = 3.0000 - 2.0000i
```

```

suma = z + z'        % = 2*real(z)
```

```
suma = 6
```

```
resta = z - z'       % = 2*i*imag(z)= -2*i*imag(z')
```

```
resta = 0.0000 + 4.0000i
```

```
producto = z*z'      % = magnitud^2 (operacion conmutativa)
```

```
producto = 13
```

```
inverso = 1/z
```

```
inverso = 0.2308 - 0.1538i
```

```
inverso = z'/r^2
```

```
inverso = 0.2308 - 0.1538i
```

de Moivre's formula

Llinks complex numbers and trigonometry

```
z = r * exp(i*theta)
```

```
z = 3.0000 + 2.0000i
```

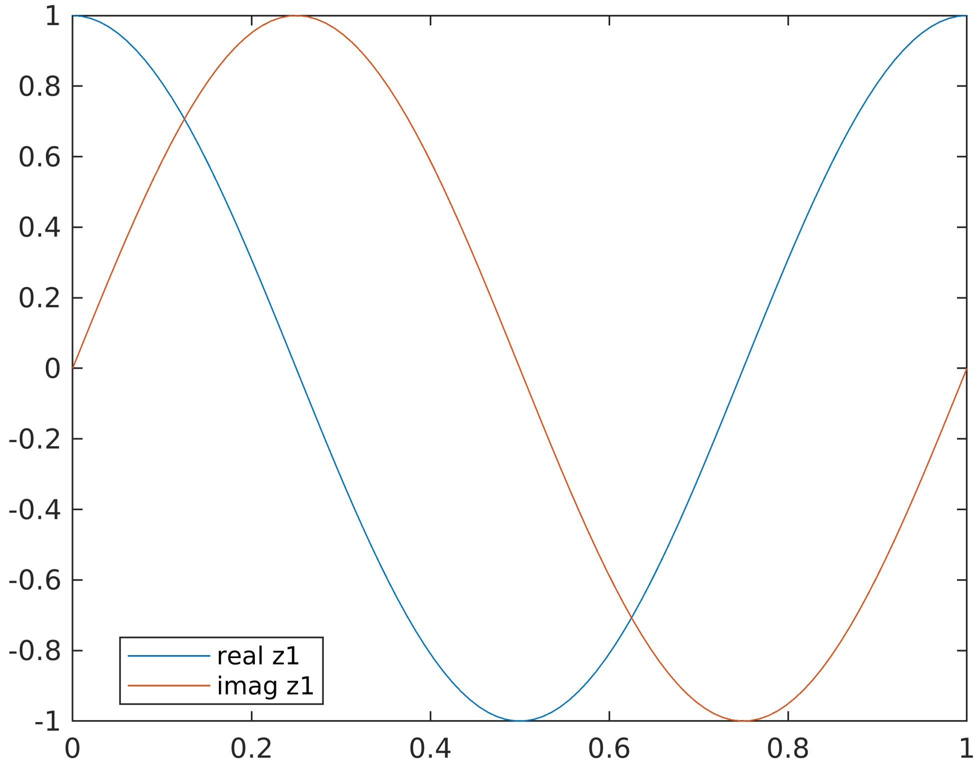
```
z = r*( cos(theta) + i*sin(theta) )
```

```
z = 3.0000 + 2.0000i
```

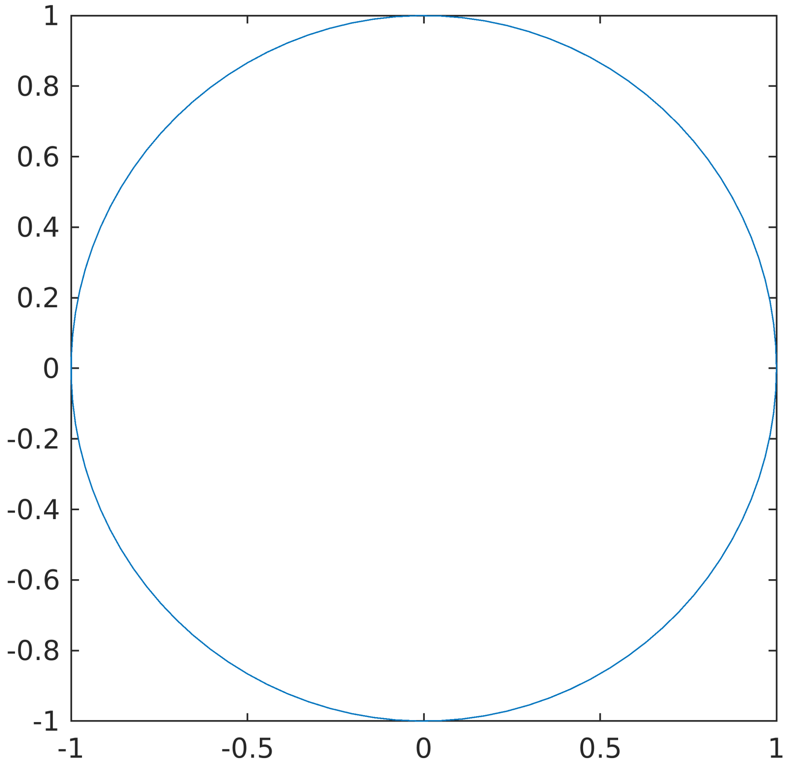
Armónicos en función del tiempo

Generemos números complejos cuyo ángulo varía en función del tiempo. Evaluemos los 2 primeros armónicos, correspondientes a frecuencias de 1 y 2 ciclos / s.

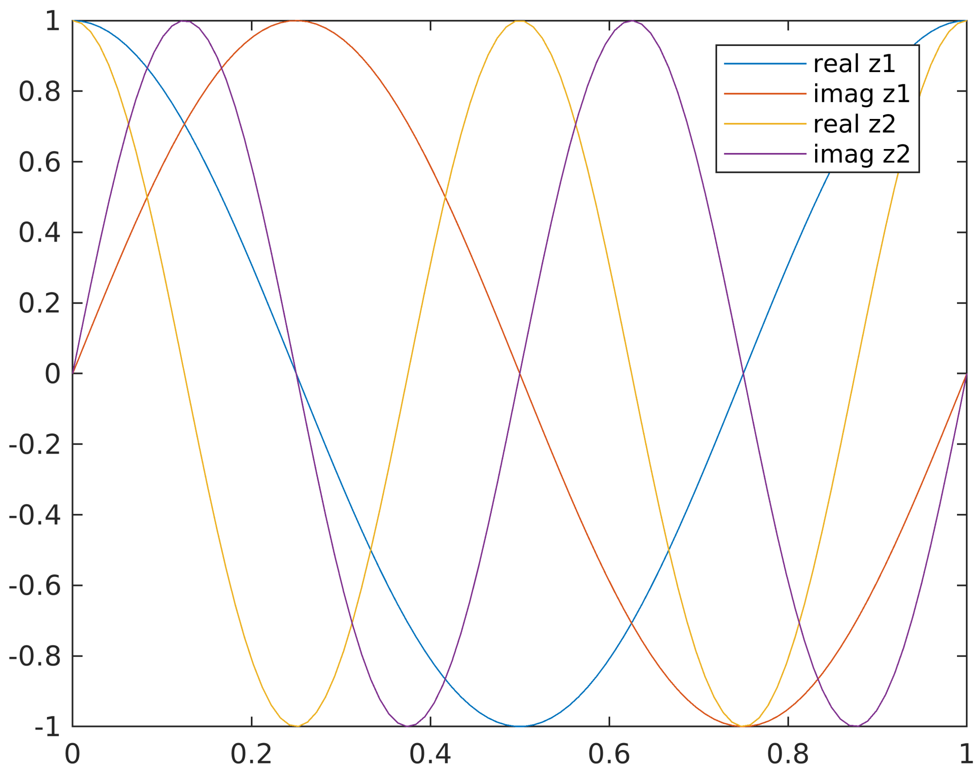
```
% r*exp(i*theta(t));
% theta = w*t (w es la velocidad angular rad/s)
% theta = 2*pi*f*t (f es la frecuencia ciclos/s)
% r=1; theta=n*(2*pi*t); n=1,2
% exp(i*n*(2*pi*t)) = exp(n*i*(2*pi*t))
% f = 1 Hz
% Graficamos 100 puntos hasta t=1s.
t=linspace(0,1);
z1 = exp(1i*(2*pi*t));
plot(t,real(z1),t,imag(z1));
legend('real z1','imag z1',"Location","best");
```




```
plot(real(z1),imag(z1))
axis equal square;
```




```
plot(t,real(z1),t,imag(z1));
hold on;
% f = 2 Hz
z2 = exp(2i*(2*pi*t));
plot(t,real(z2),t,imag(z2));
legend('real z1','imag z1','real z2','imag z2',"Location","best");
hold off;
```



Multiplicación y potencia

```
z
```

```
z = 3.0000 + 2.0000i
```

```
% (3+2i)*(3+2i) = 3*3 + 3*2+i + 2*i*3 + 2*2*i*i = 9 + 12i - 4 = 5 + 12i
za2 = z*z
```

```
za2 = 5.0000 + 12.0000i
```

```
% (3+2i)^2 = 3^2 + 2*3*2i + (2i)^2 = 9 + 12i - 4 = 5 + 12i
za2 = z^2
```

```
za2 = 5.0000 + 12.0000i
```

```
r = abs(z);
theta = angle(z);
za2 = r^2 * exp(j*2*theta)
```

```
za2 = 5.0000 + 12.0000i
```

Raíces n-ésimas

A real number or complex number has n complex roots of degree n.

While the roots of 0 are not distinct (all equaling 0), the n nth roots of any other real or complex number are all distinct.

If x is real

If n is even

If x is positive, one of its nth roots is positive, one is negative, and the rest are not real

If x is negative, none of the nth roots is real

If n is odd, one nth root is real and has the same sign as x, while the other roots are not real

If x is not real, then none of its roots is real

```
% z = r*exp(j*theta) = r*(cos(theta) + i*sin(theta))
% w = z^(1/n)   n-valued    n natural
% w = R*exp(j*phi) = R*(cos(phi) + i*sin(phi))
% w^n = (R^n)*exp(j*n*phi)
%     = z = r*exp(j*theta)
% The absolute values on both (last) rows must be equal; thus
% R^n = r y R = r^(1/n) (nonnegative)
% Equating the arguments n*phi and theta and recalling that theta is
% determined only up to integer multiples of 2pi, we obtain
% n*phi = theta + 2*pi*k and thus phi = (theta+2*pi*k)/n
% For k=0,1,...,n-1 we get n distinct values of w
% z^(1/n) = R^(1/n)*exp(j*(theta + 2*pi*k)/n); k=0,1,...,n-1
% The n values lie on a circle of radius r^(1/n) with center at the
% origin and constitute the vertices of a regular polygon of n sides.

% n = 2;
z = 3 + 2j;
za12 = z^(1/2)
```

```
za12 = 1.8174 + 0.5503i
```

```
za12^2
```

```
ans = 3.0000 + 2.0000i
```

```
r = abs(z);
theta = angle(z);
k = 0:1;  % k=0,1,n...,n-1
za12 = r^(1/2) * exp(j*(theta + 2*pi*k)/2)
```

```
za12 = 1×2 complex
   1.8174 + 0.5503i  -1.8174 - 0.5503i
```

```
za121 = r^(1/2) * exp(j*theta/2);            % k=0
za121^2
```

```
ans = 3.0000 + 2.0000i
```

```
za122 = r^(1/2) * exp(j*(theta + 2*pi)/2);   % k=1
za122^2
```

```
ans = 3.0000 + 2.0000i
```

Raíces n-ésimas de 1 (nth roots of unity)

```
z = 1;
% w = z^(1/n) = R^(1/n)*exp(j*(theta + 2*pi*k)/n); k=0,1,...,n-1
r = abs(1);          % 1 -> R=1
theta = angle(1);    % 0 -> phi = (2*pi*k)/n; k=0,...n-1
% w = exp(j*2*pi*k)/n); k=0,1,...,n-1
% For k=0,1,...,n-1
for n=2:5
    for k=0:n-1
        rnk1 = exp(j*(2*pi*k)/n);
        pre = real(rnk1);
        if abs(pre)<eps, pre=0; end
        pim = imag(rnk1);
        if abs(pim)<eps, pim=0; end
        fprintf('n=%d, k=%d, raiz = %d + j%d\n',n,k,pre,pim);
        % Graficarlos
    end
end
```

```
n=2, k=0, raiz = 1 + j0
n=2, k=1, raiz = -1 + j0
n=3, k=0, raiz = 1 + j0
n=3, k=1, raiz = -5.000000e-01 + j8.660254e-01
n=3, k=2, raiz = -5.000000e-01 + j-8.660254e-01
n=4, k=0, raiz = 1 + j0
n=4, k=1, raiz = 0 + j1
n=4, k=2, raiz = -1 + j0
n=4, k=3, raiz = 0 + j-1
n=5, k=0, raiz = 1 + j0
n=5, k=1, raiz = 3.090170e-01 + j9.510565e-01
n=5, k=2, raiz = -8.090170e-01 + j5.877853e-01
n=5, k=3, raiz = -8.090170e-01 + j-5.877853e-01
n=5, k=4, raiz = 3.090170e-01 + j-9.510565e-01
```

Raíces n-ésimas (continuación)

nthroot

```
% If w denotes the value corresponding to k=1, then the values of
% 1^(1/n) can be written as
% 1,w,w^2,...,w^(n-1)
% More generally, if a1 is any root of an arbitrary complex number z (~=0)
% then the values of z^(1/n) are
% a1,a1*w,a1*w^2,...,a1*w^(n-1)
% because multiplying a1 by w^k corresponds to increasing the argument of
% a1 by 2*pi*k/n.

n = 3;
zR = 8;
a1 = nthroot(zR,n)
```

```
a1 = 2
```

```
k = 1;
w1 = exp(j*(2*pi*k)/n)
```

```
w1 = -0.5000 + 0.8660i
```

```
a1w1 = a1*w1
```

```
a1w1 = -1.0000 + 1.7321i
```

```
(a1*w1)^n
```

```
ans = 8.0000 - 0.0000i
```

```
a1w12 = a1*w1^2
```

```
a1w12 = -1.0000 - 1.7321i
```

```
(a1*w1^2)^n
```

```
ans = 8.0000 - 0.0000i
```

```
n = 3;
zNR = 3 + 2j;
pre = real(zNR);
pim = imag(zNR);
rNR = abs(zNR);
thetaNR = angle(zNR);
R = nthroot(rNR,n)
```

```
R = 1.5334
```

```
root0 = R*exp(j*(thetaNR + 2*pi*0)/n)    % a1
```

```
root0 = 1.5040 + 0.2986i
```

```
root0^n
```

```
ans = 3.0000 + 2.0000i
```

```
root1 = R*exp(j*(thetaNR + 2*pi*1)/n)
```

```
root1 = -1.0106 + 1.1532i
```

```
root1^n
```

```
ans = 3.0000 + 2.0000i
```

```
root2 = R*exp(j*(thetaNR + 2*pi*2)/n)
```

```
root2 = -0.4934 - 1.4519i
```

```
root2^n
```

```
ans = 3.0000 + 2.0000i
```

```
k = 1;
w1 = exp(j*(2*pi*k)/n);
root1 = root0*w1
```

```
root1 = -1.0106 + 1.1532i
```

```
root2 = root0*w1^2
```

```
root2 = -0.4934 - 1.4519i
```

La ecuación más hermosa en Matemáticas

$e^{i\pi} + 1 = 0$

```
exp(i*pi)  % cos(pi) + i*sen(pi) = -1 + 0i
```

```
ans = -1.0000 + 0.0000i
```