

Robertson-Walker Metric:

$$ds^2 = -c^2 dt^2 + a(t)^2 [dr^2 + S_\kappa^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

$$S_\kappa(r) = R \sin(r/R), \kappa = +1$$

$$S_\kappa(r) = r, \kappa = 0$$

$$S_\kappa(r) = R \sinh(r/R), \kappa = -1$$

$$\text{Friedmann Equation: } \left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3c^2} \epsilon(t) = -\frac{\kappa c^2}{a(t)^2 R_0^2} + \frac{\Lambda}{3}$$

$$\text{Friedmann Equation: } \frac{H^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

$$\text{Blackbody Radiation: } \epsilon_\gamma = \alpha_{RAD} T^4, \lambda_{max} T = 0.2898 \text{ cm K}$$

$$\text{Fluid Equation: } \dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$$

$$\text{Acceleration Equation: } \frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P) + \frac{\Lambda}{3}$$

$$\text{Equation of State: } P = w\epsilon$$

$$\text{Wien's Law: } \lambda_{max} T = 0.2898 \text{ cm K}$$

$$\text{Radiation Energy Density: } \epsilon_{rad} = \alpha_{rad} T^4$$

$$\text{Deceleration Parameter: } q_0 \equiv -\left(\frac{\ddot{a}a}{\dot{a}^2}\right)_{t=t_0}$$

$$q_0 = \frac{1}{2} \sum_w \Omega_{w,0} (1 + 3w)$$

$$\text{Luminosity Distance: } d_L \equiv \sqrt{\frac{L}{4\pi F}}$$

$$\text{Angular Diameter Distance: } d_A \theta \equiv l$$

$$d_L = S_k(r)(1+z)$$

$$d_A = S_k(r)/(1+z)$$

$$d_p(t_0) \approx \frac{c}{H_0} \left[z - \left(\frac{1+q_0}{2} \right) z^2 \right] + \frac{cH_0}{2} \frac{z^2}{H_0^2}$$

$$d_L \approx \frac{c}{H_0} z \left(1 + \frac{1-q_0}{2} z \right)$$

$$d_A \approx \frac{c}{H_0} z \left(1 - \frac{3+q_0}{2} z \right)$$

$$\text{Virial Theorem: } \ddot{I} = 2W + 4K$$

$$\text{Kinetic Energy: } K = \frac{1}{2} M \langle v^2 \rangle$$

$$\text{Potential Energy: } W = -\alpha \frac{GM^2}{r_h}, \text{ where } \alpha \approx 0.4 \text{ for galaxy clusters}$$

$$\text{Rotation Speed: } v_c = \sqrt{\frac{GM(R)}{R}}$$

$$\text{Hydrostatic Equilibrium: } M(r) = \frac{kT(r)r}{G\mu} \left[-\frac{d \ln \rho}{d \ln r} - \frac{d \ln T}{d \ln r} \right]$$

$$\text{Deflection Angle: } \alpha = \frac{4GM}{c^2 b}$$

$$\text{Einstein Radius: } \theta_E = \left(\frac{4GM}{c^2 d} \frac{1-x}{x} \right)^{\frac{1}{2}}$$

Ionization Fraction:

$$\frac{1-x}{x^2} = 3.84\eta \left(\frac{kT}{m_e c^2} \right)^{\frac{3}{2}} \exp \left(\frac{Q}{kT} \right)$$

Neutron to proton ratio:

$$\frac{n(n)}{n(p)} = \exp(-1.29 \text{ MeV}/(kT))$$

Deuteron to neutron ratio:

$$\frac{n(D)}{n(n)} \approx 6.5\eta (kT/(m_n c^2))^{3/2} \exp(2.22 \text{ MeV}/(kT))$$

$$\text{Dynamical Timescale (free fall): } t_{D_{yn}} = \sqrt{\frac{1}{4\pi G \bar{\rho}}}$$

$$\text{Jeans Length: } \lambda_J = 2\pi c_s t_{dyn}$$

$$\text{Jeans Mass: } M_J = \rho_b \frac{4\pi}{3} \lambda_J^3$$

$$\text{Sound Speed: } c_s = \sqrt{w c}$$

$$\text{Sound Speed (baryons): } c_s = \sqrt{\frac{kT}{m c^2}} c$$

$$\text{Ionization rate: } \Gamma(z) = n_e(z) \sigma_e c$$

$$\text{Optical depth: } \tau(t) = \int_t^{t_0} \Gamma(t) dt$$

$$\text{Linear Growth: } \ddot{\delta} + 2H\dot{\delta} - \frac{3}{2}H^2\Omega_m(t)\delta = 0, \text{ where } \delta \equiv \frac{\epsilon - \bar{\epsilon}}{\bar{\epsilon}}$$

$$\text{Mass Fluctuations \& Power Spectrum: } \langle \left(\frac{\delta M}{\langle M \rangle} \right)^2 \rangle \approx P(k) k^3$$

Luminosity Function:

$$\phi(L) dL = \phi^*(L/L^*)^\alpha \exp(-L/L^*) dL/L^*$$

Halo Radius:

$$R_h = (3M_T / (64\pi \rho_{m,0} (1+z_c)^3))^{1/3}$$

Halo Gas Temperature:

$$T \approx 1 \times 10^6 \text{ K} (M_T / 10^{12} \text{ M}_\odot)^{2/3} ((1+z_c)/5)$$

$$\text{Cooling time: } t_c \approx 13 \text{ Gyr} (10^{-27} \text{ g cm}^{-3} / \rho_{gas}) (T / 10^6 \text{ K})^{1/2}$$

Fundamental Constants

gravitational constant: $G = 6.673 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$

speed of light: $c = 2.998 \times 10^8 \text{m s}^{-1}$

reduced Planck constant:

$$\hbar = 1.055 \times 10^{-34} \text{J s} = 6.582 \times 10^{-16} \text{eV s}$$

Boltzmann constant:

$$k = 1.381 \times 10^{-23} \text{J K}^{-1} = 8.617 \times 10^{-5} \text{eV K}^{-1}$$

radiation constant:

$$\alpha_{RAD} = 7.56 \times 10^{-16} \text{J m}^{-3} \text{K}^{-4}$$

electron rest energy: $m_e c^2 = 0.5110 \text{MeV}$

proton rest energy: $m_p c^2 = 938.272 \text{MeV}$

neutron rest energy: $m_n c^2 = 939.566 \text{MeV}$

Conversion of Units

astronomical unit: $1 \text{AU} = 1.496 \times 10^{11} \text{m}$

megaparsec: $1 \text{Mpc} = 3.086 \times 10^{22} \text{m}$

solar mass: $1 \text{M}_\odot = 1.989 \times 10^{30} \text{kg}$

solar luminosity: $1 \text{L}_\odot = 3.846 \times 10^{26} \text{J s}^{-1}$

gigayear: $1 \text{Gyr} = 3.156 \times 10^{16} \text{s}$

electron volt: $1 \text{eV} = 1.602 \times 10^{-19} \text{J}$

Absolute Magnitude: $M = m - 5 \log \left(\frac{d_L}{1 \text{Mpc}} \right) - 25$

Cosmological Parameters

Hubble constant: $H_0 = 70 \pm 7 \text{km s}^{-1} \text{Mpc}^{-1}$

Hubble time: $H_0^{-1} = (4.4 \pm 0.4) \times 10^{17} \text{s} = 14.0 \pm 1.4 \text{Gyr}$

Hubble distance: $c/H_0 = (1.32 \pm 0.13) \times 10^{26} \text{m} = 4300 \pm 400 \text{Mpc}$

critical energy density: $\varepsilon_{c,0} = 5200 \pm 1000 \text{MeV m}^{-3}$

critical mass density: $\rho_{c,0} = \varepsilon_{c,0}/c^2 = (9.2 \pm 1.8) \times 10^{-27} \text{kg m}^{-3}$

density parameter in radiation: $\Omega_{r,0} = 8.4 \times 10^{-5}$

density parameter in baryons: $\Omega_{bary,0} = 0.04$

density parameter in dark matter: $\Omega_{m,0} = 0.26$

density parameter in dark energy: $\Omega_{\Lambda,0} = 0.70$

temperature of CMB: $T_0 = 2.73 \text{K}$

Table of Integrals

$$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a \arctan \frac{\sqrt{x(a-x)}}{x-a}$$

$$\int \frac{x}{\sqrt{x+a}} dx = \frac{2}{3}(x-2a)\sqrt{x+a}$$

$$\int \frac{x}{\sqrt{x-a}} dx = \frac{2}{3}(x+2a)\sqrt{x-a}$$