MATH 119A
Practice Final
Fall 2025

Name ______________________________ Student ID #______________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of California, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:______________________________

1	25	
2	25	
3	25	
4	25	
Total	100	

- Your exam should consist of this cover sheet, followed by 4 problems. Check that you have a complete exam.
- Pace yourself. You have 75 minutes to complete the exam and there are 4 problems, each problem has two pages. Try not to spend more than 20 minutes on each problem.
- Unless otherwise indicated, show all your work and justify your answers.
- Unless otherwise indicated, your answers should be exact values rather than decimal approximations. (For example, $\frac{\pi}{4}$ is an exact answer and is preferable to its decimal approximation 0.7854.)
- You may use one 8.5×11-inch sheet of handwritten notes. All other electronic devices (including graphing and programmable calculators and calculators with calculus functions) are forbidden.
- The use of headphones or earbuds during the exam is not permitted.
- You are not allowed to use your phone for any reason during this exam. Turn your phone off and put it away for the duration of the exam.
- You have signed an honor statement, and cheating is a hassle for everyone involved. DO NOT CHEAT.
- Raise your hand if you have any questions.
- Double check your work if you have time left.

GOOD LUCK!

1. (25 points) consider the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

   (a) (5 points) Find the eigenvalues of A and state the multiplicity for each

   (b) (5 points) Find a basis for $\mathbf{R}^3$ consists of generalized eigenvectors of A (eigenvectors are generalized eigenvectors)

(c) (5 points) Using the previous part, construct a matrix T such that $T^{-1}AT$ is of the form $D + N$, where D is a diagonal matrix with the eigenvalues on the diagonal, N is a nilpotent

(d) (5 points) Find the general solution of the system $X' = AX$

(e) (5 points) Find the stable, unstable and center subspace

2. (25 points) Homework 6 problem 6

3. (25 points) Homework 6 problem 8

4. Consider the following nonlinear system: $\dot{x} = xy - 1, \dot{y} = x - y^3$.

   (a) Find all fixed points and classify them using linearization

   (b) Determine if the linearization predicts the stability of the fixed points accurately (You may check the nonlinear system using nullcines, or use known theorems)

   (c) Sketch the phase portrait using the information from the previous parts

5. Repeat the same procedure in problem 4 for the system $\dot{x} = \sin(y), \dot{y} = x - x^3$

6. Consider the nonlinear system: $\dot{x} = y - ax, \dot{y} = -ay + \frac{x}{(1+x)}$.

   (a) Find the largest subset of $\mathbf{R}^2$ for which the uniquness and existence theorem applies

   (b) Find all fixed points for the case where $a = 0$ and $a \neq 0$.

   (c) For $a \neq 0$, Determine for each fixed point if the Hartman-Grobman theorem applies. Classify all fixed points where the theorem applies.

7. (25 points)

   (a) Let $v_1, ...v_n$ be eigenvectors corresponding to distinct real eigenvalues $\lambda_1, ...\lambda_n$ of an matrix $A \in M_n(\mathbf{R})$ Prove that $v_1, ...v_n$ are linearly independent.

   (b) Let $v_1, ...v_n$ be generalized eigenvectors corresponding to distinct real eigenvalues $\lambda_1, ...\lambda_n$ of a matrix $A \in M_n(\mathbf{R})$. Prove that $v_1, ...v_n$ are linearly independent.

   (c) Suppose $A \in M_3(\mathbf{R})$. Let v an eigenvector of A with the corresponding real eigenvalue λ with multiplicity 2. Let u be a vector such that $(A - \lambda I)u = v$. Let v_3 be an eigenvector for the real eigenvalue $\lambda_3 \neq \lambda$. Define $T = [v_1, v_2, v_3]$. Prove that T is invertible and the matrix $T^{-1}AT$ is block diagonal.

EXTRA SHEET