Fundamental Constants

gravitational constant: $G = 6.673 \times 10^{-8}\mathrm{cm^3\,g^{-1}\,s^{-2}}$
speed of light: $c = 2.998 \times 10^{10}\,\mathrm{cm\,s^{-1}}$
Planck's constant: $h_P = 6.626 \times 10^{-27}\,\mathrm{erg\,s}$
Boltzmann constant: $k_B = 1.381 \times 10^{-16}\mathrm{erg\,K^{-1}}$
Blackbody constant: $a_{rad} = 7.566 \times 10^{-15}$ erg cm^{-3} K^{-4}
Stefan-Boltzmann constant:
$\sigma_{SB} = 5.671 \times 10^{-5}$ erg s^{-1} cm^{-2} K^{-4}
electron rest energy: $m_e c^2 = 0.5110\,\mathrm{MeV}$
proton rest energy: $m_p c^2 = 938.272\,\mathrm{MeV}$
neutron rest energy: $m_n c^2 = 939.566\,\mathrm{MeV}$
Hubble constant: $H_0 = 100h$ km s^{-1} Mpc^{-1}

Conversion of Units

astronomical unit: $1\,\mathrm{AU} = 1.496 \times 10^{13}\,\mathrm{cm}$
parsec: $1\,\mathrm{pc} = 3.086 \times 10^{18}\,\mathrm{cm}$
solar mass: $1\,\mathrm{M_\odot} = 1.989 \times 10^{33}\,\mathrm{g}$
solar luminosity: $1\,\mathrm{L_\odot} = 3.846 \times 10^{33}\,\mathrm{erg\,s^{-1}}$
year: $1\,\mathrm{yr} = 3.156 \times 10^7\,\mathrm{s}$
electron volt: $1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$

Table of Integrals

$$\Gamma(x) = \int_0^\infty y^{(x-1)} e^{-y} dy$$

For positive integers, $\Gamma(n+1) = n!$.

$\Gamma(p+1) = p\Gamma(p)$

$\Gamma(-0.2) = -5.821$ $\Gamma(-0.1) = -10.68$ $\Gamma(0.1) = 9.514$ $\Gamma(0.2) = 4.591$ $\Gamma(0.3) = 2.992$; $\Gamma(0.4) = 2.218$; $\Gamma(0.5) = 1.773$; $\Gamma(0.6) = 1.489$; $\Gamma(0.7) = 1.298$;

$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a \arctan \frac{\sqrt{x(a-x)}}{x-a}$
$\int \frac{x}{\sqrt{x+a}} dx = \frac{2}{3}(x-2a)\sqrt{x+a}$
$\int_0^\infty \frac{x^2 dx}{(x^2+1)^3} = \frac{\pi}{16}$
$\int \frac{x}{\sqrt{x-a}} dx = \frac{2}{3}(x+2a)\sqrt{x-a}$

Formulae

Density Distribution of Stars in the Milky Way

$$n_*(R,z) = n(0,0) e^{-R/h_R} e^{-|z|/h_z}$$

Stellar Initial Mass Funciton

$$\xi(M)\Delta M = \xi_0 \left(\frac{M}{\mathrm{M_\odot}}\right)^{-2.35} \frac{\Delta M}{\mathrm{M_\odot}}$$

Exponentially Declining Star Formation History

$$\psi(t) = \frac{1}{\tau} e^{-(t-t_f)/\tau}$$

Schecter Function

$$\phi(L) = \frac{\phi^*}{L^*}\left(\frac{L}{L^*}\right)^\alpha e^{-L/L^*}$$

Apparent magnitude

$$m_1 - m_2 = -2.5\log(f_1/f_2)$$

AB magnitude scale

$$m = 2.5\log f_\nu - 48.60$$

Absolute magnitude

$$M = m - 5\log(d/10\ \mathrm{pc})$$

Definition of Specific Flux
$F_\nu = \int I_\nu cos\theta d\Omega$, where $d\Omega = sin\theta d\theta d\phi$.

Mean Specific Intensity

$$J_\nu = \frac{1}{4\pi}\int_0^{2\pi}\int_0^\pi I_\nu(\theta,\phi) sin\theta d\theta d\phi$$

Specific Energy Density: $u_\nu = \frac{4\pi}{c} J_\nu$
Equation of Radiation Transfer

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu$$

Optical Depth: $\tau_\nu(x) = \int_0^x \kappa_\nu ds$
Gaussian Line Profile
$\phi(v_z)dv_z = \left(\frac{1}{2\pi\sigma_z^2}\right)^{1/2} \exp\left(-\frac{v_z^2}{2\sigma_z^2}\right) dv_z$, where $\sigma_z = \sqrt{\frac{kT}{\mu m_p}}$.

Color Excess: $E(B-V) = (B-V) - (V-B)_0$
Ratio of Total to Selective Extinction: $R \equiv A_V/E(B-V)$
Mean Free Path: $l \approx \frac{1}{n_p \sigma_{rec}(v)}$
Recombination Coefficient

$$\alpha(T_e) \equiv \langle\sigma_{rec}(v)v\rangle = \int_0^\infty \sigma_{rec}(v) v f(v) dv$$

Recombination Time:

$$t_{rec} \approx \frac{1}{\alpha_{rec}(T) n_p}$$

Stromgren Radius

$$R_s = \left[\frac{3}{4\pi}\frac{Q}{\alpha_{rec}(T) n_e^2}\right]^{1/3}$$

Hα Luminosity

$$L(\mathrm{H}\alpha\ ) = 10^{42}\ \mathrm{erg/s}\left(\frac{Q}{1.88 \times 10^{44}\ \mathrm{s^{-1}}}\right)$$

Collisional Cooling Rate: $L = n_e^2 \Lambda(T)$
Bremsstrahlung (Free-Free) Radiation:

$$\Lambda(T) = 3 \times 10^{-27}\left(\frac{T}{1\ \mathrm{K}}\right)^{1/2}$$

Blackbody Radiation

Source Function

$$S_\nu = B_\nu(T),\ \text{or} j_\nu = B_\nu(T)\kappa_\nu$$

Planck Function

$$B_\nu(T) = \frac{2h_P\nu^3}{c^2}\frac{1}{e^{h_p\nu/k_BT}-1}$$

Wien's Law: $\lambda_{max}T = 0.29$ cm K
Energy Density: $u = a_{rad}T^4$
Flux: $F = \sigma_{SB}T^4$

Virial Theorem: $\ddot{I} = 2W + 4K$
Poisson's Equation: $\nabla^2\Phi = 4\pi G\rho$
Poisson's Equation (Spherical Polar):
$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}\left(sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2 sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2} = 4\pi G\rho$
Poisson's Equation (Cylindrical Polar, simplified to rotational symmetry):
$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\psi}{\partial R}\right) + \frac{\partial^2\psi}{\partial z^2} = 4\pi G\rho$

Oort Equations

$v_r = R_0 \sin l\left(\frac{V}{R} - \frac{V_0}{R_0}\right)$
$v_t = \left[\frac{V}{R} - \frac{V_0}{R_0}\right]R_o\cos l - \frac{V}{R}d$

Oort Constants

$A \equiv -\frac{R_0}{2}\frac{d\Omega}{dR}|_{R_0}$ and $B \equiv A - \Omega_0$

Effective Potential: $\Phi_{eff} = \Phi(R,z) + \frac{L_z^2}{2R^2}$

Epicyclic Frequency: $\kappa^2 = \frac{1}{R^3}\frac{d}{dR}\left[(R^2\Omega)^2\right]$

Toomre Q Parameter: $Q \equiv \frac{c_s\kappa}{\pi G\mu_{gas}}$

Density Wave Theory

$\Omega - \frac{\kappa}{m} \le \Omega_P \le \Omega + \frac{\kappa}{m}$

de Vaucouleurs Profile:
$I(R) = I(R_e)\exp\left(-b[(R/R_e)^{1/n} - 1]\right)$

Collisionless Boltzmann Equation
$\frac{\partial f(\mathbf{x},\mathbf{v},t)}{\partial t} + \mathbf{v}\cdot\nabla f - \nabla\Phi\cdot\frac{\partial f}{\partial\mathbf{v}} = 0$

Jeans Equation

$\frac{\partial\langle v\rangle}{\partial t} + \langle v\rangle\frac{\partial\langle v\rangle}{\partial x} = -\frac{\partial\Phi}{\partial x} - \frac{1}{n}\frac{\partial}{\partial x}[n\sigma^2(x,t)]$

Continuity Equation (Spherical Polar):

$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho u_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\rho\sin\theta u_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(\rho u_\phi) = 0$

Continuity Equation (Cylindrical Polar):

$\frac{\partial\rho}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}(R\rho u_R) + \frac{1}{R}\frac{\partial}{\partial\theta}(\rho u_\theta) + \frac{\partial}{\partial z}(\rho u_z) = 0$

Fluid Dynamics:

$\frac{\partial\rho}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho u_r) = 0$
$\rho\frac{\partial\bar{u}}{\partial t} + \rho\bar{u}\odot\nabla\bar{u} = -\nabla P + \rho\bar{g}$
$\frac{\partial E}{\partial t} + \nabla\odot[(E+P)\bar{u}] = -\rho\dot{(Q)} + \rho\frac{\partial\Phi}{\partial t}$

Sound Speed: $c_s^2 = \frac{dP}{d\rho}$
Hydrostatic Equilibrium: $M(<r) = \frac{k}{\mu m_p}\frac{r^2}{G\rho(r)}\frac{d}{dr}(\rho(r)T(r))$
Shock Jump Conditions:

$\rho_1 u_1 = \rho_2 u_2$
$\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2$
$\frac{1}{2}u_1^2 + \epsilon_1 + \frac{P_1}{\rho_1} = \frac{1}{2}u_2^2 + \epsilon_2 + \frac{P_2}{\rho_2}$
$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)P_2 + (\gamma-1)P_1}{(\gamma+1)P_1 + (\gamma-1)P_2}$

Sedov-Taylor Solution:
$R_{sh} \approx 5.3\ \text{pc}\left(\frac{E}{10^{51}\ \text{erg}}\right)^{1/5}\left(\frac{n}{1\ \text{cm}^{-3}}\right)^{-1/5}\left(\frac{t}{1000\ \text{yr}}\right)^{2/5}$

Galaxy Scaling Relations:

$M_{BH} \approx 3\times10^{-3}M_{bulge}$
$M_{BH} = 2.0\times10^8\ \text{M}_\odot\left[\frac{\sigma}{200\ \text{km/s}}\right]^{5.2}$
Faber-Jackson Relation $L_V \approx 2\times10^{10}\ \text{L}_\odot\ \left[\frac{\sigma}{200\ \text{km/s}}\right]^4$
Fundamental Plane:

$$\log R_e = 0.34\langle\mu_e\rangle + 1.4\log\sigma_0 + const$$

Kennicutt-Schmidt Law:

$$\Sigma_{SFR}(\ \text{M}_\odot/\text{yr/kpc}^2) = (2.5\pm0.7)\times10^{-4}\left(\frac{\mu_{gas}}{\text{M}_\odot/\text{pc}^2}\right)^{1.4\pm0.15}$$