

AVERAGES

① Formula of Average

$$\text{Average} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

eg:- Find the average of 1, 2, 3, 4, 5, 6, 7

Sol:- Avg = $\frac{\text{Sum of observations}}{\text{Number of observations}}$ Avg

$$\text{Sum of observations} = 1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$$

$$\text{Number of observations} = 7$$

$$\text{Avg} = \frac{\text{Sum}}{\text{Number}} = \frac{28}{7} = 4$$

② In other words Average is a measure of central value or central concentrated value among the group of given values.

eg:- (i) 1, 2, 3, 4, 5

(ii) 1, 2, 3, 4

2.5 Avg

No	Sum	Avg
05 (odd)	15	3
04 (even)	10	2.5

NOTE: prime numbers : Having only two factors '1' and itself

eg: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,

1) Find the average of first 10 prime numbers

~~a) 12.9~~

$$\text{Avg} = \frac{\text{Sum}}{\text{Number}} = \frac{2+3+5+7+11+13+17+19+23+29}{10}$$

b) 15.8

c) 14.7

$$= \frac{129}{10} = \boxed{12.9}$$

d) 16.3

NOTE:- (i) '2' is the only even prime number

(ii) '1' is neither a prime nor a composite number

③ (i) Sum of 1st 'n' natural numbers (Σn)

$$\Sigma n = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$\rightarrow \text{Avg of first 'n' natural numbers} = \frac{\text{Sum}}{\text{Number}} = \frac{\Sigma n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{(n+1)}{2}$$

(ii) Sum of the squares of first 'n' natural numbers (Σn^2)

$$\Sigma n^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\rightarrow \text{Avg of squares of 1st 'n' natural numbers} = \frac{\text{Sum}}{\text{Number}} = \frac{\Sigma n^2}{n} = \frac{\frac{n(n+1)(2n+1)}{6}}{n} = \frac{(n+1)(2n+1)}{6}$$

(iii) Sum of the cubes of first 'n' natural numbers (Σn^3)

$$\Sigma n^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$\text{Avg} = \frac{\text{Sum}}{\text{number}} = \frac{\Sigma n^3}{n} = \frac{\frac{n^2(n+1)^2}{4}}{n} = \frac{n(n+1)^2}{4}$$

$$n = 51$$



2) Find the average of first 51 multiples of 6

a) 154

b) 158

c) 160

☒ d) 156

$$\text{Avg} = \left(\frac{n+1}{2} \right) 6 = \left(\frac{51+1}{2} \right) 6 = \boxed{156}$$

④ unit conversion

$$1 \text{ kilometer} = 1000 \text{ m} \\ (1 \text{ km})$$

$$1 \text{ Hectometer} = 100 \text{ m} \\ (1 \text{ Hm})$$

$$1 \text{ Decameter} = 10 \text{ m} \\ (1 \text{ Dm})$$

$$1 \text{ Decimeter} = \frac{1}{10} \text{ m} \\ (1 \text{ dm})$$

$$1 \text{ Centimeter} = \frac{1}{100} \text{ m} \\ (1 \text{ cm})$$

$$1 \text{ millimeter} = \frac{1}{1000} \text{ m}$$

$$1 \text{ Hectare} = 100 \text{ m} \times 100 \text{ m} = 10,000 \text{ m}^2 = 2\frac{1}{2} \text{ acres}$$

3) Find the average of 2 m, 2.4 dm and 4 Dm.

a) 14.8 m

~~b) 14.08 m~~

c) 1.408 m

d) None of these

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ 2.0\text{ m} & 2.4 \times \frac{1}{10} & 4 \times 10 = 40\text{ m} \\ & = 0.24\text{ m} & \end{array}$$

$$\begin{aligned} \text{Avg} &= \frac{\text{Sum}}{\text{Number}} = \frac{2 + 0.24 + 40}{3} \\ &= \frac{42.24}{3} \\ &= \boxed{14.08\text{ m}} \end{aligned}$$

4) The average of 5 numbers is 12. Four of them are 10, 14, 8 and 2. Find the fifth number?

a) 34

☒ b) 26

c) 18

d) 48

Let 5th number be x

$$\text{Avg} = \frac{\text{Sum}}{\text{Number}} = \frac{10 + 14 + 8 + 2 + x}{5} = 12$$

$$\text{Sum} = 34 + x = 60$$

$$5^{\text{th}} \text{ number} = x = 60 - 34 = 26$$

5) The average of 8 numbers is 22. Four numbers are 12, 24, 36 and 48. What is the average of the remaining four numbers?

a) 14
$$\text{Avg} = \frac{\text{Sum}}{\text{Number}} = \frac{12 + 24 + 36 + 48 + x_1 + x_2 + x_3 + x_4}{8} = 22$$

b) 18

c) 20
$$\text{sum} = 120 + x_1 + x_2 + x_3 + x_4 = 176$$

$$\text{sum} = x_1 + x_2 + x_3 + x_4 = 176 - 120 = 56$$

d) 22

$$\text{Avg} = \frac{\text{Sum}}{\text{Number}} = \frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{56}{4} = 14$$

6) In a cricket match, the average runs scored by five persons are 20. If the captain and vice captain scored 40 and 30. What is the average of the remaining players?

- ☒ a) 10
- b) 15
- c) 20
- d) 30

$$\text{Avg} = \frac{\text{Sum}}{\text{Number}} = \frac{40 + 30 + x_1 + x_2 + x_3}{5} = 20$$

$$\text{Sum} = 70 + x_1 + x_2 + x_3 = 100$$

$$x_1 + x_2 + x_3 = 100 - 70 = 30$$

$$\text{Avg} = \frac{\Sigma 3}{3} = \frac{x_1 + x_2 + x_3}{3} = \frac{30}{3} = 10$$

7) 30 pens and 75 pencils were purchased for Rs 510. If the average price of a pencil was Rs 2. Find the average price of a pen

a) 10

☒ b) 12

c) 14

d) 15

	<u>No's</u>		[Avg price - Price of each article]	
Pens	→ 30	→	x	} Rs 510
Pencils	→ 75	→	2	

$$30x + 75 \times 2 = 510$$

$$30x = 510 - 150$$

$$x = \frac{360}{30} = 12$$

8) A man spend on an average of Rs 250 for the first 8 months of a year and Rs 200 for the next four months. If he saves Rs 800 that year, find his monthly income.

a) 700

b) 500

c) 400

☒ d) 300

$$\text{Avg Expen} = \frac{\Sigma 8}{8} = 250$$

$$\text{Sum} = \Sigma 8 = 2000$$

$$\text{Avg Expen} = \frac{\Sigma 4}{4} = 200$$

$$\text{Sum} = \Sigma 4 = 800$$

$$\Sigma 8 = 2000$$

$$\Sigma 4 = 800$$

$$\text{Savings} = \underline{800}$$

$$\text{Yearly Income } (\Sigma 12) = 3600$$

$$\begin{aligned} \text{monthly income} &= \frac{\Sigma 12}{12} = \frac{3600}{12} \\ &= 300 \end{aligned}$$

⑤ (i) If n is odd: then the average of n consecutive numbers, n consecutive even numbers and n consecutive odd numbers is always lies in the middle

	observations	No's (odd)	Sum	Avg
consecutive → No's	1 2 3 4 5	05	15	$\frac{15}{5} = 3$
consecutive → odd No's	1 3 5 7 9	05	25	$\frac{25}{5} = 5$
consecutive → Even No's	0 2 4 6 8	05	20	$\frac{20}{5} = 4$

Avg

⑥ (ii) If n is even: then the avg of n consecutive No's, consecutive even numbers, consecutive odd No's is always lies in the avg of middle two observations

	observations	No's (even)	Sum	Avg
Consecutive → No's	1 2 3 4	04	10	$10/4 = 2.5$
Con even No's →	0 2 3 4 6	04	12	$12/4 = 3$
Con odd No's →	1 3 4 5 7	04	16	$16/4 = 4$

Avg

9) The average of five consecutive even numbers is 62. Find the highest even number?

a) 64

☒ b) 66

c) 68

d) 70

— — 62 64 66

10) The average of five consecutive odd numbers is 17. Find the least odd number?

a) 15

b) 13

c) 11

d) 9

13 15 17

$$\text{Avg} = \frac{\Sigma 5}{5} = 17$$

$$\text{Sum} = \Sigma 5 = 85$$

11) The average of five odd numbers is 17. Find the least odd number?

a) 15

$$\begin{array}{ccccc} \underline{1} & \underline{3} & \underline{5} & \underline{7} & \underline{69} \end{array}$$

b) 13

$$\begin{array}{ccccc} \underline{13} & \underline{15} & \underline{17} & \underline{19} & \underline{21} \end{array}$$

c) 11

$$\begin{array}{ccccc} \underline{11} & \underline{13} & \underline{15} & \underline{17} & \underline{25} \end{array}$$

~~d) Cannot be determined~~

$$\begin{array}{ccccc} \underline{-1} & \underline{-3} & \underline{-5} & \underline{-7} & \underline{101} \end{array}$$

6

operand	Observations			Sum 12	Avg 4
	$x_1 = 2$	$x_2 = 4$	$x_3 = 6$		
+1	3	5	7	15	5 (4+1)
-1	1	3	5	9	3 (4-1)
$\times 2$	4	8	12	24	8 (4 \times 2)
$\div 2$	1	2	3	6	2 ($\frac{4}{2} = 2$)

Change in sum \propto Change in Avg

* Independent of No of observations.

12) The average of 15 numbers is 2, if each number is multiplied by 5, What is the new average?

a) 75

b) 6

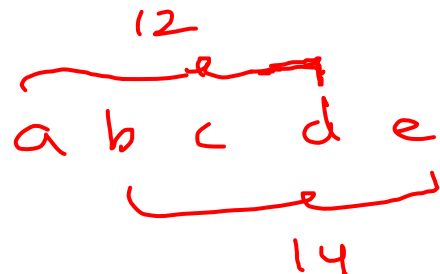
☒ c) 10

d) 15

$$\text{New avg} = \frac{x_1 \times 5 + x_2 \times 5 + \dots + x_{15} \times 5}{15} = 2 \times 5$$

$$\text{New avg} = 2 \times 5 = \boxed{10}$$

⑦ out of 5 nos Avg of 1st four is 12 and that of last four is 14 if the first Number is 20 then find Last number?

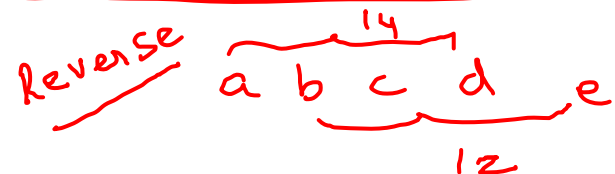


Short cut

$$a = 20$$

$$e = ?$$

$$\begin{aligned} e &= (14 - 12) \times 4 + 20 \\ &= 2 \times 4 + 20 \\ &= 28 \end{aligned}$$



$$\begin{aligned} a &= 20 \\ e &= ? \end{aligned}$$

$$\begin{aligned} (14 - 12) \times 4 &= 8 \Rightarrow e = 20 - 8 \\ &= 12 \end{aligned}$$



$$\text{Avg} = \frac{a+b+c+d}{4} = 12$$

$$\text{Sum} = a+b+c+d = 48$$

$$20 + b+c+d = 48$$

$$b+c+d = 48 - 20 = 28$$

$$\text{Avg} = \frac{b+c+d+e}{4} = 14$$

$$\text{Sum} = b+c+d+e = 56$$

$$28 + e = 56$$

$$\begin{aligned} e &= 56 - 28 \\ &= 28 \end{aligned}$$

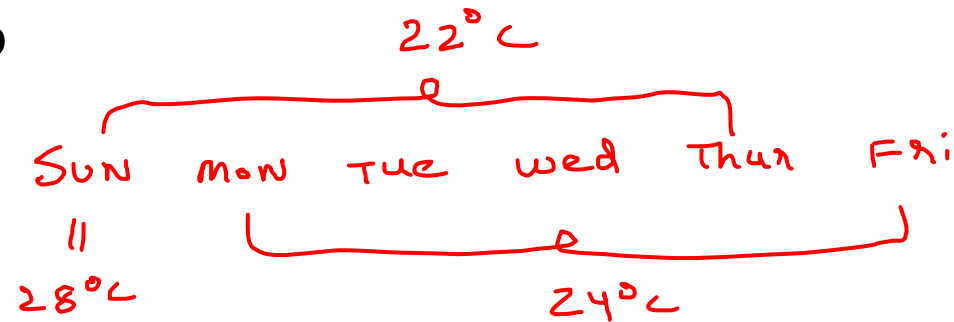
13) The average temperature, from Sunday to Thursday is 22 deg centigrade and from Monday to Friday is 24 deg centigrade the temperature on Sunday is 28 deg centigrade. Find the temperature on Friday(in deg centigrade)?

a) 37

b) 38

c) 39

d) 40



$$\begin{aligned}\text{Friday} &= (24 - 22) \times 5 + 28 \\ &= 38\end{aligned}$$

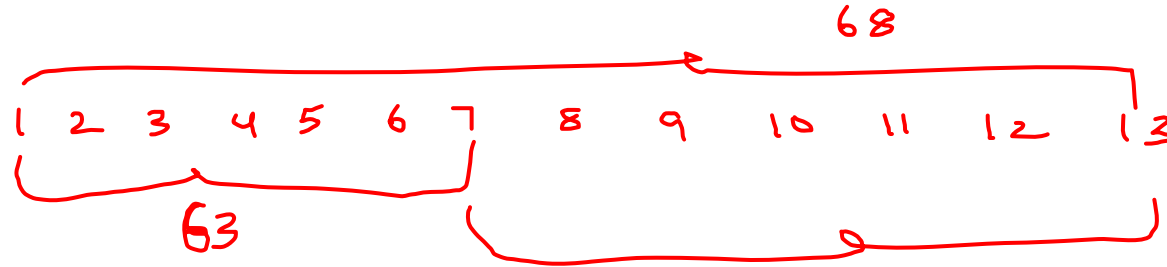
14) The average of 13 results is 68. the average of first seven is 63 and that of the last seven is 70. What is the seventh result?

a) 47

b) 72

c) 58

d) 63



Here 7th Result Repeated twice

$$\begin{aligned} \text{Avg} &= \frac{\Sigma 13}{13} = 68 \\ \text{Sum} &= \Sigma 13 = 68 \times 13 \\ &= 884 \end{aligned}$$

$$\begin{aligned} \text{Avg} &= \frac{\Sigma 7}{7} = 63 \\ \text{1st 7 Result Sum} &= 441 \end{aligned}$$

$$\begin{aligned} \text{Avg} &= \frac{\Sigma 7}{7} = 70 \\ \text{Last 7 results Sum} &= \Sigma 7 = 490 \end{aligned}$$

$$\begin{array}{r} \text{first 7} \rightarrow 441 \\ \text{last 7} \rightarrow 490 \\ \hline 931 \end{array}$$

$$\begin{array}{r} 7^{\text{th}} \text{ Result} = \frac{931}{2} \\ \hline 465.5 \end{array}$$

15) 3 years ago, the average age of a family of 5 members was 18 years, on the addition of a child the present average age is still the same. What is the present age of the child?

a) 1 year $-3 \text{ years ago} \Rightarrow \text{Avg} = \frac{\Sigma 5}{5} = 18$

b) 2 years $\text{sum} = \Sigma 5 = 90$

present age = $90 + 3 \times 5 = 105 \text{ years}$

☒ c) 3 years

$\text{Avg} = \frac{\Sigma 6}{6} = 18$

$\text{sum} = \Sigma 6 = 108 \text{ yrs}$

present age of child = $\Sigma 6 - \Sigma 5 = 108 - 105$
 $= 3 \text{ years}$

d) 4 years

16) The average age of a class of 20 students is 12 years. When the age of the teacher is also included the average will be increased by 8 months. What is the age of the teacher?

a) 24 years

☒ b) 26 years

c) 30 years

d) 32 years

Teacher age = 12 yrs (let)

$$\frac{+8}{12} \times 21 = \frac{14}{1}$$

$$\text{Teacher age} = 26 \text{ yrs}$$

17) The average weight of 15 men is increased by 1 kg. When one of them, whose weight is 60 kg is replaced by a new man. What is the weight of the new man?

a) 45 kg

b) 15 kg

c) 60 kg

☒ d) 75 kg

$$\begin{array}{rcl} \text{Wt of the New man} & = & 60 \text{ kg (Let)} \\ + 1 \times 15 & = & \underline{15} \\ \hline \text{wt of New man} & = & 75 \text{ kg} \end{array}$$

⑧ If the avg wt of 'a' students is 'x' kg and the avg wt of 'b' students is 'y' kg, then the average wt of the class is

$$\text{Avg wt} = \frac{ax + by}{a + b}$$

⑨ If the average of 'a' quantities is 'x' and the average of 'b' quantities out of them is 'y' then the average of Remaining quantities is

$$\text{Avg} = \frac{ax - by}{a - b}$$

18) The average weight of 10 boys is 20 kg and that of the 20 girls is 10 kg. What is the average weight of the class of 30 students?

a) 20 kg

b) 25 kg

~~c) 13.33 kg~~

d) 20.5 kg

$$\text{Avg wt} = \frac{10 \times 20 + 20 \times 10}{10 + 20}$$

$$= \frac{400}{30} = 13.33 \text{ kg}$$

19) The average weight of a class is 40 kg. 10 new students of average weight 38 kg, joined the class there by decreasing the average by 1 kg. What is the original strength of the class?

Original strength of the class = x

☒ a) 10

☐ b) 15

☐ c) 20

☐ d) 25

$$\text{Avg wt} = \frac{40x + 10 \times 38}{(x + 10)} = (40 - 1)$$

$$\begin{aligned} \text{Sum} &= 40x + 380 = 39x + 390 \\ x &= 10 \end{aligned}$$

20) The average temp of 10 days is 35 deg centigrade. Later it was found that one day the reading 42 deg centigrade is wrongly read as 24 deg centigrade. What is the correct average (in deg centigrade)?

a) 53

☒ b) 36.8

c) 53.7

d) 38.2

$$\text{Avg} = \frac{\Sigma 10}{10} = 35^{\circ}\text{C}$$

$$\begin{array}{ccc} 42^{\circ}\text{C} & \longrightarrow & 24^{\circ}\text{C} \\ & \nwarrow \quad \nearrow & \\ & +18^{\circ}\text{C} & \end{array}$$

$$\frac{18}{10} = 1.8$$

$$\text{Correct Avg temp} = 35 + 1.8 = 36.8^{\circ}\text{C}$$

21) In an examination the average was found to be 50 marks. After deducting computerization error the marks of the 100 candidates has to be changed from 90 to 60 each and the average came down to 45 marks. What is the number of candidates who took the examination?

a) 500
~~b) 600~~
 c) 650
 d) 680

	No of Candidates	Marks for each student	Sum	Avg
	100	90	9000	50
		60	6000	45

No of Candidates = $\frac{9000 - 6000}{50 - 45} = 600$

22) A batsman has a certain average runs for 11 innings. In the 12th innings he made a score of 90 runs and thereby decreased his average by 5. What is his average after 12 innings?

☒ a) 145

b) 160

c) 150

d) 140

$$\text{Avg after 12th innings} = x$$

$$\text{Avg} = \frac{90 + (x+5) 11}{12} = x$$

$$\begin{aligned} \text{Sum} &= 90 + 11x + 55 = 12x \\ x &= 145 \end{aligned}$$

$$\begin{aligned}\text{Passed} &= x \\ \text{Failed} &= 120 - x\end{aligned}$$

23) The average marks obtained by 120 candidates was 35. If the average marks of passed candidates was 39 and that of the failed candidates was 15. What is the number of candidates who passed the examination?

☒ a) 100

b) 20

c) 80

d) 40

$$\text{Avg} = \frac{39x + 15(120 - x)}{120} = 35$$

$$\text{Sum} = 39x + 1800 - 15x = 4200$$

$$24x = 2400$$

$$x = 100$$

⑨ If the observations $x_1, x_2, x_3, \dots, x_n$ have the following weights $w_1, w_2, w_3, \dots, w_n$ then the weighted Average is given by.

$$\text{weighted Average} = \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{w_1 + w_2 + \dots + w_n}$$

24) What is the average of a students who received 90 in English, 84 in Algebra, 75 in French and 76 in Music , if the subjects have the following weights : English 4, Algebra 3, French 3, and Music 1?

a) 81

b) 81.5

c) 82

☒ d) 83

	marks	—	weightage
Eng —	90	—	4
Alge —	84	—	3
Fren —	75	—	3
Music —	76	—	1

$$\begin{aligned}
 \text{Weighted Avg} &= \frac{90 \times 4 + 84 \times 3 + 75 \times 3 + 76 \times 1}{4 + 3 + 3 + 1} \\
 &= \frac{913}{11}
 \end{aligned}$$

25) Six men A, B, C, D, E and F agree with a seventh man G to provide a sum of money among them A, B, C, D, E and F are to subscribe Rs 10 each, and G is to pay Rs 3 more than the average of the seven. What is the whole sum to be provided?

☒ a) Rs 73.50

$$\text{Avg} = \frac{\Sigma 7}{7} = x \text{ (Let)}$$

b) Rs 74

$$\text{Avg} = \frac{10 \times 6 + x + 3}{7} = x$$

c) Rs 73

$$x = 10.50$$

d) Rs 72.50

$$\text{Sum} = 60 + 3 + 10.5 = 73.5$$

$$\text{trucks initially planned} = \frac{60}{3} = 20$$

26) A certain number of trucks were required to transport 60 tons of steel wire from the Vizag steel plant. However, it was found that since each truck could take 0.5 tons of cargo less, another 4 trucks were needed. How many trucks were initially planned to be used?

No of trucks initially planned = x (let)

a) 10

b) 5

☒ c) 20

d) 25

$$\frac{60}{x-0.5} - \frac{60}{x} = 4$$

$$15 \cancel{60} [\cancel{x} - \cancel{x} + 0.5] = \cancel{4} [x^2 - 0.5x]$$

$$x^2 - 0.5x - 7.5 = 0$$

$$10x^2 - 5x - 75 = 0$$

$$2x^2 - x - 15 = 0$$

$$2x^2 - 6x + 5x - 15 = 0$$

$$\left. \begin{array}{l} 2x(x-3) + \\ 5(x-3) = 0 \\ x = 3 \text{ or } -\frac{5}{2} \end{array} \right\}$$

$$\text{Avg} = \frac{\text{Sum}}{\text{Number}}$$

$$\text{Avg of 1st } n \text{ natural numbers} = \frac{n+1}{2}$$

27) A set of consecutive positive integers beginning with 1 is written on the black board. A student came along and erased one number. The average of the remaining numbers is $35\frac{7}{17}$. What was the number erased?

$35\frac{7}{17}$ lies between $\overline{35}$ to 36

$$\text{Avg} = \frac{n+1}{2} = 35 \quad (\text{let us assume the avg as } 35)$$

$$\text{Sum} = n+1 = 70$$

$$n = 69$$

$$\text{Number erased} = x (\text{let})$$

$$\frac{69 \times 35 - x}{(69 - 1)} = 35\frac{7}{17}$$

$$2415 - x = 2408$$

$$\text{Erased} = x =$$

$$\text{Number}$$

$$2415 - 2408$$

$$= \boxed{7}$$

a) 7

b) 8

c) 9

d) None of these