RATIO AND PROPORTION

① Ratio ÷ Comparison of two or more elements of the same kind

eg ÷ The weights of A and B are in the ratio 2:3
The heights of A and B are in the ratio 4:5
} Comparison has to be on the same ground

② Ratio is nothing but a fraction that means $a:b$ or $\frac{a}{b}$

③ Ratio is a number

④ A and B are expressed in the ratio 3:4 and they can be represented as $A = 3x$ or 3 parts or 3p
$B = 4p$ or 4 parts or 4p

⑤ $2:4:6 \Rightarrow 1\times\not{2} : \not{2}\times 2 : \not{2}\times 3 \Rightarrow 1:2:3$
Common factor can be cancelled out

(6) $\frac{1}{7} : \frac{2}{7} : \frac{3}{7} \Rightarrow \frac{1}{7} : \frac{2}{7} : \frac{3}{7} \Rightarrow 1 : 2 : 3$ $(\because)$ common denominator can be cancelled out

(7) Proportion :- (i) comparison of two or more ratios

(ii) Thus two ratios are said to be in proportion when they are equal

(8) $a : b :: c : d$

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Product of extremes = product of Means

(9) If a, b, c, d are in proportion then

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a+c}{b+d} = \frac{a}{b}$$

$$\frac{a-c}{b-d} = \frac{a}{b}$$

eg :- (i) $\frac{1}{2} = \frac{4}{8} \Rightarrow \frac{1+4}{2+8} = \frac{5}{10} = \frac{1}{2}$

(ii) $\frac{3}{5} = \frac{30}{50} \Rightarrow \frac{30+3}{50+5} = \frac{33}{55} = \frac{3}{5}$

(10) Continued proportion of a, b, c is $b^2 = ac$

(11) If a, b, c, d are in proportion then

(i) 4th proportion = $\boxed{d = \frac{bc}{a}}$

Extremes

$a : b :: c : d$

means / middle terms

$ad = bc$

$\boxed{d = \frac{bc}{a}}$

(ii) 3rd proportional = $\boxed{c = \frac{b^2}{a}}$

$a : b :: b : c$

$\boxed{c = \frac{b^2}{a}}$

(iii) mean proportional = $\boxed{x = \sqrt{ab}}$

$a : x :: x : b$

$x^2 = ab$

$\boxed{x = \sqrt{ab}}$

(12)

$1:2 \rightarrow \frac{1}{2} \times \frac{2}{2} = \frac{2}{4} \Rightarrow 2:4$

$1:2 \rightarrow \frac{1}{2} \times \frac{3}{3} = \frac{3}{6} \Rightarrow 3:6$

$1:2 \Rightarrow \frac{1}{2} \times \frac{4}{4} = 4/8 \Rightarrow 4:8$

$1:2 \Rightarrow 2:4 \Rightarrow 3:6 \Rightarrow 4:8$

1) Find the following

a) Find the third proportional to 8 and 12: → $a = 8;\ b = 12$

b) Find the Fourth proportional to 5, 6 and 8: → $a = 5,\ b = 6,\ c = 8$

c) Find the Mean proportional 9, 16: → $a = 9,\ b = 16$

(a) 3rd proportion $= \frac{b^2}{a} = \frac{\overset{3}{\cancel{12}} \times \overset{6}{\cancel{12}}}{\underset{2}{\cancel{8}}} = 18$

(b) 4th proportion $= d = \frac{bc}{a} = \frac{6 \times 8}{5} = \frac{48}{5} = 9.6$

(c) mean proportional $= x = \sqrt{ab} = \sqrt{9 \times 16} = 3 \times 4 = 12$

(13) (i) If $\frac{A}{P} = \frac{B}{Q} = \frac{C}{R}$ then $A : B : C = ?$

$$\frac{A}{P} = \frac{B}{Q} = \frac{C}{R} = K$$

$A = PK$; $B = QK$; $C = RK$

$A : B : C = PK : QK : RK = P : Q : R$

$$\boxed{A : B : C = P : Q : R}$$

(ii) If $PA = QB = RC$ then $A : B : C = ?$

$PA = QB = RC = K$

$PA = K$ | $QB = K$ | $RC = K$

$A = \frac{K}{P}$ | $B = \frac{K}{Q}$ | $C = \frac{K}{R}$

$A : B : C \rightarrow \frac{K}{P} : \frac{K}{Q} : \frac{K}{R} \Rightarrow QR : PR : PQ$

$\frac{1}{P} : \frac{1}{Q} : \frac{1}{R} \Rightarrow \frac{QR}{PQR} : \frac{PR}{PQR} : \frac{PQ}{PQR}$

2) A:B = 2:3, B:C = 4:5, C:D = 6:7, then A:B:C:D is equal to

a) 16:22:30:35

b) 16:24:15:35

c) 16:24:30:35

d) 18:24:30:35

A : B = 2 : 3 $\xrightarrow{\times 8}$ 16 : 24

B : C = 4 : 5 $\xrightarrow{\times 6}$ 24 : 30

C : D = 6 : 7 $\xrightarrow{\times 5}$ 30 : 35

A : B : C : D = 16 : 24 : 30 : 35

3) If A:B:C = 2:3:4, then A/B:B/C:C/A

a) 4:9:16

b) 8:9:12

c) 8:9:16

d) 8:9:24

$$\frac{2}{3} : \frac{3}{4} : \frac{4}{2}$$

$$\frac{2\times4}{3\times4} : \frac{3\times3}{4\times3} : \frac{4\times6}{2\times6}$$

$$\frac{8}{12} : \frac{9}{12} : \frac{24}{12}$$

$$8 : 9 : 24$$

$A:B = \frac{1}{2} : \frac{3}{8} = \frac{1\times4}{2\times4} : \frac{3}{8} = \frac{4:3}{8\ 8} = 4:3 \xrightarrow{\times2} 8:6$

$B:C = \frac{1}{3} : \frac{5}{9} = \frac{1\times3}{3\times3} : \frac{5\times1}{9\times1} = \frac{3}{9} : \frac{5}{9} = 3:5 \xrightarrow{\times2} 6:10$

4) If A:B = ½:3/8, B:C = 1/3:5/9 and C:D = 5/6:3/4 then the ratio of A:B:C:D

a) 4:6:8:10

b) 6:4:8:10

c) 6:8:9:10

d) 8:6:10:9

$C:D = \frac{5}{6} : \frac{3}{4} = \frac{5\times2}{6\times2} : \frac{3\times3}{4\times3} = \frac{10}{12} : \frac{9}{12} = 10:9 \xrightarrow{\times1} 10:9$

$A:B:C:D = 8:6:10:9$

5) If 2A=3B=4C, then A:B:C

a) 2:3:4

b) 4:3:2

c) 6:4:3

d) 20:15:2

$\frac{2A}{24} = \frac{3B}{24} = \frac{4C}{24}$

$\frac{A}{12} = \frac{B}{8} = \frac{C}{6}$

$A:B:C = 12:8:6 = 6:4:3$

6) If A/3=B/4=C/5, then A:B:C = 3:4:5

a) 4:3:5

b) 5:4:3

c) 3:4:5

d) 20:15:2

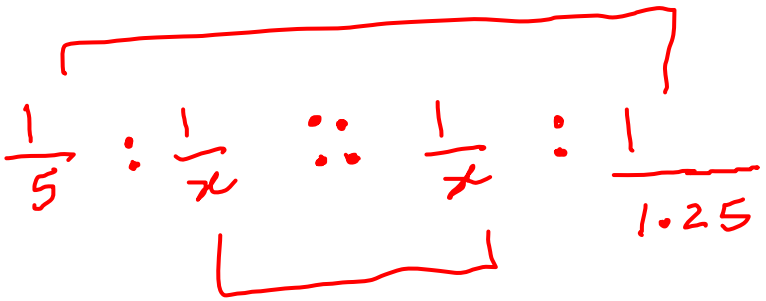
7) If 1/5:1/X :: 1/X:1/1.25, then the value of X is

a) 1.5

b) 2

c) 2.5

d) 3.5



$$\frac{1}{x \times x} = \frac{1}{5 \times 1.25}$$

$$x^2 = 6.25$$

$$x = \pm 2.5$$

$$x = 2.5$$

8) Rs 3115 is divided among A, B and C so that if Rs 25, Rs 28 and Rs 52 be diminished from their respective shares, the remainders will be in the ratio 8:15:20. Find the share of C?

a) 1452

b) 585

c) 1348

d) 1078

Rs 3115

Rs 3115
− 105

Rs 3010

A B C

−25 − 28 − 52

A : B : C

8 : 15 : 20

8p + 15p + 20p = 43p

43p → Rs 3010

1p → Rs 70

C → 20 × 70

= 1400

+ 52

1452

25paise → Rs 0.25

1 Rupee — 100 paise

9 — 50 paise

$\frac{50 \times 1}{100} = Rs\ 0.50$

9) A bag contains one rupee, 50 paise and 25 paise coins in the ratio 2:3:5. Their total value is Rs 114. Find the value of 50 paise coins?

a) 24

b) 30

c) 36

d) 42

1 Rs : Rs 0.50 : Rs 0.25

2 : 3 : 5

$2 \times 1.00 = Rs\ 2.00$

$3 \times 0.50 = Rs\ 1.50$

$5 \times 0.25 = Rs\ 1.25$

4.75

Rs 4.75 → Rs 114

1P → ?

$1P = \frac{1 \times 114}{4.75} = 24$

Value of 50 paise = $3 \times 24 = 72 \times \frac{1}{2} =$ Rs 36

10) A money bag contains coins of one rupee, 50 paise and 25 paise in the ratio of 4:5:8. If there are 170 coins in the bag, find the amount?

a) Rs 85

b) Rs 170

c) Rs 175

d) None of these

1Rs : Rs 0.50 : Rs 0.25

4 : 5 : 8

17P → 170

1P → 10

1Rs → 4×10 = 40 × 1.00 = Rs 40.00

Rs 0.50 → 5×10 = 50 × $\frac{1}{2}$ = Rs 25.00

Rs 0.25 → 8×10 = 80 × $\frac{1}{4}$ = Rs 20.00

Rs 85.00

11) Rs 94 is divided in to two parts in such a way that fifth part of the first and eighth part of the second are in the ratio 3:4. What is the first part ?

a) Rs 30

b) Rs 64

c) Rs 40

d) Rs 54

Rs 94

$\frac{I}{5} : \frac{II}{8} :: 3 : 4$

$\frac{4I}{5} = \frac{3II}{8}$

$\frac{I}{II} = \frac{15}{32}$

$I : II = 15 : 32$

47p → 94

1p → 2

I st part = 2×15

= Rs 30

12) Five mangoes and four oranges cost as much as three mangoes and seven oranges. What is the ratio of the cost of one mango to the cost of one orange?

a) 3:2

b) 2:3

c) 1:4

d) 4:1

$5m + 4o = 3m + 7o$

$5m - 3m = 7o - 4o$

$2m = 3o$

$\frac{1m}{1o} = \frac{3}{2}$

$m:o = 3:2$

Salary of B = 600 (let)

100% → 600

65% → 390

35% → 210

21p → 210

1p → 10

13) A, B, C together earn Rs 1450 and spend 60%, 65% and 70% of their salaries respectively. If their savings are in the ratio 14:21:15. What is the salary of B?

a) 350

b) 500

c) 660

d) 600

Savings A

14p → 14 × 10 = 140

40% → 140

100% → ?

$\frac{100 \times 140}{40} = 350$

Savings C

15p → 15 × 10 = 150

30% → 150

100% → ?

$\frac{100 \times 150}{30} = 500$

Salary → A → 350

" → B → 600

" → C → 500

Rs 1450

Let No of | Heads | Legs

(x) Hens → 01 → 02

(y) Cows → 01 → 04

14) A man has some hens and cows. If the number of heads be 50 and the number of feet be 136. What is the number of hens?

a) 28

b) 32

c) 18

d) 23

$2x + 4y = 136$

$x + y = 50 \xrightarrow{\times 2}$

$2x + 4y = 136$

$2x + 2y = 100$

$2y = 36$

$y = 18$

$x = 50 - y$

$x = 50 - 18$

$x = 32$

	A.C	1st class	sleeper
Passengers →	1 :	2 :	7
Fare →	5 :	4 :	2

15) In an express train, the passengers travelling in A.C sleeper class, first class and sleeper class are in the ratio 1:2:7 and rate for each class in the ratio 5:4:2. If the total income from this train is Rs 54000. Find the income of Indian Railways from A.C Sleeper class.

a) 10000
b) 11000
c) 12000
d) 14000

Income (pass x Fare)

$1 \times 5 : 2 \times 4 : 7 \times 2$

$5 : 8 : 14$

27p → 54000

1p → 2000

Income A.C Sleeper class $= 5 \times 2000$

$= Rs\ 10,000$

Fraction = $\frac{x}{y}$ (let) $\rightarrow \frac{x-5}{y-5} = \frac{1}{2}$

$\rightarrow \frac{x+2}{y+2} = \frac{2}{3} \Rightarrow 3x+6 = 2y+4$

$2x-10 = y-5$

$3x-2y = -2 \rightarrow$ (II)

$2x-y = 5 \rightarrow$ (I) $\times 2$

16) When 5 is subtracted from both the numerator and the denominator of a fraction, the fraction reduces to ½. When 2 is added to the numerator and the denominator, the fraction reduces to 2/3. Find the fraction?

a) ~~12/19~~

b) 19/12

c) 8/12

d) 11/8

$4x - 2y = 10$

$3x - 2y = -2$

$(-) \quad (+) \quad = \quad +$

$x = 12$

$3(12) - 2y = -2$

$36 - 2y = -2$

$2y = 36 + 2$

$y = \frac{38}{2} = 19$

$\frac{x}{y} = \frac{12}{19}$

Rs 117

A B C

$\frac{1}{2} : \frac{1}{3} : \frac{1}{4} \Rightarrow \frac{6}{12} : \frac{4}{12} : \frac{3}{12} \Rightarrow 6:4:3$

17) By mistake, instead of dividing Rs 117 among A, B and C in the ratio of ½:1/3:1/4. It was divided in the ratio of 2:3:4. Who gains the most?

a) C

b) B

c) A

d) No one.

13p → 117

1p → 9

A → 6 × 9 = Rs 54

B → 4 × 9 = Rs 36

C → 3 × 9 = Rs 27

By mistake it was divided

A : B : C = 2 : 3 : 4

9p → Rs 117

1p → Rs 13

A → 2 × 13 = Rs 26

B → 3 × 13 = Rs 39

C → 4 × 13 = Rs 52

C gains most by 52 − 27 = Rs 25

Let Boys = x

Girls = y

$B : G = x : y = B \Rightarrow \frac{x}{y} = B$

18) If the ratio of boys to girls is B, and the ratio of girls to boys is G, then B + G is

a) Greater than or equal to 1

b) Equal to 1

c) Greater than 1

d) Less than 1.

$G : B = y : x = G \Rightarrow \frac{y}{x} = G$

$B + G = \frac{x}{y} + \frac{y}{x} = \frac{x^2 + y^2}{xy}$

Minimum Ratio = B : G or G : B is 1 : 1

$(x : y)$ (or) $(y : x)$

$(x-y)^2 = x^2 + y^2 - 2xy$

$(1-1)^2 = x^2 + y^2 - 2xy$

$0 = x^2 + y^2 - 2xy$

$x^2 + y^2 = 2xy$

$\frac{x^2 + y^2}{xy} = 2 > 1$

That means the ratio will be equal to '1' and greater than '1'.

5 cows = 7 Bull | 21 Bull = 10 Hor | 16 Hor = 9 cam

1 cow = $\frac{7}{5}$ Bull | 1 Bull = $\frac{10}{21}$ Hor | 1 Hor = $\frac{9}{16}$ cam

19) 5 Cows cost as much as 7 Bullocks ; 21 Bullocks cost as much as 10 Horses ; 16 Horses cost as much as 9 Camels. If the cost of one Camel is Rs 3200, find the cost of one Cow?

1 cow = $\frac{7}{5} \times \frac{10}{21} \times \frac{9}{16} \times$ (1 camel)

1 camel = Rs 3200

a) Rs 1200

b) Rs 2100

c) Rs 1700

d) Rs 1400.

1 cow = $\frac{7}{5} \times \frac{10}{21} \times \frac{9}{16} \times 3200$ = Rs 1200

Distance covered by 1 leap of a cat = x

" " " " " " , Dog = y

Distance (Cat) = $5x$

Distance (Dog) = $4y$

20) A cat takes 5 leaps for every 4 leaps of a dog, but 3 leaps of the dog are equal to 4 leaps of the cat. What is the ratio of the speed of the cat to that of the dog?

a) 11:15

b) 15:11

c) 16:15

d) 15:16

$3y = 4x$

$y = \frac{4}{3}x$

Ratio of Speeds of Cat & Dog = Distance covered by cat : Distance covered by dog

$= 5x : 4y$

$= 5x : 4\left[\frac{4}{3}x\right]$

$= \frac{15x}{3} : \frac{16x}{3}$

$= 15 : 16$

27 can be expressed as a perfect cube as 3^3

In 3^3 — Base will represent weighing pans & No of balls of equal division

Exponent will give attempts to find heavier Ball.

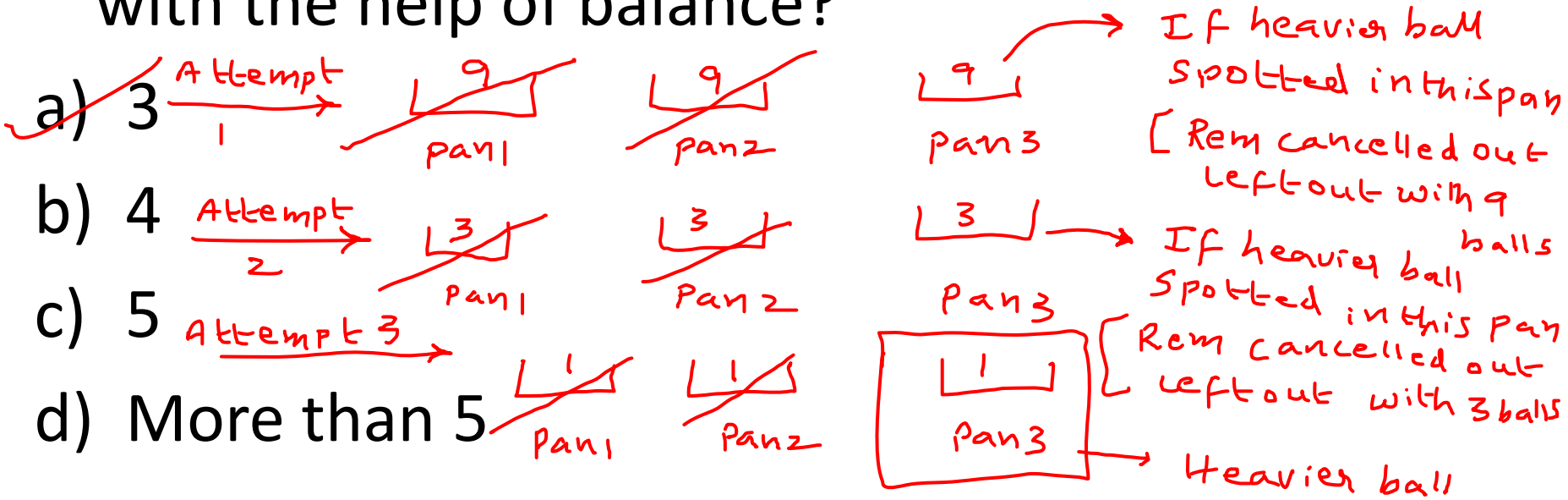
21) Odd one out: You are given twenty seven balls of similar size, but one of these is heavier than all the others which weigh the same. In how many number of least weighings can you identify the heavier ball with the help of balance?

a) 3

b) 4

c) 5

d) More than 5



22) Concentrations of three wines A, B and C are 10%, 20% and 30% respectively. They are mixed in the ratio 2:3:X resulting in a 23% concentration solution. Find X

a) 7

b) 6

c) 5

d) 4

$$\frac{10}{100} \times 2 + \frac{20}{100} \times 3 + \frac{30}{100} \times x = \frac{23}{100}\left[2+3+x\right]$$

$$\frac{20+60+30x}{100} = \frac{23[5+x]}{100}$$

$$80+30x = 115+23x$$

$$7x = 35$$

$$x = 5$$

23) There are two alloys of gold and silver. In the first alloy, there is twice as much gold as silver, and in the second alloy there is 5 times less gold than silver. How many times more must we take of the second alloy than the first in order to obtain a new alloy in which there would be twice as much silver as gold?

a) 2 times
b) 3 times
c) 4 times
d) 10 times

I st alloy :- Silver = x (Let)
Gold = 2x + x = 3x
⇒ G : S
3x : x
3 : 1
(Total = 4 parts)

II nd alloy :- Silver = 4x
Gold = 5x − 4x = x
⇒ G : S
x : 4x
1 : 4
Total parts = 5 parts

I alloy + II alloy = III alloy [If this has to happen the equal quantities of alloys has to be taken through out]

	G : S	Total parts		New mixture	
I alloy –	3 : 1	4P	×5 → 4p×5 = 20p ⇒	I alloy →	G : S
II alloy –	1 : 4	5p	×4 → 5p×4 = 20p		15 : 5
				II alloy →	4 : 16

III^nd alloy

G = x (let)

S = 2x + x = 3x ⇒

G : S

x : 3x

1 : 3

Let the no of times more to be taken of 2nd alloy from 1st alloy be 'x'

Gold: 4x + 15

Silver: 16x + 5

III^nd alloy

G : S = 1 : 3

$\frac{G}{S} = \frac{1}{3}$

$\frac{4x+15}{16x+5} = \frac{1}{3}$

12x + 45 = 16x + 5

4x = 40

x = 10 times

(14) **Proportion** ÷ Is nothing but equal

(15) Proportion

Direct Proportion

Indirect (or) Inversely Proportion

(16) **Direct proportion**

$$x \propto y$$

$$x = Ky$$

$$\frac{x}{y} = K \text{ (Constant)}$$

(17) Inversely proportion ÷

$$x \propto \frac{1}{y}$$

$$x = \frac{k}{y}$$

$$xy = k \text{ (constant)}$$

24) The speed of the Rajadhani Express is 42 kmph when no compartment is attached, and the reduction in speed is directly proportional to the square root of the number of compartments attached. If the speed of the train carried by this engine is 24 kmph when 9 compartments are attached, the maximum number of compartments that can be carried by the engine is

a) 49
b) 48
c) 146
d) 47

(Reduction in speed) $\propto$ $\sqrt{\text{No of compartments}}$

$S \propto \sqrt{c}$

$S = k\sqrt{c}$

Reduction in speed $(42 - 24) = k\sqrt{9}$

$18 = k \times 3$

$k = \frac{18}{3} = 6$

$(42 - 0) = 6\sqrt{\text{compartments}}$

$\sqrt{\text{No of compartments}} = \frac{42}{6} = 7$

No of compartments $= 7^2 = 49$

No of Books $\propto$ length of the shelf
No of Books $\propto \frac{1}{\text{thickness}}$
$\Rightarrow B \propto \frac{l}{t} \Rightarrow B = k\frac{l}{t}$

25) The number of books kept on a shelf varies directly as the length of the shelf and inversely as the thickness of the books. If the length of the shelf is 2 m and the thickness of each book is 25 cm, there will be 8 books on a shelf. If the length of a shelf is 4 m and the thickness of each book is 20 cm, then how many books can be arranged on it?

a) 25
b) 40
c) 35
d) 20 ✓

$l = 2\,m$
$t = 25\,cm = 0.25\,m$
$B = 8$
$B = k \cdot \frac{l}{t}$
$8 = k \cdot \frac{2}{0.25}$
$k = \frac{8 \times 0.25}{2} = 1$

$l = 4\,m$
$t = 20\,cm = 0.20\,m$
$B = ?$
$B = k \cdot \frac{l}{t}$
$= 1 \times \frac{4}{0.2} = \frac{40}{2} = 20$

	A		B
Last year	3	:	4
Last : present	4 : 5		2 : 3
(individual) $\frac{P}{L}$	$\Rightarrow 5/4$		$3/2$

A last year $= 3x$
B last year $= 4x$

$\frac{P}{L} \times L = P$

26) Last year the ratio between the salaries of A and B was 3:4. But the ratio of their individual salaries between last year and this year were 4:5 and 2:3 respectively. If the sum of their present salaries is Rs 4160, then how much is the salary of A now?

present salaries $(A + B) = 4160$

a) Rs 1040

b) Rs 1600

c) Rs 2560

d) Rs 3120

$\frac{5}{4} \times 3x + \frac{3}{2} \times 4x = 4160$

$\frac{15x}{4} + \frac{12x}{2} = 4160$

$\frac{15x + 24x}{4} = 4160$

$x = \frac{4160 \times 4}{39}$

A present salary $= \frac{5}{4} \times 3x$

$\frac{5}{4} \times 3 \times \frac{4160 \times 4}{39}$

$= Rs\ 1600$