EXAMPLE 2 : $R = \mathbb{C}[[t]]$

$y^2 = (X^3 - t)(X^3 - t^2) = f(x)$; $f_1(x) = X^3 - t$, $f_2(x) = X^3 - t^2$

ζ: primitive 3rd root of unity

Roots of $f = \{t^{\frac{1}{3}}, \zeta t^{\frac{1}{3}}, \zeta^2 t^{\frac{1}{3}}, t^{\frac{2}{3}}, \zeta t^{\frac{2}{3}}, \zeta^2 t^{\frac{2}{3}}\}$

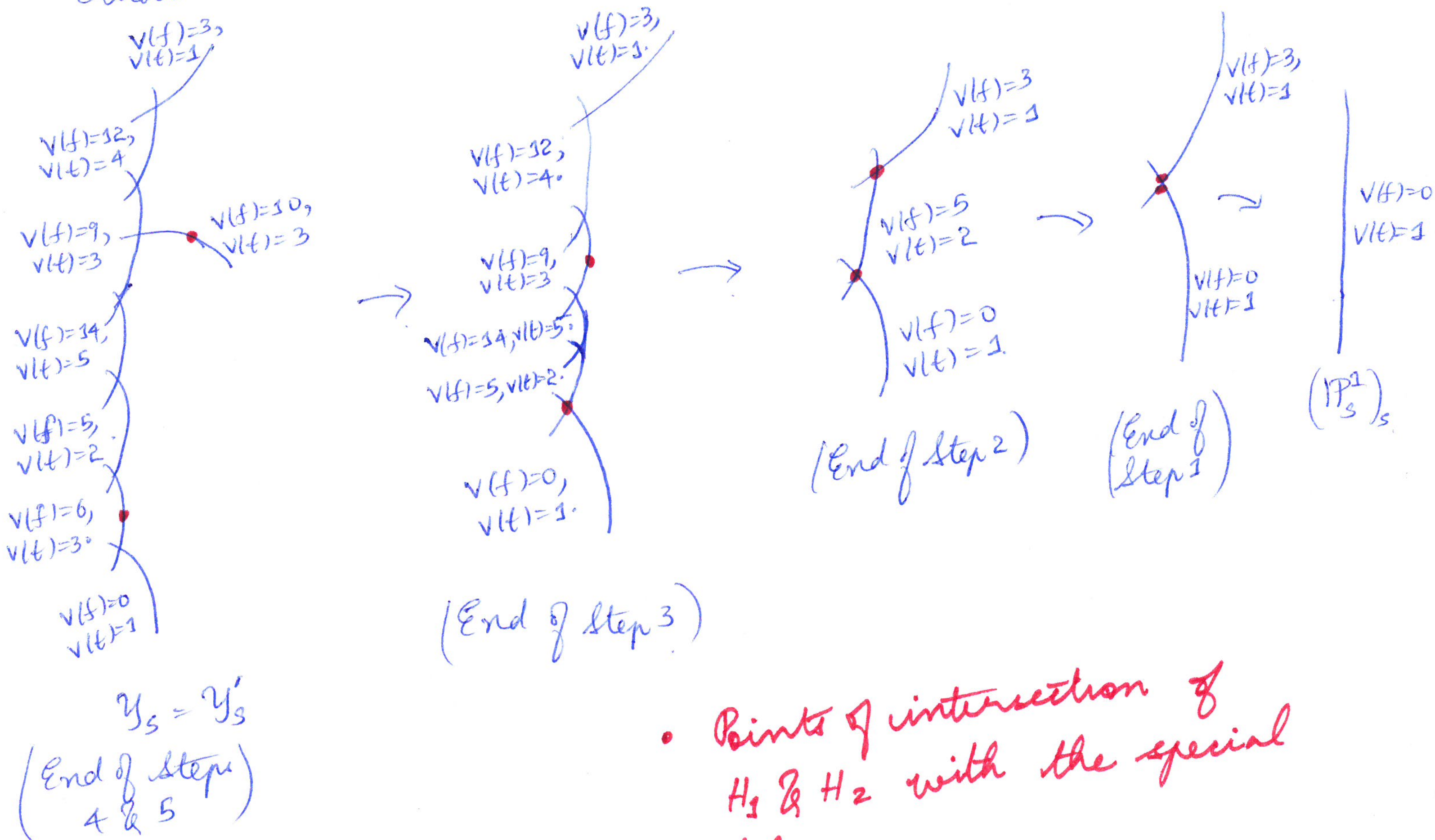
$\Delta_{min} = v_t(disc(f_1)) + v_t(disc(f_2)) + v_t(Resultant(f_1, f_2))$

$= 2\left(\frac{3}{2}\right)\left(\frac{1}{3}\right) + 2\left(\frac{3}{2}\right)\left(\frac{2}{3}\right) + 2 \cdot 3 \cdot 3 \cdot \frac{1}{3} = 12$

$div(f_1) = H_1 - 3(\infty)$, $div(f_2) = H_2 - 3(\infty)$

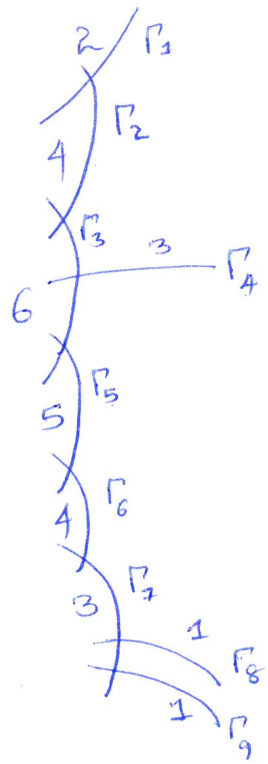
H_1 is regular, H_2 is not.

We will draw the special fibers of the models of $\mathbb{P}^1_K$ that are obtained at the end of each of the steps 1 – 5.



• Points of intersection of H_1 & H_2 with the special fiber.

$\mathcal{X}'_s = \mathcal{X}_s$



$$a_i = (1-m_{\Gamma_i})\,\chi(\Gamma_i) + \sum_{j\neq i}(m_{\Gamma_j}-1)\,\Gamma_i\cdot\Gamma_j$$

$$-\mathrm{Art}(\mathcal{X}'/S) = \sum_i a_i + \sum_{i<j}\Gamma_i\cdot\Gamma_j$$

$$= \sum_i a_i + 8$$

$$a_1 = -2+3 = 1$$

$$a_2 = a_5 = a_6 = 0$$

$$a_3 = -10+3+2+4 = -1$$

$$a_4 = -4+5 = 1$$

$$a_7 = -4+3 = -1$$

$$a_8 = a_9 = 2$$

(End of Step 6)

$$\therefore \sum a_i = 4$$

$$\Rightarrow \boxed{-\mathrm{Art}(\mathcal{X}'/S) = 12 = \Delta_{min}.}$$