$R = \mathbb{C}[[t]]$

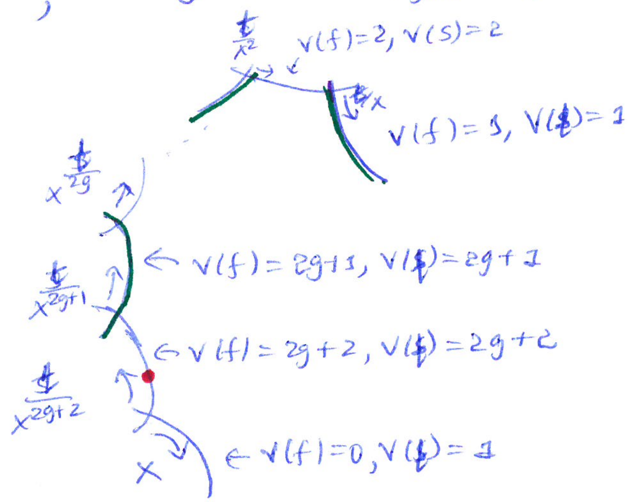
EXAMPLE 1: $y^2 = \underbrace{x^{2g+2} - t}_{= f(x)}$, ζ primitive $2g+2^{th}$ root of unity

$$\text{Roots of } f = \left\{ t^{\frac{1}{2g+2}}, \zeta t^{\frac{1}{2g+2}}, \ldots, \zeta^{2g+1} t^{\frac{1}{2g+2}} \right\}$$

$$\Delta_{min} = 2 \binom{2g+2}{2} \frac{1}{2g+2} = 2g+1$$

- $\operatorname{div}(f)$ on $\mathbb{P}^1_S$ is regular but its intersection with the special fiber is not transverse.
- $\mathcal{Y}' = \mathbb{P}^1_S$, $\mathcal{Y} = Bl_{2g+2} \mathbb{P}^1_S$

$\mathcal{Y}_s =$ (special fiber of the model $\mathcal{Y}$)

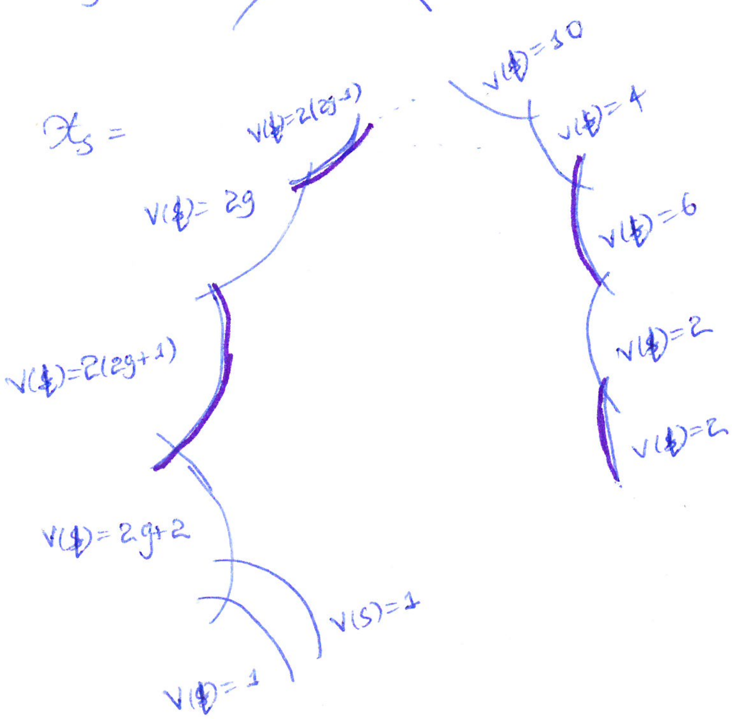


— : component that is part of the branch locus

• - pt. where the horizontal divisor $x^{2g+2}-t$ intersects the special fiber.

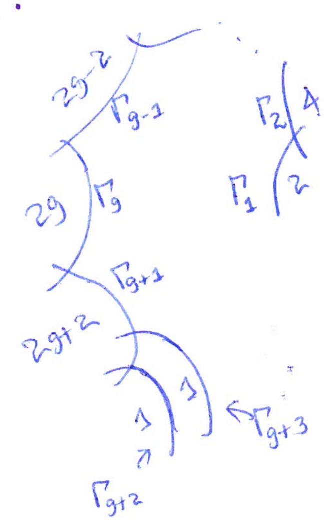
- $\mathcal{X}'_s =$

$\mathcal{X}_s =$



Contract — components in $\mathcal{X}$ to obtain a model (snc) $\mathcal{X}''$.

$\mathcal{X}''_s$:



For $1 \le i \le g+1$, m_{Γ_i} $(:= m_i) = 2i$.

$$\chi(\Gamma_i) = 2$$

$$m_{\Gamma_{g+2}} = m_{\Gamma_{g+3}} = 1.$$

$$\chi(\Gamma_{g+2}) = \chi(\Gamma_{g+3}) = 2.$$

For $1 \le i \le g+1$, $\Gamma_i \cdot \Gamma_{i+1} = 1$.

$$\Gamma_{g+1} \cdot \Gamma_{g+3} = 1.$$

$$-\mathrm{Art}(\mathcal{X}''/S) = \sum_i \left\{ (1-m_i)\,\chi(\Gamma_i) + \sum_{j \ne i} (m_j - 1)\Gamma_j \cdot \Gamma_i \right\} + \sum_{i<j} \Gamma_i \cdot \Gamma_j$$

For $2 \le i \le g$,

$$(1-m_i)\chi(\Gamma_i) + (m_{i-1}-1)\Gamma_{i-1}\cdot\Gamma_i + (m_{i+1}-1)\Gamma_{i+1}\cdot\Gamma_i = 0$$

$$\therefore\ -\mathrm{Art}(\mathcal{X}''/S) = (1-m_1)\chi(\Gamma_1) + (m_2-1)\Gamma_1\cdot\Gamma_2 + (1-m_{g+1})\chi(\Gamma_{g+1}) + (m_g-1)\Gamma_g\cdot\Gamma_{g+1}$$

$$+ (m_{g+2}-1)\Gamma_{g+2}\cdot\Gamma_{g+1} + (m_{g+3}-1)\Gamma_{g+3}\cdot\Gamma_{g+1} + (1-m_{g+2})\chi(\Gamma_{g+2})$$

$$+ (m_{g+1}-1)\Gamma_{g+2}\cdot\Gamma_{g+1} + (1-m_{g+3})\chi(\Gamma_{g+3}) + (m_{g+1}-1)\Gamma_{g+3}\cdot\Gamma_{g+1}$$

$$+ \sum_{i<j} \Gamma_i \cdot \Gamma_j$$

$$= -2 + 3 - \cancel{(2g+1)}2 + 2g-1 + \cancel{(2g+1)} + \cancel{(2g+1)} + g+2$$

$$= 2g + g + 2$$

# of contractions to get from $\mathcal{X}''$ to $\mathcal{X}' = g+1$

$$\therefore\ \boxed{-\mathrm{Art}(\mathcal{X}'/S) = 2g+g+2-(g+1) = 2g+1 = \Delta_{min}}$$

Remark:

1) If $R = \mathbb{Q}_p$ & $p \nmid 2g+2$, $y^2 = x^{2g+2} - p$, then same calculation shows $-\mathrm{Art}(\mathcal{X}/S) = \Delta_{min}$ since $\delta = 0$ (Swan term) by Saito's criterion.

If $p \mid 2g+2$, then $\Delta_{min} > 2g+1$ & $-\mathrm{Art}(\mathcal{X}/S) > 2g+1$; not clear what happens to the inequality in this case.

If $p \neq 2$, one possibility is to use Stefan Wewers' & Irene Bouw's calculations for δ via semi-stable reduction to check if inequality still holds.

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2) The same calculation works with any Eisenstein polynomial in place of $x^{2g+2} - t$.