Assignment-4

(1) w_h = worked hard for the course

g_a = got an A.

Given that from previous offerings of the course

$P(w_h) = 0.85$

$P(g_a) = 0.95$

$P(g_a/w_h) = 0.99$

(a) We need to calculate $P(w_h/g_a)$

$$P(w_h/g_a) = \frac{P(g_a/w_h)\,P(w_h)}{P(g_a)}$$

$$= \frac{0.99 \times 0.85}{0.95}$$

$$= 0.8857$$

(b) Given that student didn't work hard for the course, but got A.

$P(g_a/\overline{w_h})$ should be calculated.

(or) $P(g_a/w_h')$

$$P(g_a/w_h') = \frac{P(w_h'/g_a)\,P(g_a)}{P(w_h')}$$

$P(w_h) = 0.85 \Rightarrow P(w_h') = 0.15$

$P(w_h/g_a) = 0.8857 \Rightarrow P(w_h'/g_a) = 1 - 0.8857 = 0.1143$

$$P(g_a/w_h') = \frac{P(w_h'/g_a)\,P(g_a)}{P(w_h')}$$

$$= \frac{0.1143 \times 0.95}{0.15}$$

$$= 0.7239$$

(2) $P(\text{seengreen}/\text{green}) = 0.75$

$P(\text{seen blue}/\text{green}) = 0.25$

$P(\text{seen blue}/\text{blue}) = 0.75$

$P(\text{seengreen}/\text{blue}) = 0.25$

[Since discrimination between blue and green is 75% reliable, probability that the seen taxi colour is blue given that originally taxi colour is blue is 0.75]

(a) According to the baye's theorem

$$P(A/B) = \frac{P(B/A)\,P(A)}{P(B)}$$

Here we don't know P(blue) and P(green). We don't know how many taxis are blue coloured and how many of them are coloured green. Therefore, it is not possible to calculate the most likely colour for the taxi.

$$P(\text{Blue}|\text{seen blue}) = \frac{P(\text{seenblue}|\text{blue})\,P(\text{blue})}{P(\text{seenblue}|\text{blue})\,P(\text{blue}) + P(\text{seenblue}|\text{green})\,P(\text{green})}$$

(According to Baye's theorem)

we don't know P(blue) and P(green).

(b) $P(\text{green}) = \frac{9}{10} = 0.9$ $\quad P(\text{blue}) = 0.1$

By using above formula

$$P(\text{blue}|\text{seenblue}) = \frac{0.75\times0.1}{0.25\times0.9+0.75\times0.1} = \frac{0.075}{0.3} = 0.25$$

$$P(\text{green}|\text{seenblue}) = \frac{P(\text{seen blue}|\text{green})\,P(\text{green})}{P(\text{seenblue}|\text{green})\,P(\text{green}) + P(\text{seenblue}|\text{blue})\,P(\text{blue})}$$

$$= \frac{0.25\times0.9}{0.25\times0.9+0.75\times0.1}$$

$$= \frac{0.225}{0.225+0.075}$$

$$= \frac{0.225}{0.3} = 0.75$$

Since P(green|seen blue) > P(blue|seen blue). The most probable or most likely colour is green.

13 (a) Given W = good, S = pass and N = out.

Using naive bayes classifier, we need to predict his happiness.

P(Weather = W | Happy = H)

Weather	Happy = yes	Happy = No
Good	1/3	3/5
Bad	2/3	2/5

P(Study = S / Happy = H)

Study	Happy = yes	Happy = no
Pass	3/3	$\frac{1}{5}$
fail	0	4/5

P(Neighbour = N / Happy = H)

Neighbour	Happy = yes	Happy = NO
Home	2/3	$\frac{2}{5}$
Out	1/3	$\frac{3}{5}$

(a) $x' \Rightarrow (W = good,\ S = pass,\ and\ N = out)$

$P(W = good / Happy = yes) = \frac{1}{3}$

$P(W = good / Happy = No) = \frac{3}{5}$

$P(S = pass / Happy = yes) = 1$

$P(S = pass / Happy = No) = \frac{1}{5}$

$P(N = out / Happy = yes) = \frac{1}{3}$

$P(N = out / Happy = No) = \frac{3}{5}$

$P(Happy = yes) = \frac{3}{8}$

$P(Happy = No) = \frac{5}{8}$

Using MAP rule,

$P(yes / x') = [P(good/yes)\ P(pass/yes)\ P(out/yes)]\ P(Happy = yes)$

$= \frac{1}{3} \times 1 \times \frac{1}{3} \times \frac{3}{8}$

$= 0.0416$

$P(No / x') = \frac{3}{5} \times \frac{1}{5} \times \frac{3}{5} \times \frac{5}{8}$

$= 0.045$

Since $P(No / x') > P(yes / x')$

We label $x' \Rightarrow (W = good,\ S = pass,\ N = out) = No.$

(b) $X' = (W = good, S = pass, N = out)$ using bayes classifier.

For representation, $H \Rightarrow$ Happy = yes
$H' \Rightarrow$ Happy = No.

$$P(H \mid w = good, s = pass, N = out) = \frac{P(W = good/H)\ P(S = pass/H)\ P(N = out/H)\ P(H)}{P(W = good/H)\ P(S = pass/H)\ P(N = out/H)\ P(H) + P(W = good/H')\ P(S = pass/H')\ P(N = out/H')\ P(H')}$$

$$= \frac{\frac{1}{3} \times 1 \times \frac{1}{3} \times \frac{3}{8}}{\left[\frac{1}{3} \times 1 \times \frac{1}{3} \times \frac{3}{8}\right] + \left[\frac{3}{5} \times \frac{1}{5} \times \frac{3}{5} \times \frac{5}{8}\right]}$$

[∵ values are taken from table(s)]

$$= \frac{0.0416}{0.0416 + 0.045}$$

$$= \frac{0.0416}{0.0866} = 0.48036.$$

$$P(H'|W=good, S=pass, N=out) = \frac{P(W=good|H')\,P(S=pass|H')\,P(N=out|H')\,P(H')}{P(W=good|H)\,P(S=pass|H)\,P(N=out|H)\,P(H) + P(W=good|H')\,P(S=pass|H')\,P(N=out|H')\,P(H')}$$

$$= \frac{\frac{3}{5} \times \frac{1}{5} \times \frac{3}{5} \times \frac{5}{8}}{\left[\frac{1}{3} \times 1 \times \frac{1}{3} \times \frac{3}{8}\right] + \left[\frac{3}{5} \times \frac{1}{5} \times \frac{3}{5} \times \frac{5}{8}\right]}$$

$$= \frac{0.045}{0.0416 + 0.045}$$

$$= \frac{0.045}{0.0866} = 0.519.$$

Since $P(H'|W=good, S=pass, N=out) > P(H|W=good, S=pass, N=out)$

We classify the given condition W = good, S = pass, N = out as No.

∴ We predict that Jim is not Happy.

(4) $P(C{+}{+}) = 0.5$

$P(Java) = 0.4$

$P(M) = 0.01$ [M - Mocrosoft Employees]

$P(C{+}{+}|M) = 0.99$

$P(Java|M) = 0.98$

We need to find $P(M \mid C{+}{+}, Java)$

According to naive bayes theorem,

$$P(M \mid C{+}{+}, Java) = P(C{+}{+}/M)\, P(Java/M)\, P(M)$$

$$= 0.99 \times 0.98 \times 0.01$$

$$= 0.009702$$

$$P(M' \mid C{+}{+}, Java) = P(C{+}{+}/M')\, P(Java/M')\, P(M')$$

$$P(M) = 0.01 \Rightarrow P(M') = 1 - 0.01 = 0.99$$

We know that $P(C{+}{+}/M) = 0.99$, $P(Java/M) = 0.98$

$P(C{+}{+}) = 0.5$ $P(Java = 0.4)$ $P(M) = 0.01$

$\cancel{P(M/C{+}{+})}$

$$P(C{+}{+}) = P(C{+}{+}/M)\, P(M) + P(C{+}{+}/M')\, P(M')$$

$\cancel{0.99 \rightarrow}$

$$0.5 = 0.99 \times 0.01 + P(C{+}{+}/M') \times 0.99$$

$$0.5 = 0.0099 + P(C{+}{+}/M') \times 0.99$$

$$P(C{+}{+}/M') \times 0.99 = 0.5 - 0.0099$$

$$= 0.4901$$

$$P(C{+}{+}/M') = \frac{0.4901}{0.99} = 0.495$$

$$P(\text{Java}) = P(\text{Java}/M)\, P(M) + P(\text{Java}/M')\, P(M')$$

$$0.4 = 0.98 \times 0.01 + P(\text{Jova}/m') \times 0.99$$

$$P(\text{Jova}/m') = 0.394$$

$$\therefore \quad P(M'/ C++, \text{Jova}) = P(\text{Java}/m')\, P(C++/m')\, P(M')$$

$$= 0.394 \times 0.495 \times 0.99$$

$$= 0.19307$$

$$P(M/C++, \text{Javo}) < P(M'/C++, \text{Java})$$

A programmer who knows both C++ and Java is more probable to be non macrosoft employee.

(5) $P(X_i|Y=1) = \theta_{i1}^{x_i}(1-\theta_{i1})^{(1-x_i)}$

$$P(Y=1|X_i) = \frac{P(X_i|Y=1)\,P(Y=1)}{P(X_i|Y=1)\,P(Y=1)+P(X_i|Y=0)\,P(Y=0)}$$

$$= \frac{1}{1+\frac{P(X_i|Y=0)\,P(Y=0)}{P(X_i|Y=1)\,P(Y=1)}}$$

$$= \frac{1}{1+\exp\left(\ln\left(\frac{P(Y=0)}{P(Y=1)}\right)+\sum_i \ln\left(\frac{P(X_i|Y=0)}{P(X_i|Y=1)}\right)\right)}$$

Let $P(Y=1) = x$

$$\Rightarrow P(Y=1|X_i) = \frac{1}{1+\exp\left[\left(\frac{1-x}{x}\right)+\sum_i \ln\left(\frac{P(X_i|Y=0)}{P(X_i|Y=1)}\right)\right]}$$

$$\sum_i \ln\left(\frac{P(X_i|Y=0)}{P(X_i|Y=1)}\right) = \sum_i \ln \frac{\theta_{i0}^{x_i}(1-\theta_{i0})^{(1-x_i)}}{\theta_{i1}^{x_i}(1-\theta_{i1})^{(1-x_i)}}$$

$$= \sum_i \ln\left(\frac{\theta_{i0}}{1-\theta_{i0}}\,\frac{(1-\theta_{i1})}{\theta_{i1}}\right)^{x_i}\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)$$

$$= \sum_i \left\{ x_i \ln \frac{\theta_{i0}(1-\theta_{i1})}{\theta_{i1}(1-\theta_{i0})} + \ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right\}$$

$$P(Y=1|X) = \frac{1}{1+\exp\left\{\ln\left(\frac{1-x}{x}\right)+\sum_i x_i \ln\left(\frac{\theta_{i0}(1-\theta_{i1})}{\theta_{i1}(1-\theta_{i0})}\right)+\ln\left(\frac{1-\theta_{i0}}{1-\theta_{i1}}\right)\right]}$$

Let $w_0 = \ln\left(\frac{1-x}{x}\right) + \sum_i \ln\left(\frac{1-\theta_{i0}}{1+\theta_{i1}}\right)$

$w_i = \sum_i \ln\left(\frac{\theta_{i0}(1-\theta_{i1})}{\theta_{i1}(1-\theta_{i0})}\right)$

$$P(Y=1|X) = \frac{1}{1+\exp(w_0+\sum_i x_i w_i)} \qquad P(Y=0|X) = \frac{\exp(w_0+\sum_i x_i w_i)}{1+\exp(w_0+\sum_i x_i w_i)}$$

The obtained expressions are same as those given by Logistic Regression
$\Rightarrow$ LR is the discriminative counter part of Naive Bayes classifier over Boolean features