COMP417 Artificial Intelligence
Exercise Set 3

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Exercise 1

Sentence A

$Smoke$	$Smoke \implies Smoke$
$true$	$true$
$false$	$true$

The sentence is *valid* (reflexive property, see truth table).

Sentence B

$Smoke$	$Fire$	$Smoke \implies Fire$
$true$	$true$	$true$
$false$	$true$	$true$
$false$	$false$	$true$
$true$	$false$	$false$

The sentence is *satisfiable* (see truth table).

Sentence C

Let $B \equiv (Smoke \implies Fire)$ and $C \equiv (\neg Smoke \implies \neg Fire)$.

$Smoke$	$Fire$	B	C	$B \implies C$
$true$	$true$	$true$	$true$	$true$
$false$	$true$	$true$	$false$	$false$
$false$	$false$	$true$	$true$	$true$
$true$	$false$	$false$	$true$	$true$

The sentence is *satisfiable* (see truth table).

Sentence D

$Fire \vee \neg Fire \equiv true$ (law of excluded middle), therefore the sentence is *valid*:

$$Smoke \vee Fire \vee \neg Fire \equiv$$
$$Smoke \vee true \equiv$$
$$true$$

Sentence E

$$
\begin{gathered}
((Smoke \land Heat) \implies Fire) \iff ((Smoke \implies Fire) \lor (Heat \implies Fire)) \equiv \\
(\neg(Smoke \land Heat) \lor Fire) \iff ((Smoke \implies Fire) \lor (Heat \implies Fire)) \equiv \\
(\neg Smoke \lor \neg Heat \lor Fire) \iff ((Smoke \implies Fire) \lor (Heat \implies Fire)) \equiv \\
(\neg Smoke \lor \neg Heat \lor Fire) \iff ((\neg Smoke \lor Fire) \lor (\neg Heat \lor Fire)) \equiv \\
(\neg Smoke \lor \neg Heat \lor Fire) \iff (\neg Smoke \lor \neg Heat \lor Fire) \equiv \\
true
\end{gathered}
$$

Therefore the sentence is *valid*.

Sentence F

$$
\begin{gathered}
(Smoke \implies Fire) \implies ((Smoke \land Heat) \implies Fire) \equiv \\
(\neg Smoke \lor Fire) \implies ((Smoke \land Heat) \implies Fire) \equiv \\
(\neg Smoke \lor Fire) \implies (\neg(Smoke \land Heat) \lor Fire) \equiv \\
\neg(\neg Smoke \lor Fire) \lor (\neg(Smoke \land Heat) \lor Fire) \equiv \\
(Smoke \land \neg Fire) \lor (\neg(Smoke \land Heat) \lor Fire) \equiv \\
(Smoke \land \neg Fire) \lor \neg Smoke \lor \neg Heat \lor Fire \equiv \\
((Smoke \lor \neg Smoke) \land (\neg Fire \lor \neg Smoke)) \lor \neg Heat \lor Fire \equiv \\
(true \land (\neg Fire \lor \neg Smoke)) \lor \neg Heat \lor Fire \equiv \\
\neg Fire \lor \neg Smoke \lor \neg Heat \lor Fire \equiv \\
(\neg Fire \lor Fire) \lor \neg Smoke \lor \neg Heat \equiv \\
true \lor \neg Smoke \lor \neg Heat \equiv \\
true
\end{gathered}
$$

Therefore the sentence is *valid*.

Sentence G

$$
\begin{gathered}
Big \lor Dumb \lor (Big \implies Dumb) \equiv \\
Big \lor Dumb \lor \neg Big \lor Dumb \equiv \\
(Big \lor \neg Big) \lor (Dumb \lor Dumb) \equiv \\
true \lor Dumb \equiv \\
true
\end{gathered}
$$

Therefore the sentence is *valid*.

Sentence H

$$
\begin{gathered}
(Big \wedge Dumb) \vee \neg Dumb \equiv \\
(Big \vee \neg Dumb) \wedge (Dumb \vee \neg Dumb) \equiv \\
\neg Dumb \vee Big \equiv \\
Dumb \implies Big
\end{gathered}
$$

$Dumb$	Big	$Dumb \implies Big$
$true$	$true$	$true$
$false$	$true$	$true$
$false$	$false$	$true$
$true$	$false$	$false$

Therefore the sentence is *satisfiable*.

Exercise 2

Let A be a sentence, and $W_1, ..., W_n$ the worlds in which A would be false. Then the given observation is:

$$A \equiv \neg W_1 \wedge ... \wedge \neg W_n \tag{1}$$

Each world W_i can be described as a conjunction of sentences, namely:

$$W_i \equiv W_{i,1} \wedge ... \wedge W_{i,m} \tag{2}$$

From (1)$\wedge$(2):

$$
\begin{gathered}
A \equiv \\
\neg(W_{1,1} \wedge ... \wedge W_{1,k}) \wedge ... \wedge \neg(W_{n,1} \wedge ... \wedge W_{n,l}) \equiv \\
(\neg W_{1,1} \vee ... \vee \neg W_{1,k}) \wedge ... \wedge (\neg W_{n,1} \vee ... \vee \neg W_{n,l})
\end{gathered}
$$

which is in CNF.

Exercise 3

The vocabulary of the first order logic expressions contains the following predicates and constants:

- $Student(x)$: x is a student
- $Takes(x, c, s)$: student x takes course c in semester s (used in both present and past tense)
- $French$, $Greek$: constants for the corresponding courses
- $Spring2001$: constant for the corresponding semester
- $Passes(x, c, s)$: student x passes course c in semester s
- $Score(x, c, s)$: the score obtained by student x in course c in semester s
- $> (a, b)$: a is greater than b
- $Person(x)$: x is a person
- $Policy(x)$: x is a policy

- $Buys(x, p, a)$: person x buys policy p from agent a
- $Smart(x)$: x is smart
- $Expensive(x)$: x is expensive
- $Sells(a, p, x)$: agent a sells policy p to person x
- $Agent(x)$: x is an agent
- $Insured(x)$: x is insured

Sentence A

$$\exists x \; Student(x) \land Takes(x, French, Spring2001)$$

Sentence B

$$\forall x, s \; Student(x) \land Takes(x, French, s) \implies Passes(x, French, s)$$

Sentence C

$$\exists x \forall y \; Student(x) \land Takes(x, Greek, Spring2001) \land \neg(y = x) \implies \neg Takes(y, Greek, Spring2001)$$

Sentence D

$$\exists x \; \forall y, s \; > (Score(x, Greek, s), Score(y, French, s))$$

Sentence E

$$\forall x \; Person(x) \land (\exists p, a \; Policy(p) \land Buys(x, p, a)) \implies Smart(x)$$

Sentence F

$$\forall x, p, a \; Person(x) \land Expensive(p) \land Policy(p) \implies \neg Buys(x, p, a)$$

Sentence G

$$\exists a \; \forall p, x \; Agent(a) \land Policy(p) \land Sells(a, p, x) \implies (Person(x) \land \neg Insured(x))$$

Exercise 4

The pseudo-code can be described by the following first order logic expression:

$$\exists j \; Object(j) \land Politician(j) \land Honest(j) \land Incorruptible(j)$$

where:

- $Object(j)$: j is an element of an Objects array
- $Politician(j)$: j is a politician
- $Honest(j)$: j is honest
- $Incorruptible(j)$: j is incorruptible