COMP417 Artificial Intelligence
Exercise Set 3

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Exercise 1

Sentence A

$Smoke$	$Smoke \implies Smoke$
$true$	$true$
$false$	$true$

The sentence is *valid* (reflexive property, see truth table).

Sentence B

$Smoke$	$Fire$	$Smoke \implies Fire$
$true$	$true$	$true$
$false$	$true$	$true$
$false$	$false$	$true$
$true$	$false$	$false$

The sentence is *satisfiable* (see truth table).

Sentence C

Let $B \equiv (Smoke \implies Fire)$ and $C \equiv (\neg Smoke \implies \neg Fire)$.

$Smoke$	$Fire$	B	C	$B \implies C$
$true$	$true$	$true$	$true$	$true$
$false$	$true$	$true$	$false$	$false$
$false$	$false$	$true$	$true$	$true$
$true$	$false$	$false$	$true$	$true$

The sentence is *satisfiable* (see truth table).

Sentence D

$Fire \vee \neg Fire \equiv true$ (law of excluded middle), therefore the sentence is *valid*:

$$Smoke \vee Fire \vee \neg Fire \equiv$$
$$Smoke \vee true \equiv$$
$$true$$

Sentence E

$$
\begin{gathered}
((Smoke \wedge Heat) \implies Fire) \iff ((Smoke \implies Fire) \vee (Heat \implies Fire)) \equiv \\
(\neg(Smoke \wedge Heat) \vee Fire) \iff ((Smoke \implies Fire) \vee (Heat \implies Fire)) \equiv \\
(\neg Smoke \vee \neg Heat \vee Fire) \iff ((Smoke \implies Fire) \vee (Heat \implies Fire)) \equiv \\
(\neg Smoke \vee \neg Heat \vee Fire) \iff ((\neg Smoke \vee Fire) \vee (\neg Heat \vee Fire)) \equiv \\
(\neg Smoke \vee \neg Heat \vee Fire) \iff (\neg Smoke \vee \neg Heat \vee Fire) \equiv \\
true
\end{gathered}
$$

Therefore the sentence is *valid*.

Sentence F

$$
\begin{gathered}
(Smoke \implies Fire) \implies ((Smoke \wedge Heat) \implies Fire) \equiv \\
(\neg Smoke \vee Fire) \implies ((Smoke \wedge Heat) \implies Fire) \equiv \\
(\neg Smoke \vee Fire) \implies (\neg(Smoke \wedge Heat) \vee Fire) \equiv \\
\neg(\neg Smoke \vee Fire) \vee (\neg(Smoke \wedge Heat) \vee Fire) \equiv \\
(Smoke \wedge \neg Fire) \vee (\neg(Smoke \wedge Heat) \vee Fire) \equiv \\
(Smoke \wedge \neg Fire) \vee \neg Smoke \vee \neg Heat \vee Fire \equiv \\
((Smoke \vee \neg Smoke) \wedge (\neg Fire \vee \neg Smoke)) \vee \neg Heat \vee Fire \equiv \\
(true \wedge (\neg Fire \vee \neg Smoke)) \vee \neg Heat \vee Fire \equiv \\
\neg Fire \vee \neg Smoke \vee \neg Heat \vee Fire \equiv \\
(\neg Fire \vee Fire) \vee \neg Smoke \vee \neg Heat \equiv \\
true \vee \neg Smoke \vee \neg Heat \equiv \\
true
\end{gathered}
$$

Therefore the sentence is *valid*.

Sentence G

$$
\begin{gathered}
Big \vee Dumb \vee (Big \implies Dumb) \equiv \\
Big \vee Dumb \vee \neg Big \vee Dumb \equiv \\
(Big \vee \neg Big) \vee (Dumb \vee Dumb) \equiv \\
true \vee Dumb \equiv \\
true
\end{gathered}
$$

Therefore the sentence is *valid*.

Sentence H

$$(Big \wedge Dumb) \vee \neg Dumb \equiv$$
$$(Big \vee \neg Dumb) \wedge (Dumb \vee \neg Dumb) \equiv$$
$$\neg Dumb \vee Big \equiv$$
$$Dumb \implies Big$$

$Dumb$	Big	$Dumb \implies Big$
$true$	$true$	$true$
$false$	$true$	$true$
$false$	$false$	$true$
$true$	$false$	$false$

Therefore the sentence is *satisfiable*.

Exercise 2

Let A be a sentence, and $W_1, ..., W_n$ the worlds in which A would be false. Then the given observation is:

$$A \equiv \neg W_1 \wedge ... \wedge \neg W_n \tag{1}$$

Each world W_i can be described as a conjunction of sentences, namely:

$$W_i \equiv W_{i,1} \wedge ... \wedge W_{i,m} \tag{2}$$

From (1)∧(2):

$$A \equiv$$
$$\neg(W_{1,1} \wedge ... \wedge W_{1,k}) \wedge ... \wedge \neg(W_{n,1} \wedge ... \wedge W_{n,l}) \equiv$$
$$(\neg W_{1,1} \vee ... \vee \neg W_{1,k}) \wedge ... \wedge (\neg W_{n,1} \vee ... \vee \neg W_{n,l})$$

which is in CNF.

Exercise 3

The vocabulary of the first order logic expressions contains the following predicates and constants:

- $Student(x)$: x is a student
- $Takes(x, c, s)$: student x takes course c in semester s (used in both present and past tense)
- $French$, $Greek$: constants for the corresponding courses
- $Spring2001$: constant for the corresponding semester
- $Passes(x, c, s)$: student x passes course c in semester s
- $Score(x, c, s)$: the score obtained by student x in course c in semester s
- $>(a, b)$: a is greater than b
- $Person(x)$: x is a person
- $Policy(x)$: x is a policy

- $Buys(x, p, a)$: person x buys policy p from agent a
- $Smart(x)$: x is smart
- $Expensive(x)$: x is expensive
- $Sells(a, p, x)$: agent a sells policy p to person x
- $Agent(x)$: x is an agent
- $Insured(x)$: x is insured

Sentence A

$$\exists x \; Student(x) \wedge Takes(x, French, Spring2001)$$

Sentence B

$$\forall x, s \; Student(x) \wedge Takes(x, French, s) \implies Passes(x, French, s)$$

Sentence C

$$\exists x \forall y \; Student(x) \wedge Takes(x, Greek, Spring2001) \wedge \neg(y = x) \implies \neg Takes(y, Greek, Spring2001)$$

Sentence D

$$\exists x \; \forall y, s \; > (Score(x, Greek, s), Score(y, French, s))$$

Sentence E

$$\forall x \; Person(x) \wedge (\exists p, a \; Policy(p) \wedge Buys(x, p, a)) \implies Smart(x)$$

Sentence F

$$\forall x, p, a \; Person(x) \wedge Expensive(p) \wedge Policy(p) \implies \neg Buys(x, p, a)$$

Sentence G

$$\exists a \; \forall p, x \; Agent(a) \wedge Policy(p) \wedge Sells(a, p, x) \implies (Person(x) \wedge \neg Insured(x))$$