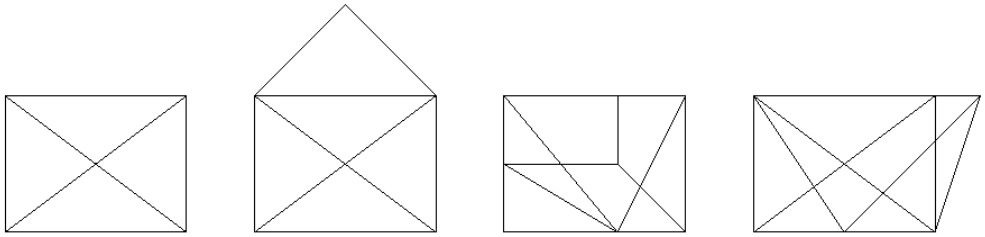
Problems. . .

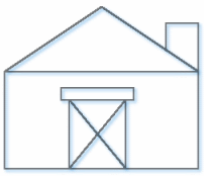
Problems bellow can be solved using graphs. Some solutions are, in disorder, in the section 2 (some suit for several problems). Others are in previous/futur tutorials/exams!

1 Problems

Problem 1 (Without lifting the pencil)



Among these figures, which ones can be drawn with only one continuous line?



Problem 2 (Arthur's Knights)

The legend states that Arthur's knights are all united by the power of friendship. In reality, everything isn't that perfect in *Kaamelott*. Here are the "incompatibilities" between the knights.

- Sagamor can't stand Tristan nor Mordred ;
- same goes for Tristan with Lancelot or Perceval ;
- Lamorak and Yvain don't trust Lancelot ;
- Perceval refuses to speak to Yvain and Keu ;
- Keu and Lamorak hate Mordred ;
- finally, Yvain and Tristan never got on well together.

1. The Quest of the Grail: Arthur wants to send teams of knights to search for it.
   (a) Suggest to Arthur a solution with 4 teams of 2.
   (b) Can Arthur do with less teams?
2. The round table: Arthur wants to do a table plan allowing the knights to meet without him and to be able to spend a good time by having compatible people on their left and right.
   (a) Suggest a table plan to Arthur.
   (b) To give himself the image of a modern manager, Arthur suggests a U-shaped table where the knights will be sitting on the outside part of the U. Suggest a new table plan to Arthur.

Problem 3 (Fox, goose and bag of beans)

Once upon a time a farmer went to a market and purchased a fox, a goose, and a bag of beans. On his way home, the farmer came to the bank of a river and rented a boat. But in crossing the river by boat, the farmer could carry only himself and a single one of his purchases: the fox, the goose, or the bag of beans.

If left unattended together, the fox would eat the goose, or the goose would eat the beans.

The farmer's challenge was to carry himself and his purchases to the far bank of the river, leaving each purchase intact. How did he do it?

Problem 4 (Murderer)

One day Sherlock Holmes meets up with his friend Watson, who was in charge of the investigation on a mysterious murder three years ago.
At the time, the Duke of Densmore was killed by a bomb, which had completely flattened the castle where Densmore had retired. The news had at the time explained that the destruction of his testament during the blast was much inconvenient for one of his seven ex-wives. However, before he died, the Duke had all of them invited over **a few days** to his scottish castle.

Holmes: I remember this case; what is strange, is that the bomb was build precisely to fit and be hidden in the armor of the bedroom, which means that the murderer necessarily visited the castle a few times!

Watson: Certainly, and for this reason, I have questioned all the women: Ann, Betty, Charlotte, Edith, Félicia, Georgia and Helen. They all swore they only came once to the castle in their life.

Holmes: Hum! Have you asked them at which time they had respectively visited the castle?

Watson: Alas! None of them remembered the exact dates, it was more than 3 years ago! Notwithstanding, I have asked them who they met:

> Ann met Betty, Charlotte, Félicia and Georgia.
>
> Betty met Ann, Charlotte, Edith, Félicia and Helen.
>
> Charlotte met Ann, Betty and Edith.
>
> Edith met Betty, Charlotte and Félicia.
>
> Félicia met Ann, Betty, Edith and Helen.
>
> Georgia met Ann and Helen.
>
> Helen met Betty, Félicia and Georgia.

You see, my dear Holmes, the clues are concordant!

It was at that moment that Holmes took a pencil and drew a strange scheme, with points and lines connecting them. Then, in less than thirty seconds, he declared:

Holmes: Well well! What you just gave me allows me to determine with exactness the murderer.

Who is the murderer?

Problem 5 (Eat Brownies)

How long it takes to make brownies ?
Here is a recipe with the time for each task.

Ref.	recipe	time (mn)
A	Put the chocolate and butter to melt in bain-marie.	7
B	Quickly stir in eggs, sugar and flour in a bowl.	5
C	Chop nuts into chunks.	3
D	Sift flour with salt.	1
E	Add the chocolate (A) to the mix B and stir together well.	4
F	Add to the mix flour (D) and nuts (C).	2
G	Grease and flour the cake pan.	1
H	Preheat the oven.	10
I	Pour the brownie mix (F) into the cake pan.	1
J	Put in the oven.	35

1. **The cook is alone:** the recipe will take the sum of all durations (assuming the cook watches its oven while it heats up...). However he needs help with the task order.

   How do you get all the possible orders?

2. The cook has found assistants, some tasks can thus be done simultaneously: What is the minimum duration to perform the recipe?

Problem 6 (Air Traffic)

Consider four towns V_1, V_2, V_3, V_4 in a country with very poor air traffic: there are only direct flies from V_1 to V_2 and to V_4, from V_2 to V_3, from V_3 to V_1 and V_4, and from V_4 to V_2.

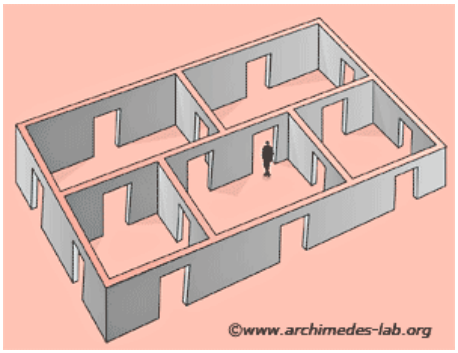
The only airline has solutions to travel from any town V_i to any town V_j with minimum stopovers. How many stopovers are there at most in a travel?

Problem 7 ("Sculptures")

How many ballons are needed to build this sculpture?

Problem 8 (Five room puzzle)

Can you find a path that goes trough each door only once?

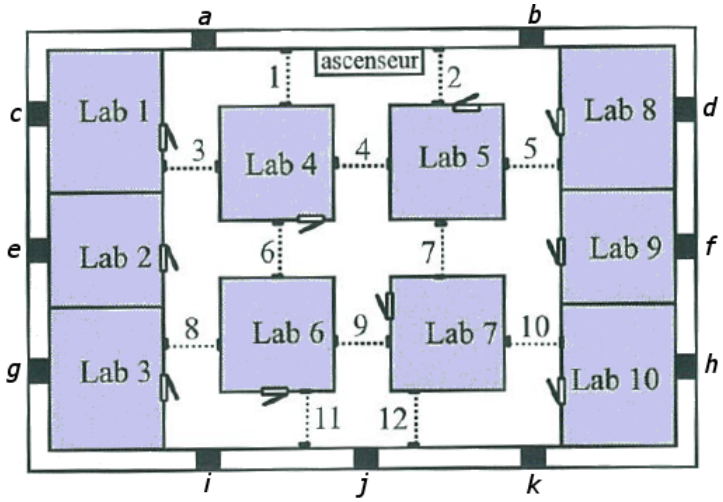


Problem 9 (Saving Algernon – *mid-term 2016*)

Algernon, the famous mouse, has been kidnapped by a research laboratory member, who brings it to his lab. But Algernon is clever and has managed to escape, taking advantage of the opening of the laboratory door. Algernon has to be found: during the night it might have gone anywhere and now it can be in a lab or stuck in an air vent.

Here is a map of the research laboratory:

- 10 labs (Lab 1 to Lab 10), one for each researcher;
- the air vents, represented by the black squares (from a to k): a mouse can go in but cannot go out and each air vent takes to a dead end ;
- the elevator ("ascenseur"), which takes directly outside ;
- and, in dashed lines, the motion detectors.



Motion detectors are rather archaic: simple mechanical indicators that count passage number (one cannot know when they have been triggered).

Here are the numbers of detections during the night:

detector #	1	2	3	4	5	6	7	8	9	10	11	12
nb detections	2	1	3	3	1	3	0	2	2	3	2	1

Which researcher is the kidnapper? Where is Algernon now?

Problem 10 (Bandwidth Allocation)

Some telecom networks are made up of transmitters that transmit each on its own frequency. When two transmitters are too close, they cannot have the same frequency assigned, it may cause interference.

How to determine a doable allocation with a minimum of frequencies?

Problem 11 (Nim)

Find a winning strategy to this classical Nim game version:

At the beginning, there are n matches. Each player takes in turn 1, 2 or 3 matches. The player to take the last match loses.

What about this "reversed" version ?

This time, the goal of the game is to be the player to remove the last match. Moreover, one can take up to 4 matches per turn.

Problem 12 (Doublets)

Extract from **Doublets A Word Puzzle** by Lewis Caroll (1879):

> The rules of the Puzzle are simple enough/ Two words are proposed, of the same length ; and the Puzzle consists in linking these together by interposing other words, each of which shall differ from the next word *in one letter only.* That is to say, one letter may be changed in one of the given words, then one letter in the word so obtained, and so on, till we arrive at the other given word. The letters must not be interchanged among themselves, but each must keep to its own place.

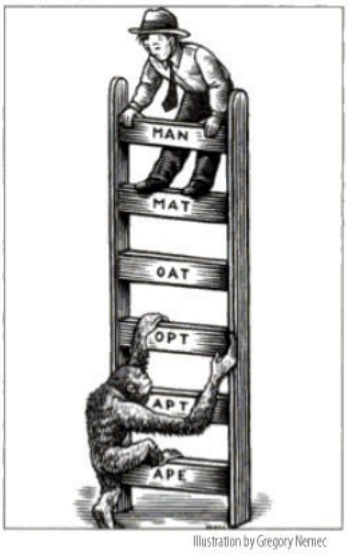


Illustration by Gregory Nemec

Doublets set in "Vanity Fair" (British weekly magazine):

- Change TEARS into SMILE
- Turn POOR into RICH
- Make FLOUR into BREAD
- Raise FOUR to FIVE
- Prove PITY to be GOOD
- Change BLACK to WHITE
- Prove a ROGUE to be a BEAST

1. From two words, and a glossary, find the shortest solution: called *ladder*.
2. How to find the most difficult problem (the longer *ladder*) from a glossary?

Problem 13 (Concert)

A charity festival is organised in a big theater. Seven world-renowned artists are invited. The festival organizer has to plan several concerts for the different invited musicians refuse to play with some others:

- Luther Allunison do not want to play at the same time as Jimi Endisque or Bob Ditlane ;
- John Biaise will not come if Phil Colline is present ;
- Phil Colline refuse to play with Rory Garaguerre ;
- Only Bob Ditlane can stand Jimi Endisque ;
- Robert Fripe is big-headed, he will come only if he plays alone ;
- Rory Garaguerre and Luther Allunison cannot play together.

1. How many concerts the organizer has to plan?
2. Propose a sharing out for each concert.

. . . and Graphs

2 Solutions?

Exercise 2.1 (Path)

1. How to find a path (a chain) between two vertices in a graph? Give two different methods and compare them.
2. Write a function that searches for a path between two vertices. If a path is found, it has to be returned (a vertex list).

Exercise 2.2 (Euler)

An Eulerian path is a path in a graph which visits every edge exactly once. Similarly, an Eulerian cycle is an Eulerian path which starts and ends on the same vertex.

1. **Test:**
   (a) What are the properties of an Eulerian graph (a graph with an Eulerian path or cycle)?
   (b) Deduce a function that checks if a graph is Eulerian.
2. **Path:**
   (a) Write a function that tests whether a vertex list can be an Eulerian path for a graph.
   (b) How to build the Eulerian path (or cycle) of an Eulerian graph?

Exercise 2.3 (Sous-graphe)

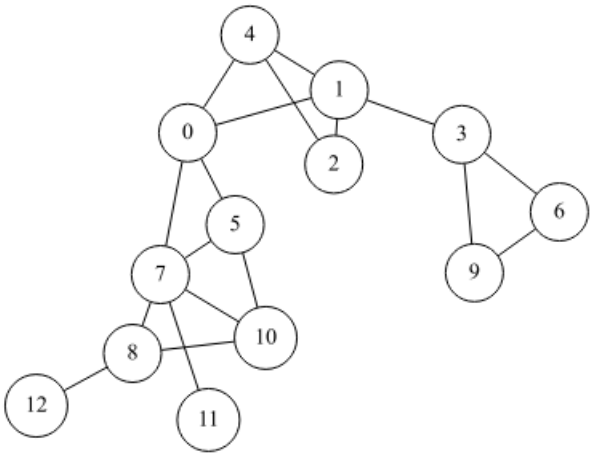


Figure 1: Graphe G

1. Which vertices are at a maximum distance of 2 from the vertex 0 in G?
2. Build the subgraph from S', set of vertices from question 1.
3. Which traversal allows us to obtain those vertices?
4. Write a function that builds a subgraph of a graph (or a digraph) from a subset of vertices that are at a maximum distance of a source vertex.

Exercise 2.4 (Algo)

Considere the following "algorithm" on a digraph:

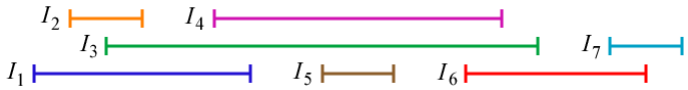
> While the graph is non empty
>
> - find a vertex without predecessor
> - delete this vertex from the graph

1. What property should have the graph so that the algorithm stops?
2. How to implement this algorithm without actually removing the vertices?
3. Write the function that returns the list of vertices in the chosen order.

Exercise 2.5 (Interval Graph)

An interval graph is an undirected graph formed from a family of intervals $I_1, \cdots, I_n$, by creating one vertex i for each interval I_i, and connecting two vertices i and j, $i \neq j$ by an edge whenever the corresponding two sets have a nonempty intersection ($I_i \cap I_j \neq \varnothing$).

1. Draw the graph from the following intervalle set:



2. What are the properties of an interval graph?

Exercise 2.6 (Coloring - Chromatic Number)

In graph theory, graph coloring is a way of coloring the vertices of a graph such that no two adjacent vertices share the same color. The smallest number of colors needed to color a graph is called its chromatic number.

1. Write a function that tests if a coloring, given by a list that represents for each vertex its color (an integer), is a proper coloring for a graph.
2. What is the connection between the chromatic number and the maximum vertex degree?
3. How to find the chromatic number of a graph?

Exercise 2.7 ("Kernel")

Let G be the graph $G = <S, A>$

☐ An *stable set* (*independent set*) is a set of vertices in a graph, no two of which are adjacent. That is, it is a set S_1 of vertices (subset of S) such that for every two vertices in S_1, there is no edge connecting the two.

☐ A *dominating set* is a subset S_2 of S such that every vertex of S not in S_2 is adjacent to at least one member of S_2.

☐ We will call a subset of S that is both stable and dominating a *graph's kernel*.

1. Build the 12-order graph to represent the last problem.
2. (a) Give the kernel of the graph (there is only one here).
   (b) How can the kernel help us find a solution?
3. (a) A digraph without circuit has only one kernel. How to build the kernel of such a graph?
   (b) And for any graph?