Betweenness-accessibility: Estimating impacts of accessibility on networks

Georgios Sarlas[*,a], Antonio Paez[b], Kay W. Axhausen[a]

[a] *ETH Zurich, Institute for Transport Planning and Systems, Stefano-Franscini-Platz 5, 8093 Zurich, Switzerland*

[b] *McMaster University, School of Geography and Earth Sciences, 1280 Main St W, Hamilton, Ontario, L8S 1S4 Canada*

Abstract

Accessibility is a central concept in transport geography research that has been described as a holistic measure of transportation and land use systems. This concept has numerous implementations, but virtually all share the way accessibility is measured as an attribute of pairs of origins and destinations. Borrowing from concepts in network science, in this paper we propose a new centrality measure called betweenness-accessibility. This measure couples the familiar betweenness indicator of social networks with the geographical concept of accessibility. Betweenness-accessibility is useful to estimate the impacts of accessibility on networks as potential for interaction is reflected on them. The new measure is illustrated using a reproducible example. In addition, an application to vulnerability analysis in the city of Zurich, Switzerland, provides an empirical case study to showcase the usefulness of betweenness-accessibility.

Key words: Accessibility; Centrality; Betweenness; Network analysis; Vulnerability analysis

Introduction

Accessibility is a central concept in transport geography with importance that stems from its ability to provide a holistic view of the way transportation systems interact with the spatial distribution of opportunities in a region (Handy and Niemeier 1997). Multiple approaches to derive operational measures of accessibility exist in the literature that belong to two classes, depending on whether they are based on concepts of gravitation/spatial interaction or consumer utility (Ben-Akiva and Lerman 1977; Kwan 1998; Páez, Scott, and Morency 2012). Of these two, gravity-based measures are certainly the most commonly used in practice, due to their ease of implementation, interpretation, and communicability (Geurs and Wee 2004).

[*] Corresponding Author

Email addresses: `georgios.sarlas@ivt.baug.ethz.ch` (Georgios Sarlas), `paezha@mcmaster.ca` (Antonio Paez), `axhausen@ivt.baug.ethz.ch` (Kay W. Axhausen)

A key aspect of gravity-based accessibility measures is that they measure the potential for interaction between origins and destinations. More formally, the accessibility of location i is the weighted sum (using transportation cost c_{ij}) of the opportunities W $\forall j$, possibly excluding i itself to eliminate self-potential (Frost and Spence 1995):

$$A_i^W = \sum_{j \neq i}^{n} g(W_j) f(c_{ij}) \tag{1}$$

Measures of this kind are related to the gravity model for human interaction (see Carrothers 1956):

$$T_{ij} = g_1(W_i) g_2(W_j) f(c_{ij}) \tag{2}$$

It can be seen that gravity-based accessibility measures are the summation of $g_2(W_j)f(c_{ij})$ over j for a given i. Essentially, this represents the potential of i for interaction with the rest of the system. Indicators of this type have found application in many fields, including, inter alia, in regional economic analysis (e.g., Frost and Spence 1995; Vickerman, Spiekermann, and Wegener 1999), in the study of food access (e.g., Apparicio, Cloutier, and Shearmur 2007; A. Páez, Mercado, et al. 2010b; Widener 2017), in the distribution and potential use of urban amenities (e.g., Dony, Delmelle, and Delmelle 2015; Reyes, Páez, and Morency 2014), and access to health care facilities (e.g., Apparicio et al. 2017; Guagliardo 2004; A. Páez, Mercado, et al. 2010a; Páez, Higgins, and Vivona 2019).

Besides the concept of gravity, an alternative approach to understand spatial interaction originated with Stouffer (1940), who was interested in intervening opportunities (also see Black and Conroy 1977; and Cheung and Black 2008). In his work, Stouffer paid attention to pass-through flows, or what opportunities were available for trips between origin i and destination j. This aspect of the system is important not only for moderating trip length (captured in gravity-based accessibility measures by a properly calibrated cost function $f(c_{ij})$; see Sarlas and Axhausen 2018 for a relevant discussion), but also as a measure of travelers' exposure to opportunities. Accordingly, a strand of the accessibility literature has evolved along the lines of accounting for varying levels of exposure, mainly by exploiting the space-time prism concept of time-geography (Hägerstrand 1970; Miller 1991). There, the underlying idea is the way mobility-related attributes and constraints (e.g., time budget, commuting behavior, trip chains, etc.), influence the potential for mobility. Commuter-based variants of accessibility in this literature are conceptually close to Stouffer's (1940) intervening opportunities definition since they specify the relevant interaction space based on both home, work, and relevant locations in-between (e.g., Farber et al. 2013; Widener et al. 2013; Fransen et al. 2015).

A prerequisite for spatial interaction is the existence of (physical) channels to support it. This component of spatial interaction is accounted for in gravity-based accessibility indicators by means of cost estimates embedded in the distance decay function $f(c_{ij})$. More generally, the interdependent nature between the potential for interaction and the role of networks to support interaction has been widely acknowledged, and network analysis rightly holds a prominent place in geographical research. Examples of network analysis include vulnerability analysis (e.g., Demšar, Špatenková, and Virrantaus 2008; López et al. 2017), the study of spatial inequality issues (e.g., Liu, Dai, and Derudder 2017), categorization of city structures (e.g., Crucitti, Latora, and Porta 2006), understanding of urban traffic capacity (e.g., Loder et al. 2019), and connectivity analysis of global cities (e.g., Hennemann and Derudder 2014).

In this paper, we are interested in the potential impacts of accessibility on networks. More concretely, we are interested in the potential distribution of flows in a network, conditional on a certain level of accessibility in the system. We draw inspiration from the literature on social networks, and particularly the concept of network centrality, to propose a new measure we call *betweenness-accessibility*. By allowing an analyst to estimate the allocation of potential flows to a network, betweenness-accessibility provides a rich picture of the ways a transportation system and the landscape of opportunities interact. Given its connections to network centrality, betweenness-accessibility provides a convenient way of quantifying the importance of each network element with respect to its ability to support spatial interaction, and hence accessibility. In this regard, apart from its complementary role in accessibility analysis, betweenness-accessibility can play a role in vulnerability analysis. This is an emerging topic that has drawn attention to accessibility as a tool to understand the ability of a network to operate under strain or interdiction (Liao and Wee 2017).

The rest of this paper is organized as follows. In the following section, we briefly review the centrality concept and introduce our measures of betweenness-accessibility based on a general weighted centrality indicator formulation. Next, we present a simple, worked-out example of betweenness-accessibility implementation using a synthetic dataset that helps illustrate how betweenness-accessibility is calculated and what it represents. This section is followed by a case study of Zurich where the corresponding betweenness-accessibility results are utilized for vulnerability analysis purposes, as a potential use case. We conclude the paper by discussing the potential and limitations of our proposed measures and some directions for future research.

Betweenness centrality and betweenness-accessibility

A key concern in network analysis is the study of centrality. In general terms, centrality is a measure of the importance of elements in a network, although "importance" is a flexible concept that hinges on specific assumptions about the way that traffic flows through a network. (Freeman, Roeder, and Mulholland 1979; Degenne and Fors 1999). A popular measure of centrality, for example, is the degree centrality of a node, which quantifies the number of edges that incide on a node (Freeman, Roeder, and Mulholland 1979). Another measure, betweenness centrality, evaluates the importance of a node in a network by considering its flow-through traffic, that is, the number of paths connecting other nodes that pass through it (Freeman 1977).

Seen from this perspective, centrality measures are similar to accessibility indicators in that they take into account the potential for interaction between different elements in a system. For example, degree centrality is similar to a cumulative opportunities measure with a threshold of a single edge:

$$C_{\mathrm{D}}(p_i) = \sum_j p_j I(l_{ij} \leq 1) \tag{3}$$

In the expression above p_i and p_j are nodes in the network, $I(\cdot)$ is an indicator function that takes the value of one if the argument is true and zero otherwise; l_{ij} is the number of intervening edges between p_i and p_j.

Betweenness centrality, in contrast, is concerned with the pathways for interaction - and therefore with pass-through flows, similar to Stouffer's (1940) intervening opportunities model. It offers, as will be seen, a different perspective of centrality that serves as a useful complement to accessibility analysis. A thorough discussion on the topic of centrality is given by Borgatti (2005), and the interested reader is directed to Wasserman, Faust, and Iacobucci (1994) for further details about various definitions of centrality. For the purpose of this paper we focus exclusively on the concept of betweenness centrality.

Betweenness centrality

Suppose that there is a graph representation V of a network composed of nodes and edges that connect some, or all, of said nodes. Under this setup, the betweenness of a node p_i is a measure of the frequency with which the node is part of the shortest paths that connect all pairs of nodes in the network (Freeman 1977). Since this index depends on the number of paths $\sigma(p_j, p_k \mid p_i)$ where p_i acts as a bridge between nodes p_j and p_k, it can be conceptualized as a measure of flow control (Wasserman, Faust, and Iacobucci 1994). Freeman (1977) defines betweenness as follows:

$$C_{\mathrm{B}}(p_i) = \sum_{j,k \in V, j \neq k \neq i} \frac{\sigma(p_j, p_k \mid p_i)}{\sigma(p_j, p_k)} \tag{4}$$

where $C_{\mathrm{B}}(p_i)$ corresponds to the betweenness of node p_i, $\sigma(p_j, p_k \mid p_i)$ is the number of shortest paths between nodes p_j and p_k that pass through node p_i, $\sigma(p_j, p_k)$ is the total number of shortest paths between p_j and p_k, $j, k = \{1, 2, ..N\}$, and N is the total number of nodes in the network. Subsequently, a normalization term $\frac{1}{(N-1)(N-2)}$ can be included to turn the measure into the system-wide proportion of shortest paths that pass through node p_i.

A similar measure was proposed by Girvan and Newman (2002), where betweenness represents the number of paths between nodes that pass through a specific edge l_i .

$$C_{\mathrm{B}}(l_i) = \sum_{j,k \in V, j \neq k} \frac{\sigma(p_j, p_k \mid l_i)}{\sigma(p_j, p_k)} \tag{5}$$

Normalization can be implemented in this case by multiplying the measure by the term $\frac{1}{N(N-1)}$ to obtain the system-wide proportion of paths that traverse l_i.

Apart from the betweenness centrality measures described above, variants of the original formulation have emerged over the years. Brandes (2008) reviews them, also discussing their algorithmic implementations. One prominent example of such measures is the so-called *distance-scaled betweenness*, where shortest paths receive weights that are inversely proportional to their length (Borgatti and Everett 2006). Essentially, this weighting operationalizes the idea that longer paths should count less towards the betweenness calculation than shorter paths. A further modification on the betweenness measure was introduced by Geisberger, Sanders, and Schultes (2008), where weights are defined based on relative position of a node p_i in relevant shortest paths.

Likewise but in the context of transportation networks, Lowry (2014) modified the stress-centrality indicator by applying a weighting on the basis of trip production and attraction rates at the ends of shortest paths. It should be noted that stress-centrality was originally proposed by Shimbel (1953); although similar to the betweenness measure, it differs from C_{B} by using the absolute number of shortest paths instead of the fraction. Nevertheless, in cases where a single shortest path exists per pair of nodes (normally the case for weighted transportation networks), both measures yield identical results.

Betweenness-accessibility

For transportation networks, use of betweenness indicators such as those above has helped to understand useful aspects of the *topology* of networks (e.g., Reggiani et al. 2009; Lin and Ban 2013), but their relationship with the *function* of networks, i.e., the flows, remains somewhat underexplored (e.g., Lowry 2014; López et al. 2017). In this regard, Gao et al. (2013) highlighted the incapability of betweenness centrality to serve as a predictor of traffic flow since it only accounts for a network's topological properties. To demonstrate that, they conducted a correlation analysis of various betweenness indicators and simulated traffic data.

The inherent limitations of betweenness can be summarized into two key issues (see Gao et al. 2013; Sarlas and Axhausen 2015). First, despite the fact that weights on links are taken into account in the identification of paths, the potential interaction between nodes is also affected by the mass of the nodes: in other words, their population or number of opportunities, as seen in the gravity model. And secondly, since population - and most opportunities - are not uniform across space, this gives rise to flow production and attraction patterns that are generally not spatially uniform.

For this reason, a more general formulation of betweenness can be obtained by including a weighting component that allows the analyst to account for the non-uniform impact on centrality of each pair of nodes. This idea is formalized as follows:

$$C_{\mathrm{B_W}}(l_i) = \sum_{j,k \in V, j \neq k} \frac{\sigma(p_j, p_k \mid l_i)}{\sigma(p_j, p_k)} w_{jk} \tag{6}$$

where w_{jk} is a measure of interaction between nodes. In the case of transportation networks, this could be the number of trips produced or attracted by nodes, whereas in social networks, it could be the number of emails, frequency of communication, or transfers between agents in the network.

As before, a normalized indicator can be obtained by dividing the measure by the system-wide sum of the weights:

$$C^{\mathrm{N}}_{\mathrm{B_W}}(l_i) = \frac{1}{\sum\limits_{j,k \in V, j \neq k} w_{jk}} \sum_{j,k \in V, j \neq k} \frac{\sigma(p_j, p_k \mid l_i)}{\sigma(p_j, p_k)} w_{jk} \tag{7}$$

In practical terms, a restriction on the number of interacting nodes can be imposed by defining a subset of interacting nodes such as $N_S \subset N$. In its most general form, betweenness would account for interaction between all pairs of nodes. As an alternative, only shortest paths $\{p_j, p_k\}$ $\forall p_j, p_k \in N_S$ can be considered instead, resulting in a betweenness indicator denoted as $C^{\mathrm{N}}_{\mathrm{B_{OD}}}$. Furthermore, similar to the gravity model, we assume that masses of the nodes W_j and W_k are useful proxies for trip production and attraction levels. The weights are specified based on the distance decay function of the gravity model. More specifically, the interaction between nodes p_j and p_k is assumed to be determined based on the cost of interaction, as modulated by a distance decay function $f(c_{jk})$. In this fashion, we can relate the weights needed to calculate betweenness to the gravity-based indicator of accessibility, in such a way that accessibility now provides insights into the centrality of elements in the network.

As is the case more generally in accessibility analysis, the parameter(s) of the distance decay function must be assumed or estimated (Páez, Scott, and Morency 2012). If estimated, a data-driven approach can be employed, forming essentially a broader analytical framework requiring O-D and cost matrices (e.g., Páez et al. 2013; Liao and Wee 2017) or time use data (e.g., Neutens et al. 2014; Lopez and Páez 2017), for example. In particular, having identified the set of shortest paths between all interacting nodes, the connection cost per O-D pair can be easily calculated and the corresponding trip ends can be matched to representative nodes. Subsequently, the distance decay function can be calibrated using a model-based approach (e.g., Sarlas and Axhausen 2018).

Two variables typically used as indicators of the masses of nodes are population (*pop*) and employment (*empl*). Based on this, two types of betweenness-accessibility indicators can be specified. The first set of indicators quantifies the potential level of a link's exposure in terms of trip production (population) and attraction (employment), respectively:

$$C_{\mathrm{pop}}(l_i) = \sum_{j,k, j \neq k} \frac{\sigma(p_j, p_k \mid l_i)}{\sigma(p_j, p_k)} pop_j \frac{empl_k f(tt_{jk})}{Acc^{\mathrm{empl}}_j} \tag{8}$$

$$C_{\text{empl}}(l_i) = \sum_{j,k,j\neq k} \frac{\sigma(p_j, p_k \mid l_i)}{\sigma(p_j, p_k)} empl_j \frac{pop_k f(tt_{jk})}{Acc_j^{\text{pop}}} \tag{9}$$

with:

$$Acc_j^{\text{pop}} = \sum_{k,k\neq j} pop_j f(tt_{jk}) \tag{10}$$

$$Acc_j^{\text{empl}} = \sum_{j,k\neq j} empl_j f(tt_{jk}) \tag{11}$$

The normalized version of these indicators (i.e., $C^{\text{N}}_{\text{pop}}(l_i)$ and $C^{\text{N}}_{\text{empl}}(l_i)$) is obtained by dividing the resulting values by the system-wide population (i.e., $TotalPop = \sum_j pop_j$) and employment (i.e., $TotalEmpl = \sum_j empl_j$), respectively.

Given the weights' normalization, the interpretation of $C^{\text{N}}_{\text{pop}}(l_i)$ is as the proportion of total population allocated to link l_i when the potential for interaction across the system is considered. Conversely, the interpretation of $C^{\text{N}}_{\text{empl}}(l_i)$ is the proportion of jobs serviced by link l_i when the potential for interaction across the system is considered. The unnormalized variants, in contrast, refer to the corresponding absolute values.

The second indicator quantifies the importance of elements in a network with respect to their contributions to the generation of accessibility. This indicator focuses on the potential to reach opportunities through each element of the network:

$$C_{A_{\text{pop}}}(l_i) = \sum_{j,k,j\neq k} \frac{\sigma(p_j, p_k \mid l_i)}{\sigma(p_j, p_k)} pop_k f(tt_{jk}) \tag{12}$$

$$C_{A_{\text{empl}}}(l_i) = \sum_{j,k,j\neq k} \frac{\sigma(p_j, p_k \mid l_i)}{\sigma(p_j, p_k)} empl_k f(tt_{jk}) \tag{13}$$

In this case, the normalized counterparts (i.e., $C^{\text{N}}_{A_{\text{pop}}}(l_i)$ and $C^{\text{N}}_{A_{\text{empl}}}(l_i)$) are obtained by dividing the resulting values by the system-wide accessibility to population (i.e., $TotalAcc^{pop}$) and employment (i.e., $TotalAcc^{empl}$), respectively. The interpretation of $C^{\text{N}}_{A_{\text{pop}}}(l_i)$ is as the proportion of system-wide accessibility to population that is supported by link l_i; on the other hand, $C^{\text{N}}_{A_{\text{empl}}}(l_i)$ is the proportion of system-wide accessibility to employment supported by link l_i. As before, the unnormalized versions correspond to absolute values of accessibility.

A reproducible example is useful to illustrate the mechanics required to implement the indicators of betweenness-accessibility defined above.

Reproducible example

In this section we present a small reproducible example to illustrate the concept of betweenness-accessibility; the reproducible example (data and code in the form of an R Notebook) can be downloaded from this link:

https://github.com/paezha/Betweenness-Accessibility

The example uses: 1) a small subset of Dissemination Areas, the smallest publicly available geography in Canada[1]; and 2) a small section of road network publicly available from Statistics Canada[2].

The network obtained from the Census does not have speeds on links. Instead, it has rank and class codes describing links, that can be used to impute speed. For instance, a class 10 link is a highway and a class 23 link is a local street. Speed values can be imputed based on the link class and speed limits obtained from the road classification system for Toronto[3]. Table 1 shows that the sample system consists of 9 zones with (simulated) employment and population data; Table 2 shows that the network consists of 13 links. Key information about the zoning system and network is shown in Fig. 1. Panel (a) of this figure shows population by zone, not including people who work in the same zone; Panel (b) shows employment by zone, not including jobs taken by people who live in the same zone, and last, Panel (c) shows the free flow travel time by link. In the example, population tends to be high to the east, whereas employment is highest in the northwest of the region.

Table 1: Summary of zones in reproducible example

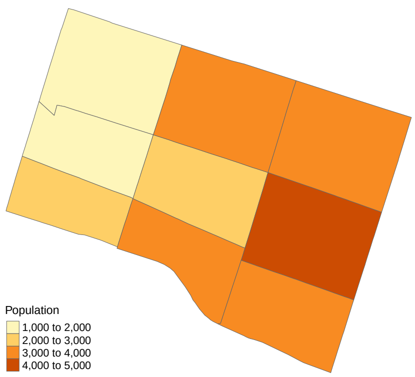
Zone ID	Population	Employment (jobs)	Accessibility to employment
1	1658	8000	283.96
2	1858	4082	479.56
3	2464	750	505.99
4	3243	1250	294.66
5	2473	3250	1058.34
6	4341	1250	757.02
7	3669	1250	386.15
8	3381	5400	734.95
9	3395	1250	644.96

Accessibility is calculated for this system based on Equation (11), using free flow travel time ($fftt$) on the network as a measure of cost, and the following

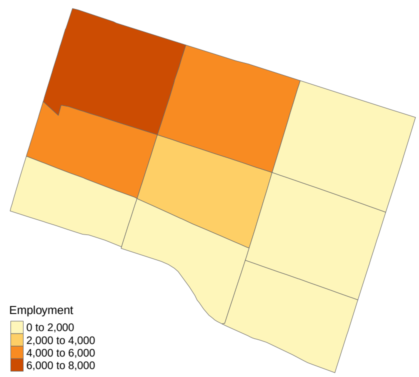
[1] https://www12.statcan.gc.ca/census-recensement/2011/geo/bound-limit/bound-limit-eng.cfm

[2] https://www12.statcan.gc.ca/census-recensement/2011/geo/RNF-FRR/index-eng.cfm

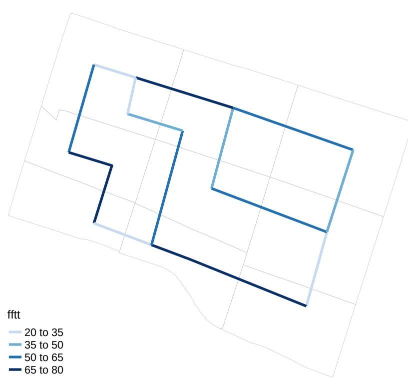
[3] https://www.toronto.ca/wp-content/uploads/2018/01/950a-Road-Classification_Summary-Document.pdf



(a) Population



(b) Employment (jobs)



(c) Free flow travel time (seconds)

Figure 1: Key information for reproducible example

Table 2: Summary of links in network in reproducible example

Node ID	Link class	Link length (km)	Link speed (km/h)	Free flow travel time (s)
1	23	0.31	40	27.90
2	20	0.59	60	35.36
3	23	0.65	40	58.08
4	23	0.75	40	67.31
5	20	0.44	60	26.20
6	20	0.54	60	32.64
7	20	0.87	60	52.50
8	20	0.90	60	54.15
9	20	0.61	60	36.55
10	20	1.18	60	70.90
11	20	0.84	60	50.20
12	23	0.41	40	36.86
13	23	0.72	40	65.21
14	23	0.26	40	23.68

negative exponential impedance function:

$$f(fftt_{ij}) = exp(-0.05 \cdot fftt_{ij})$$

Accessibility calculations results are shown in Fig. 2, where it can be seen that accessibility is high in more central locations and low in peripheral locations, especially when network travel time to reach other zones is long.

Calculation of betweenness-accessibility involves finding the shortest paths from each origin to every potential destination. Shortest paths are found using travel time in the network. Fig. 3 shows the shortest path trees from each node. Once the shortest paths have been identified, it is possible to allocate the potential for interaction to the network, according to levels of employment accessibility, as described in formula (8).

Consider, for example, Panel (a) in Fig. 3. This panel displays the shortest path tree from Node 1 (i.e., the centroid of Zone 1) in the northwest corner of the system. This zone, with a population of $1,658$, "pushes" $1,314.55$ people on the link between Nodes 1 and 3. Of these, 8.29 continue on the link to Node 4. Put another way, Zone 3 is "pulling" $1,306.26$ workers from Zone 1, and Zone 4 is pulling 8.29 workers from Zone 1. Similarly, Zone 1 is pushing 340.46 people on the first segment of the network in the Node 2 direction. Of these, 336.13 continue to Node 2 and 7.33 to Node 8. Of those that continued to Node 8, only 0.21 proceed to Node 5. Similarly, 4.63 of those who continued to Node 2 advance to Node 7 and 31.66 to Node 6. Finally, only 0.86 of those that reached Node 6 advance to Node 11 - or alternatively, the number of workers "pulled" by Node 11 (i.e., Zone 9) from Zone 1 is only 0.86. As seen in this panel, network links between Nodes 7 and 11, between Nodes 5 and 11, and between 4 and 8, do not accommodate any potential flow-through originating in Zone 1.

The rest of the panels in Fig. 3 provide a similar breakdown of the way flow-throughs are allocated for each zone of origin. Betweenness-accessibility is

Figure 2: Accessibility in reproducible example

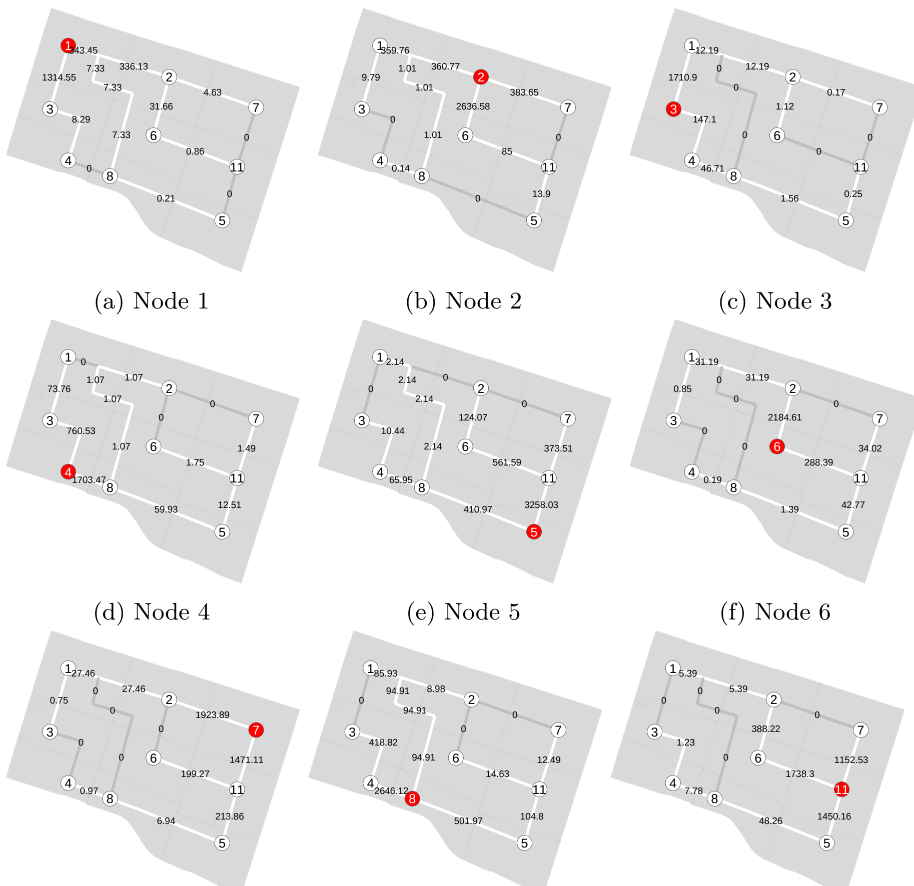
the total sum of all network flows after this process is completed for every origin zone. Results are shown in Fig. 4, where potential volume for each network link can be seen as an incidental effect of accessibility.

Case study

Context and data sources

In this section, we present an application of betweenness-accessibility in a real-life urban network. This allows us to examine different aspects of the proposed measures, along with their potential utility in practice. The case study is the city of Zurich - the largest city in Switzerland with a population of over 400,000 at the center of a metropolitan area with a population of almost 1.4 million[4]. In addition to its role as a major city in Switzerland, Zurich is one of the main economic hubs in central Europe. We use a detailed network of navigation-quality, commercially available from Tom-Tom. The network includes all links and nodes within city boundaries. In addition, to minimize the impact of boundary effects, the case study includes a buffer area of two kilometers. The resulting network consists of over $48,000$ links and $23,000$ nodes (see Fig. 5).

[4]Statistical Data on Switzerland 2018, Federal Statistical Office



(a) Node 1 (b) Node 2 (c) Node 3

(d) Node 4 (e) Node 5 (f) Node 6

(g) Node 7 (h) Node 8 (i) Node 11

Figure 3: Shortest paths and contributions to betweenness-accessibility by node

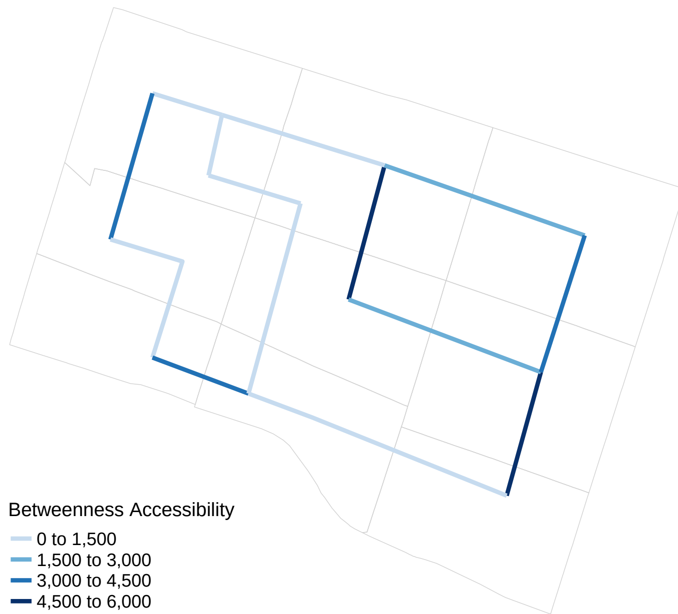


Figure 4: Betweenness-accessibility in reproducible example

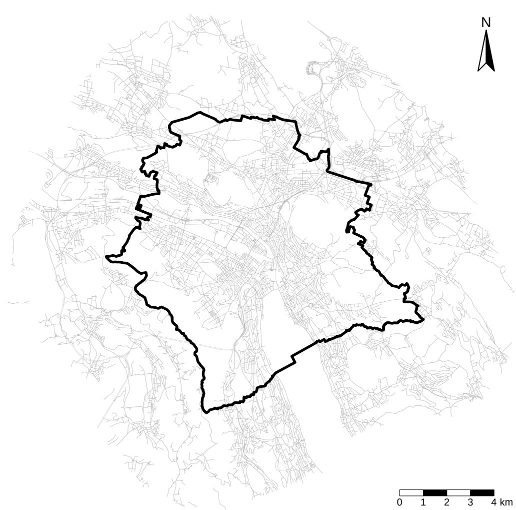


Figure 5: Network overview and city boundaries

As discussed, betweenness-accessibility provides a bridge between the concepts of betweenness (from social network analysis) and accessibility (from transport geography). Therefore, in addition to network data, we also need population and employment data for the relevant accessibility calculations. It is important to note that more disaggregated data increases the computational cost of the indicator, since the number of shortest paths grows quadratically with the number of origins/destinations (i.e., $N(N-1)$ shortest paths for a directed network with N nodes). For the case study, we use the zoning system of the Swiss National Transport Model[5]. The city of Zurich is divided into 308 zones with sizes ranging from 0.02 to 3.89 km^2 (mean value of 0.30 km^2). The population and employment data are for the year 2015; identification and processing of $308 \times 307 = 94'556$ shortest paths is required for construction of introduced indicators. To account for spatial opportunities distribution within each zone, we randomly assign a node within each zone as the starting and ending node. All shortest path calculations use free-flow travel time; these are used to construct the origin-destination (O-D) travel time matrix, a prerequisite for accessibility calculations.

The distance decay function is calibrated using the survival analysis method presented in Sarlas and Axhausen (2018). For this, we employ data from a household survey[6], using observations of individuals living and working in Zurich and commuting by car. Individuals' home and work locations are matched to the corresponding zones. Subsequently, we utilize the previously constructed O-D travel time matrix to obtain the survival function of trip length within the boundaries of the city. In the next step, we fit a negative exponential function to the trip length function to give the following calibrated distance decay function:

$$f(tt_{ij}) = exp(\beta \cdot tt_{ij}^{\alpha})$$

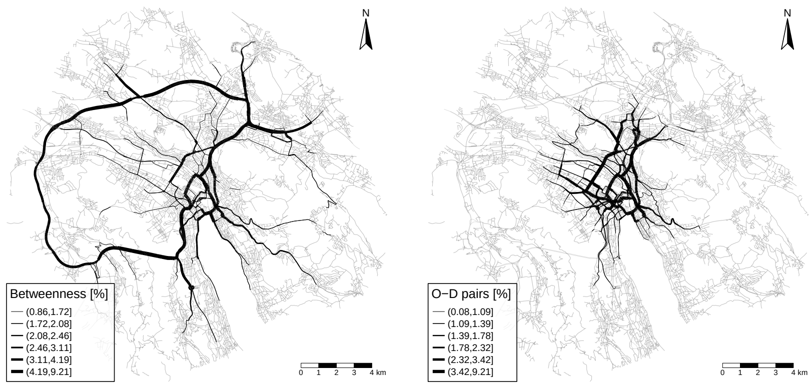
with $\alpha = 3.5875$, $\beta = -6.164 * 10^{-4}$, and tt the free flow travel time in minutes.

After completing the analysis of accessibility and identification of the required shortest paths, the next step is to calculate the normalized betweenness-accessibility indicators (Equations 8,9,12,and 13). For comparison purposes, we also calculate and present results of the conventional betweenness indicator $C_{\mathrm{B}}^{\mathrm{N}}$. Apart from that, the $C_{\mathrm{B_{OD}}}^{\mathrm{N}}$ indicator is also calculated. Recall that $C_{\mathrm{B_{OD}}}^{\mathrm{N}}$ is the equivalent to $C_{\mathrm{B}}^{\mathrm{N}}$ but only for a subset of interacting nodes; in this particular case we use the 308×307 shortest paths, and normalize them by means of the total number of shortest paths (i.e., $308 \times 307 = 94'556$).

All analysis described was performed using `R` (R 2013), and utilizing the *igraph* package (Csardi and Nepusz 2006), customized with additional functions to calculate the various indicators. This analysis is conducted for links, but similar analysis can be conducted for nodes with only minor adjustments to the calculations.

[5]NPVM 2015, ARE

[6]Swiss Micro-census 2015, ARE



(a) $C_{\mathrm{B}}^{\mathrm{N}}$ (b) $C_{\mathrm{B_{OD}}}^{\mathrm{N}}$

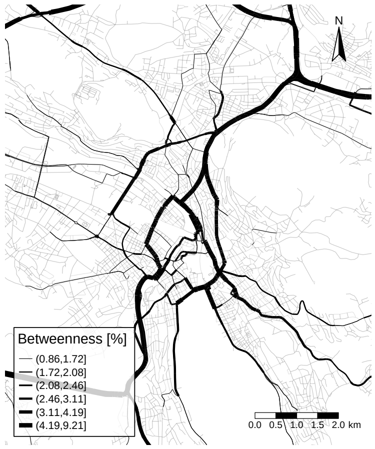
Figure 6: Comparison of $C_{\mathrm{B}}^{\mathrm{N}}$ and $C_{\mathrm{B_{OD}}}^{\mathrm{N}}$ indicators' results

Calculation of indicators

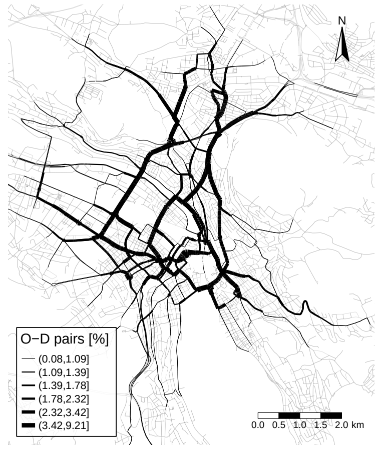
The indicators are shown in Figures 6 and 7). It is interesting to see that the different indicators show disparate pictures of the importance of links. A comparison between $C_{\mathrm{B}}^{\mathrm{N}}$ and $C_{\mathrm{B_{OD}}}^{\mathrm{N}}$ (Fig. 6) for the whole network shows that, in the former, links on the periphery - along with some in the central area of Zurich - are relatively more important. For $C_{\mathrm{B_{OD}}}^{\mathrm{N}}$, however, important links are fully concentrated in the central parts of the city. The range of values is also interesting, since it indicates that 1% of links with the highest values per case have relative shares of $4.19 - 9.21\%$ and $3.42 - 9.21\%$ of the total shortest paths passing through them.

To better appreciate the differences between $C_{\mathrm{B_{OD}}}^{\mathrm{N}}$ and betweenness-accessibility indicators we focus on the central area of Zurich (Fig. 7). At first, all indicators appear to yield similar results. However, differences can be identified especially with respect to the corresponding shares. For $C_{\mathrm{pop}}^{\mathrm{N}}$, the highest 1% of links have a a potential number of users in a range of $3.22 - 7.5\%$ of the total city population. For $C_{\mathrm{empl}}^{\mathrm{N}}$, the corresponding relative share is $3.01 - 8.21\%$, revealing a higher share of total employment positions. The results of indicators $C_{A_{\mathrm{pop}}}^{\mathrm{N}}$ and $C_{A_{\mathrm{empl}}}^{\mathrm{N}}$ are similar, but slightly lower. Interestingly, it appears that connectivity generated through specific links produces almost 6.8% and 6.4%, respectively, of total accessibility per case.

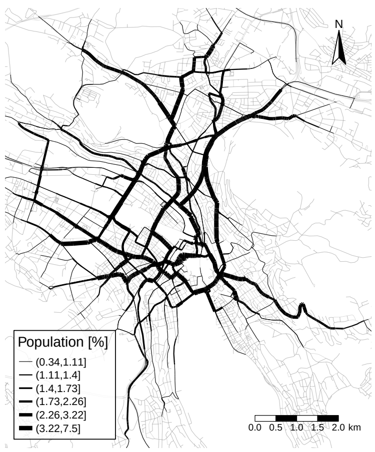
Last, correlation analysis is used to explore how various indicators relate to each other (see Table 3). As seen in the table, the conventional betweenness indicator has the lowest degree of correlation with other indicators. This is not surprising, given the substantial differences in their formulations - betweenness only considers network topology, while other measures also consider flows on the links (also see López et al. 2017). Other indicators tend to exhibit higher correlations, despite using different weighting schemes. This can perhaps be attributed to the fact that they all utilize the same sets of shortest paths.



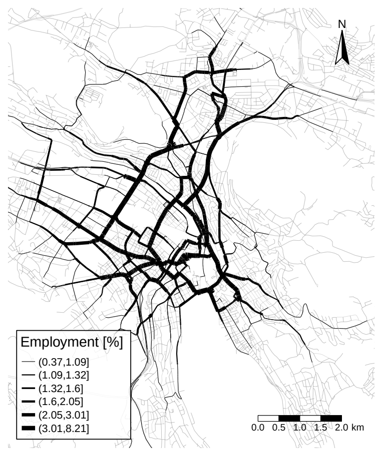
(a) $C_{\mathrm{B}}^{\mathrm{N}}$



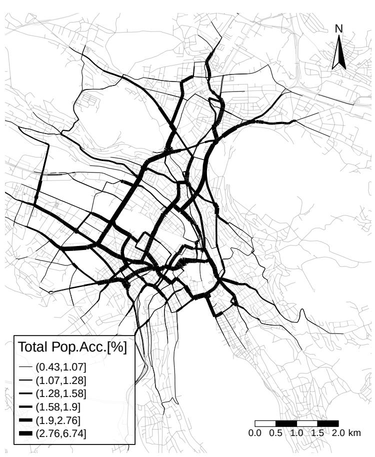
(b) $C_{\mathrm{B_{OD}}}^{\mathrm{N}}$



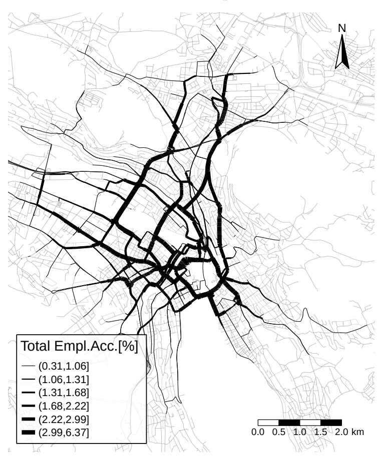
(c) $C_{\mathrm{pop}}^{\mathrm{N}}$



(d) $C_{\mathrm{empl}}^{\mathrm{N}}$



(e) $C_{A_{\mathrm{pop}}}^{\mathrm{N}}$



(f) $C_{A_{\mathrm{empl}}}^{\mathrm{N}}$

Figure 7: Various centrality indicators' results

Table 3: Correlation matrix of betweenness accessibility measures

	$C^{\mathrm{N}}_{\mathrm{B}}$	$C^{\mathrm{N}}_{\mathrm{B_{OD}}}$	$C^{\mathrm{N}}_{\mathrm{pop}}$	$C^{\mathrm{N}}_{\mathrm{empl}}$	$C^{\mathrm{N}}_{\mathrm{A_{empl}}}$	$C^{\mathrm{N}}_{\mathrm{A_{pop}}}$
$C^{\mathrm{N}}_{\mathrm{B}}$	1	0.63	0.55	0.55	0.54	0.57
$C^{\mathrm{N}}_{\mathrm{B_{OD}}}$	0.63	1	0.89	0.91	0.92	0.96
$C^{\mathrm{N}}_{\mathrm{pop}}$	0.55	0.89	1	0.69	0.93	0.78
$C^{\mathrm{N}}_{\mathrm{empl}}$	0.55	0.91	0.69	1	0.83	0.97
$C^{\mathrm{N}}_{\mathrm{A_{empl}}}$	0.54	0.92	0.93	0.83	1	0.86
$C^{\mathrm{N}}_{\mathrm{A_{pop}}}$	0.57	0.96	0.78	0.97	0.86	1

Application of betweenness-accessibility to vulnerability analysis

As noted in the introduction, the use of accessibility as a tool to understand robustness/vulnerability is an emerging topic of research (Liao and Wee 2017). In this section, we turn our attention to the application of betweenness-accessibility to assess the vulnerability of links in a network - that is, the degree to which performance of the network degrades when links are subjected to stress, or interdicted. Previous research has measured the performance of networks using network connectivity and travel time (e.g., Jenelius, Petersen, and Mattsson 2006; Scott et al. 2006). As well, different strategies have been proposed to approach both network performance and interdiction, including Holme et al. (2002), who use four different attack strategies based on topological indicators. The use of betweenness-accessibility sees the role of the network in terms of allowing users to reach destinations, and therefore allows us to study the socio-economic impacts of a network's degradation (see Taylor, Sekhar, and D'Este 2006,Chen et al. (2007); Sohn 2006; Taylor 2017).

To test the ability of betweenness-accessibility to evaluate vulnerability, we study the performance of the network subject to attacks on its links (edges). As highlighted by Holme et al. (2002), this approach (so-called "attack vulnerability") quantifies how network performance declines when specific elements of the network are removed (Barabási and Réka 1999). For the present case, the attack strategy uses the ranking of the links in terms of six indicators (i.e., two betweenness centrality and four betweenness-accessibility indicators). In addition, each attack happens in increments of five links; in total, we run fifteen steps (i.e., attacks) in this experiment. In pseudo-code, the experiment is presented in Algorithm 1.

As seen there, the performance of the network is assessed after each attack. To that end, two approaches are used for the evaluation. The first approach is accessibility-based; accordingly, we measure changes in system-wide accessibility. The second approach measures changes in system-wide travel time; optimal travel times are calculated using the well-known transportation problem (Horner 2002; Scott, Kanaroglou, and Anderson 1997). To implement the transportation problem, the number of residents per zone is scaled up proportionally to match the total number of employment positions in Zurich. Subsequently, an optimization process that minimizes total travel time system-wide is used to calculate the

Algorithm 1 Attack vulnerability experiment

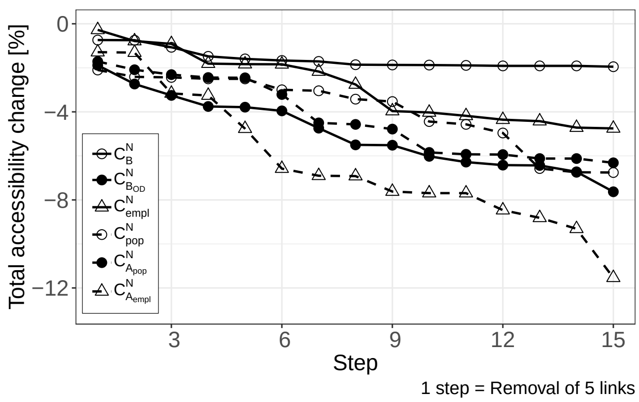
for $i = 1$ to 6 indicators **do**
  Initialize full network
  Calculate indicator i
  for $j = 1$ to 15 attacks **do**
    Remove current network's five most important links b.o. indicator i
    Calculate performance P_i^j of current network
  end for
end for

corresponding demand matrix. To account for demand dimensionality, the optimization problem is formulated as two distinct problems. In the first one (denoted as population-employment scenario), zonal population serves as trip production proxy and employment positions as trip attraction one, and vice versa (denoted as employment-population scenario). The results of various evaluation criteria for the six measures are presented in Figures 8 and 9.

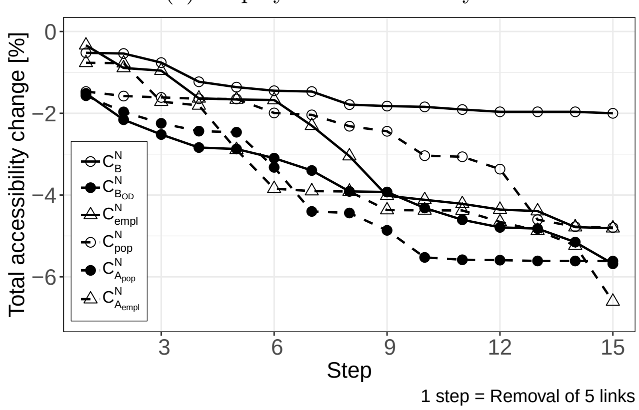
As seen in Figure 8, the network deteriorates much faster - and to a greater extent - when attacks target links with high values of betweenness-accessibility. Removing links based on the formulation with embedded employment accessibility weights ($C^{\mathrm{N}}_{A_{\mathrm{empl}}}$) has the highest impact on total accessibility reduction. Particularly, for the employment accessibility case (Fig. 8a), reduction in accessibility levels is substantial, almost 12%, with a difference of about 4% from the second best performing criterion. In addition, we note that removal of the 10 highest links results in a reduction of more than 2% of serviceability for both cases, revealing a network that is highly susceptible to the interdiction of a few links. Among other ranking indicators, the $C^{\mathrm{N}}_{\mathrm{B_{OD}}}$ indicator outperforms most centrality indicators in both cases. Furthermore, it is noteworthy that the simple betweenness indicator ($C^{\mathrm{N}}_{\mathrm{B}}$) yields - by far - the worst results in identifying critical links.

With respect to performance according to total travel time (see Fig. 9), attack strategies that target the demand dimension are most effective. For instance, in the first variation (Fig. 9a), the transportation problem is set up to minimize total travel time from residence to employment zones. Thus, it is interesting to see that targeting links based on their accessibility to employment (i.e., $C^{\mathrm{N}}_{A_{\mathrm{empl}}}$ and $C^{\mathrm{N}}_{\mathrm{pop}}$) has the highest impact. Between these two indicators, addition of the trip production variable (i.e., $C^{\mathrm{N}}_{\mathrm{pop}}$) helps to identify the most critical links. Similar patterns can be seen when population accessibility weights are used (Fig. 9b). Interestingly, the worst ranking results are identified for formulations involving weights based on accessibility of corresponding trip production variables.

The results demonstrate how betweenness-accessibility can be effectively used to explore the potential impacts of interdicting elements in a network.



(a) Employment accessibility



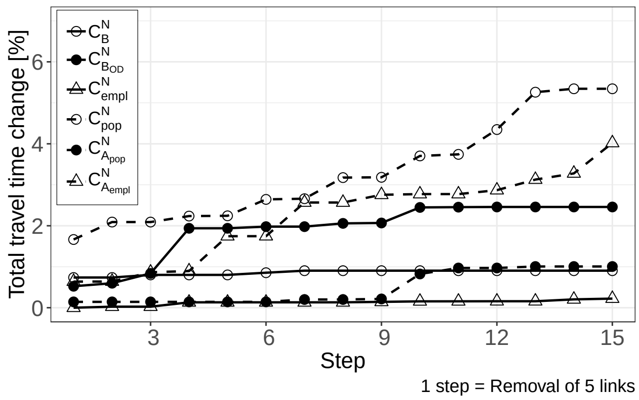
(b) Population accessibility

Figure 8: Accessibility-based performance indicator

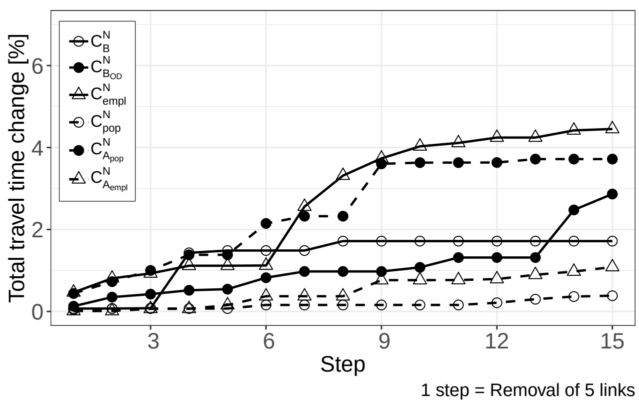
Conclusions

In this paper, a new set of indicators combining the concepts of centrality and gravity-based accessibility was introduced. In particular, the indicators expanded the potential of both centrality analysis and accessibility analysis. From the social network analysis perspective, betweenness-accessibility introduces a weight that measures interaction potential, whereas from the accessibility perspective, the new measure allows a researcher to estimate incidental accessibility impacts in a network, as well as how a network helps generate accessibility. In conclusion, the new centrality formulation allows overcoming previously identified shortcomings of traditional betweenness measures, resulting in a measure tailored for networks with heterogeneous interaction levels.

Usefulness of betweenness-accessibility was illustrated with a case study of Zurich, in Switzerland. Analysis showcased how the measure can help to identify



(a) Population-employment scenario



(b) Employment-population scenario

Figure 9: Total travel-time performance indicator

important network elements when system-wide interaction potential is considered. The technique can be used to allocate potential flows to network elements in a relatively straightforward way, as shown by its application to the Zurich case. Further, utility of betweenness accessibility was also tested using vulnerability analysis. The application demonstrated the concept's ability to identify critical links whose interdiction results in substantial losses of network serviceability. Performance evaluation shows that betweenness-accessibility is potentially an effective tool to identify network vulnerability.

Overall, the concept of betweenness-accessibility helps to provide a richer picture of the ways a transportation system operates to generate connectivity, compared to accessibility and betweenness measures alone. That said, it allows a quantification of accessibility to take place in a network-based way; a perspective which can find application in areas such as spatial- and social-equity analysis, vulnerability analysis, cost-benefit analysis, and land-use and transport interaction models.

It is important to acknowledge that one limitation of the approach is the use of a single shortest path for each pair of nodes, which is equivalent to an all-or-nothing assignment procedure that ignores capacity constraints. Relaxation of this limitation is certainly a direction for future research. Nevertheless, an apparent advantage of the presented methodology is that it can easily accommodate different weighting schemes, as its formulation is flexible in that sense. For instance, upon the availability of either actual route choice data, or simulated ones, more than one shortest paths can be considered per case along with the respective route choice rates as weights. Similarly, modal considerations can be taken into account in various ways. One such way would be through tuning in the distance decay function parameters to account for that (e.g., an embedded simplistic mode choice model, or using (exogenously) estimated mode choice rates). Another possible pathway towards that, is to incorporate such aspects solely within the normalization step, even though such ways would fail to capture spatial variability of mode choice considerations. Utilizing multi-modal networks constitutes also a possibility.

Last, the value of the newly introduced family of indicators, especially on its general weighted version, can potentially extend beyond the scope of transport geography research as it can pave the way for examining different aspects of various kinds of networks (e.g., social) where interaction between network elements happens in a disproportional way.

Acknowledgments

This research was supported by the Swiss National Science Foundation as part of the project "Models without (personal) data" (project number: 144134) to Georgios Sarlas and Kay W. Axhausen. The following `R` packages were used in the course of this investigation and the authors wish to acknowledge their developers: `kableExtra` (Zhu 2019), `knitr` (Xie 2014), `igraph` (Csardi and Nepusz 2006), `maptools` (Bivand and Lewin-Koh 2019), `plyr` (Wickham 2011), `reshape` (Wickham 2007), `rgdal` (Bivand, Keitt, and Rowlingson 2019), `rticles` (Allaire et al. 2018), `sp` and `spdep` (Bivand, Pebesma, and Gómez-Rubio 2008), `spstat` (Baddeley, Rubak, and Turner 2015), `tidyverse` (Wickham 2017), and `tmap` (Tennekes 2018).

References

Allaire, JJ, Yihui Xie, R Foundation, Hadley Wickham, Journal of Statistical Software, Ramnath Vaidyanathan, Association for Computing Machinery, et al. 2018. *Rticles: Article Formats for R Markdown.* https://CRAN.R-project.org/package=rticles.

Apparicio, P., M. S. Cloutier, and R. Shearmur. 2007. "The Case of Montreal's Missing Food Deserts: Evaluation of Accessibility to Food Supermarkets." Journal Article. *International Journal of Health Geographics* 6: 1–13.

Apparicio, P., J. Gelb, A. S. Dube, S. Kingham, L. Gauvin, and E. Robitaille.

2017. “The Approaches to Measuring the Potential Spatial Access to Urban Health Services Revisited: Distance Types and Aggregation-Error Issues.” Journal Article. *International Journal of Health Geographics* 16. doi:10.1186/s12942-017-0105-9.

Baddeley, A., E. Rubak, and R. Turner. 2015. *Spatial Point Patterns: Methodology and Applications with R*. London: Chapman; Hall/CRC Press. http://www.crcpress.com/Spatial-Point-Patterns-Methodology-and-Applications-with-R/Baddeley-Rubak-Turner/9781482210200/.

Barabási, A.L., and A. Réka. 1999. “Emergence of Scaling in Random Networks.” *Science* 286 (5439). American Association for the Advancement of Science: 509–12.

Ben-Akiva, M., and S.R. Lerman. 1977. *Disaggregate Travel and Mobility Choice Models and Measures of Accessibility*. Conference Paper.

Bivand, R., and N. Lewin-Koh. 2019. *Maptools: Tools for Handling Spatial Objects*. https://CRAN.R-project.org/package=maptools.

Bivand, R., T. Keitt, and B. Rowlingson. 2019. *Rgdal: Bindings for the 'Geospatial' Data Abstraction Library*. https://CRAN.R-project.org/package=rgdal.

Bivand, R.S., E.J. Pebesma, and V. Gómez-Rubio. 2008. *Applied Spatial Data Analysis with R*. Vol. 747248717. Springer.

Black, J., and M. Conroy. 1977. “Accessibility Measures and the Social Evaluation of Urban Structure.” Journal Article. *Environment and Planning A* 9 (9): 1013–31.

Borgatti, S.P. 2005. “Centrality and Network Flow.” *Social Networks* 27 (1): 55–71.

Borgatti, S.P., and M.G. Everett. 2006. “A Graph-Theoretic Perspective on Centrality.” *Social Networks* 28 (4): 466–84.

Brandes, U. 2008. “On Variants of Shortest-Path Betweenness Centrality and Their Generic Computation.” *Social Networks* 30 (2): 136–45.

Carrothers, Gerald AP. 1956. “An Historical Bedew of the Gravity and Potential Concepts of Human Interaction.” *Journal of the American Institute of Planners* 22 (2). Taylor & Francis: 94–102.

Chen, A., C. Yang, S. Kongsomsaksakul, and M. Lee. 2007. “Network-based acessibility measures for vulnerability analysis of degradable transportation networks.” *Networks and Spatial Economics* 7 (3): 241–56. doi:10.1007/s11067-006-9012-5.

Cheung, C., and J. Black. 2008. “A Reappraisal of the Intervening Opportunities Model of Commuter Behaviour.” *Road & Transport Research: A Journal of Australian and New Zealand Research and Practice* 17 (2). ARRB Group Ltd: 3.

Crucitti, Paolo, Vito Latora, and Sergio Porta. 2006. “Centrality Measures in Spatial Networks of Urban Streets.” *Physical Review E* 73 (3). APS: 036125.

Csardi, G., and T. Nepusz. 2006. “The Igraph Software Package for Complex Network Research.” *InterJournal, Complex Systems* 1695 (5): 1–9.

Degenne, A., and M. Fors. 1999. *Introducing Social Networks*. Book.

Introducing Statistical Methods. London: SAGE Publications.

Demšar, U., O. Špatenková, and K. Virrantaus. 2008. "Identifying critical locations in a spatial network with graph theory." *Transactions in GIS* 12 (1): 61–82. doi:10.1111/j.1467-9671.2008.01086.x.

Dony, C. C., E. M. Delmelle, and E. C. Delmelle. 2015. "Re-Conceptualizing Accessibility to Parks in Multi-Modal Cities: A Variable-Width Floating Catchment Area (Vfca) Method." Journal Article. *Landscape and Urban Planning* 143: 90–99. doi:10.1016/j.landurbplan.2015.06.011.

Farber, Steven, Tijs Neutens, Harvey J Miller, and Xiao Li. 2013. "The Social Interaction Potential of Metropolitan Regions: A Time-Geographic Measurement Approach Using Joint Accessibility." *Annals of the Association of American Geographers* 103 (3). Taylor & Francis: 483–504.

Fransen, Koos, Tijs Neutens, Philippe De Maeyer, and Greet Deruyter. 2015. "A Commuter-Based Two-Step Floating Catchment Area Method for Measuring Spatial Accessibility of Daycare Centers." *Health & Place* 32. Elsevier: 65–73.

Freeman, L.C. 1977. "A Set of Measures of Centrality Based on Betweenness." *Sociometry*. JSTOR, 35–41.

Freeman, L.C., D. Roeder, and R.R. Mulholland. 1979. "Centrality in Social Networks: II. Experimental Results." *Social Networks* 2 (2): 119–41.

Frost, M.E., and N.A. Spence. 1995. "The Rediscovery of Accessibility and Economic Potential: The Critical Issue of Self-Potential." Journal Article. *Environment and Planning A* 27 (11): 1833–48.

Gao, Song, Yaoli Wang, Yong Gao, and Yu Liu. 2013. "Understanding Urban Traffic-Flow Characteristics: A Rethinking of Betweenness Centrality." *Environment and Planning B: Planning and Design* 40 (1). SAGE Publications Sage UK: London, England: 135–53.

Geisberger, R., P. Sanders, and D. Schultes. 2008. "Better Approximation of Betweenness Centrality." In *Proceedings of the Meeting on Algorithm Engineering & Expermiments*, 90–100. Society for Industrial; Applied Mathematics.

Geurs, K.T., and B. van Wee. 2004. "Accessibility Evaluation of Land-Use and Transport Strategies: Review and Research Directions." *Journal of Transport Geography* 12 (2): 127–40.

Girvan, M., and M. E. J. Newman. 2002. "Community structure in social and biological networks." *Proceedings of the National Academy of Sciences* 99 (12): 7821–6.

Guagliardo, M. F. 2004. "Spatial Accessibility of Primary Care: Concepts, Methods and Challenges." Journal Article. *International Journal of Health Geographics* 3 (3): 1–13.

Handy, S., and D. Niemeier. 1997. "Measuring Accessibility: An Exploration of Issues and Alternatives." Journal Article. *Environment and Planning A* 29 (7): 1175–94.

Hägerstrand, T. 1970. "What About People in Regional Science?" Journal Article. *Papers of the Regional Science Association* 24: 7–21.

Hennemann, Stefan, and Ben Derudder. 2014. "An Alternative Approach to the Calculation and Analysis of Connectivity in the World City Network."

Environment and Planning B: Planning and Design 41 (3). SAGE Publications Sage UK: London, England: 392–412.

Holme, P., B.J. Kim, Yoon C.N., and S.K. Han. 2002. "Attack vulnerability of complex networks." *Physical Review E - Statistical Physics, Plasmas, Fluids, and Related Interdisciplinary Topics* 65 (5): 14. doi:10.1103/PhysRevE.65.056109.

Horner, M.W. 2002. "Extensions to the Concept of Excess Commuting." *Environment and Planning A: Economy and Space* 34 (3). SAGE Publications Sage UK: London, England: 543–66.

Jenelius, E., T. Petersen, and L.G. Mattsson. 2006. "Importance and exposure in road network vulnerability analysis." *Transportation Research Part A: Policy and Practice* 40 (7): 537–60. doi:10.1016/j.tra.2005.11.003.

Kwan, M.P. 1998. "Space-Time and Integral Measures of Individual Accessibility: A Comparative Analysis Using a Point-Based Framework." Journal Article. *Geographical Analysis* 30 (3): 191–216.

Liao, F. X., and B. van Wee. 2017. "Accessibility Measures for Robustness of the Transport System." Journal Article. *Transportation* 44 (5): 1213–33. doi:10.1007/s11116-016-9701-y.

Lin, J. Y., and Y. F. Ban. 2013. "Complex Network Topology of Transportation Systems." Journal Article. *Transport Reviews* 33 (6): 658–85. doi:10.1080/\-01441647.2013.848955.

Liu, Xingjian, Liang Dai, and Ben Derudder. 2017. "Spatial Inequality in the Southeast Asian Intercity Transport Network." *Geographical Review* 107 (2). Wiley Online Library: 317–35.

Loder, Allister, Lukas Ambühl, Monica Menendez, and Kay W Axhausen. 2019. "Understanding Traffic Capacity of Urban Networks." *Scientific Reports* 9 (1). Nature Publishing Group: 1–10.

Lopez, Fernando A., and Antonio Páez. 2017. "Spatial Clustering of High-Tech Manufacturing and Knowledge-Intensive Service Firms in the Greater Toronto Area." Journal Article. *Canadian Geographer-Geographe Canadien* 61 (2): 240–52. doi:10.1111/cag.12326.

Lowry, M. 2014. "Spatial Interpolation of Traffic Counts Based on Origin–destination Centrality." *Journal of Transport Geography* 36: 98–105.

López, F.A., A. Páez, J.A. Carrasco, and N.A. Ruminot. 2017. "Vulnerability of nodes under controlled network topology and flow autocorrelation conditions." *Journal of Transport Geography* 59: 77–87. doi:10.1016/j.jtrangeo.2017.02.002.

Miller, Harvey J. 1991. "Modelling Accessibility Using Space-Time Prism Concepts Within Geographical Information Systems." *International Journal of Geographical Information System* 5 (3). Taylor & Francis: 287–301.

Neutens, T., S. Daniels, J. Minnen, I. Glorieux, P. De Maeyer, and N. Van de Weghe. 2014. "Spatial and Temporal Fluctuations in Individual Accessibility: A Comparative Analysis Among Subgroups of the Population." Journal Article. *Geografisk Tidsskrift-Danish Journal of Geography* 114 (2): 119–31. doi:10.1080/21662282.2013.863547.

Páez, A., C.D. Higgins, and S.F. Vivona. 2019. "Demand and Level of Service Inflation in Floating Catchment Area (Fca) Methods." Journal Article.

PloS One 14 (6): e0218773. doi:10.1371/journal.pone.0218773.

Páez, A., R. G. Mercado, S. Farber, C. Morency, and M. Roorda. 2010a. "Accessibility to Health Care Facilities in Montreal Island: An Application of Relative Accessibility Indicators from the Perspective of Senior and Non-Senior Residents." Journal Article. *International Journal of Health Geographics* 9. doi:10.1186/1476-072x-9-52.

Páez, A., R.G. Mercado, S. Farber, C. Morency, and M. Roorda. 2010b. "Relative Accessibility Deprivation Indicators for Urban Settings: Definitions and Application to Food Deserts in Montreal." Journal Article. *Urban Studies* 47 (7): 1415–38. doi:10.1177/0042098009353626.

Páez, A., D. M. Scott, and C. Morency. 2012. "Measuring Accessibility: Positive and Normative Implementations of Various Accessibility Indicators." Journal Article. *Journal of Transport Geography* 25: 141–53. doi:10.1016/j.jtrangeo.2012\-.03.016.

Páez, Antonio, Steven Farber, Ruben Mercado, Matthew Roorda, and Catherine Morency. 2013. "Jobs and the Single Parent: An Analysis of Accessibility to Employment in Toronto." Journal Article. *Urban Geography* 34 (6): 815–42. doi:10.1080/02723638.2013.778600.

R, Core Team. 2013. "R: A Language and Environment for Statistical Computing." Vienna, Austria.

Reggiani, A., S. Signoretti, P. Nijkamp, and A. Cento. 2009. "Network Measures in Civil Air Transport: A Case Study of Lufthansa." Journal Article. *Lecture Notes in Economics and Mathematical Systems* 613: 257–82. ISI:000263416000014 C:/Papers/Lecture Notes in Economics and Mathematical Systems/Lecture Notes in Economics and Mathematical Systems (2009) 613 257-282.pdf.

Reyes, M., A. Páez, and C. Morency. 2014. "Walking Accessibility to Urban Parks by Children: A Case Study of Montreal." Journal Article. *Landscape and Urban Planning* 125: 38–47. doi:10.1016/j.landurbplan.2014.02.002.

Sarlas, G., and K.W. Axhausen. 2015. "Prediction of AADT on a nationwide network based on an accessibility-weighted centrality measure." *Arbeitsberichte Verkehrs-und Raumplanung, IVT, ETH Zurich* 1094. IVT, ETH Zurich.

Sarlas, Georgios, and Kay W Axhausen. 2018. "Commuting Distance and Individual Accessibility." *Arbeitsberichte Verkehrs-Und Raumplanung* 1376. IVT, ETH Zurich.

Scott, D.M., P.S. Kanaroglou, and W.P Anderson. 1997. "Impacts of Commuting Efficiency on Congestion and Emissions: Case of the Hamilton Cma, Canada." *Transportation Research Part D: Transport and Environment* 2 (4): 245–57.

Scott, D.M., D.C. Novak, L. Aultman-Hall, and F. Guo. 2006. "Network Robustness Index: A new method for identifying critical links and evaluating the performance of transportation networks." *Journal of Transport Geography* 14 (3): 215–27.

Shimbel, A. 1953. "Structural Parameters of Communication Networks." Journal Article. *The Bulletin of Mathematical Biophysics* 15 (4): 501–7.

Sohn, J. 2006. "Evaluating the significance of highway network links under the flood damage: An accessibility approach." *Transportation Research Part A: Policy and Practice* 40 (6): 491–506. doi:10.1016/j.tra.2005.08.006.

Stouffer, S.A. 1940. "Intervening Opportunities: A Theory Relating Mobility and Distance." Journal Article. *American Sociological Review* 5 (6): 845–67.

Taylor, M.A.P. 2017. *Accessibility Methods.* doi:10.1016/B978-0-12-811010-2.00005-8.

Taylor, M.A.P., S.V.C. Sekhar, and G.M. D'Este. 2006. "Application of accessibility based methods for vulnerability analysis of strategic road networks." *Networks and Spatial Economics* 6 (3-4): 267–91. doi:10.1007/s11067-006-9284-9.

Tennekes, M. 2018. "tmap: Thematic Maps in R." *Journal of Statistical Software* 84 (6): 1–39. doi:10.18637/jss.v084.i06.

Vickerman, R., K. Spiekermann, and M. Wegener. 1999. "Accessibility and Economic Development in Europe." Journal Article. *Regional Studies* 33 (1): 1–15. ISI:000078687200001.

Wasserman, S., K. Faust, and D. Iacobucci. 1994. *Social Network Analysis: Methods and Applications.* Book. Cambridge: Cambridge University Press.

Wickham, H. 2007. "Reshaping Data with the Reshape Package." *Journal of Statistical Software* 21 (12). http://www.jstatsoft.org/v21/i12/paper.

———. 2011. "The Split-Apply-Combine Strategy for Data Analysis." *Journal of Statistical Software* 40 (1): 1–29. http://www.jstatsoft.org/v40/i01/.

Wickham, Hadley. 2017. *Tidyverse: Easily Install and Load the 'Tidyverse'.* https://CRAN.R-project.org/package=tidyverse.

Widener, M. J. 2017. "Comparing Measures of Accessibility to Urban Supermarkets for Transit and Auto Users." Journal Article. *Professional Geographer* 69 (3): 362–71. doi:10.1080/00330124.2016.1237293.

Widener, Michael J, Steven Farber, Tijs Neutens, and Mark W Horner. 2013. "Using Urban Commuting Data to Calculate a Spatiotemporal Accessibility Measure for Food Environment Studies." *Health & Place* 21. Elsevier: 1–9.

Xie, Yihui. 2014. "Knitr: A Comprehensive Tool for Reproducible Research in R." In *Implementing Reproducible Computational Research*, edited by Victoria Stodden, Friedrich Leisch, and Roger D. Peng. Chapman; Hall/CRC. http://www.crcpress.com/product/isbn/9781466561595.

Zhu, H. 2019. *KableExtra: Construct Complex Table with 'Kable' and Pipe Syntax.* https://CRAN.R-project.org/package=kableExtra.