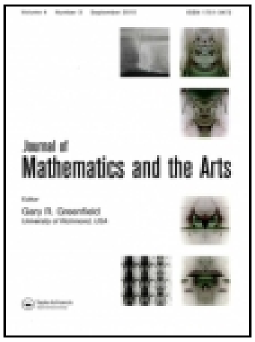


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# Duotone Truchet-like tilings

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# Duotone Truchet-like tilings

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This paper explores methods for colouring Truchet-like tiles, with an emphasis on the resulting visual patterns and designs. The methods are extended to non-square tilings that allow Truchet-like patterns of noticeably different character. Underlying parity issues are briefly discussed and solutions presented for parity problems that arise for tiles with odd numbers of sides. A new tile design called the arch tile is introduced and its artistic use demonstrated.

**Keywords:** Truchet tile; duotone colourings; semi-regular tiling; arch tile

**AMS Subject Classifications:** 05A05; 52C20; 52C30

## 1. Truchet-like tiles

In 1704, French clergyman Sebastien Truchet (1657–1729) described a simple half-coloured square tile that could be arranged to produce a number of interesting geometrical patterns [9], the artistic possibilities of which were further explored by Gombrich in 1979 [6]. Figure 1 (left) shows a development of Truchet's tile, consisting of two circular arcs at alternating corners with radius equal to half the tile edge length, that produce attractive sets of continuous closures even when the two unique rotations are randomly tiled (right).

This Truchet-like tile first appeared in board games such as Trax [12], Meander and the Black Path Game from 1960 onwards [1] and was linked to Truchet's original tile by metallurgist Cyril Stanley Smith in 1987 [11]. Gale et al. [5] demonstrated some interesting mathematical properties of closed contours formed by these modified Truchet tiles, which are now generally referred to simply as 'Truchet tiles'.

This paper explores the visual opportunities offered by colourings of Truchet-like tilings that distinguish adjoining regions. Some analysis is made of the underlying parity issues, including parity problems that arise when the method is extended to some non-square tilings. No attempt is made to catalogue or enumerate potential colourings or to systematically explore symmetries within the resulting contours; this is more a visual exploration of new colourings for some known tilings. Lastly, a development of the hexagonal Truchet-like design called the arch tile is introduced and its artistic use discussed briefly.

## 2. Duotone Truchet-like tiles

The disjoint regions within each tile may be coloured in different shades to help visually segregate the regions that form within random tilings, or simply for artistic effect. For example, Bosch gives a striking demonstration of multi-toning using a Truchet-like tiling with seven shades of colour [2].

This paper, however, will focus on duotone Truchet-like tiles coloured in two shades – light and dark – as shown in Figure 2. There are two such duotone patterns – light dominant and dark dominant – each with two unique rotations (left) that form alternating light and dark regions when tiled (right). Such duotone tiles may be labelled LD and DL according to the colours of the top left and top right corners respectively; note that each of the LD and DL tile types have light dominant and dark dominant versions. As an aside, it is interesting to note that a rectilinear form of this duotone tiling was abundantly used in Islamic art for Kufic writing [10] and more recently for three-dimensional artistic applications [8].

Duotone Truchet-like tilings are not entirely random as the duotone colouring imparts parity to the underlying grid, as shown in Figure 3 (left). Given a square lattice, the vertices of the lattice can alternately be labelled L and D in such a way that every L is adjacent to four Ds and vice versa. With this labelling in place, every square cell of the lattice can be assigned one of two possible tiles. The choice for each cell can be made independently, making it easy to generate tilings by either making use of a random selection process or proceeding according to a cell-by-cell rule.

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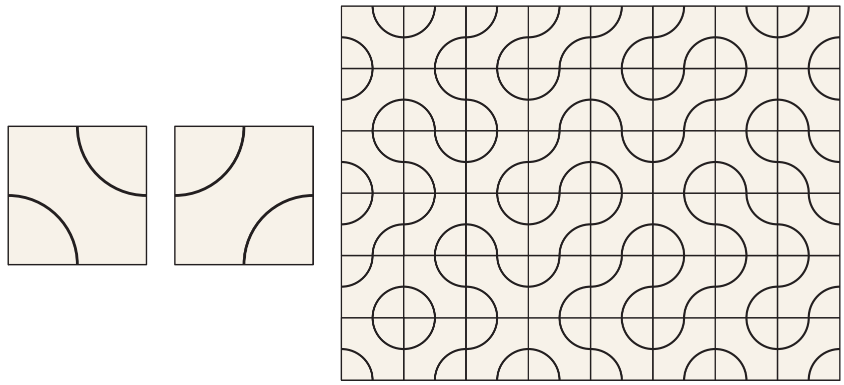

Figure 1. Truchet-like tile and a Truchet-like tiling.

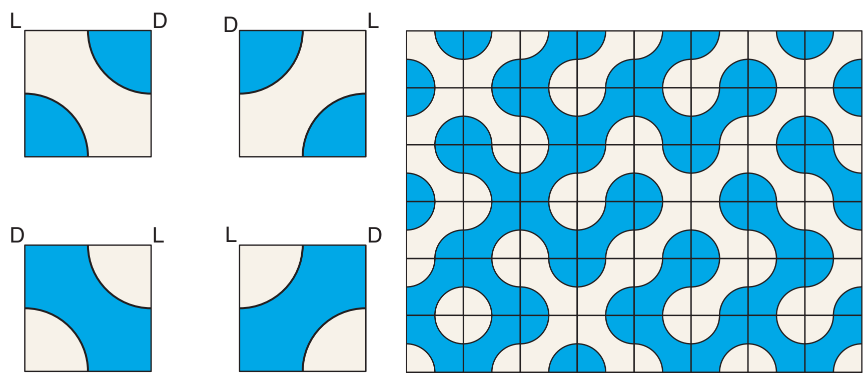


Figure 2. Duotone Truchet-like tiles.

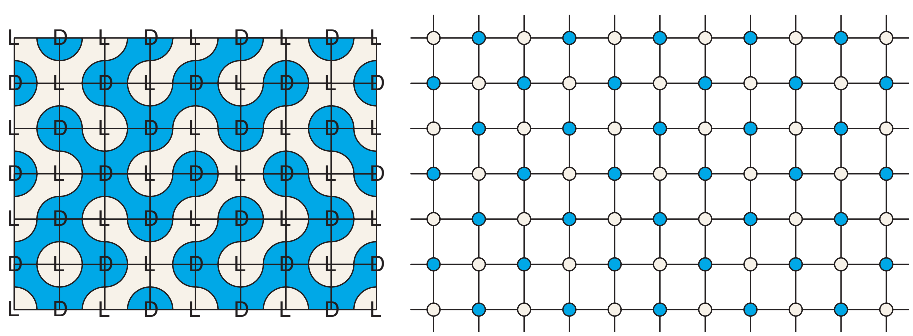

Figure 3. Parity labels and parity graph.

The first tile placed sets the parity and each subsequent tile must respect that parity. For example, if the tile in the top left corner position ($i=0$, $j=0$) is of type LD then all positions ($i$, $j$) such $i+j$ is even must be of type LD and all positions ($i$, $j$) such $i+j$ is odd must be of type DL. However, some randomness is still possible within this parity constraint, as each LD and DL position may be randomly filled with either the light or dark dominant tile with the appropriate corner labels. That is, the matching conditions implied by the duotone colouring allow for an uncountable infinity of possible tilings, in which a coin may be flipped to decide which tile to place at each step. A similar principle is applied in the use of Wang tiles for procedural texture generation [7].

## 3. Other even-sided tiles

The general design principle of the Truchet-like tile may be readily applied to other tile shapes with even numbers of sides. For example, Figure 4 shows

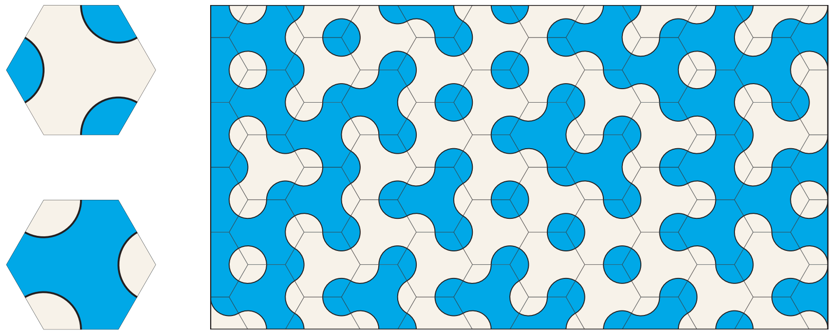

Figure 4. Duotone Truchet-like hexagons.

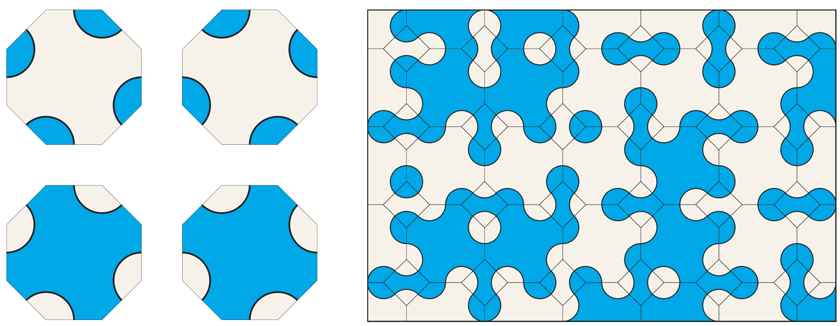

Figure 5. Duotone Truchet-like octagons (4.4.8 tiling with squares).

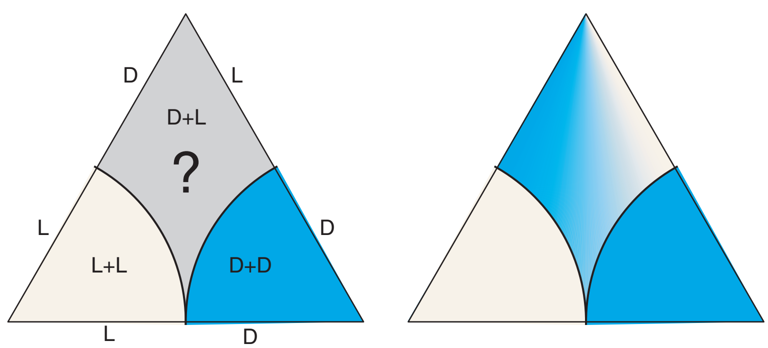


Figure 6. Parity problem with duotone triangles.

duotone Truchet-like hexagons and Figure 5 shows duotone Truchet-like octagons. Note that the octagonal tiling (semi-regular tiling 4.8.8) contains square-shaped gaps that must be filled with square duotone tiles of the appropriate parity.

In both the hexagonal and octagonal cases there again exist two distinct tile patterns, light dominant and dark dominant. However, the hexagonal version only has one unique rotation per pattern due to parity constraints; once the initial tile is placed, all subsequent tiles must be of the same parity. The octagonal version has two unique rotations per pattern, and like the square case parity alternates with each neighbouring octagon.

## 4. Odd-sided tiles

Applying the same design principle to tiles with odd numbers of sides is problematic, as the concept of alternating corner colours degenerates. For example, consider the triangular tile shown in Figure 6 (left). If the bottom left corner is declared ‘light’ and the bottom right corner declared ‘dark’, then what shade for the top corner constitutes the alternative of both?

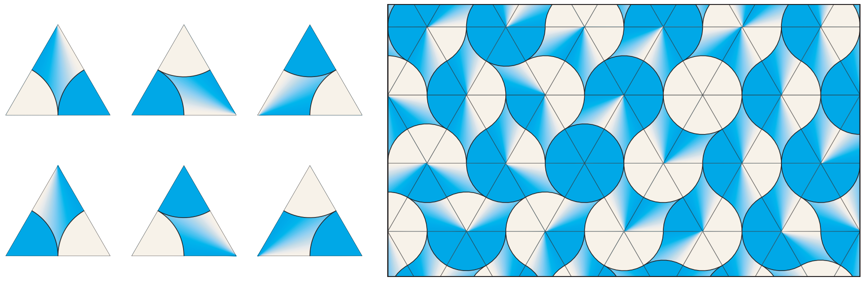

Figure 7. Duotone Truchet-like triangles.

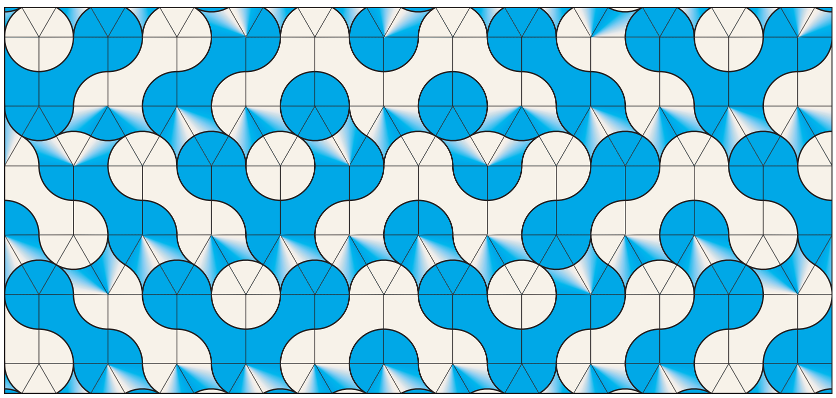

Figure 8. Duotone triangles and squares (3.3.3.4.4 tiling).

One solution is to adapt the colouring rules to incorporate a colour blend between the two shade constraints (Figure 6, right).

This produces two distinct patterns depending on whether the blend flows clockwise or anticlockwise light-to-dark around its corner pivot. Figure 7 (left) shows six of the 12 unique pattern/rotation combinations that allow a triangular tiling (the other six combinations point down).

## 5. Mixed tilings

The definition of both even-sided and odd-sided duotone Truchet-like tiles allows a wider range of tilings. For example, Figures 8–11 show semi-regular tilings composed of triangles and squares (3.3.3.4.4 and 3.3.4.3.4) and triangles and hexagons (3.6.3.6 and 3.3.3.3.6).

The semi-regular tiling 3.4.6.4 consisting of triangles, squares and hexagons – arguably the most attractive of the semi-regular tilings – presents another parity problem as shown in Figure 12 (left). Each triplet of hexagons may be joined on two sides by square tiles that maintain the appropriate parity; however, there is no square tile that will complete the third join with the correct parity for both hexagons. The solution is to again adapt the colouring rules to define a new square tile with opposite corners of different shade and a colour blend to satisfy the relevant shade constraints (middle and right).

This new tile allows a successful duotone Truchet-like 3.4.6.4 tiling that is probably the most aesthetically interesting of the duotone tilings shown so far (Figure 13). It can be seen that the resulting regions have more variety in size, shape and colour within a well-defined overall structure.

## 6. Arch tiles

Figure 14 shows the derivation of a further design called the *arch tile* in which two arcs at alternating corners of a hexagonal tile are ‘snapped together’ to form a connecting arch. The resulting tile (right) is similar to even-sided duotone Truchet-like tiles in that tile corners are of alternating colour, even if the internal topology is different.

Unlike the square duotone Truchet-like which requires two distinct patterns with two rotations each in order to tile successfully, the arch tile only requires a single pattern in three rotations (Figure 15). Once the

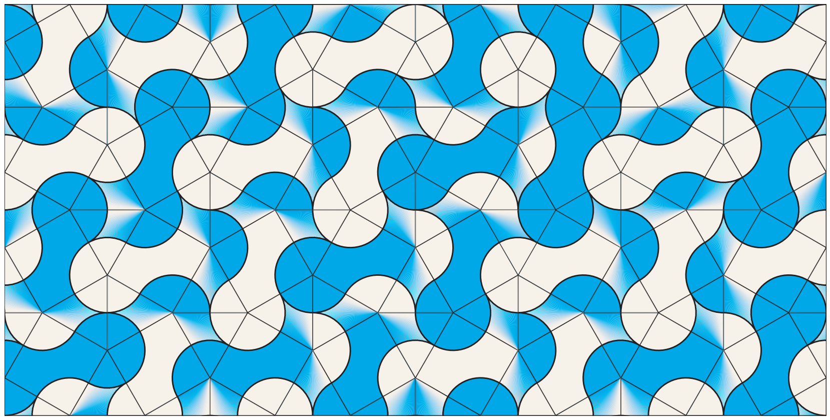

Figure 9. Duotone triangles and squares (3.3.4.3.4 tiling).

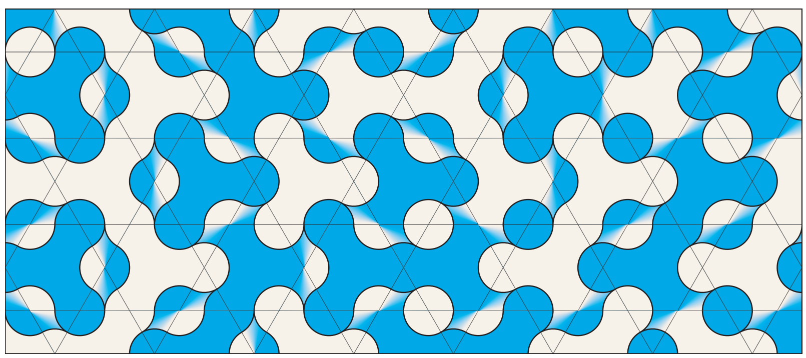

Figure 10. Duotone triangles and hexagons (3.6.3.6 tiling).

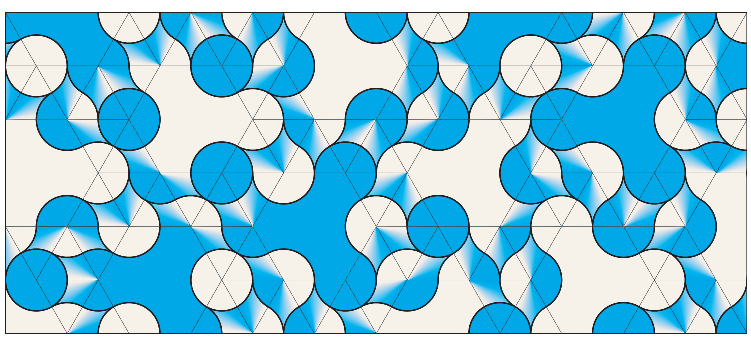

Figure 11. Duotone triangles and hexagons (3.3.3.3.6 tiling).

initial parity is decided then any of the three rotations may be placed at any position, resulting in the formation of interesting geometric shapes that suggest petroglyphs or even glyphs from an alien language. The combination of line and arc segments allows a wider variety of shapes than arc segments alone. These properties make arch tiles ideally suited to artistic applications such as large-scale mosaics in which visual texture for minimal effort is desirable.

Figure 16 shows a finer arch tiling with the tile borders removed for clarity. It can be an intriguing exercise to look for familiar shapes in such random tilings (much like looking for shapes in clouds) or attempt to reconstruct known shapes with the tiles as a form of tangram-like creative puzzle. A number of anthropomorphic and zoomorphic arch tile shapes, such as those shown in Figure 17, have been collected under a concept named Palagonia [4].

The arch tile design was first invented by the author in 2007 for the board game Mambo [3] and revised in 2008 to its duotone form for the related game Palago [4]. The rules for Palago are simple; two players, Light

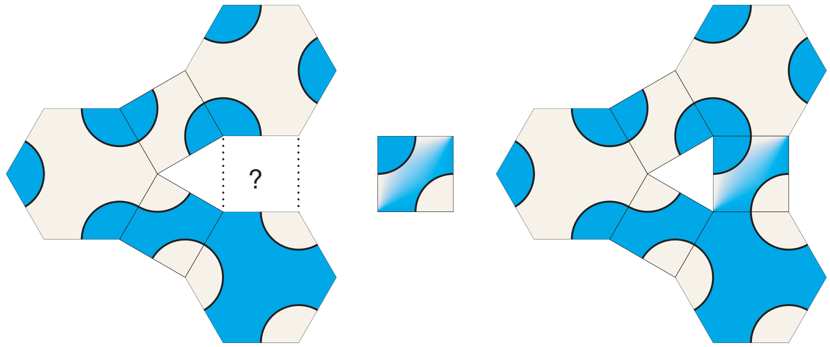


Figure 12. Parity problem with the 3.4.6.4 tiling.

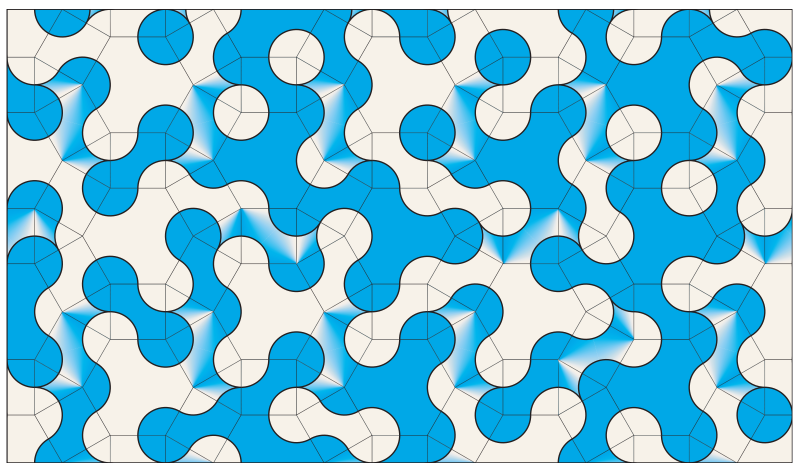

Figure 13. Duotone hexagons, squares and triangles (3.4.6.4 tiling).

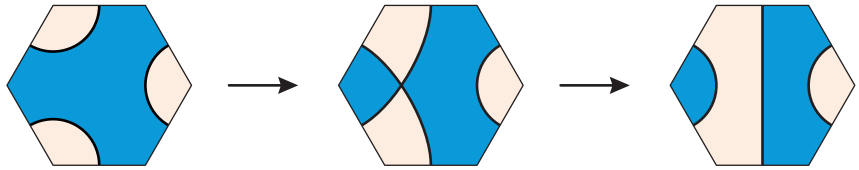

Figure 14. Derivation of the arch tile.

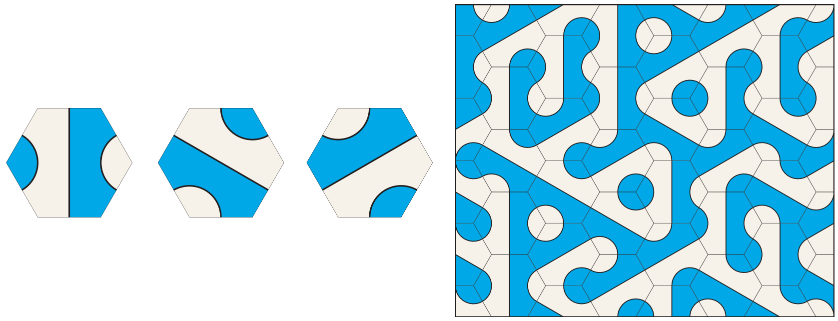

Figure 15. Three rotations of an arch tile and an arch tiling.

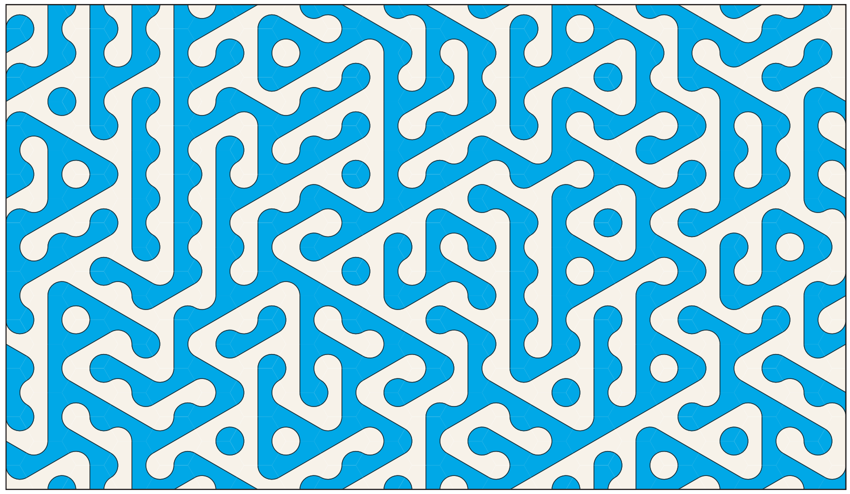

Figure 16. Arch tiling with tile borders removed.

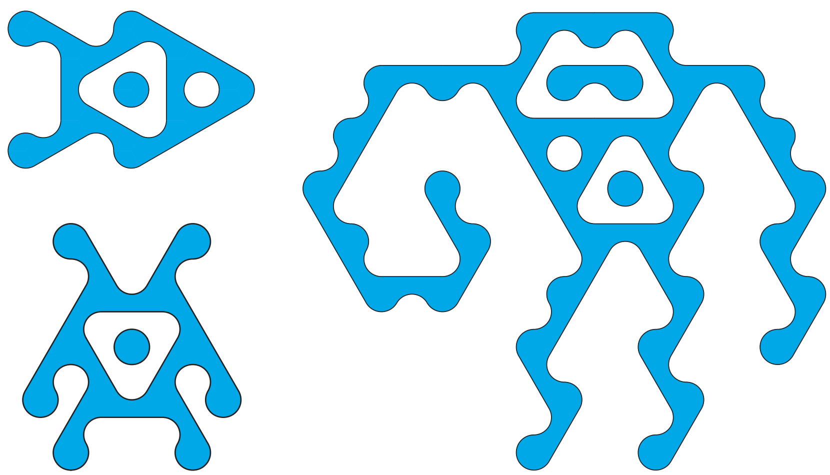

Figure 17. Arch tile artwork: fish, monkey and Palagonian.

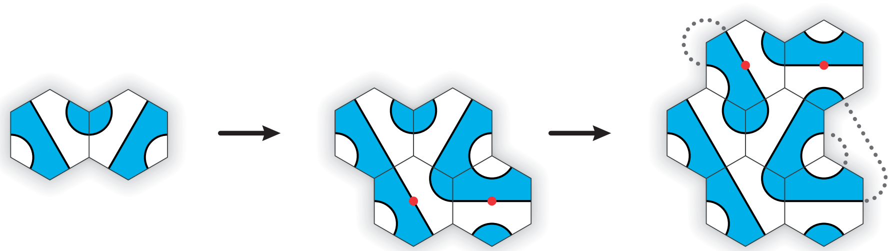

Figure 18. A short game of Palago.

and Dark, take turns placing adjacent pairs of tiles in an effort to form a closed group of their colour containing at least one arch. For example, Figure 18 shows a short game of Palago in which Dark starts (left), Light makes a poor reply (middle) and Dark sets up a winning fork indicated by the dotted lines (right).

The fact that each tile has both strong and weak regions of each colour has interesting combinatorial implications for each move. The tile's game-related roots are further emphasized by observing its coincidental similarity to one of the hexagonal tiles found in the board game Tantrix [13].

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