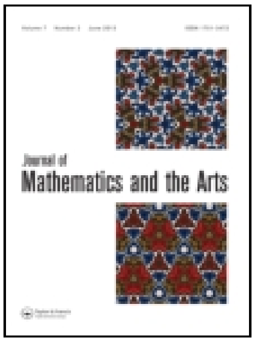

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Figurative mosaics from flexible Truchet tiles

Robert Bosch* and Urchin Colley

Department of Mathematics, Oberlin College, Oberlin, OH 44074, USA

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In 1704 Father Sébastien Truchet published an article, 'Mémoire sur les combinaisons', that describes his mathematical and artistic investigations into how a simple set of square tiles, each divided by a diagonal into a white half and a black half, can be arranged to form an infinity of pleasing designs. In this paper, we describe how to modify Truchet's tiles so that a collection of them can be used for halftoning, the reproduction of user-supplied greyscale target images in pure black and white. We do this by allowing the diagonals of the tiles to 'flex' or bend at their midpoints in accordance with the brightness of an individual pixel, or a collection of pixels, from the target image. We also present hexagonal variations, a similar scheme for the Truchet-like tiles – each decorated with two quarter-circle arcs centred at opposite corners of the square – proposed by Cyril Stanley Smith in 1987, and an extension that can be applied to all regular and semiregular tilings.

Keywords: Truchet tiles; mosaics; patterns

AMS Subject Classification: F1.1; F4.3

1. Introduction

The amazing Father Sébastien Truchet (1657–1729) was both a Carmelite clergyman and King Louis XIV's favourite hydraulic engineer. He invented a machine for transporting whole trees without damaging them, he designed scalable fonts, and in 1704 he published the article for which he is remembered today, 'Mémoire sur les combinaisons' [14] The article describes his mathematical and artistic investigations into how a simple set of square tiles, each divided by a diagonal into a white half and a black half, can be arranged to form an infinity of pleasing designs [1,2].

Figure 1 displays the *Truchet tiles* arranged in their four possible orientations and marked with the letters given to these orientations by Father Dominique Doüat, a Carmelite colleague of Truchet, in his 1722 book *Methode pour faire une infinité de desseins différens, avec des carreaux mi-partis de deux couleurs par une ligne diagonale* (Method for making an infinite number of different designs with squares divided into two colours by a diagonal line) [7]. Figure 2 displays six of the simplest designs or orientation patterns presented by Truchet (left-hand and middle columns) and Doüat (right-hand column), and Figure 3 displays two of Doüat's most complicated designs. Truchet's Design A uses orientation A exclusively, so it can be said to have a 1×1 *generator*, denoted by (A). Truchet's Designs C, D and E can be constructed from the 2×2 tilings of orientations

$$\begin{pmatrix} A & C \\ C & A \end{pmatrix}, \quad \begin{pmatrix} B & A \\ C & D \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} C & B \\ D & A \end{pmatrix},$$

respectively, but not from any smaller tilings of orientations, so these designs can be said to have 2×2 generators. In the same fashion, Doüat's Design 1 has a 2×4 generator, Design 6 has a 4×4 generator, and Designs 71 and 72 have 12×12 generators. The size of the generator is one simple measure of the complexity of a design.

It is clear that Doüat hoped that his book would be an inspiration to artists and craftsmen. In his preface, he wrote,

> In this book you will find an inexhaustible source of designs for paving churches and other buildings, for tiling floors, and for making very beautiful compartments. The painter will find inspiration. Marquetry workers, carpenters, marble cutters, and other workers will find it very useful. Embroiderers, upholsterers, weavers—all those who work on canvas or use the needle—will learn to make beautiful works. [7]

He therefore must have been at least somewhat disappointed that it failed to have an immediate impact. There is no evidence to suggest that it was widely read by the artists and craftsman of his era. In fact, few copies have survived. Fortunately, in 1979 the eminent art historian Ernst Gombrich published a volume [11] that included reproductions of some of the figures from Doüat's book, and in 1987 the metallurgist and historian of science Cyril Stanley Smith published an article [13] that included an English translation of Truchet's article and reproductions of all of Truchet's designs (as well as one of Doüat's). Toward the end of his article, Smith displayed the *Truchet-like tiles* shown in

*Corresponding author. Email: rbosch@oberlin.edu

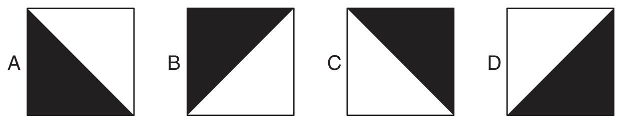

Figure 1. Truchet tiles.

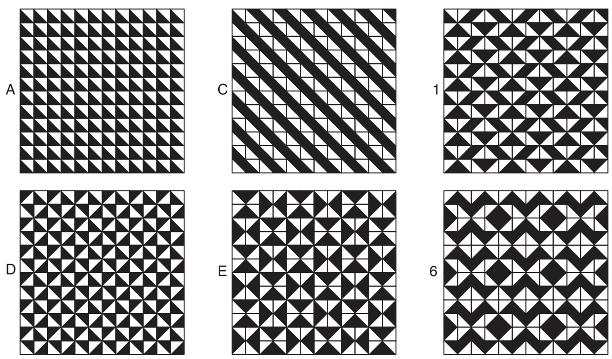

Figure 2. Truchet's Designs A, C, D and E and Doüat's Designs 1 and 6.

Figure 4. Truchet-like tiles are decorated with two quarter-circle arcs centred at opposite corners of the square. The arcs' endpoints terminate at the midpoints of the sides of the square. When Truchet-like tiles are arranged in a grid, their arcs join together to form aesthetically pleasing curves.

Pickover [11] proposed using Truchet-like tiles to visualize patterns found in random matrices of 0s and 1s. Each 0 is drawn as a Type-0 Truchet-like tile, and each 1 is drawn as a Type-1 Truchet-like tile as displayed in Figure 5.

Browne reported on, among other things, positioning Truchet-like tiles on the faces of a cube [5]. Bosch presented an integer programming formulation for arranging a collection of shaded Truchet-like tiles so that they approximate user-supplied greyscale target images [3]. Browne described how to build mosaics with duotone (two-colour

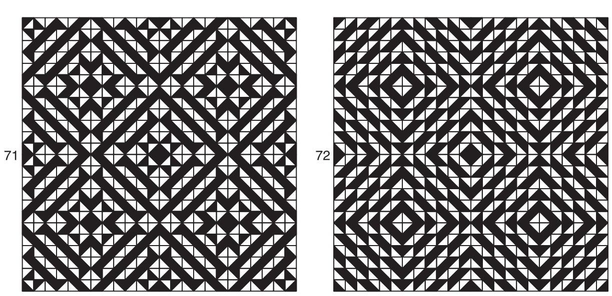

Figure 3. Doüat's Designs 71 and 72.

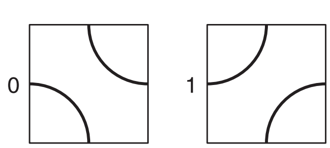

Figure 4. Truchet-like tiles.

1	0	1	0	0
1	0	1	0	1
0	0	0	1	0
0	0	0	1	1
1	1	1	0	0

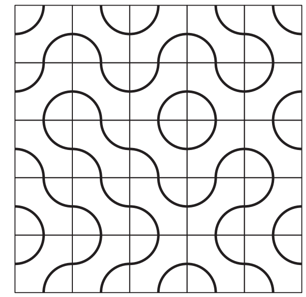

Figure 5. Using a 5×5 matrix of 0s and 1s to generate an orientation pattern for Truchet-like tiles.

shaded) Truchet-like tiles and with similarly decorated triangles and hexagons [6]. Reimann used duotone Truchet-like tiles to render text [12]. Krawczyk, in addition to providing a very detailed history, examined various tile systems in which the edges of the square tiles are subdivided into more than two equal segments [10].

In this paper, we describe how to modify Truchet's original tiles so that a collection of them can be used for halftoning, the reproduction of user-supplied greyscale target images in pure black and white. We also present hexagonal variations, a similar scheme for Truchet-like tiles, and an extension that can be applied to all regular and semiregular tilings. Dunham [8] investigated hyperbolic Truchet tilings.

2. Flexible Truchet tiles

Here we introduce *flexible Truchet tiles*, a simple modification of Truchet's original tiles. Our explanation of their construction is tied to Figure 6. To create a flexible Truchet tile, we allow the diagonal that separates black from white to flex (or bend) at its midpoint M, and we allow M to slide along the tile's other diagonal (drawn in light grey) anywhere from a point D to a point B. If we slide M toward D, we are expanding the tile's black part, making the tile

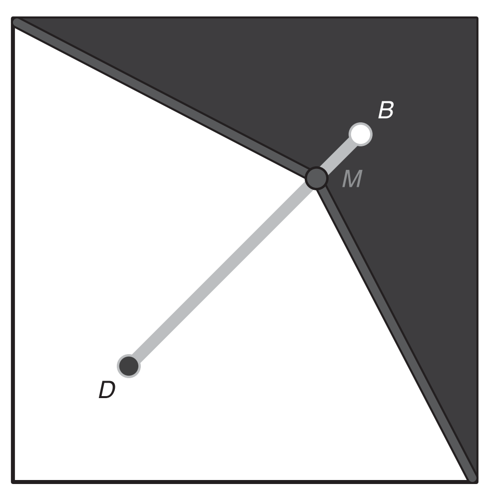

Figure 6. A flexible Truchet tile in orientation *C*.

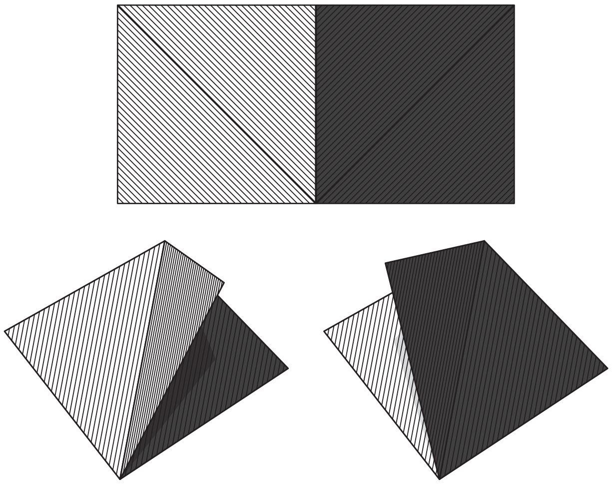

Figure 7. Flexible Truchet tiles from folded paper.

darker. If we slide M toward B, we are expanding the tile's white part, making the tile brighter. To ensure that the tile can be made as dark as it can be made bright, we choose D and B so that they are equidistant from the tile's centre. To ensure that the tile will always have a black part and a white part, we keep D and B away from the corners. In Figure 6, the point D is halfway between the square's centre and its bottom left-hand corner, and the point B is halfway between the centre and the top right-hand corner.

As shown in Figure 7, it is possible to make our flexible Truchet tiles out of folded paper. We start with a 1×2 piece of paper and make it look like the folding pattern at the top of Figure 7. The vertical black line that separates the bright square from the dark square is a mountain fold, and the two bold diagonal lines are valley folds. (The thin black lines are for shading.) After folding, we end up with a triangular flap. By tilting the flap one way or the other, we can make it so the tile, when viewed from above, appears as bright or as dark as we like.

With our flexible Truchet tiles, we can produce less bright or more bright versions of any of Truchet's patterns, any of Doüat's, or any other possible arrangement of Truchet tiles. By appropriately flexing the diagonals, we can produce any desired brightness. Figure 8 displays five

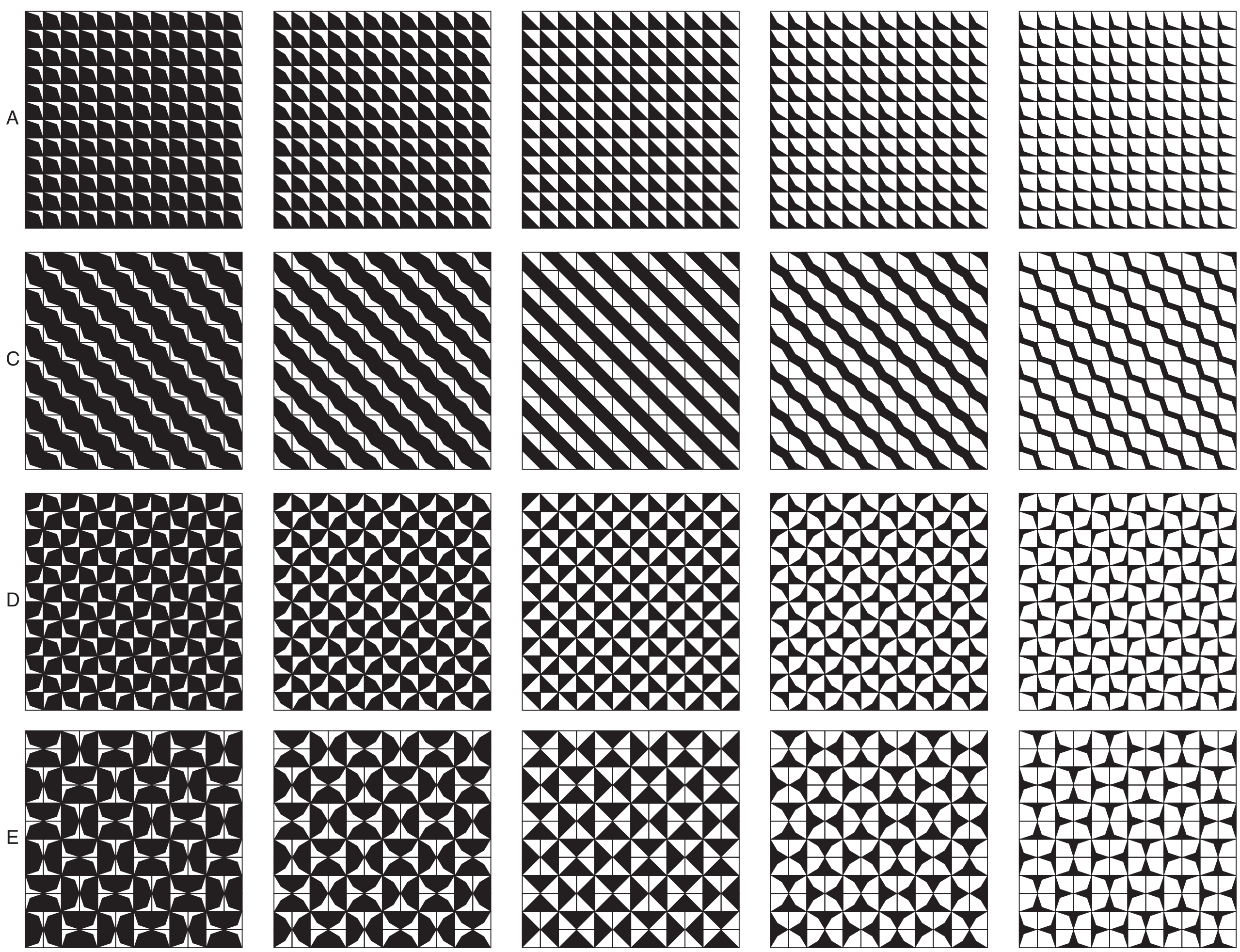

Figure 8. Flexible-Truchet-tile versions of Truchet's Designs A, C, D and E (the rows) arranged by brightness (the columns).

flexible-Truchet-tile versions of each of Truchet's Designs A, C, D and E. Each row corresponds to a different design and each column to a different brightness. The tiles in the leftmost column are as dark as we chose to make them, three-fourths black and one-fourth white; for each one, we moved the point M until it rested on top of the point D. The tiles in the rightmost column are as bright as we chose to make them, one-fourth black and three-fourths white; for each one, we moved M to B. The tiles in the middle column are Truchet's original tiles, half black and half white; for each one, we kept M at the centre.

Of course, we need not flex all the tiles by exactly the same amount. If we flex different tiles differently, we can produce a flexible-Truchet-tile mosaic that will closely resemble a user-selected greyscale target image. If the target image's row-i, column-j pixel, pixel (i, j), has brightness $b_{ij} \in [0, 1]$, where 0 stands for black, 1 stands for white and intermediate values stand for various shades of grey, then we can flex the row-i, column-j tile, tile (i, j), by forcing the midpoint of its diagonal, $M(i, j)$, to be the following convex combination of its dark point $D(i, j)$ and its bright point $B(i, j)$:

$$M(i, j) = (1 - b_{ij})D(i, j) + b_{ij}B(i, j). \qquad (1)$$

If we do this, $M(i, j)$ will end up at $D(i, j)$ when pixel (i,j) is black ($b_{ij} = 0$), $M(i,j)$ will end up at $B(i,j)$ when pixel (i,j) is white ($b_{ij} = 1$), and $M(i,j)$ will stay at the centre of the tile, its original or 'home-base' position, when pixel (i, j) is a shade of grey midway between black and white ($b_{ij} = 0.5$).

Figures 9–12 display various flexible-Truchet-tile mosaics, each constructed from the same target image, a 1726 engraving by Henri-Simon Thomassin entitled *Portrait of Father Sébastien Truchet*, which was itself based on a painting by Elisabeth Cheron le Hay. The mosaics in Figures 9 and 10 use Truchet's Designs A, C, D and E for their underlying orientation patterns. The mosaics in Figure 11 use Doüat's Designs 1, 6, 71 and 72, and the mosaic in Figure 12 uses randomly generated orientations.

The mosaics in Figure 9 are of lower resolution (36 tiles × 36 tiles), while the mosaics in Figures 10–12 are of higher resolution (72 tiles × 72 tiles). We present both lower and higher resolution mosaics for Truchet's Designs A, C, D and E to enable readers to compare the two resolutions.

Each of the flexible-Truchet-tile mosaics can be thought of as a blend or mash-up of the target image and the design or orientation pattern. From afar, viewers will be able to recognize the target image, but be unable to see the individual tiles or to identify the design. Up close, viewers will be able to examine the individual tiles and the design, but will lose sense of the target image. From an intermediate distance, they will be able to see both.

Of course, the ease with which a viewer can identify the target image depends on the resolution of the mosaic. Certainly, any reasonable measure of this 'readability' will judge a higher resolution 72 × 72 mosaic to be better than a lower resolution 36 × 36 mosaic. However, it is also apparent that within a resolution class, not all mosaics are equally readable. We believe that the readability of a flexible-Truchet-tile mosaic depends on the design in at least two ways. Although we have conducted no formal studies to test this, we believe that the human visual processing system has easier time processing designs that have small generators (like Truchet's Designs A and D) than designs that have larger generators (like Doüat's Designs 6, 8, 71 and 72) or designs formed by randomly selecting orientations, designs that are likely to have no finite generator. For designs with large generators or no finite generator, the human visual system may be forced to expend a great deal of effort to comprehend the orientation pattern, and this may distract it from, or conflict with, its efforts to blend the individual tiles into a single comprehensible image.

In addition, we believe that the readability of a mosaic is inversely related to $r_{\max}$, the radius of the largest circle that can be drawn within a black or white region of its underlying (half black, half white) orientation pattern. It is easy to show that if the tiles' sides are of unit length, then Truchet's Designs A and D have $r_{\max} = (2 - \sqrt{2})/2 \approx 0.2929$, the smallest possible value, that Truchet's Design C and Doüat's Design 3 have $r_{\max} = \sqrt{2}/4 \approx 0.3536$, that Truchet's Design E and Doüat's Design 1 have $r_{\max} = \sqrt{2} - 1 \approx 0.4142$, and that all of the remaining designs shown have $r_{\max} = \sqrt{2}/2 \approx 0.7071$, the largest possible value.

To our eyes, Truchet's Designs A and D produced the best mosaics in terms of readability, followed (in descending order) by Truchet's Designs C and E, Doüat's Designs 1, 6, 71 and 72, and in last place the random design.

Some designs produce geometric motifs that are, in our opinion, aesthetically pleasing. When we look at the mosaic formed from Truchet's Design D, we see lovely curved 'chips', some of them white with black hourglass markings and some of them black with white hourglass markings. We are also fond of the diagonal stripes that emerge from Truchet's Design C, and perhaps surprisingly, due to their large generators and their large values for $r_{\max}$, we are very pleased with the mosaics formed from Doüat's Designs 71 and 72. They remind us of certain 'Op Art' pieces and of the work of Chuck Close.

We close this section by mentioning that flexible Truchet tiles are well suited for animations. With them, we can continuously change one mosaic into another. If target image 0's and target image 1's row-i, column-j pixels have brightness values b^0_{ij} and b^1_{ij}, respectively, then at time $t \in [0, 1]$ we force the midpoint of tile (i, j)'s diagonal to satisfy

$$\begin{aligned} M(i, j) = \big(1 - \big[(1-t)b^0_{ij} + tb^1_{ij}\big]\big)D(i, j) \\ + \big[(1-t)b^0_{ij} + tb^1_{ij}\big]B(i, j). \end{aligned} \qquad (2)$$

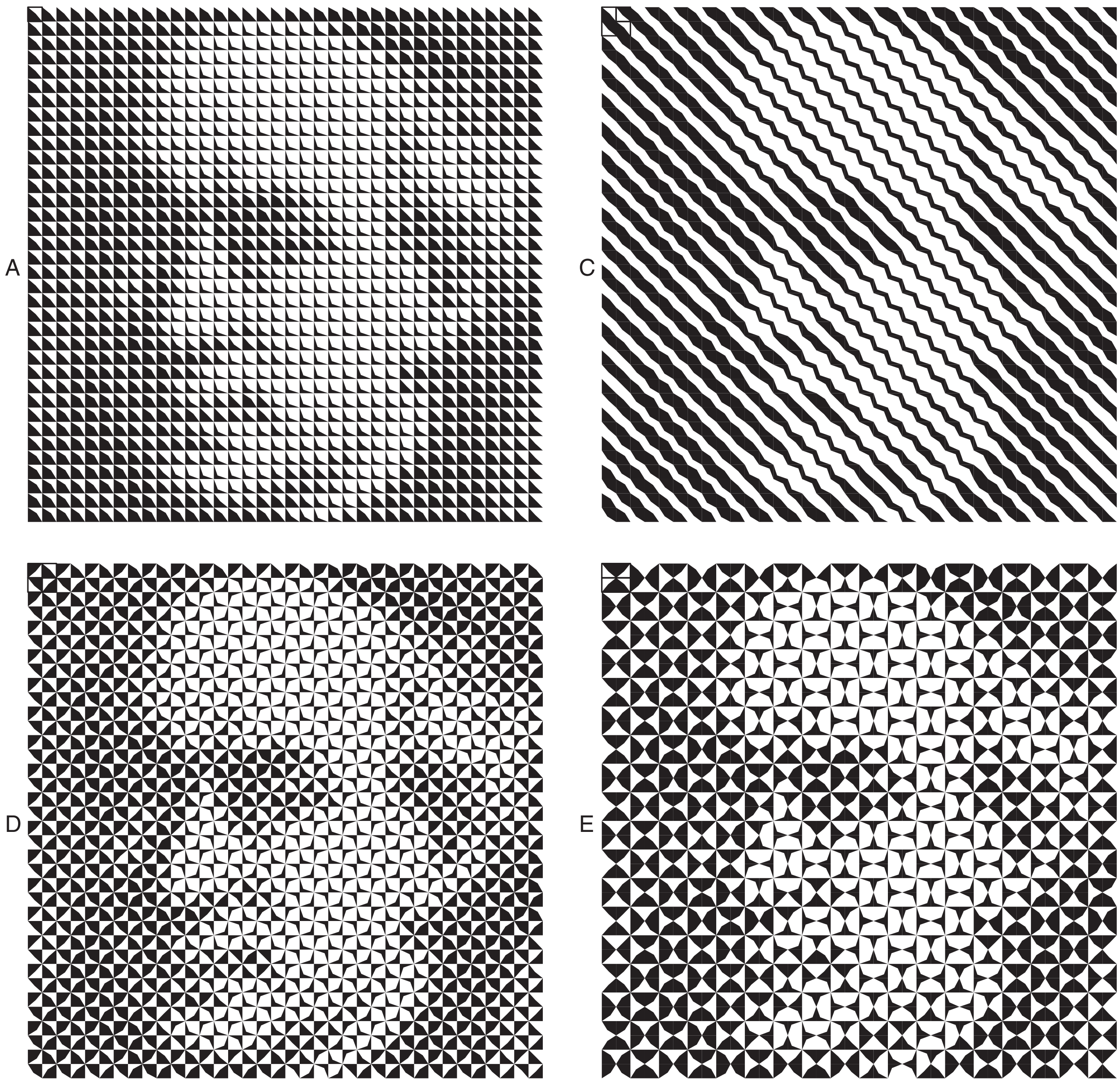

Figure 9. 36 × 36 flexible-Truchet-tile mosaics of Truchet (using Designs A, C, D and E from his article).

When $t = 0$, Equation (2) is Equation (1) but with $b_{ij} = b^0_{ij}$, and when $t = 1$, Equation (2) is Equation (1) but with $b_{ij} = b^1_{ij}$. As a result, when $t = 0$, the mosaic will resemble target image 0, and when $t = 1$, the mosaic will resemble target image 1. For intermediate values of t, it will be a blend of the two images. An example of such an animation can be found online (see Supplementary Material section at the end of this paper).

3. Flexhex tiles

In this section, we present *flexhex tiles*, hexagonal analogues of our flexible Truchet tiles. There are numerous ways in which they can be constructed. One of the simplest – and most similar in spirit to the flexible Truchet tile – involves dividing a regular hexagon into three congruent rhombuses, splitting each rhombus into two triangles along its long diagonal, and then colouring the inner triangles black and the outer triangles white, as shown in Figure 13. We call this tile the *long-diagonal rhombic flexhex tile*. (It is also possible to construct a short-diagonal version.) In its unflexed half-black-half-white home position, it looks like a black downward-pointing triangle on a white background. If we rotate it by 60°, the triangle will point upward. We call the non-rotated triangle-pointing-down form 'Orientation ▽' and the rotated triangle-pointing-up form 'Orientation △'.

To prepare for flexing a long-diagonal rhombic flexhex tile, we mark the midpoint of each long diagonal with a

Figure 10. 72 × 72 flexible-Truchet-tile mosaics of Truchet (using Designs A, C, D and E from his article).

point M. In each black triangle, we place a bright point B halfway between M and the centre of the hexagon, and in each white triangle, a dark point D halfway between M and the opposite vertex. To flex the tile, we move the three M points toward either their dark D points or their bright B points, bending the long diagonals. Moving the three M points toward the three dark D points makes the tile darker. Moving the three grey points toward the three bright B points makes the tile brighter.

As with our flexible Truchet tiles, we can produce less bright or more bright versions of any orientation pattern. Figure 14 displays five long-diagonal rhombic flexhex versions of three orientation patterns: $(\triangledown)$ (top row), $(\triangledown\vartriangle)$ (middle row) and $\begin{pmatrix}\triangledown\triangledown\vartriangle\\ \vartriangle\triangledown\triangledown\end{pmatrix}$ (bottom row). The flexhex tiles in the leftmost column are as dark as we chose to make them; for each one, we moved the three grey points until they rested on top of the black points. The flexhex tiles in the rightmost column are as bright as we chose to make them; for each one, we moved the three grey points until they were on top of the white points. The flexhex tiles in the middle column are half black and half white; for each one, we kept the grey points at the centre.

Long-diagonal rhombic flexhex tiles are very well suited for figurative mosaics. Mosaics built from them are characterized by triangular motifs. Figure 15 displays four

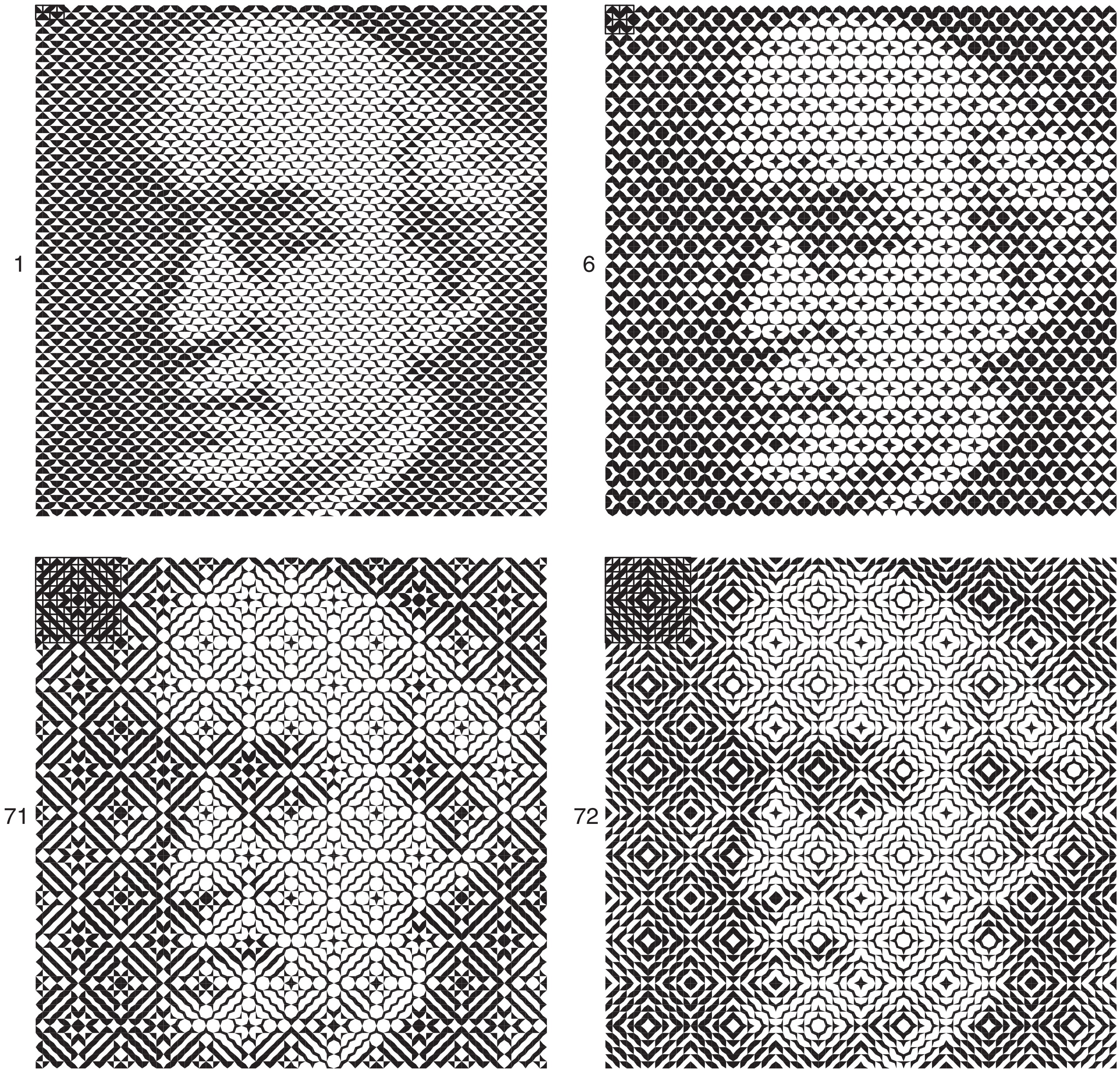

Figure 11. 72×72 flexible-Truchet-tile mosaics of Truchet (using Designs 1, 6, 71 and 72 from Doüat's book).

45×39 flexhex mosaics of Truchet, each based on the Thomassin engraving. The mosaics in the left-hand column use the long-diagonal rhombic flexhex tiles and, going from top to bottom, orientation patterns $(\triangledown)$, $(\triangledown\vartriangle)$ and $\binom{\triangledown\triangledown\vartriangle}{\vartriangle\triangledown\triangledown}$. The mosaics in the right-hand column use short-diagonal rhombic flexhex tiles. We chose the resolution to be 45×39 to make the mosaics comparable to the 72×72 flexible-Truchet-tile mosaics. A 45×39 flexhex mosaic has $45 \times 39 = 1755$ tiles and would have $6 \times 1755 = 10530$ triangles if all its tiles were kept unflexed. A 72×72 flexible-Truchet-tile mosaic has $72 \times 72 = 5184$ tiles and would have $2 \times 5184 = 10368$ triangles if all its tiles were kept unflexed.

Recall that for flexible Truchet tiles, some orientation patterns produce much more readable mosaics than do others. In the hexagonal case, there is less variability. We have yet to find a hexagonal design pattern that performs nearly as badly as the worst of the square designs. Even the random-orientation-pattern mosaics displayed in Figure 16 are quite readable (and even appealing).

One possible reason for this is that with the rhombic flexhex tiles, $r_{\max}$ does not vary much. In fact, for long-diagonal rhombic flexhex tiles, $r_{\max}$ does not depend on the orientation pattern at all. It is easy to show that it is always 0.5, the radius of the unflexed tile's black triangle. For short-diagonal rhombic flexhex tiles, when all tiles have

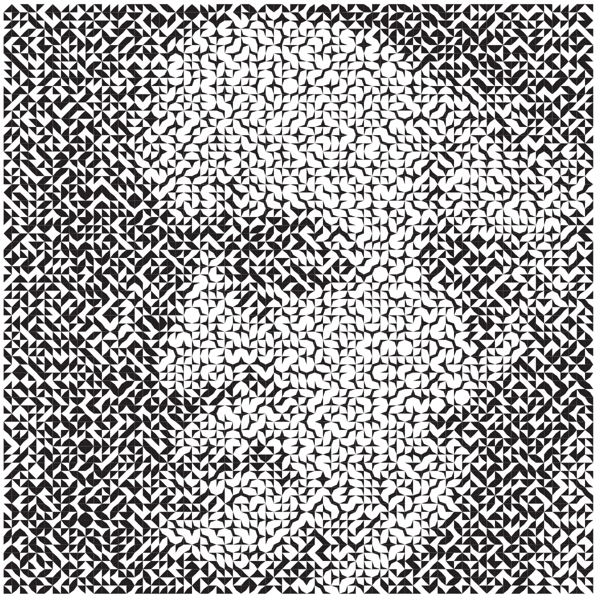

Figure 12. A 72×72 flexible-Truchet-tile mosaic of Truchet (using randomly selected orientations).

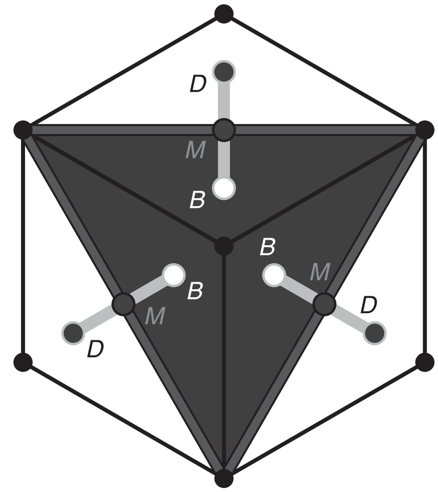

Figure 13. A long-diagonal rhombic flexhex tile in its unflexed half-black-half-white home-base position (orientation $\triangledown$).

the same orientation (as, for example, in the top mosaic in the right-hand column of Figure 15), $r_{\max} = \sqrt{3}/6 \approx 0.2887$. For all other orientation patterns, short-diagonal rhombic flexhex tiles have $r_{\max} = \sqrt{3}/4 \approx 0.4330$.

4. Flexible duotone Truchet-like tiles

Now we turn to the Truchet-like tiles. The arcs on Truchet-like tiles divide each tile into three regions, two of them quarter circles. If we alternately colour these regions black and white, we get the duotone Truchet-like tiles displayed in Figure 17. Tiles A and B come from the Type 0 Truchet-like tile, and Tiles C and D come from the Type 1 Truchet-like tile. Browne [6] described how to build mosaics with duotone Truchet-like tiles. Bosch [4] described how to use them to render user-supplied target images. Tiles A and C are dark, each having an average brightness of $\pi/8 \approx 0.39$. Tiles B and D are bright, each having an average brightness of $1 - \pi/8 \approx 0.61$. Tiles A and D share the corner colour pattern black–white–black–white

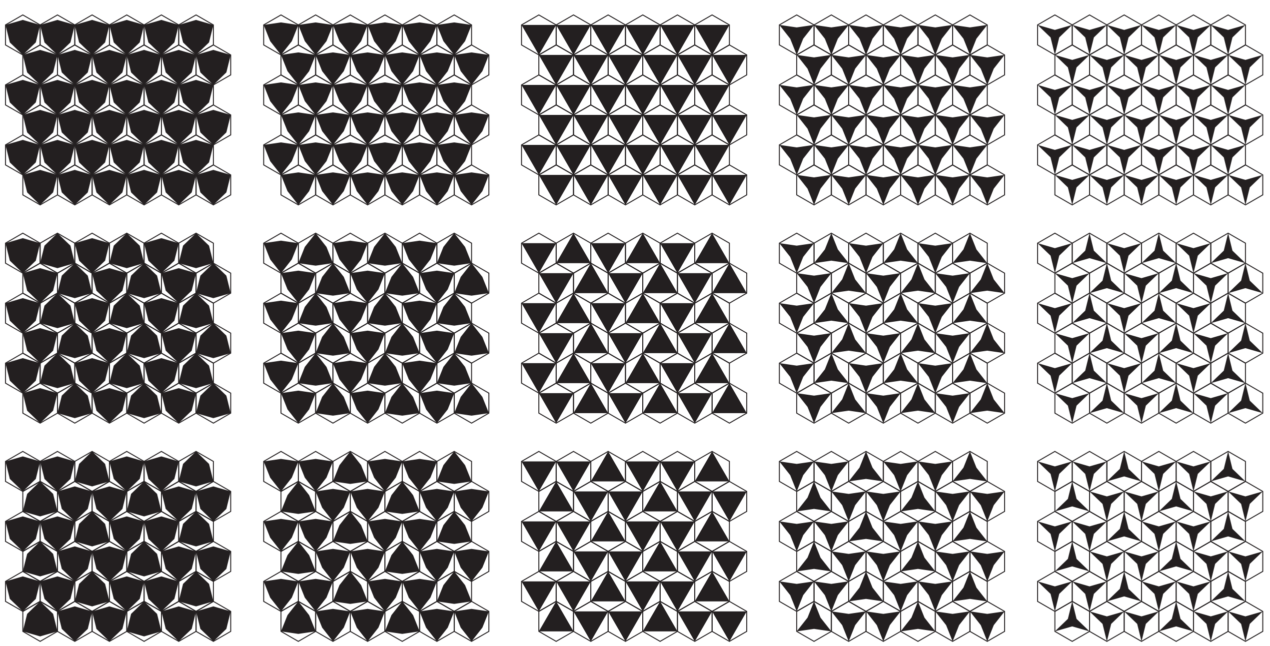

Figure 14. Long-diagonal rhombic flexhex-tile renditions of the orientation patterns $(\triangledown)$, $(\triangledown\triangle)$ and $\binom{\triangledown\triangledown\triangle}{\triangle\triangledown\triangledown}$.

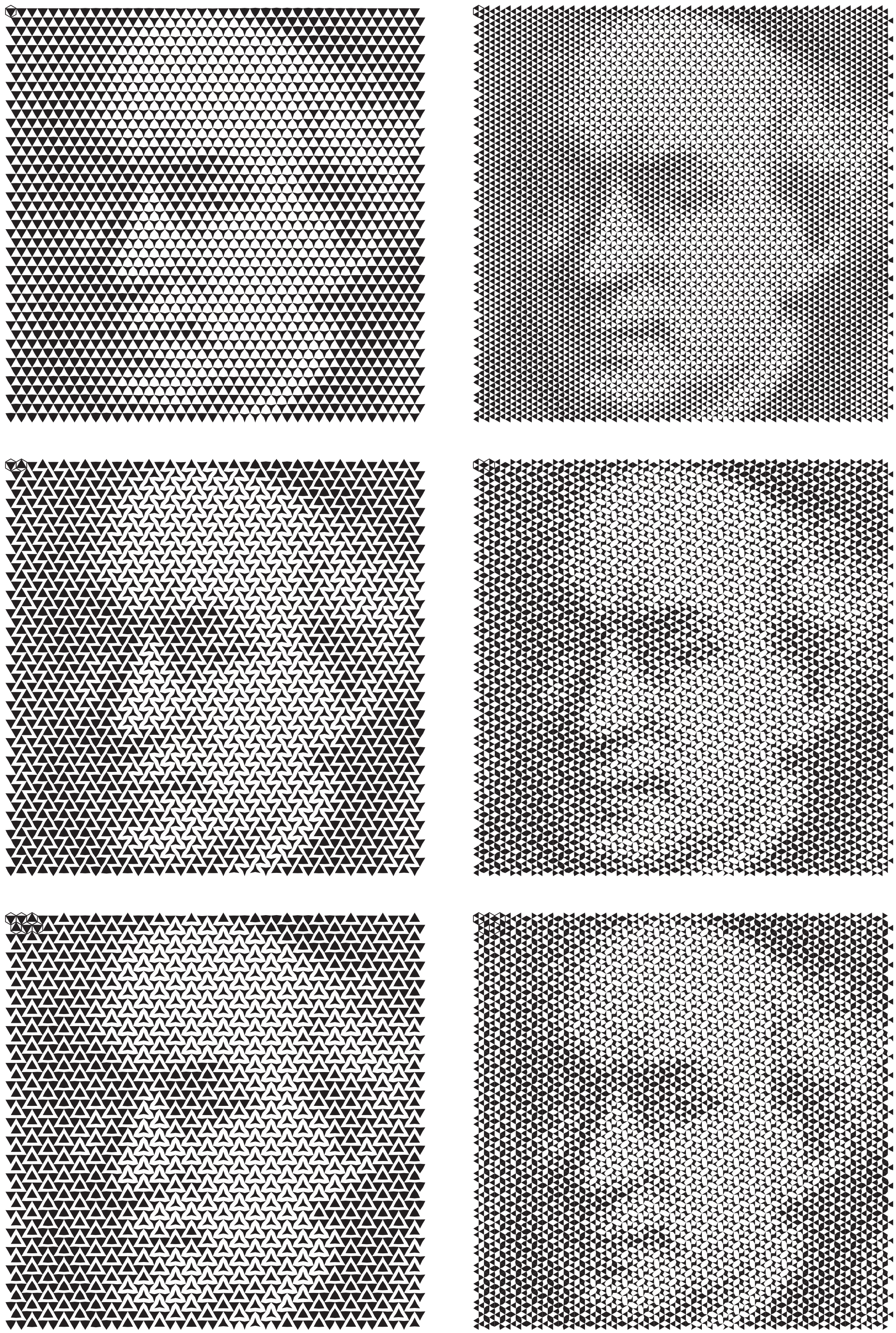

Figure 15. 45 × 39 long-diagonal (left-hand column) and short-diagonal (right-hand column) rhombic flexhex mosaics of Truchet.

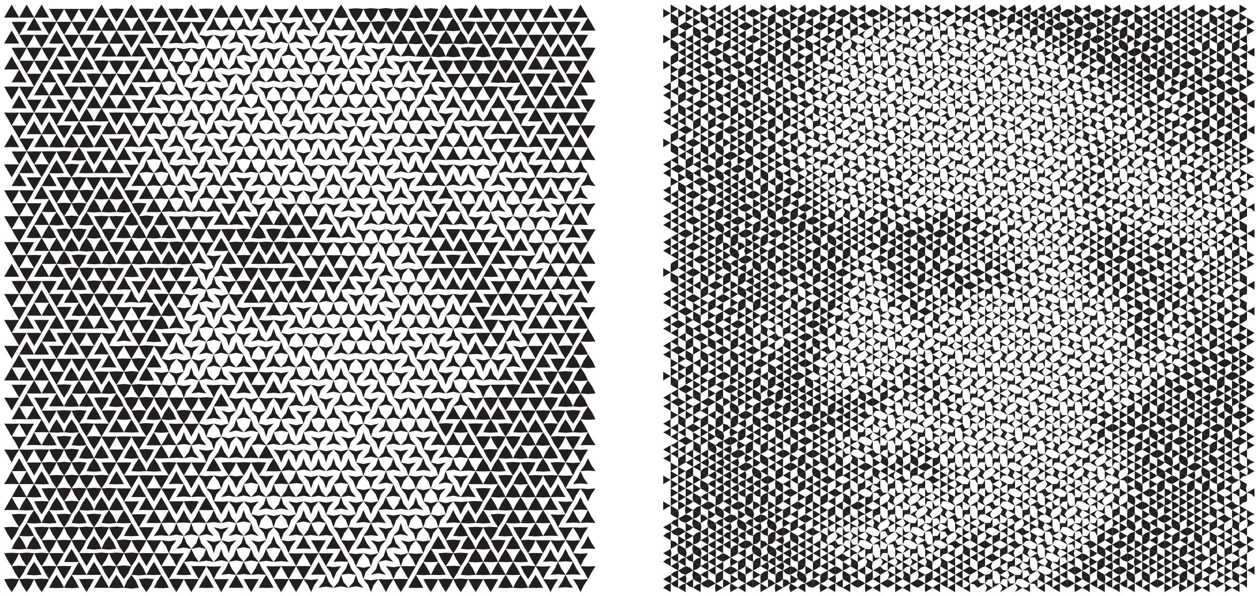

Figure 16. 45 × 39 long-diagonal (left-hand panel) and short-diagonal (right-hand panel) rhombic flexhex mosaics of Truchet (using randomly selected orientations).

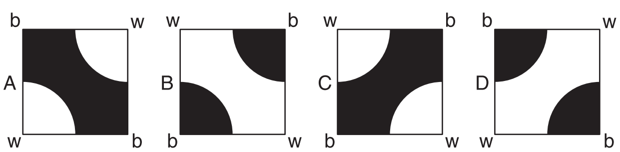

Figure 17. Duotone Truchet-like tiles.

(bwbw), and Tiles B and C have the corner colour pattern white–black–white–black (wbwb).

Bosch suggested using Pickover's matrices of 0s and 1s to generate brightnesses instead of orientations. Each 0 is drawn as a dark tile (either an A or a C), and each 1 is drawn as a bright tile (either a B or a D). In order for the colour patterns on adjacent tiles to match, the tiles must be placed so that the corner colour patterns (bwbw vs. wbwb) form a checkerboard pattern. There are two ways to do this. In the middle of Figure 18, a D tile occupies the top left-hand square of the mosaic. In the right-hand side of the figure, a B tile occupies the top left-hand square.

There are two simple ways to use this scheme to convert a greyscale image into a mosaic formed from duotone Truchet-like tiles. As before, we assume that the target image's row-i, column-j pixel has brightness $b_{i,j} \in [0, 1]$, where 0 stands for black, 1 stands for white, and intermediate values stand for various shades of grey. The first method is thresholding, rounding the brightness values to 0s and 1s. The second method is to use a form of dithering (for example, Floyd–Steinberg [8] dithering) to convert the brightness values to 0s and 1s. Figure 19 displays two mosaics formed from duotone Truchet-like tiles. For the mosaic on the left-hand side, thresholding was used. The mosaic on the right-hand side was produced using Floyd–Steinberg dithering. In each case, it is clear that the subject is Father Sébastien Truchet. But neither mosaic is as aesthetically pleasing as the best of the flexible Truchet mosaics or the

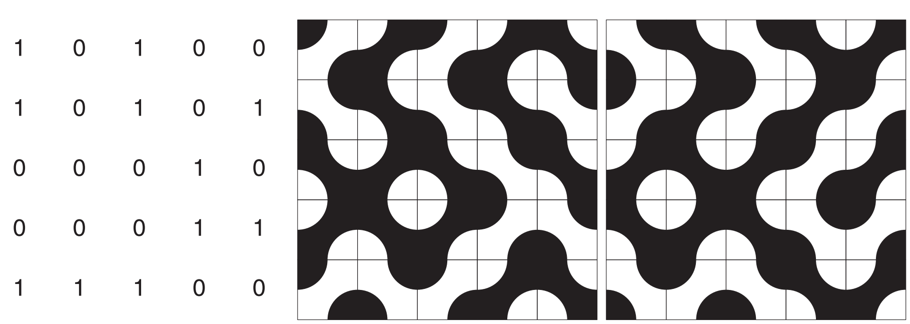

Figure 18. Using a 5 × 5 matrix of 0s and 1s to generate brightnesses for duotone Truchet-like tiles.

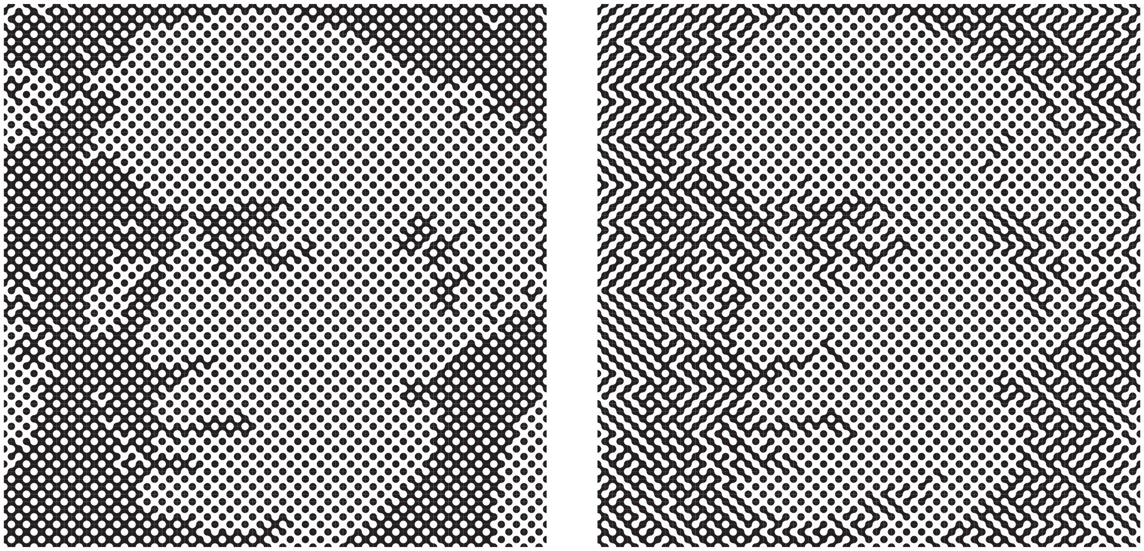

Figure 19. 72 × 72 mosaics of Truchet from duotone Truchet-like tiles using thresholding (left-hand panel) and Floyd–Steinberg dithering (right-hand panel).

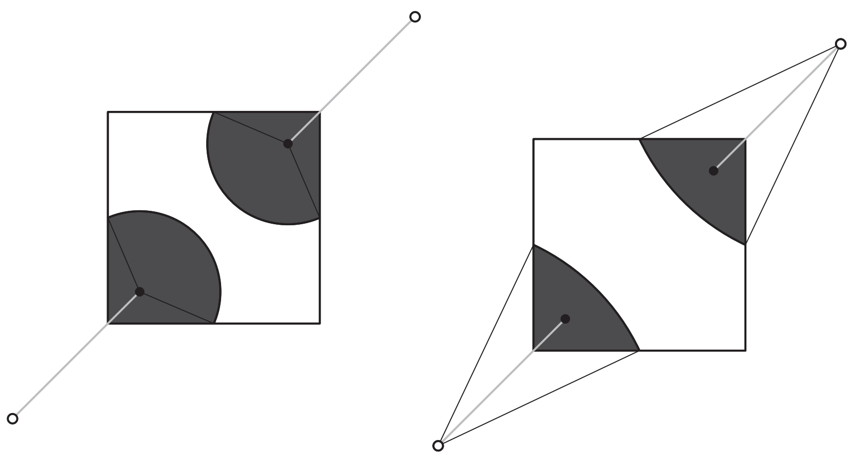

Figure 20. Type *B* flexible duotone Truchet-like tiles.

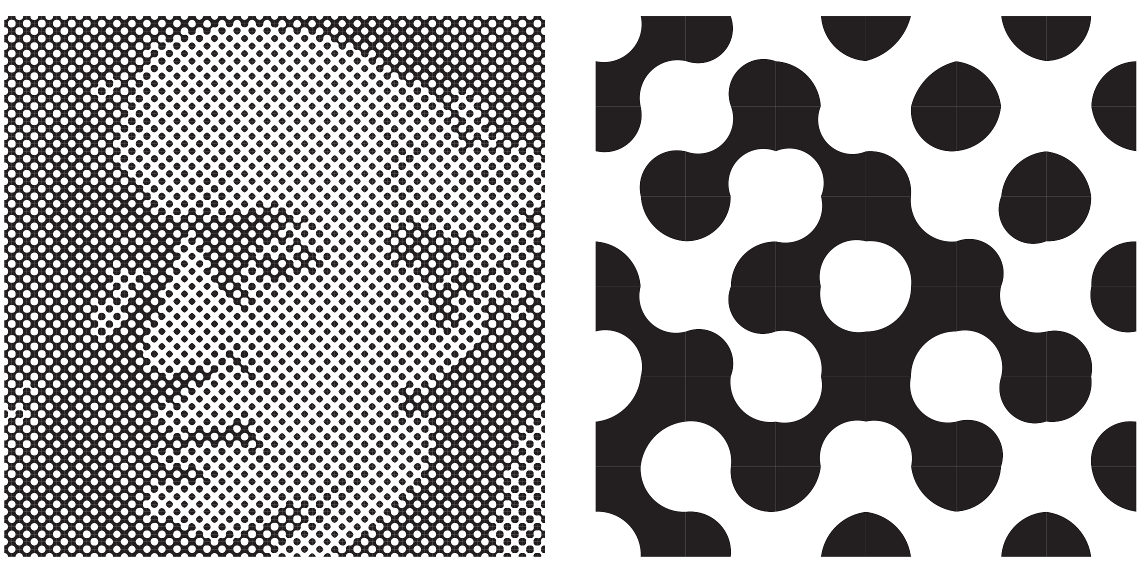

Figure 21. A 72 × 72 mosaic of Truchet from flexible duotone Truchet-like tiles (left) and a 6 × 6 detail (right).

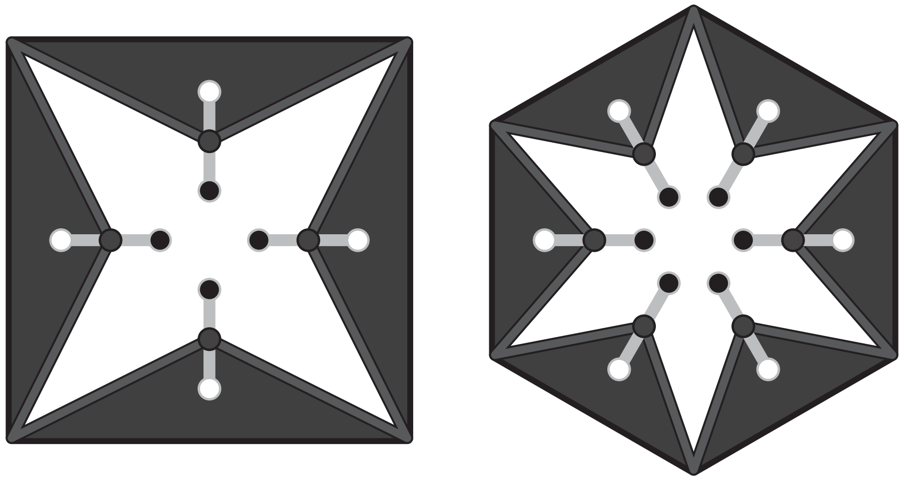

Figure 22. Four-pointed and six-pointed star tiles.

best of the flexhex mosaics. This is not surprising, as there are just two brightness values: $\pi/8 \approx 0.39$ and $1 - \pi/8 \approx 0.61$.

To flex a duotone Truchet-like tile, we move the centres of the quarter-circle arcs. We keep the arcs' endpoints at the midpoints of the sides of the square. Figure 20 displays the extreme cases of our flexing of a type B duotone Truchet-like tile.

On the left-hand side, we moved the centres of the arcs into the square (as far into the square as we allow ourselves),

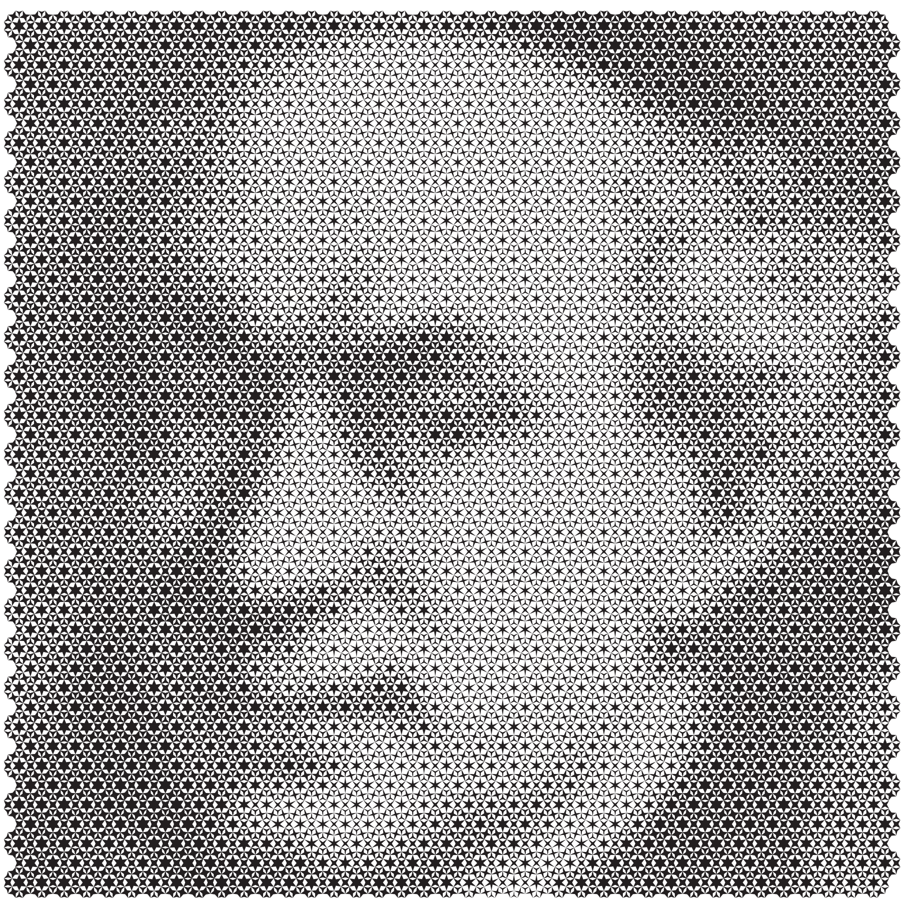

Figure 23. An Archimedean star-tile mosaic of Truchet.

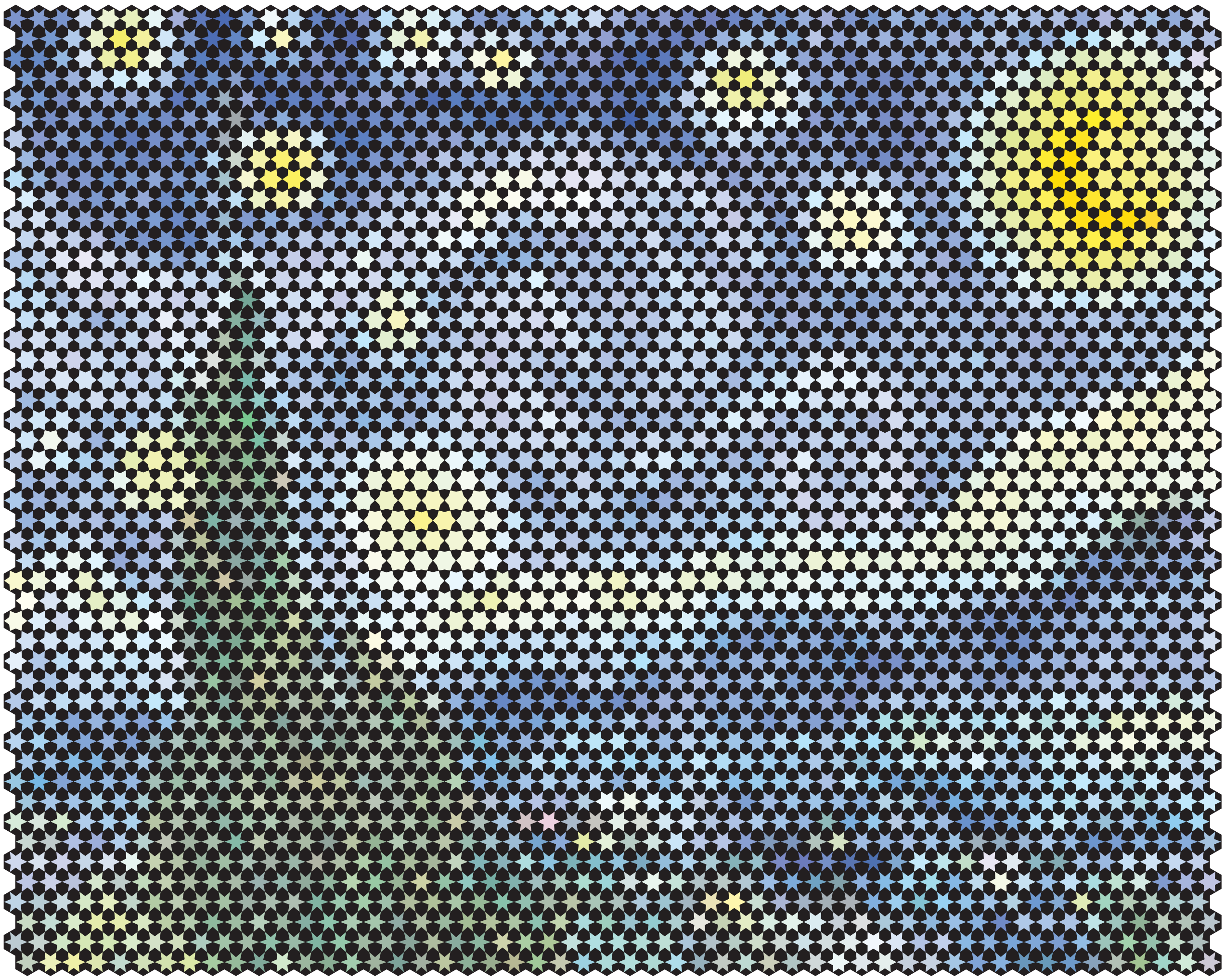

Figure 24. *Starry Starry Night (after van Gogh)*. © 2013 [Robert Bosch].

forcing the arcs to bulge and the white region to shrink to the point at which its area is one-half the area of the square. On the right-hand side, we moved the centres of the arcs outside the square (again, as far as we allow ourselves), forcing the arcs to flatten and the white region to grow to the point at which its area is approximately 70% of the area of the square. Thus, by carefully moving the centres, we are able to flex Type *B* or Type *D* tiles so that they can achieve any desired brightness between 0.50 and 0.70. In a similar fashion, we are able to flex Type *A* or Type *C* tiles so that they can attain any desired brightness between 0.30 and 0.50.

This means that if we map the brightnesses of our greyscale target image into the interval [0.30, 0.70], we can produce a very high quality mosaic using flexible duotone Truchet-like tiles. We start by replacing each $b_{i,j}$ with $b'_{i,j} = 0.3 + 0.4b_{i,j}$. If $b'_{i,j} \le 0.5$, we use a dark tile (either an *A* or a *C*) and flex it to the desired brightness. If $b'_{i,j} > 0.5$, we use a bright tile (either a *B* or a *D*) and flex it to the desired brightness. As with the unflexed duotone Truchet-like tiles, we make sure that we place the tiles in such a way that the corner colour patterns form a checkerboard pattern.

The left half of Figure 21 displays a 72×72 mosaic of Truchet formed from flexible duotone Truchet-like tiles, and the right half displays a 6×6 detail (part of his left eye). When viewed from a distance, the mosaic appears to be formed from standard duotone Truchet-like tiles (but it looks much better than the mosaics displayed in Figure 19). From afar, the human eye cannot pick up the many bulges and dents that are responsible for the increased variation in brightness.

5. Star tiles

In this final section, we present *star tiles*, which are similar in spirit to flexible Truchet tiles. Figure 22 displays the four-pointed and six-pointed star tiles in their unflexed half-black-half-white home-base positions. As with flexible

Truchet tiles and flexhex tiles, to make a star tile brighter, we slide its grey points along their tracks (drawn in light grey) toward their white points. To make a star tile darker, we slide its grey points along their tracks toward their dark points.

Figure 23 displays a star-tile mosaic of Truchet made of three-, four- and six-pointed star tiles. Here the tiles are positioned in accordance with a 3.4.6.4 semiregular (Archimedean) tiling and appear in inverted form, with black stars on a white background instead of white stars on a black background.

Figure 24 displays a modified star-tile rendition of Vincent van Gogh's *Starry Night*, a preliminary attempt at bringing colour into a flexible-tile environment. All of the tiles are hexagons containing six-pointed stars. Here the points of the star touch the midpoints of the edges of the hexagon, and the colour of the star is obtained via averaging the RGB values of the underlying pixels from the target image.

Supplementary material

The supplementary material for this paper is available online at http://dx.doi.org/10.1080/17513472.2013.838830

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