CSE 575: Advanced Cryptography
Fall 2024
Lecture 16

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Agenda for this lecture

- Announcements
- Collision-resistant hashing
- Merkle-Damgard transform
- Domain extension for signatures
- Analyzing domain extension

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Announcements

- HW3 is out! Due 11/5

News


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ABOUT
Research > App Privacy and Controls
Should We Chat, Too? Security Analysis of WeChat's MMTLS Encryption Protocol
By Mona Wang, Pellaeon Lin, and Jeffrey Knockel
October 15, 2024
閱讀繁體中文摘要

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Collision-resistant hashing

- File integrity
- authenticated data structures
- MAC HMAC
- Domain extension

Function family $h_s : \{0,1\}^m \longrightarrow \{0,1\}^n$

h_s is CRH if

- $m > n$
- $\forall$ nuPPT $\mathcal{A}$,

$$\Pr_{s \leftarrow Gen}[(x_1, x_2) \leftarrow \mathcal{A}(s) : h_s(x_1) = h_s(x_2) \wedge x_1 \neq x_2] = negl(n)$$

Collision-resistant hashing

Pedersen

$H_h : \mathbb{Z}_q \times \mathbb{Z}_q \to \mathbb{G}$

$\mathbb{G} = \langle g \rangle$ $\text{ord}\,\mathbb{G} = q$.

DLog: $\forall$ nuPPT A,

$\Pr_{y \leftarrow \mathbb{G}}[x = A(y) : g^x = y] = \text{negl}$

- Gen:
  Return $h \leftarrow \mathbb{G}$
- $H_h(a, b)$:
  Ret $g^a \cdot h^b$

- compressing? Yes
- CR: Thm: If DLog in $\mathbb{G}$, Pedersen is CR.

Proof idea:

Use DLog challenge y as hash key of Pedersen.

Collision in PH $\Rightarrow$ get $\text{DLog}_g(y)$

Collision-resistant hashing

$\mathcal{A}$

$\mathcal{B}(y)$:

$(x_1, x_2), (a_1, a_2) \leftarrow A(y)$

Ret $\frac{a_1 - x_1}{x_2 - a_2}$

Argue coll. $\Rightarrow$ $\log_g y = \frac{a_1 - x_1}{x_2 - a_2}$.

$g^{x_1} y^{x_2} = g^{a_1} y^{a_2}$

$y^{x_2 - a_2} = g^{a_1 - x_1}$

$y = g^{\frac{a_1 - x_1}{x_2 - a_2}}$

If $x_1 = a_1$ but $x_2 \neq a_2$,

$y^{x_2 - a_2} = g^0 = 1$

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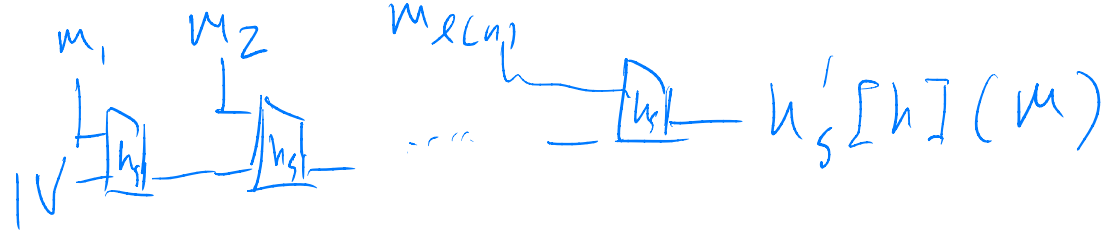
Merkle-Damgård transform

Thm: If $\exists$ CRH h_s with 1-bit compr.,
$\exists$ CRH h'_s with poly-n compr.

Proof idea:



$$h(m_{\ell(n)} \| h(\cdots h(m_1 \| IV) \cdots)) = h(m'_{\ell(n)} \| h(\cdots h(m'_1 \| IV) \cdots))$$

Either $m_{\ell(n)} = m'_{\ell(n)}$ or not. If not, win!

If so, and pref. hashes are equal, remove last bits and iterate.

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Extending the domain of a signature scheme

DS = Gen, Sign, Ver

h_S CRH

DESIG[h, DS].Gen: $(pk, sk) \leftarrow$ DS.Gen, $S \leftarrow \mathcal{H}.\text{Gen}$

Ret $((pk, S), (sk, S))$

DESIG[h, DS].Sign$((sk, S), m)$: Ret DS.Sign$(sk, h_S(m))$

DESIG[h, DS].Ver$((pk, S), m, \sigma)$: Ret DS.Ver$(pk, h_S(m), \sigma)$

Thm: If h_S CRH and DS UF-CMA, then DESIG[h, DS] is UF-CMA

Proof idea:

DESIG UF-CMA win $\Rightarrow$ either DS UF-CMA win or collision in h_S

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Analyzing DESig