

EECS 498/CSE 598: Zero-Knowledge Proofs

Winter 2026

Lecture 6



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Agenda for this lecture

- Announcements
- What is a “backend”?
- Interactive proofs
- The sumcheck protocol
- Analyzing sumcheck
- (If time) Applications of sumcheck

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Announcements

- Project 1 is online
- Autograder is working

• P2 release delayed
→ Friday

• P1 grace period

due date
pushed back

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“Backends”

Question: How does ZKP software get used to solve a real problem?

First, use frontend component to:

1. Write human-readable program in ZK programming language.
 - captures requirements of problem in typical programming abstractions
 - Program P takes public input x and private witness w , outputs 0/1.
2. Run a compiler on P to transform it into an algebraic representation C .
 - Also need to generate (or have programmer write) an auxiliary program E that does “witness extension”: take x, w inputs for P , turn them into x', w' inputs for C . Need property that for all x, w ,
 $P(x, w)=1 \Leftrightarrow C(x', w')=1$

The frontend's output is P , C , and E .

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The frontend’s output is P , C , and E .

Given concrete x, w so that $P(x, w)=1$, use backend to generate and verifies proofs about P, x, w .

Implements interfaces:

$$\begin{aligned} &Verify(pp, P, C, E, x, \pi) \\ &\rightarrow \{0,1\} Prove(pp, P, C, E, x, w) \rightarrow \pi \end{aligned}$$

The program *Prove* first runs E on x, w to get x', w' , then runs a ZK prover to generate a proof that C, x' has a corresponding w' .

The program *Verify* first uses E to turn x into x' , then verifies proof π using ZK verifier.

How to build a backend

Question: How is the backend protocol built?

1. Assume we already have C, x', w' and C is in “convenient” algebraic form like R1CS
2. Express C, x', w' as polynomials (via eg univariate or multilinear extensions)
3. Come up with a relationship between, or property of, these polynomials such that:
 - property holds if and only if $C(x', w') = 1$

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4. Design protocol between prover and verifier in an “ideal” model
 - Interactive oracle proofs (IOPs): like IPs but force honest prover behavior
 - Protocol lets prover convince the verifier these particular polynomials have the property hold
5. Use cryptography to “compile” this IOP to a real protocol:
 1. Compile IOP to interactive argument of knowledge via polynomial commitments
 2. Remove interaction using fiat-shamir transform
 3. Hide witness (ZK) by blinding “leaky” parts of proof with random data

ZK SNARK

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Right now this is all (intentionally) very abstract. The next two projects and several weeks of class will make this very concrete.

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Interactive proofs

Protocol b/w two parties P V

IP is "for" a language L

$P(x)$

$V(x)$

$x \in L$

✓

no output

→ $\{0/1\}$

(Public-coin!

(V sends
random messages

Round: pair of \rightleftarrows messages

P sends last message

$Out_V [P(x) \leftrightarrow V(x)]$

$view_V [P(x) \leftrightarrow V(x)]$

Properties of interactive proofs

IP for L
 $= \langle P, V \rangle$

Completeness $\forall (x, w) \in R_L$
 $Pr[Out_V[P(x, w) \leftrightarrow V(x)] = 1] = 1$

ϵ -soundness $\forall x \notin L \forall P^*(x)$

$$Pr[Out_V[P^*(x) \leftrightarrow V(x)] = 1] \leq \epsilon$$

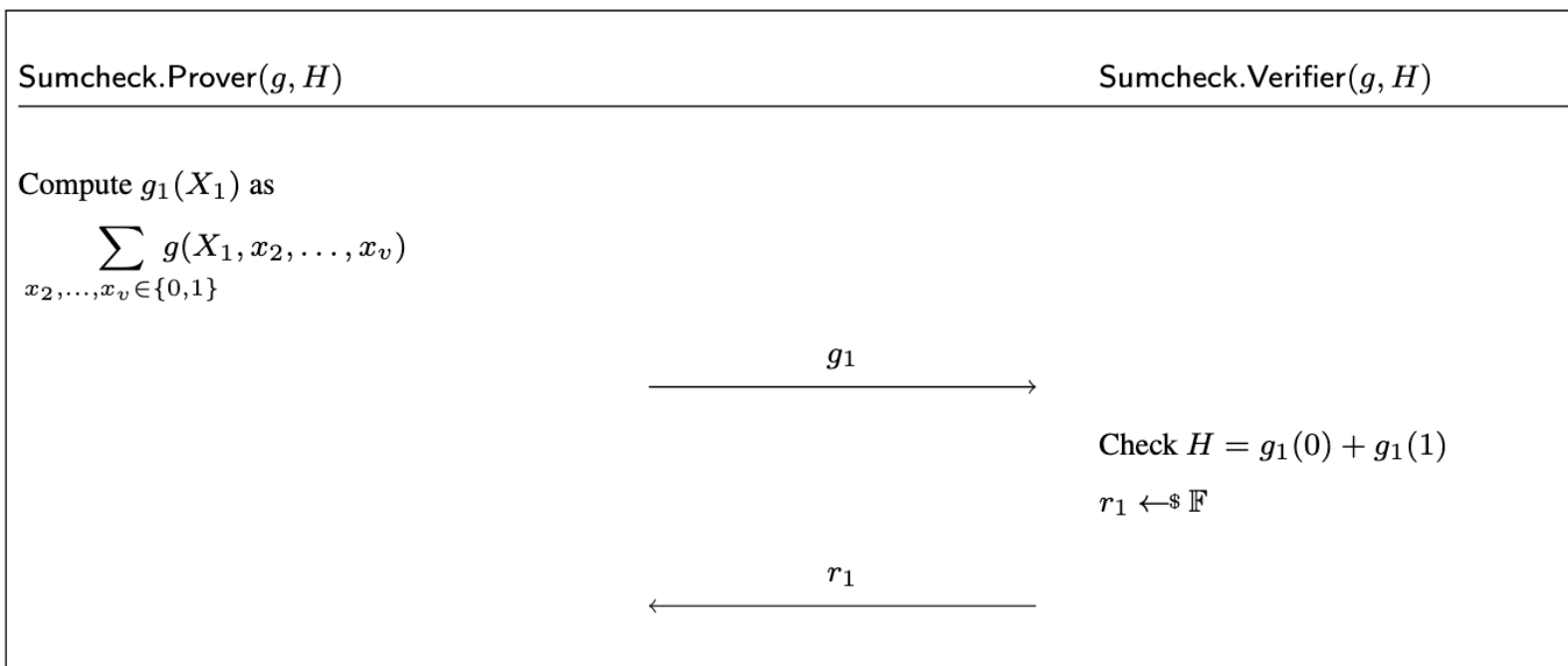
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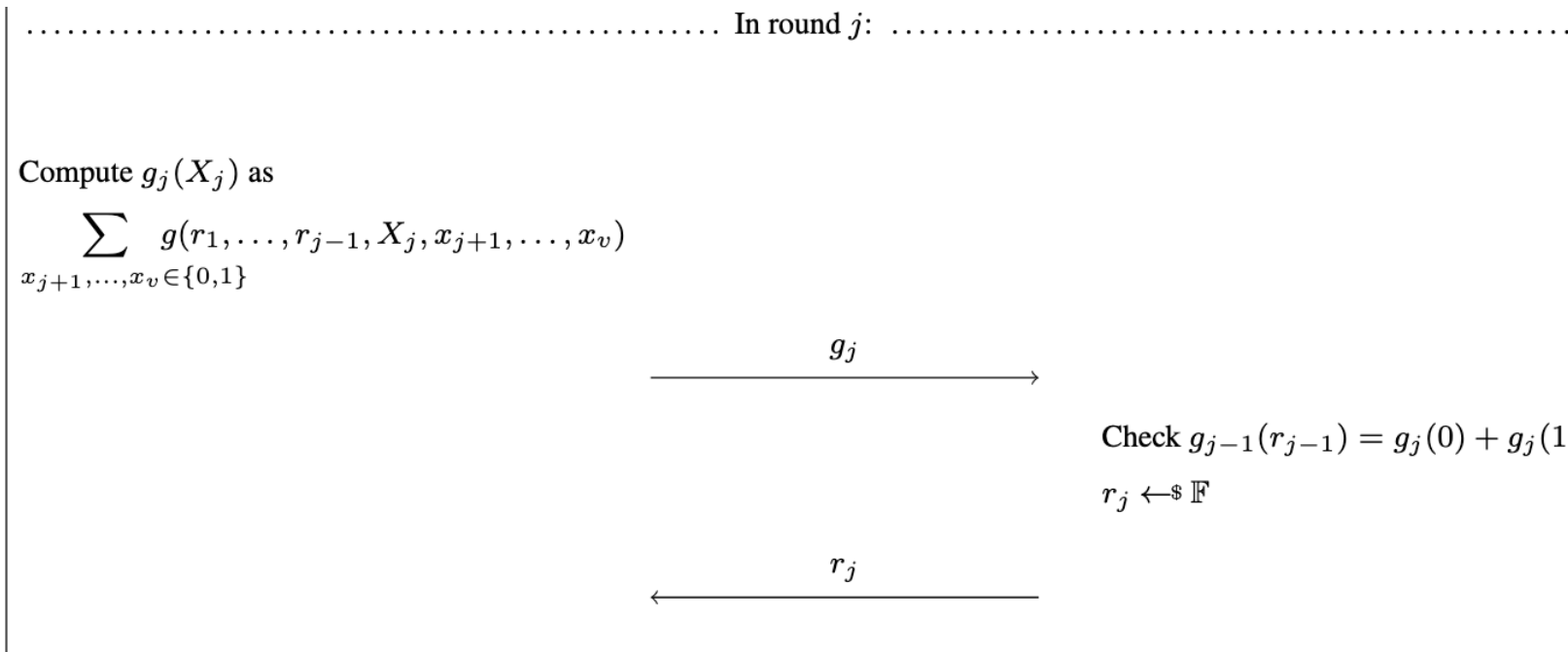
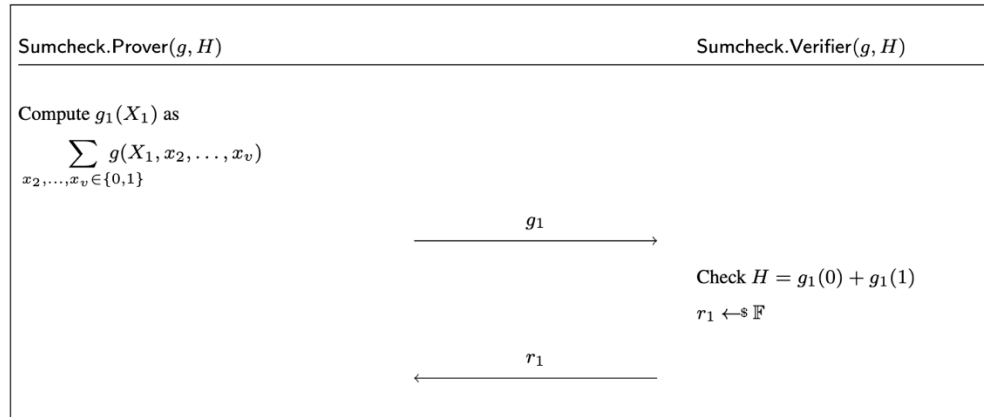
The sumcheck protocol

g is
 ℓ -variate
over \mathbb{F}
 d is
maximal
degree

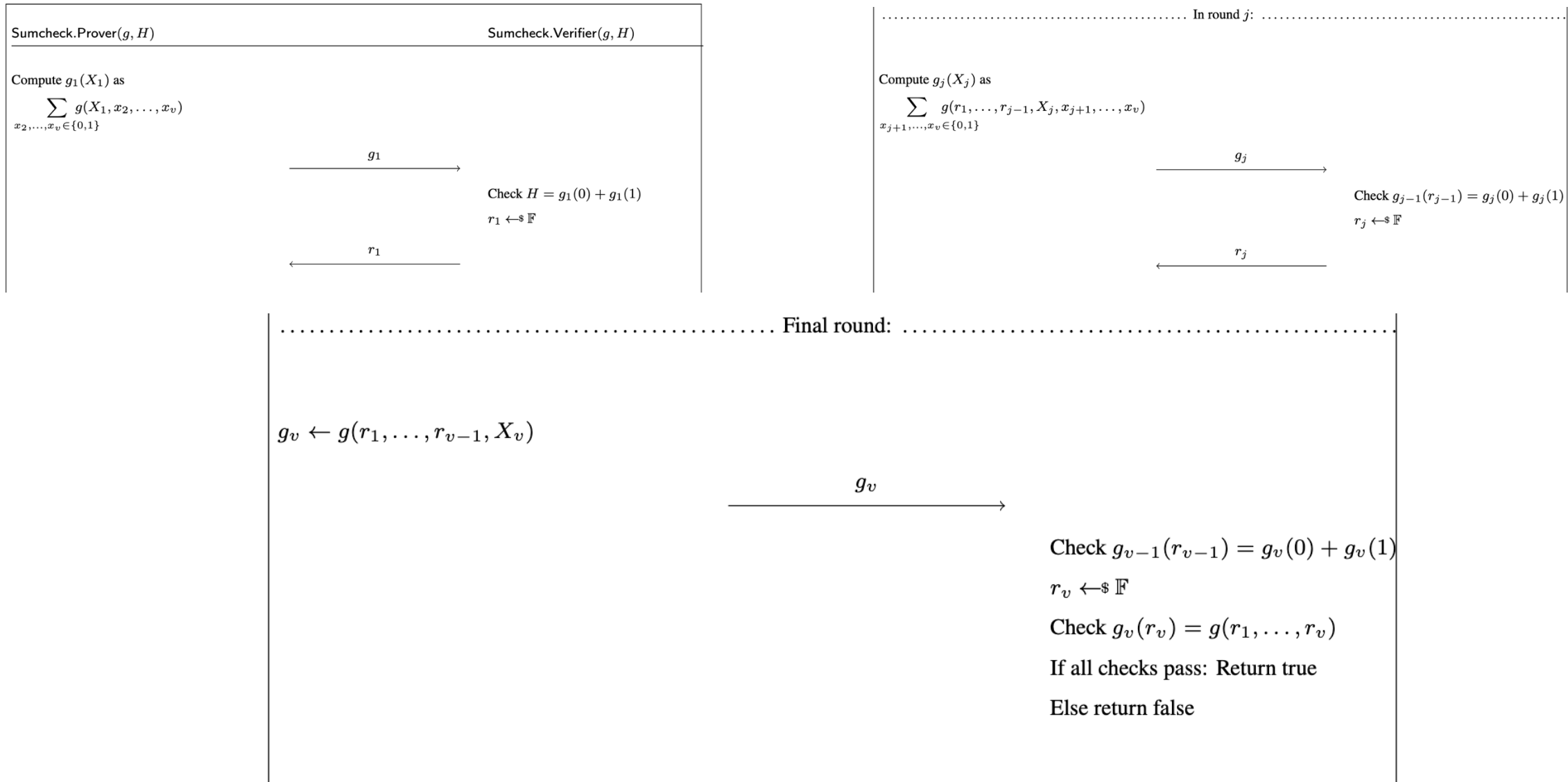
$$\text{Claim : } \sum_{c \in \{0,1\}^\ell} g(c) = H$$



The sumcheck protocol



The sumcheck protocol



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Analyzing sumcheck

Theorem 1. *Let g be an ℓ -variate polynomial of maximal degree d over the field \mathbb{F} . Then for any malicious prover \mathbb{P}^* and value $H \neq \sum_{i \in \{0,1\}^\ell} g(i)$,*

$$\Pr[\text{out}_{\mathbb{V}} [\mathbb{P}^*(g, H) \leftrightarrow \mathbb{V}(g, H)] = 1] \leq \frac{\ell d}{|\mathbb{F}|}$$

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Applications of sumcheck