

# EECS 498/CSE 598: Zero-Knowledge Proofs

## Winter 2026

### Lecture 6



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# Agenda for this lecture

- Announcements
- What is a “backend”?
- Interactive proofs
- The sumcheck protocol
- Analyzing sumcheck
- (If time) Applications of sumcheck

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# Announcements

- Project 1 is online
- Autograder is working

- P2 release delayed
  - Friday
- PI grace period

due date  
pushed back

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# “Backends”

Question: How does ZKP software get used to solve a real problem?

First, use frontend component to:

1. Write human-readable program in ZK programming language.
  - captures requirements of problem in typical programming abstractions
  - Program P takes public input  $x$  and private witness  $w$ , outputs 0/1.
2. Run a compiler on P to transform it into an algebraic representation C.
  - Also need to generate (or have programmer write) an auxiliary program E that does “witness extension”: take  $x, w$  inputs for P, turn them into  $x', w'$  inputs for C. Need property that for all  $x, w$ ,  
 $P(x, w)=1 \Leftrightarrow C(x', w')=1$

The frontend’s output is P, C, and E.

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The frontend’s output is P, C, and E.

Given concrete x,w so that  $P(x, w)=1$ , use backend to generate and verifies proofs about P, x, w.

Implements interfaces:

$$\begin{aligned} & Verify(pp, P, C, E, x, \pi) \\ & \rightarrow \{0,1\}Prove(pp, P, C, E, x, w) \rightarrow \pi \end{aligned}$$

The program *Prove* first runs E on x,w to get x',w', then runs a ZK prover to generate a proof that C,x' has a corresponding w'.

The program *Verify* first uses E to turn x into x', then verifies proof  $\pi$  using ZK verifier.

# How to build a backend

Question: How is the backend protocol built?

1. Assume we already have  $C$ ,  $x'$ ,  $w'$  and  $C$  is in “convenient” algebraic form like R1CS
2. Express  $C$ ,  $x'$ ,  $w'$  as polynomials (via eg univariate or multilinear extensions)
3. Come up with a relationship between, or property of, these polynomials such that:
  - property holds if and only if  $C(x', w') = 1$

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4. Design protocol between prover and verifier in an “ideal” model
  - Interactive oracle proofs (IOPs): like IPs but force honest prover behavior
  - Protocol lets prover convince the verifier these particular polynomials have the property hold
5. Use cryptography to “compile” this IOP to a real protocol:
  1. Compile IOP to interactive argument of knowledge via polynomial commitments
  2. Remove interaction using fiat-shamir transform
  3. Hide witness (ZK) by blinding “leaky” parts of proof with random data

ZKSNARK

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Right now this is all (intentionally) very abstract. The next two projects and several weeks of class will make this very concrete.

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# Interactive proofs

Protocol b/w two parties P V

P is "for" a language L

P(x)

↙

no output

V(x)

→

{0/1}

x ∈ L

(Public coin)

(V sends)

(Random messages)

Round: pass of  $\geq$  messages

P sends last message

out<sub>V</sub> [P(x)  $\leftrightarrow$  V(x)]

view<sub>V</sub> [P(x)  $\leftrightarrow$  V(x)]

IP for L  
=  $\langle P, V \rangle$

# Properties of interactive proofs

Completeness  $\forall (x, w) \in R_L$

$$\Pr[\text{out}_V [P(x, w)] \leftrightarrow V(x)] = 1$$

$\epsilon$ -soundness  $\forall x \notin L \quad \forall P^*(x)$

$$\Pr[\text{out}_V [P^*(x)] \leftrightarrow V(x)] = 0 \leq \epsilon$$

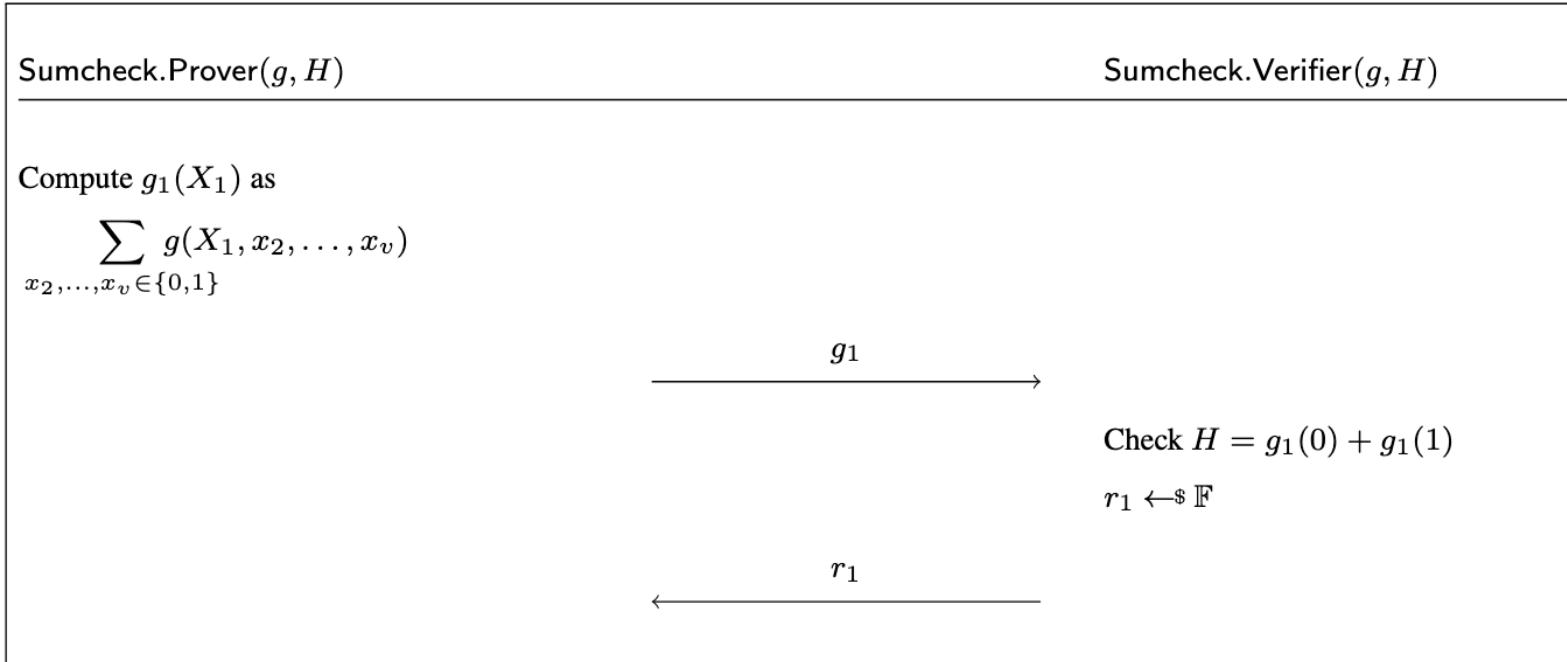
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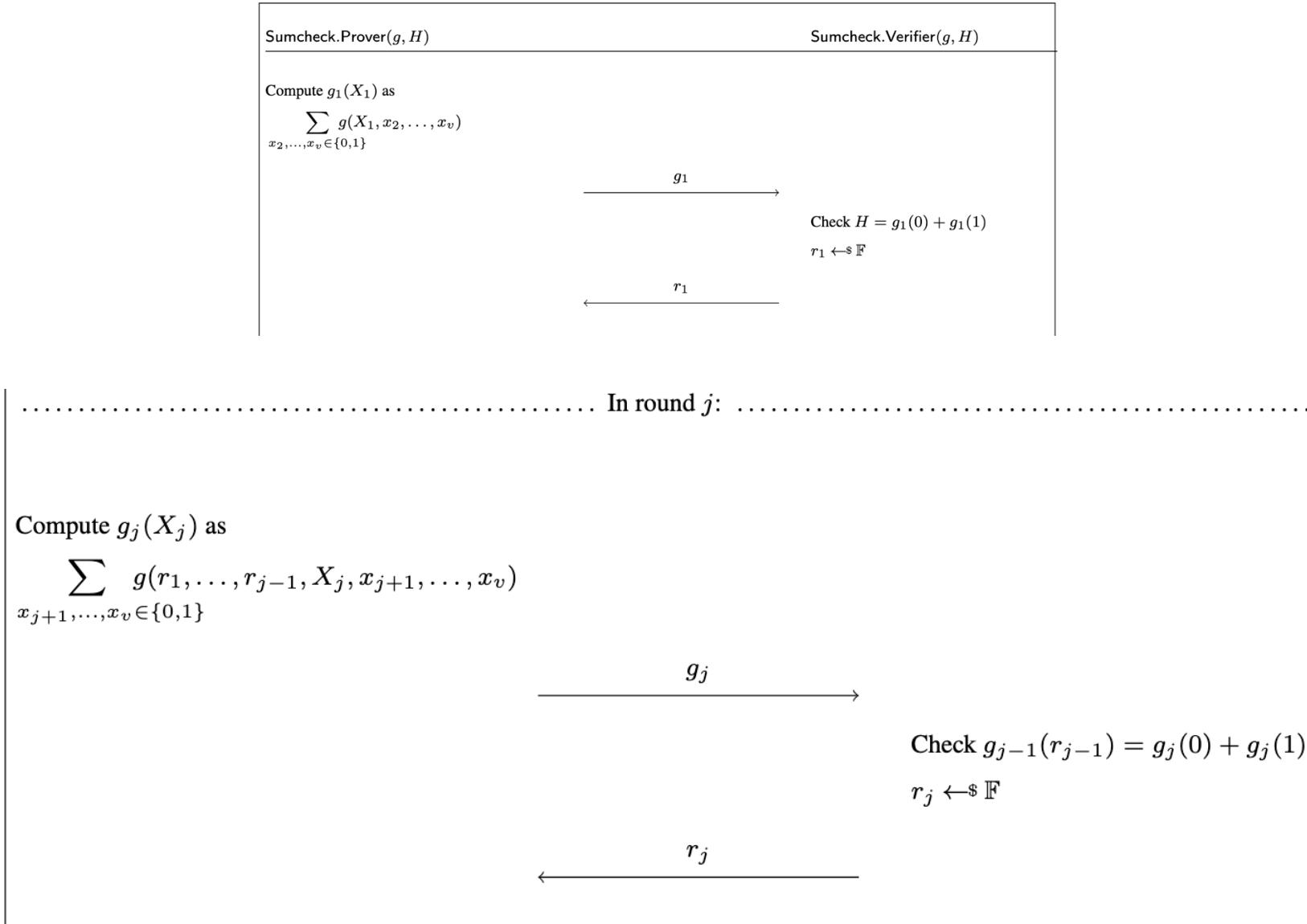
# The sumcheck protocol

Claim :  $\sum_{i \in \{0,1\}^l} g(i) = H$

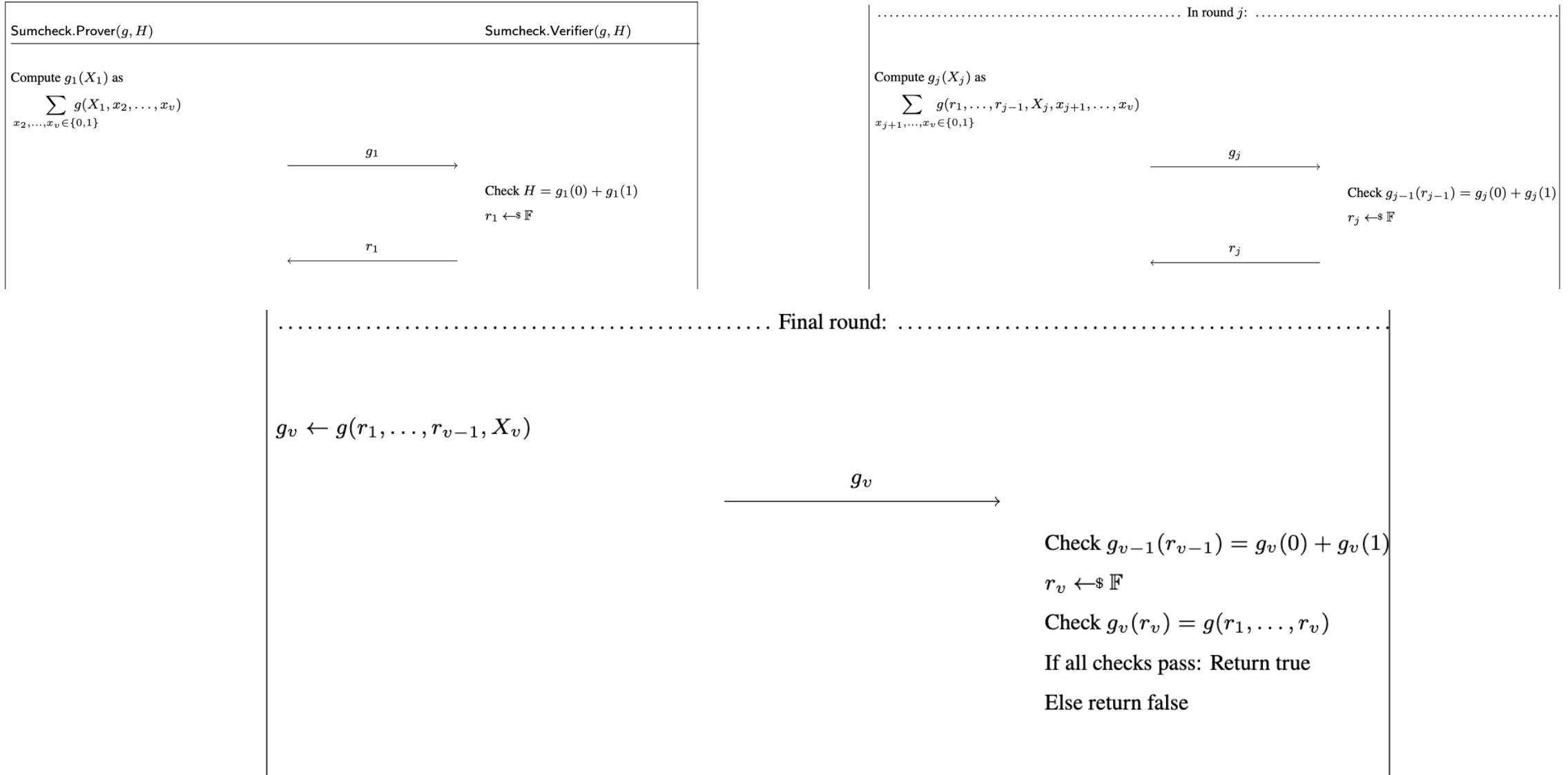
$g$  is  
 $l$ -variate  
over  $\mathbb{F}$   
 $d$  is  
maximal  
degree



# The sumcheck protocol



# The sumcheck protocol



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# Analyzing sumcheck

**Theorem 1.** *Let  $g$  be an  $\ell$ -variate polynomial of maximal degree  $d$  over the field  $\mathbb{F}$ . Then for any malicious prover  $\mathbb{P}^*$  and value  $H \neq \sum_{i \in \{0,1\}^\ell} g(i)$ ,*

$$\Pr[\text{out}_{\mathbb{V}} [\mathbb{P}^*(g, H) \leftrightarrow \mathbb{V}(g, H)] = 1] \leq \frac{\ell d}{|\mathbb{F}|}$$

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# **Applications of sumcheck**