Optimum Hierarchical Bezier Parameterisation of Arbitrary Curves and Surfaces

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Abstract

Accurate geometric representation of a general three dimensional surface by means of a few parameters is a challenging problem, that is relevent in the context of aerodynamic shape optimization. Methods like PARSEC, Hicks-Henne functions for airfoil type of surfaces and Bezier parameterization for arbitrary curves and surfaces are generally used for smooth representation in parametric form. Bezier curves are extremely sensitive to the choice of the control points. In shape optimization tasks, one generally starts with a given surface and then change the shape of surface by changing the control point locations, whose coordinates serve as optimization parameters. In this paper, the problem of finding a set of given number of control points for an arbitrary two dimensional curve is formulated as an optimization problem and solved using Genetic algorithm. The optimized locations of the control points can be used to initiate the optimal design process. As an example RAE 2822 airfoil is selected and the set of given number of control points are found for both upper and lower profiles. By the technique of degree elevation, the number of control points for the same profiles can be increased, if necessary. The method formulated in this paper can be easily extended to general three dimensional surfaces like wings, in a straight forward manner.

Keywords: Parametric Representation, Bezier curves and surfaces, MDO, Genetic Algorithm, Aerodynamic Shape Optimization.

1 Introduction

Multidisciplinary Design Optimization (MDO) is an emerging discipline in the context of engineering design in the present day modern world. With the types of simulation tools available in various areas, it is important to ensure optimal design of any system, by considering all the design aspects in the initial stages itself, before the *metal is cut* . In the context of shape optimization, a need arises to represent a general three dimensional surface by means of a minimum number of geometric parameters, which can serve as the optimization parameters to arrive at the optimum shape.

2 Methods of Representations

A general geometry can be represented using three basic approaches namely, partial differential equation approach, discrete points approach and polynomial approach. In *partial differential equation* approach, the surface is obtained as the solution of the boundary value problem governed by elliptic partial differential equations. Complex geometries like complete aircraft can be defined by

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a compact set of parameters. But this approach is time consuming and not suitable for multidisciplinary design optimization (MDO) applications. In *discrete points* approach, all the boundary points of the surface are treated as design variables. So the number of design variables becomes large. It is easy to implement but difficult to maintain geometrical smoothness. In the *polynomial approach* , the number of design parameters depends on the degree of the polynomial chosen. The sensitivity derivatives needed for the optimisation, are the coefficients of the polynomial themselves and hence they are fixed during optimization. This approach is highly suitable for multidisciplinary design optimization (MDO) applications.

Basically, a general three dimensional surface can be represented in three basic forms, namely,

$$z = f(x, y) \tag{1}$$

which is the *explicit form.* There is only one z value for each pair of x, y value; not vice versa. The surface can also be written as

$$f(x, y, z) = 0 \tag{2}$$

which is the *implicit form.* Here the equation may have more than one solution. The *parametric form* can be written as

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} f(u, v) \\ g(u, v) \\ h(u, v) \end{pmatrix} \tag{3}$$

where f,g,h are bivariate functions of u and v. For two dimensional curves, the above representation reduces to

$$x = f(t) \tag{4}$$

$$y = g(t) \tag{5}$$

where f and g are arbitrary functions of the parameter t. For airfoil type of geometries, the following popular parametric representation methods are generally used.

2.1 PARSEC

The concept of PARSEC is to select eleven parameters as the control variables in such a way that the shape of the airfoil can be computed from these control variables. The eleven control variables are, leading edge radius (R_{le}), upper crest point (Y_{up}), position of upper crest (X_{up}), upper crest curvature (YXX_{up}), lower crest point (Y_{lo}), position of lower crest (X_{lo}), lower crest curvature (YXX_{lo}), trailing edge direction angle (α_{TE}), trailing edge wedge angle (β_{TE}), trailing edge thickness (T_{TE}), trailing edge offset (T_{off}) as shown in Figure 1. The mathematical formulation for PARSEC is given by the polynomial,

$$y_u = \sum_{i=1}^{6} a_i x^{i-(1/2)} \tag{6}$$

for the upper surface and

$$y_l = \sum_{i=1}^{6} b_i x^{i-(1/2)} \tag{7}$$

for the lower surface. a_i and b_i are coefficients to be solved from the eleven control variables.

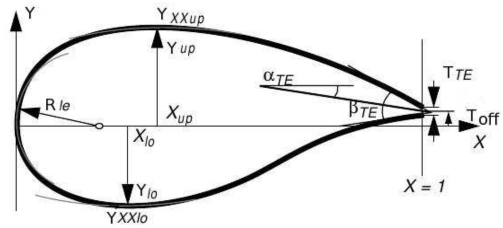

Figure 1: Design parameters for PARSEC

2.2 Hicks - Henne Bump Functions

In this method the shape of the curve is assumed to be the sum of a basis shape and sum of suitably defined and weighted sine functions. This is given by,

$$y = y_{basis} + \sum_{j=1}^{M} \alpha_j f_j(x) \tag{8}$$

$$f_j(x) = \left(sin \left(\pi x^{log 0.5 / log t_1} \right) \right)^{t_2}, 0 \leq x \leq 1 \tag{9}$$

where t_1 locates the maximum point of bump and t_2 controls the width of the bump. The design variables are the weights α_j multiplying each Hicks-Henne bump functions. This flexibility allows one to place the bump at strategic points where a redesign is preferred while leaving other parts of the airfoil intact. Figure 2 shows a set of typical Hicks-Henne Bump functions with parmeter $t_2 = 10$.

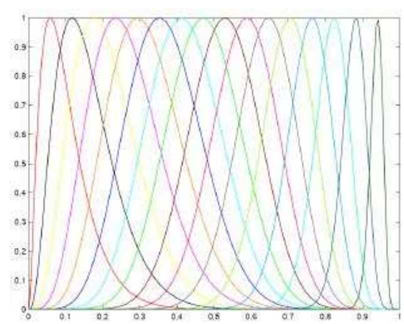

Figure 2: Hicks-Henne Bump functions

2.3 Bezier Curves and Surfaces

Bezier curves and surfaces are polynomial parametric curves and surfaces. These are defined by a set of control points. The degree of representation depends on the number of control points and in case of curves it is always one less than the number of control points. A simple illustration of bezier curve of degree 3 is given in Figure 3, where the four control points are $b(0)$ to $b(3)$. Here, Bezier curve is $b(t)$, Bezier control points are b_i, Bezier polygon is (b_0, b_1, b_2, b_3), Convex hull is $(b_0, b_1, b_3, b_2, b_0)$. The curve mimics the shape of the polygon as t varies in the range $[0, 1]$, between the end control points $b(0)$ and $b(1)$.

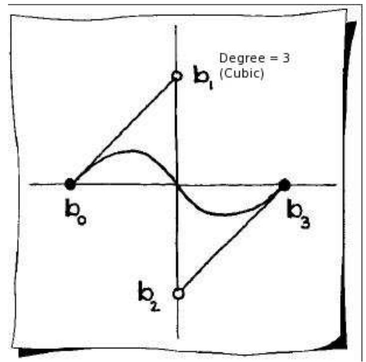

Figure 3: Concept of Bezier Curves

3 Method of Bezier Parameterization

Let $P = P_0, P_1, ..., P_n$ be a set of points $P_i \epsilon \Re^d$, $d = 2, 3$. Then the Bezier curve associated with the set P is defined by,

$$P(t) = \sum_{i=0}^{n} P_i B_i^n(t) \tag{10}$$

where $B_i^n(t)$ represents the Bernstein polynomials, which are given by,

$$B_i^n(t) = \binom{n}{i} (1-t)^{n-i} t^i \tag{11}$$

$i = 0, ..., n$. where n being the polynomial degree and,

$$\binom{n}{i} = n!/(i!(n-i)!) \tag{12}$$

The above parameterization form (eqn 10) can be extended to a general three dimensional surface in terms of two parameters u and v as

$$P(u,v) = \sum_{j=0}^{m} \left(\sum_{i=0}^{n} P_{i,j} B_{i,n}(u) \right) B_{j,m}(v) \tag{13}$$

The surface is defined as u and v vary from 0 to 1. It may be noted that if we define

$$Q_j(u) = \sum_{i=0}^{n} P_{i,j} B_{i,n}(u) \tag{14}$$

then we can write $P(u,v)$ as

$$P(u,v) = \sum_{j=0}^{m} Q_j(u) B_{j,m}(v) \tag{15}$$

It is seen that $Q_j(u)$ as given by equation 14 form the control points of another Bezier curve and together for all values of u and v, they define the surface.

3.1 Degree Elevation

In degree elevation the number of control points defining the control polygon is increased. By which the degree of the Bezier curve is increased. Given a Bezier curve of degree n, defined by the parameterization based on the $n+1$ control points $P_k = (x_k, y_k)(k = 0, 1, ..., n)$, the new set of $n+2$ control points $P_k^I = (x_k^I, y_k^I)(k = 0, 1, ..., n+1)$ given by $P_0^I = P_0$,$P_{n+1}^I = P_n$ and for $1 \leq k \leq n$,

$$P_k^I = ((k/n+1))\, P_{k-1} + (1 - (k/n+1))\, P_k \tag{16}$$

defines the same geometrical curve, now viewed as a Bezier curve of degree $n+1$. As n increases the control polygon tends to curve itself.

4 Proposed Method

In the present work, the best coordinates of given number of control points to represent an arbitrary two dimensional curve $y = f(x)$ is found using genetic algorithm approach. The curve can also be defined by a set of numerical pair of values of x and y in the form of table. The number of control points which defines the degree of the Bezier polygon is given as an input. The x and y coordinates of these control points are treated as the parameter for the optimization. A brief description on genetic algorithm is given in the next paragraph.

4.1 Genetic Algorithm

Genetic algorithm is based on the computer model of the natural evolution process. It starts with a population which consists of sets of parameters to be optimized. These are coded as binary strings called *chromosomes*. The evolution is performed through three principal operators selection, crossover, and mutation. Selection operator operates on the best individuals in every generation. A random number called *crossover probability* is selected. For each bit, in a bit stream, the probability test is performed. If passed, then it combines the good characteristics from two best individuals. For mutation, a random number called *mutation probability* is selected. Based on this, the bits in the bit stream are flipped. The two important qualities of genetic algorithm namely, *inheritance* (off springs must retain some good features that made their parents fitter than average) and *variability* (at any given time individuals with varying fitness must co-exist in the population) are maintained by various techniques such as niching and micro ga.

5 Illustration of the Proposed Method

The above method is demonstrated by finding the coordinates of the n Bezier control points, for the RAE 2822 airfoil. The non-dimensional quantity $'x/c'$ varies from 0 to 1. The numerical values of x and y coordinates in non-dimensional is available in the form of a table at 100 points. These are used as the target values for computing the objective function defined below. In order to reduce the number of parameters, the x- coordinates of the control points both for upper and lower surfaces are treated as fixed between 0 and 1. For these fixed values of $'x/c'$, the corresponding $'y/c'$ values are treated as variables.

5.1 Fitness Evaluation

In the present study, eight control points each for the upper and lower surface are used. Thus there are sixteen parameters in te present problem. Each generation produced by genetic algorithm has the best set of sixteen Bezier control points. The corresponding Bezier curves for both the upper and lower profiles are generated by evaluating equations $(10), (11), (12)$ in steps of 0.01 between 0 and 1. Now the generated Bezier curves are compared with the RAE 2822 coordinates over the range of the values from 0 to 1. The following objective function is defined as the basis for optimization.

$$Obj = -\left(\left(\sum_{i=1}^{n}((y/c)_{s_i} - (y/c)_{g_i})^2\right)_{up} + \left(\sum_{i=1}^{n}((y/c)_{s_i} - (y/c)_{g_i})^2\right)_{lo}\right) \tag{17}$$

Since genetic algorithm always maximises the objective function a negative sign has been added to minimize it. Where $(y/c)_{s_i}$ is the standard non-dimensional value and $(y/c)_{g_i}$ is the generated non-dimensional value from Bezier evaluation. If the error value is less it will have higher fitness value. Higher the fitness closer is the generated Bezier curve with the target curve. The genetic algorithm in the end will lead to the best set of control point coordinates that minimises the objective function within the range of the variables specified.

6 Analysis of Results

The initial Bezier control points has been given approximately by just looking at the standard coordinates of the curve. There is no need for specifying this accurately. The range for these variables is given in Table 1. The final Bezier control points obtained through this proposed method represents the orginal curve quite accurately, with overall error as defined above being less than 10^{-7}. These coordinates are given in Table 2. The comparison is shown in Figure 4. The control polygon for the upper and lower profiles of the airfoil is given in Figure 5. We can increase the number of control points as needed by the degree elevation procedure described in section 3.1. The result will define the same geometrical curve with increase in degree of the Bezier curve.

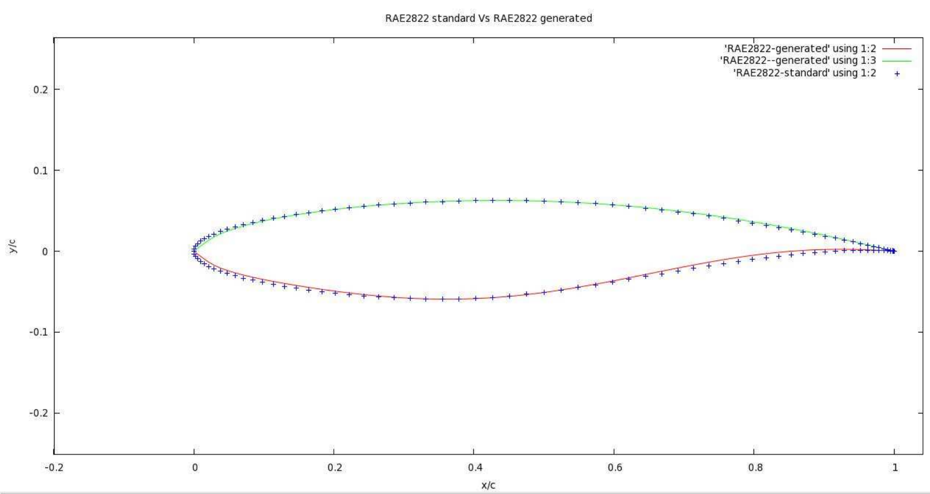

Figure 4: Comparison of RAE 2822 airfoil

Table 1: Range of y-coordinate of Bezier control points

Control point	Lower bound value	Upper bound value
$y(1)_{up} - y(8)_{up}$	-0.25	0.25
$y(1)_{low} - y(6)_{low}$	-0.25	0.10
$y(7)_{low} - y(8)_{low}$	-0.25	0.15

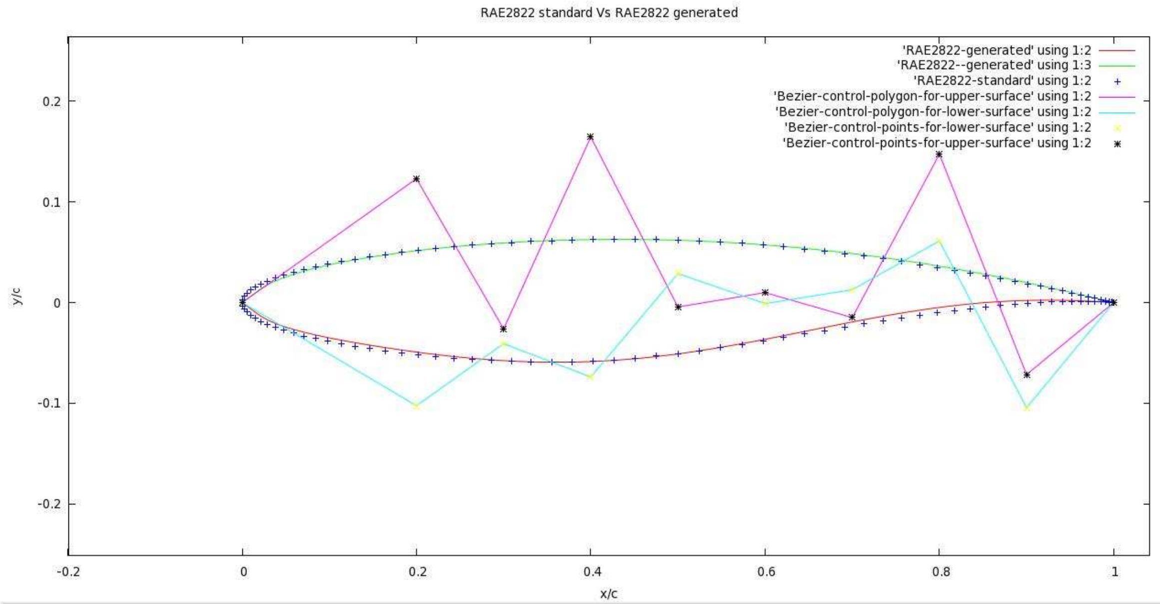

Figure 5: Control polygon of RAE 2822 airfoil

Table 2: Final Bezier control points

Control point	Value
$y(1)_{up}$	0.123022
$y(2)_{up}$	-0.026143
$y(3)_{up}$	0.164412
$y(4)_{up}$	-0.004353
$y(5)_{up}$	0.009922
$y(6)_{up}$	-0.014889
$y(7)_{up}$	0.147391
$y(8)_{up}$	-0.071439
$y(1)_{low}$	-0.102347
$y(2)_{low}$	-0.040796
$y(3)_{low}$	-0.073870
$y(4)_{low}$	0.028830
$y(5)_{low}$	-0.000997
$y(6)_{low}$	0.012819
$y(7)_{low}$	0.061180
$y(8)_{low}$	-0.104374

7 Conclusions

A new method has been proposed for finding a set of $'n'$ control points for Bezier parameterization of arbitrary curves and surfaces. The proposed method is demonstrated, taking the upper and the lower profiles of RAE 2822 airfoil and finding a set of eight control points for each of these profiles. Results obtained for this case, show that the overall error, defined as the difference in the coordinates between the Bezier generated curve and the target curve, taken in the least square sense, at all the points on the curve, is less than 10^{-7}. The coordinates of these control points can be taken as the initial values of the parameters for the shape optimization problem of maximising the aerodynamic characteristics, such as lift. The method can in principle, be extended in a straight-forward manner, to the parameterization of general three dimensional surfaces, such as wings.

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