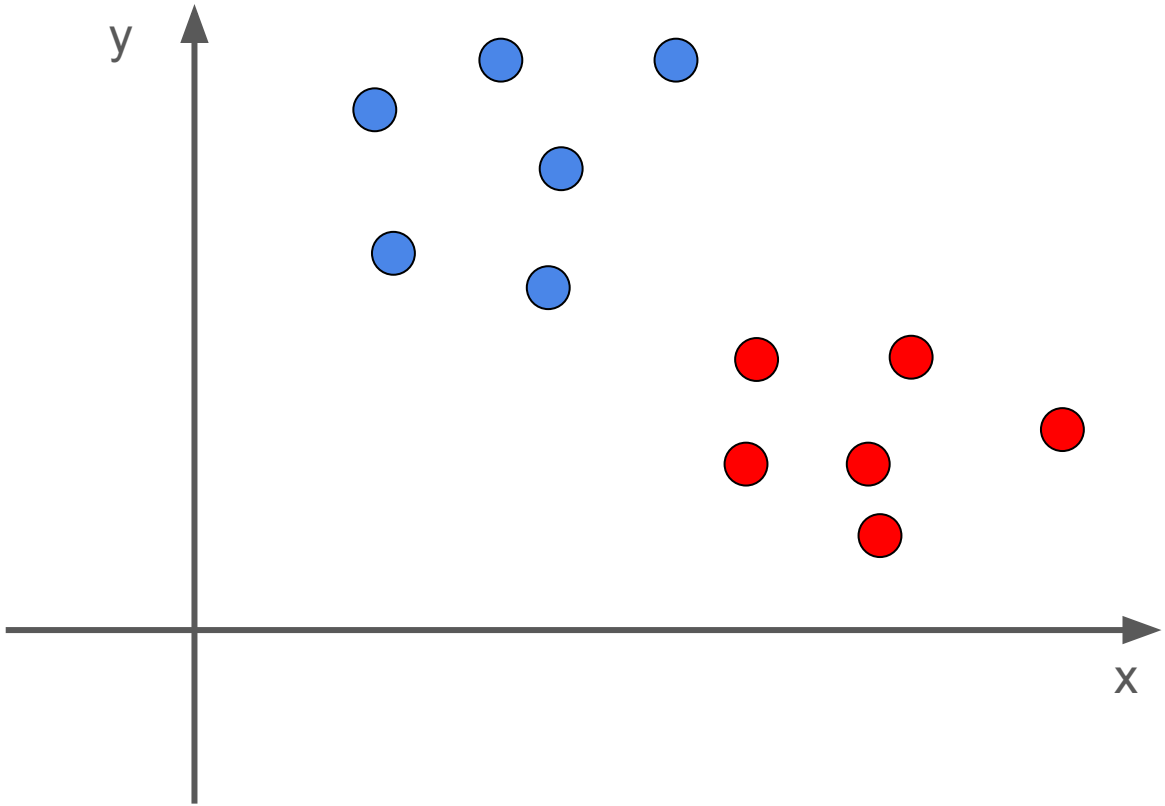
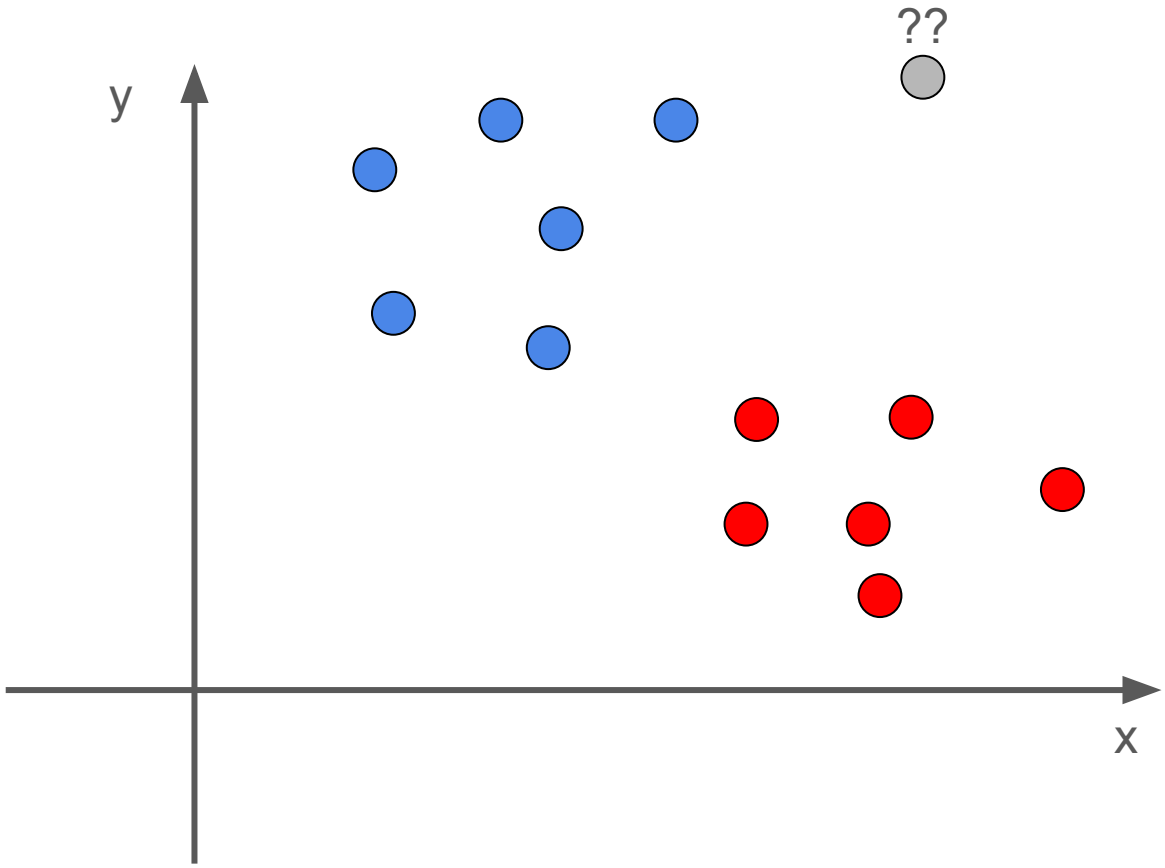
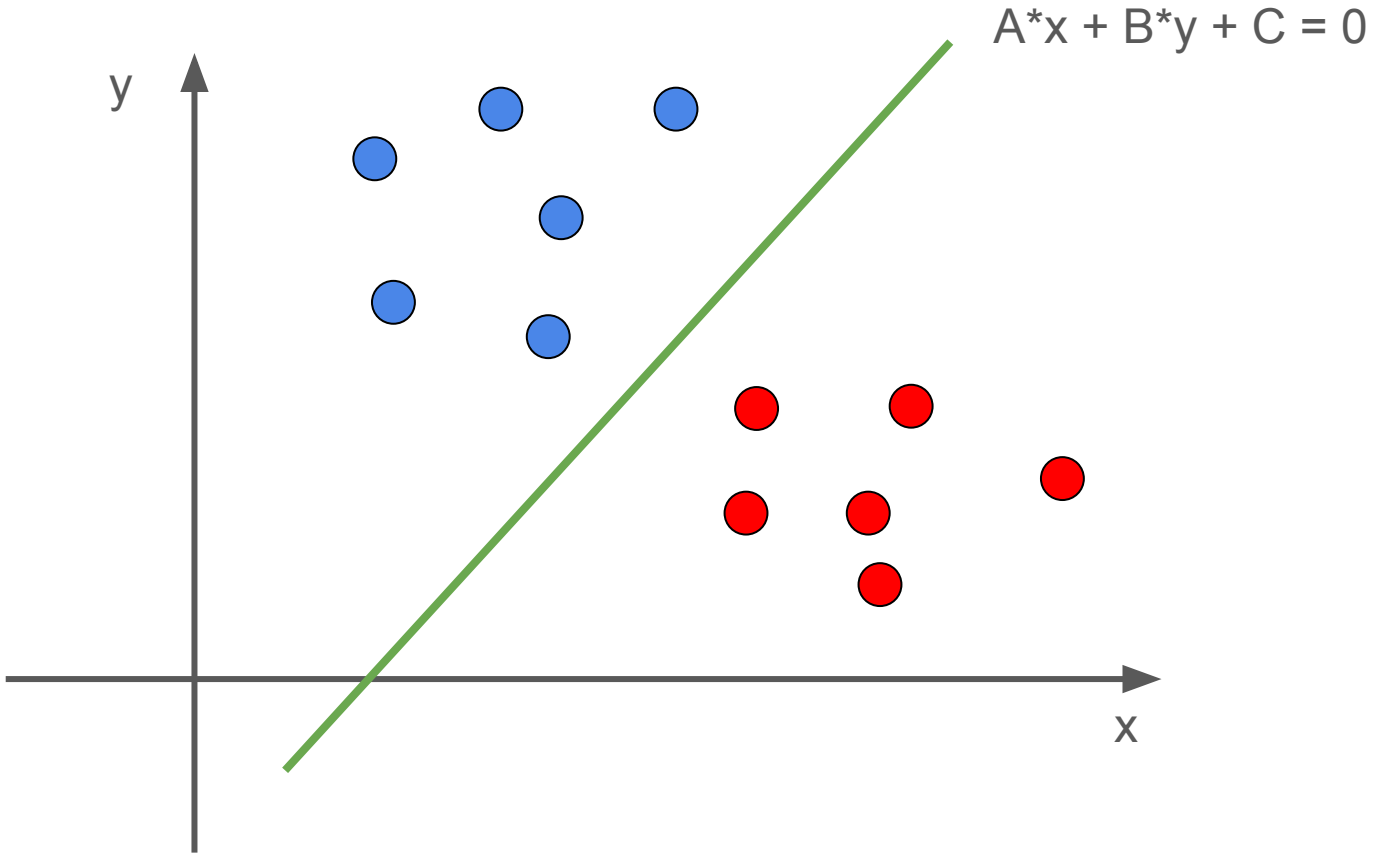
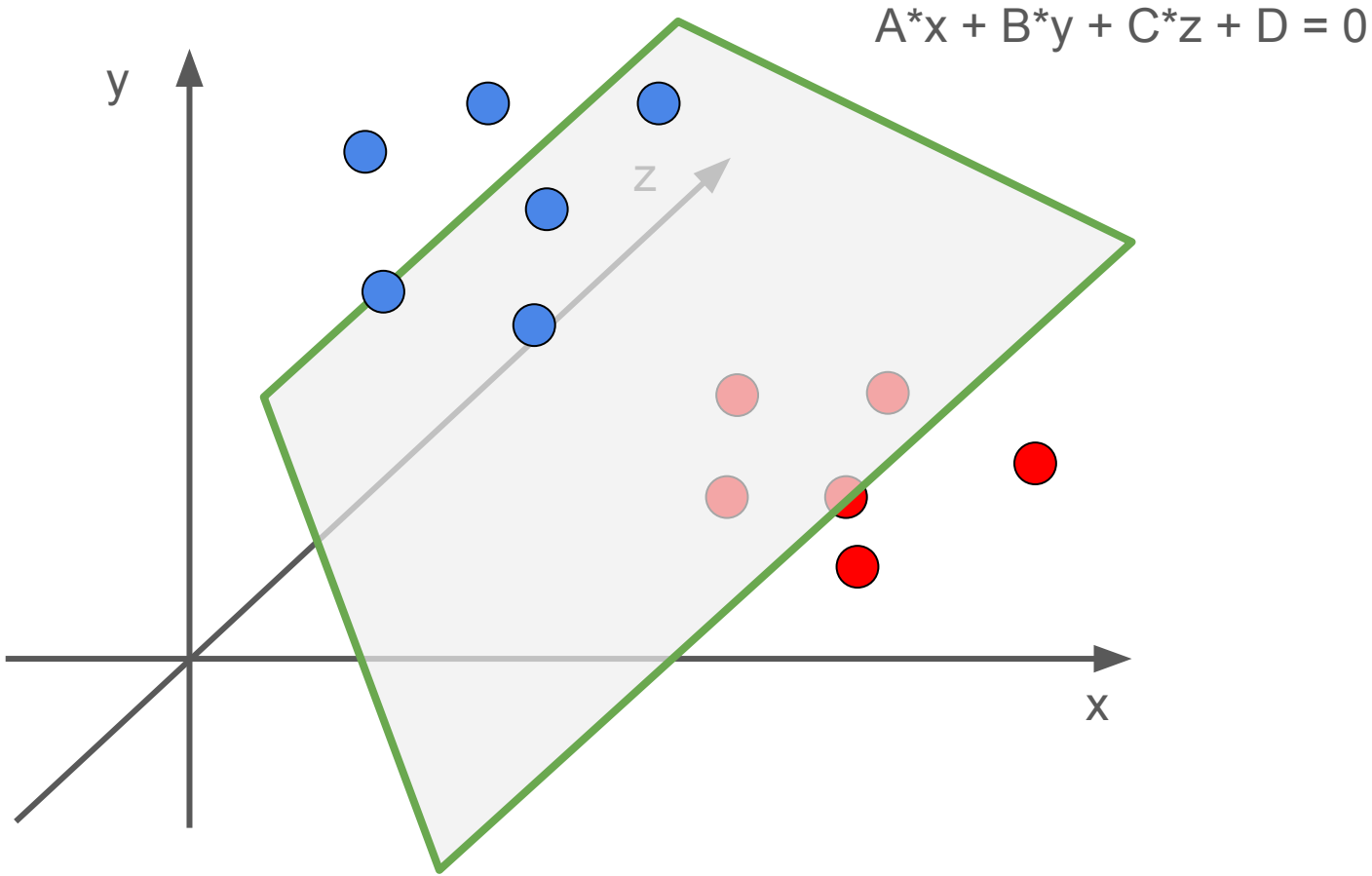
SVM
(Support Vector Machine)


y
x


??
y
x


A*x + B*y + C = 0
y
x

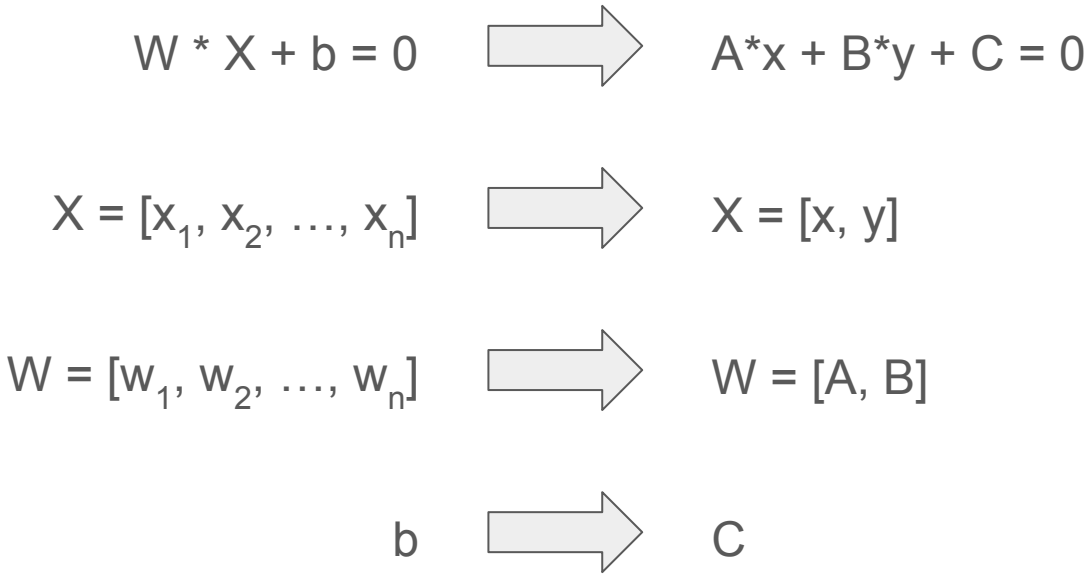

A*x + B*y + C*z + D = 0
y
z
x

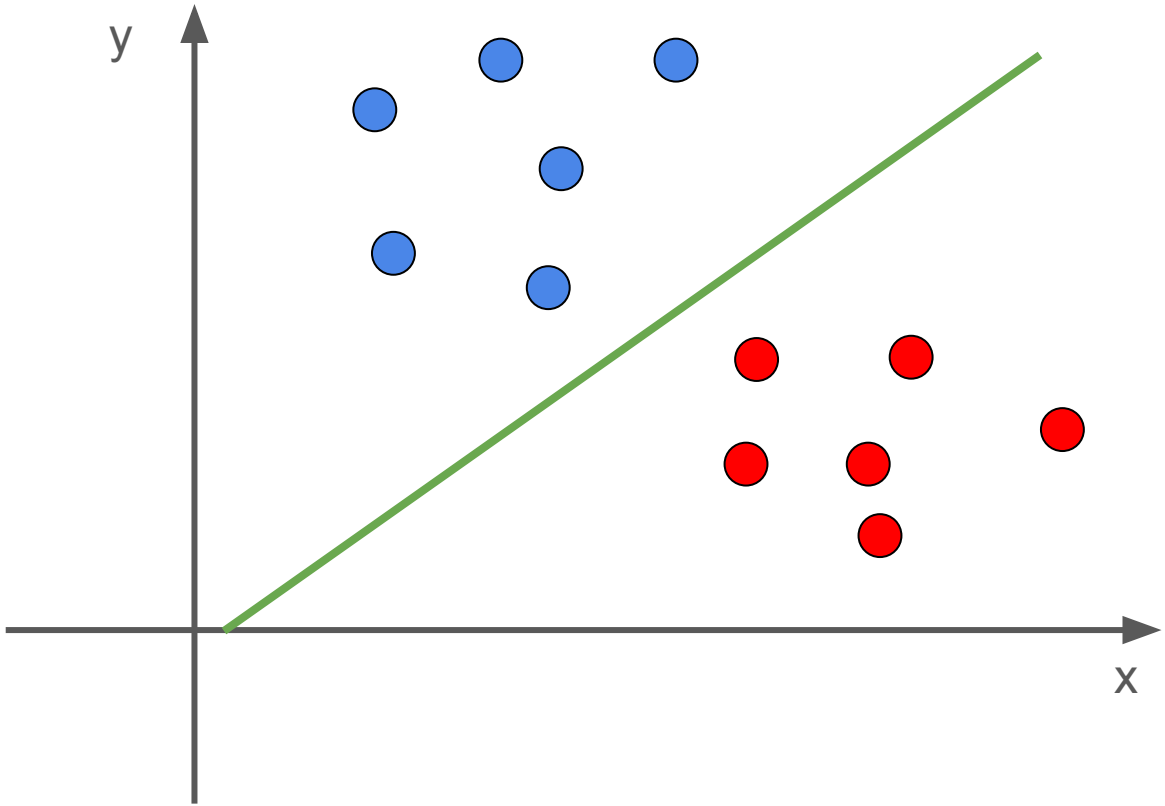
$$W * X + b = 0$$

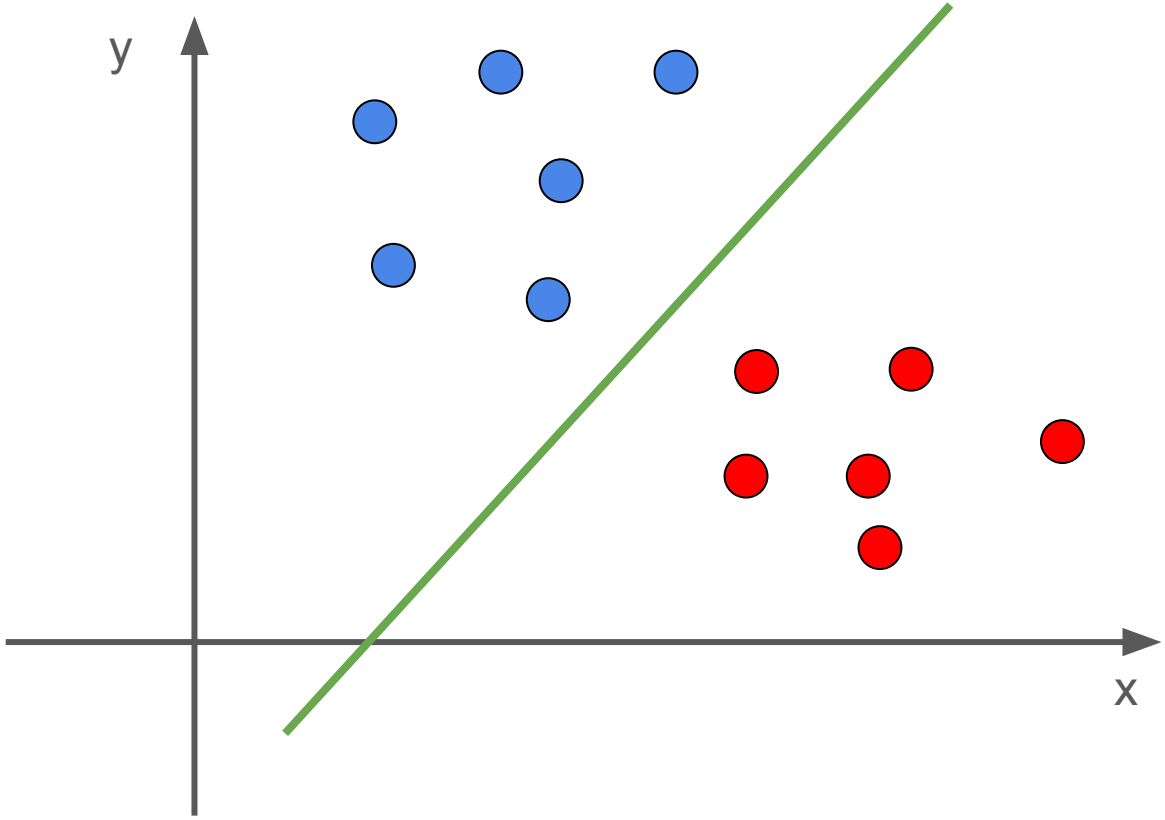
weights: $W = [w_1, w_2, \ldots, w_n]$

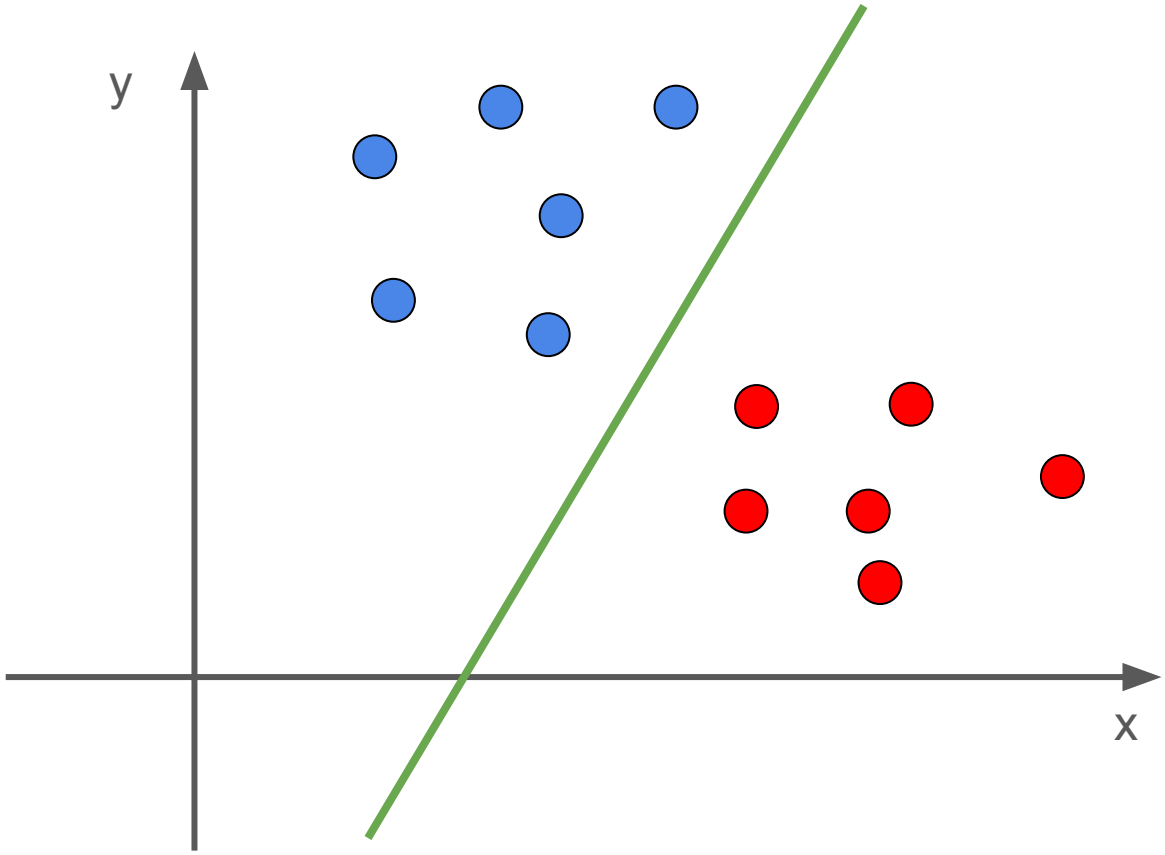
points: $X = [x_1, x_2, \ldots, x_n]$

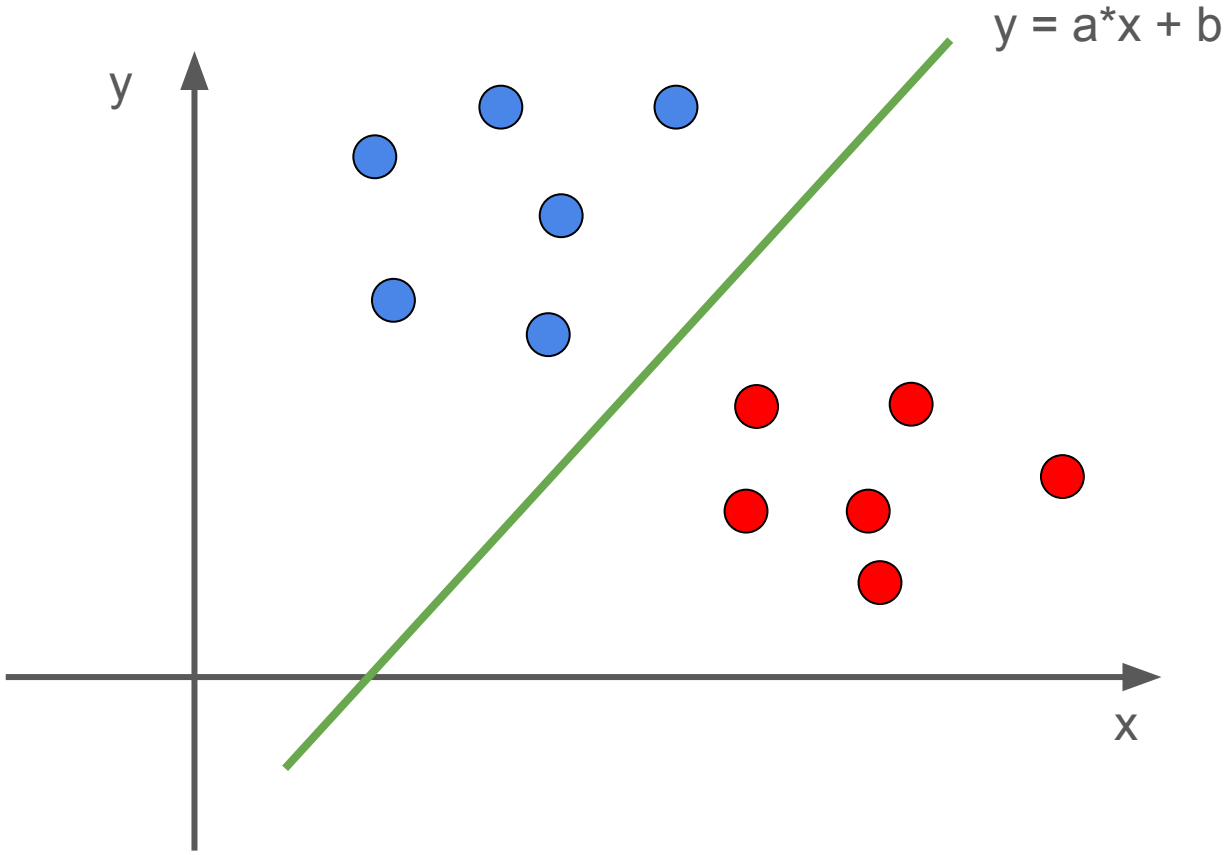
bias: b


W * X + b = 0
A*x + B*y + C = 0
X = [x1, x2, …, xn]
X = [x, y]
W = [w1, w2, …, wn]
W = [A, B]
b
C


y
x


y
x


y
x

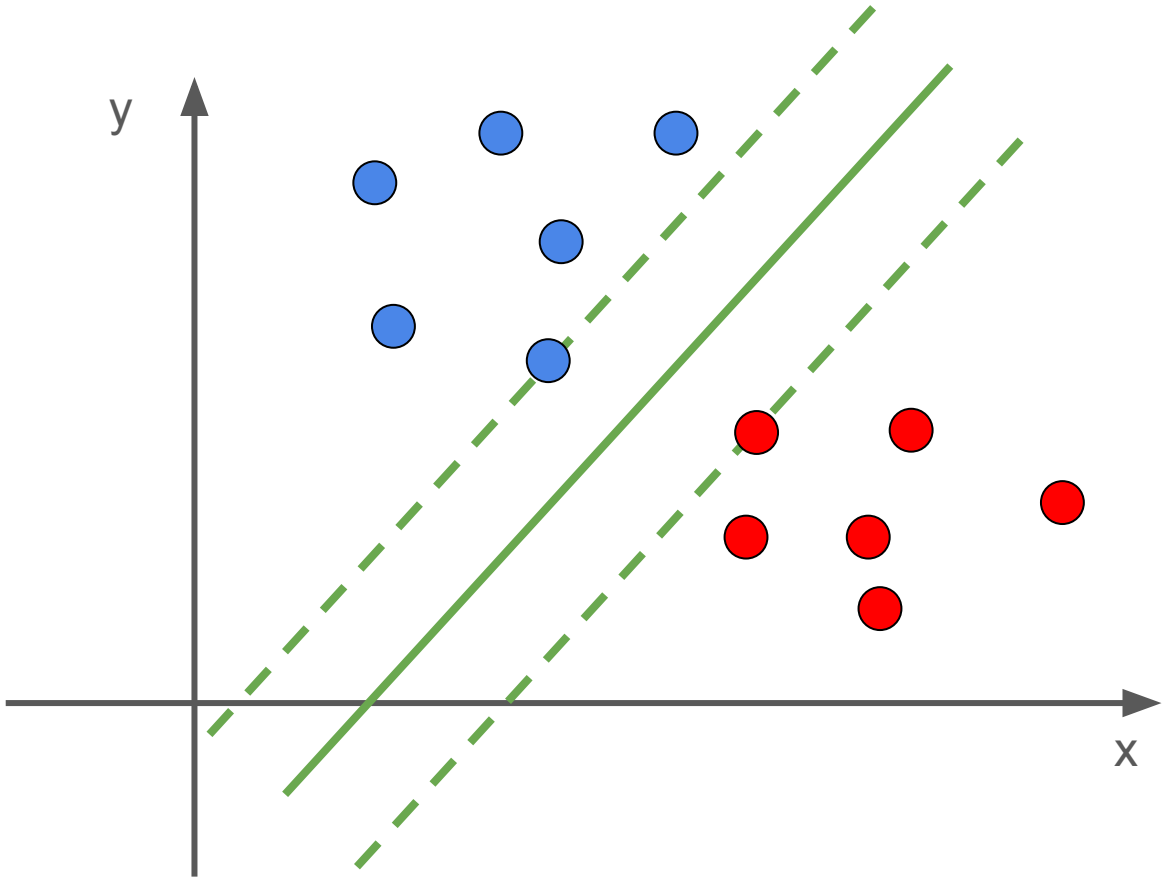

y = a*x + b
y
x

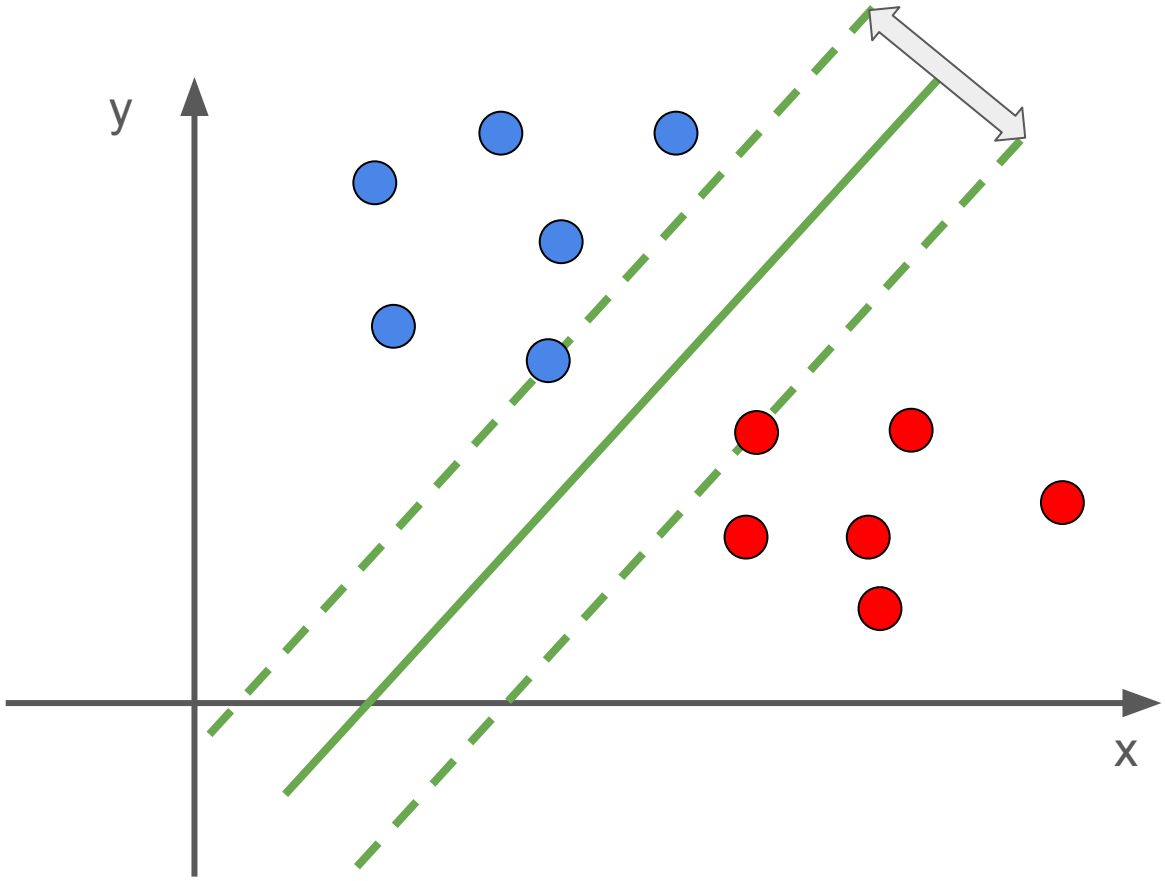
$$y = a*x + b$$

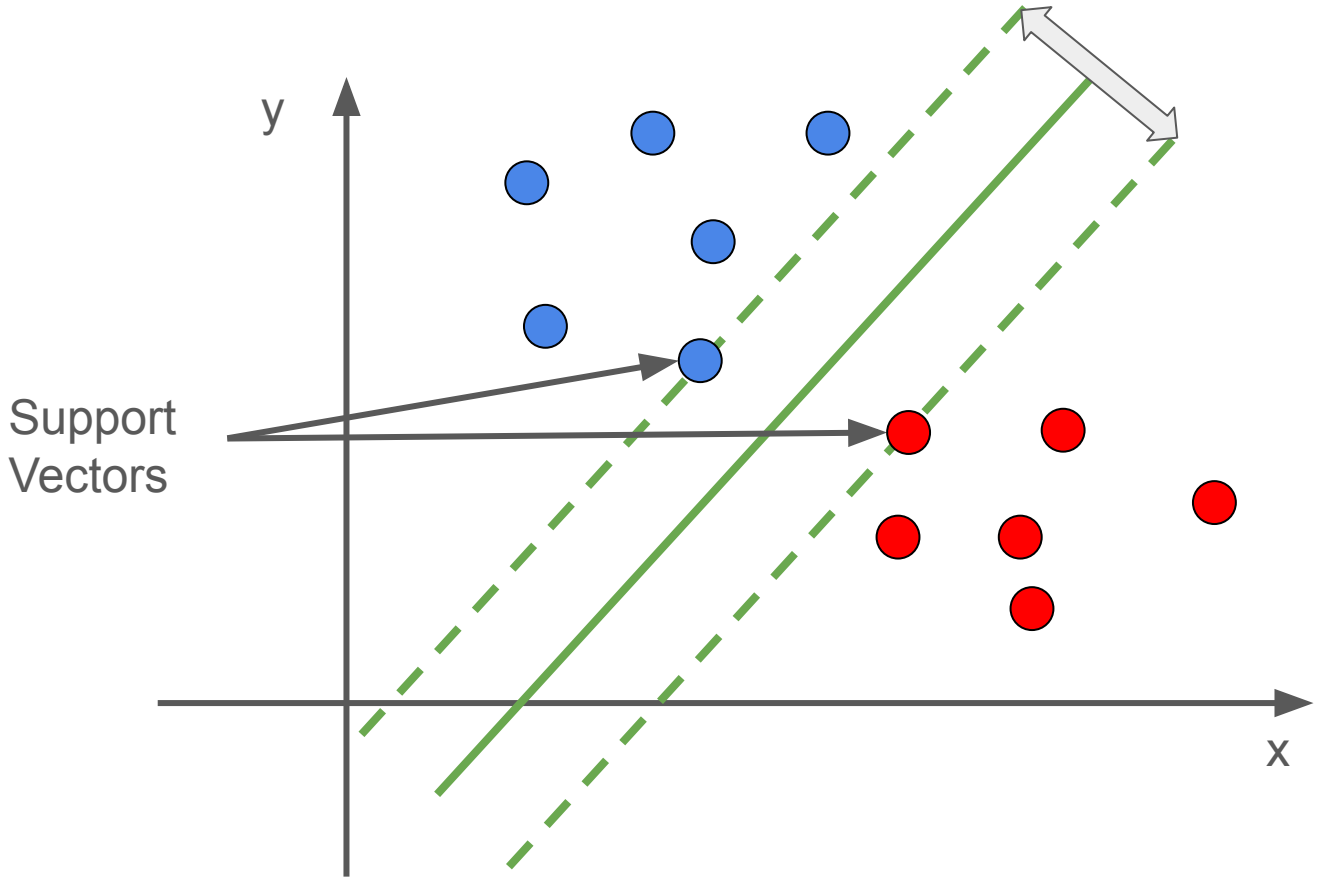
$$A*x+ B*y + C = 0$$

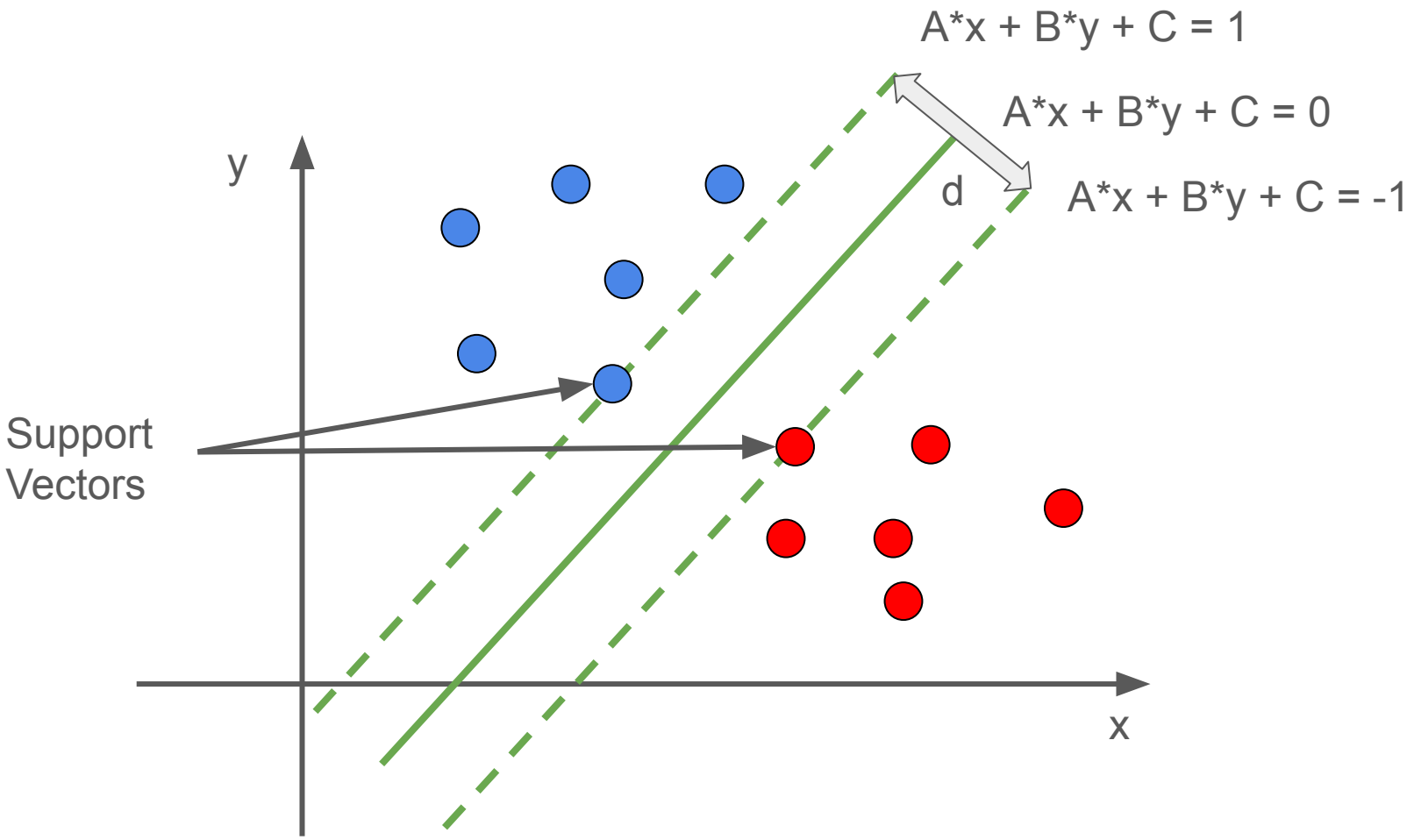
$$y = A/B * x + C/B$$

$$a = A/B$$
$$b = C/B$$


y
x


y
x


y
Support
Vectors
x


A*x + B*y + C = 1
A*x + B*y + C = 0
A*x + B*y + C = -1
d
y
x
Support
Vectors

Objective function

$$L(w,\ b)\ =\ \frac{1}{2}\lambda||w||^2\ +\ \sum_{i=1}^{n} max(0,\ 1\ -\ y_i(w^T x_i\ -\ b))$$

Line1: $A*x + B*y + C_1 = 0$

Line2: $A*x + B*y + C_2 = 0$

$$d = \frac{\left|C_2 - C_1\right|}{\sqrt{A^2 + B^2}}$$

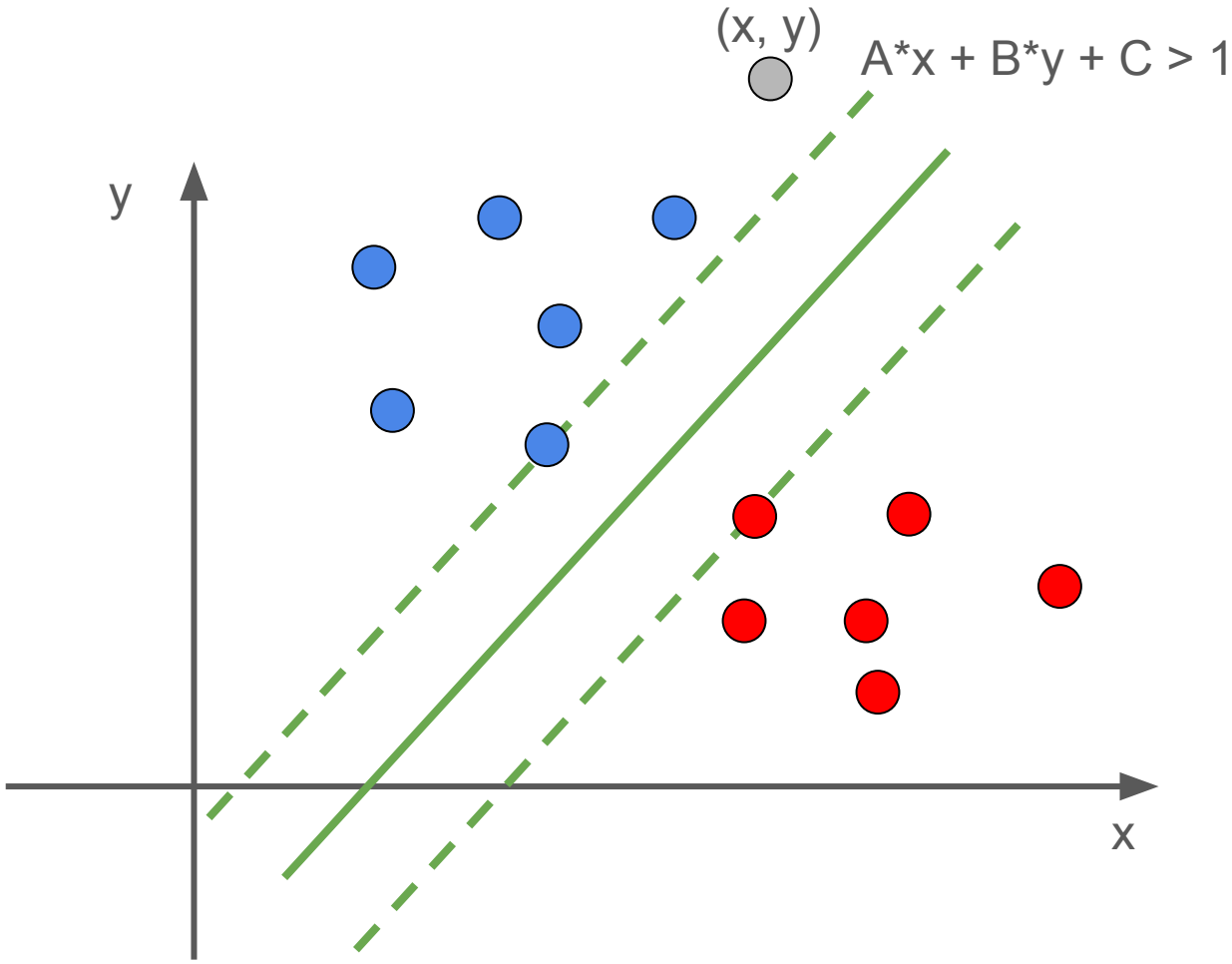
Line1: A*x + B*y + C = -1

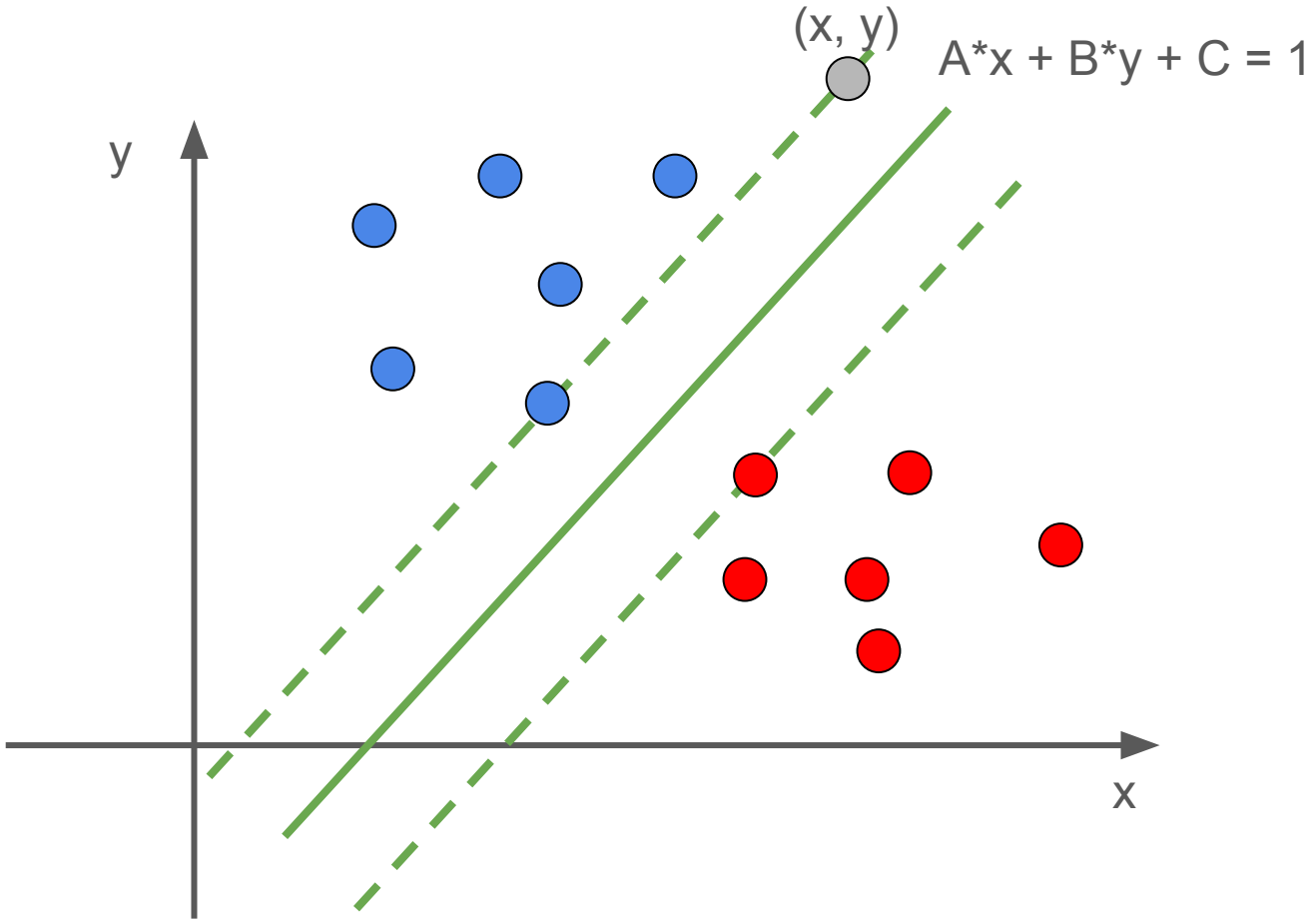
Line2: A*x + B*y + C = 1

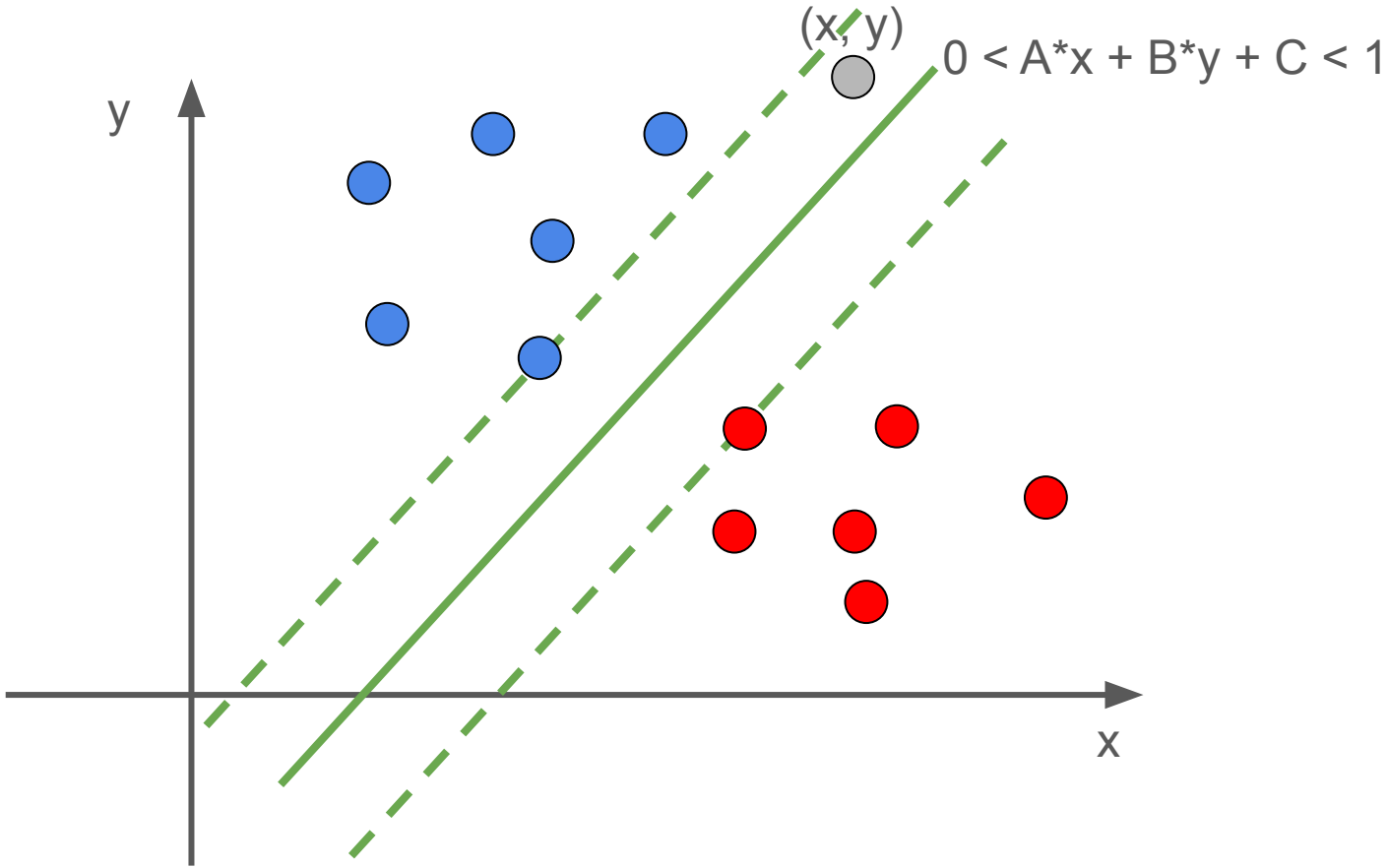
$$d = \frac{\left|C - 1 - (C + 1)\right|}{\sqrt{A^2 + B^2}} = \frac{2}{\sqrt{A^2 + B^2}} = \frac{2}{||W||}$$

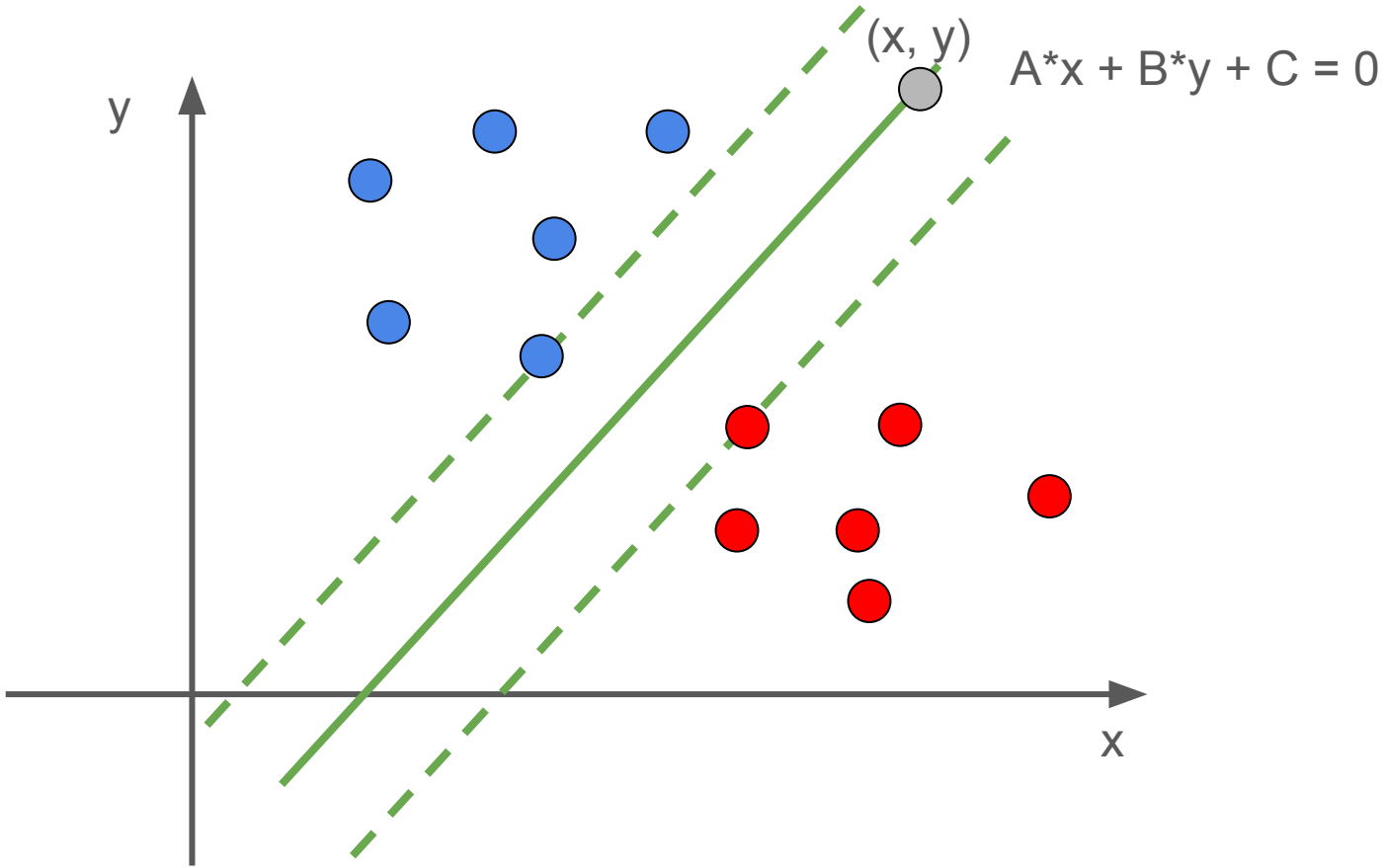
Hinge loss

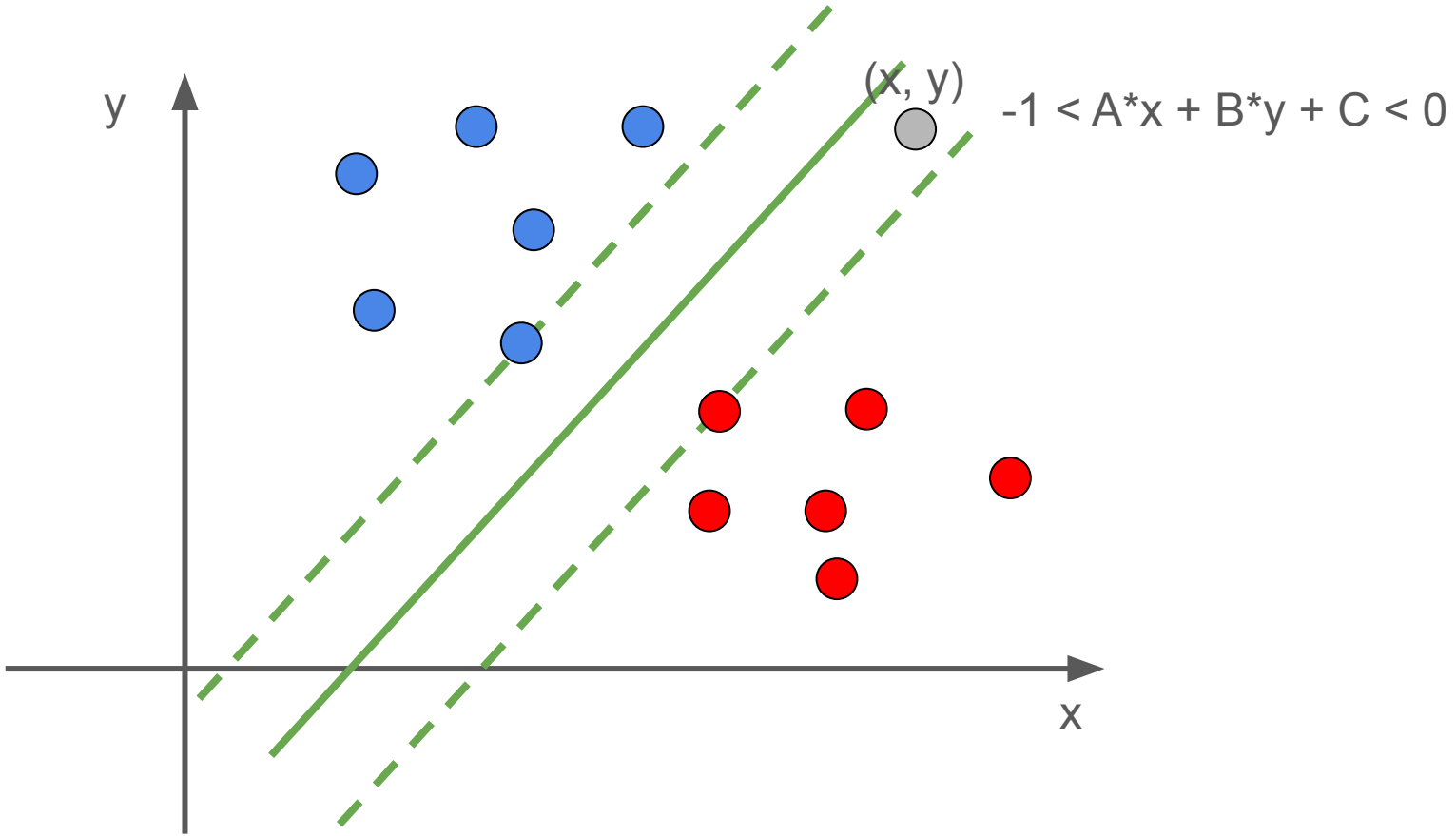
$$HingeLoss_i = max(0, 1 - y_i(w^T x_i - b))$$

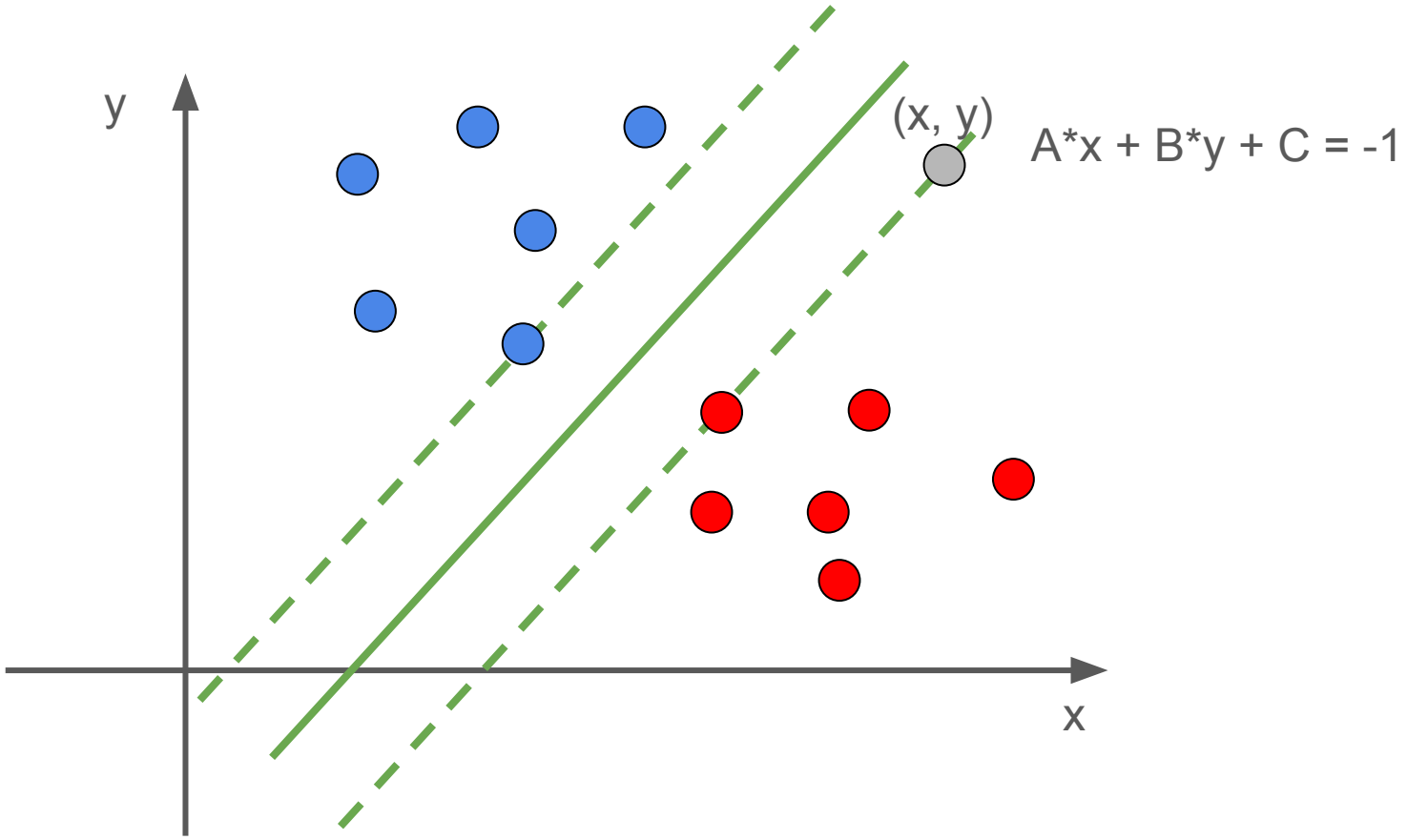

(x, y)
A*x + B*y + C > 1
y
x

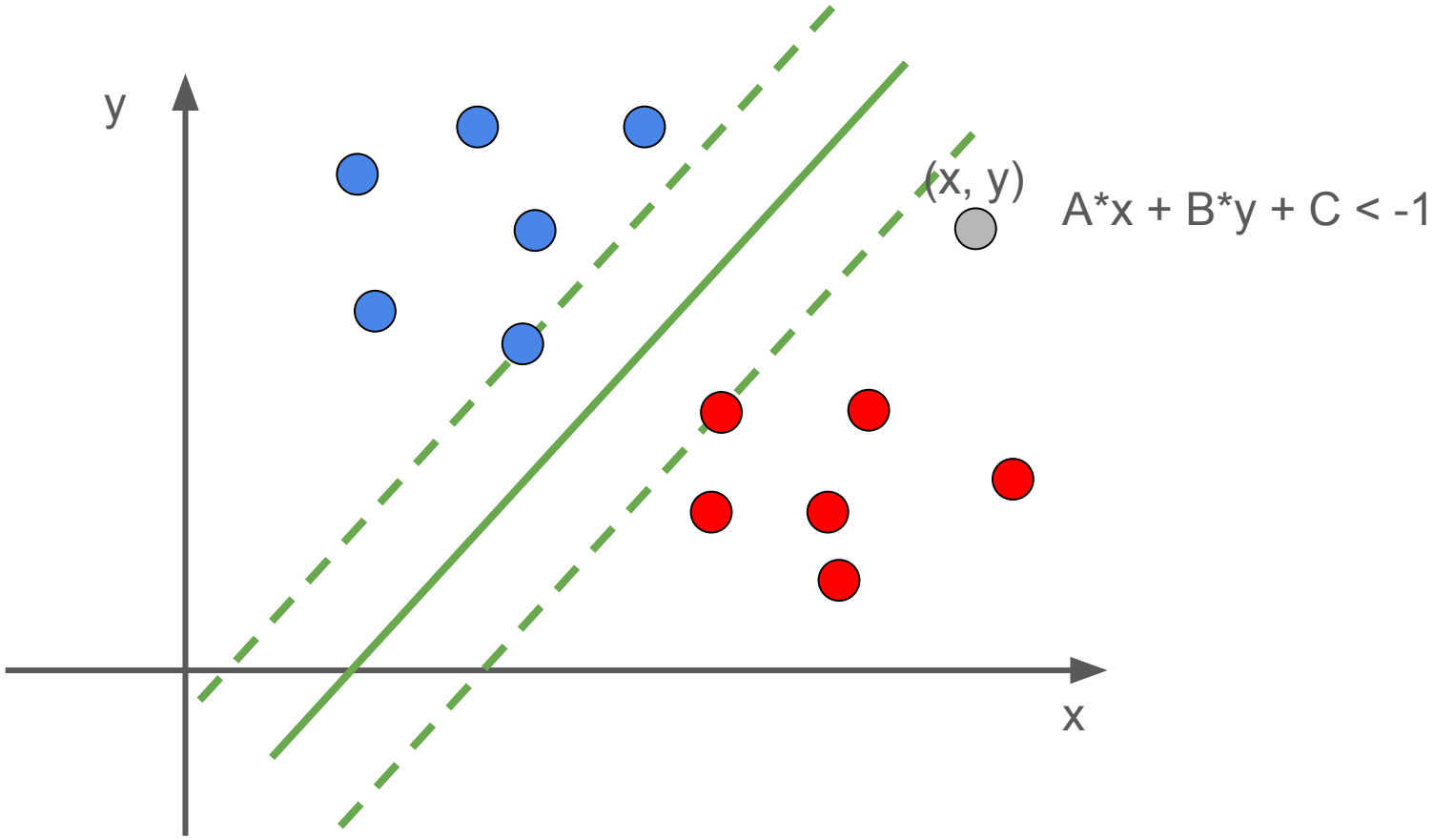

(x, y)
A*x + B*y + C = 1
y
x


(x, y)
0 < A*x + B*y + C < 1
y
x


(x, y)
A*x + B*y + C = 0
y
x


y
x
(x, y)
-1 < A*x + B*y + C < 0


y
x
(x, y)
A*x + B*y + C = -1


y
x
(x, y)
A*x + B*y + C < -1

$$y_i(w^T x_i - b) \geq 1$$

$$min_{w,\,b}\left(\frac{1}{2}\lambda||w||^2 + \sum_{i=1}^{n} max(0,\ 1 - y_i(w^T x_i - b))\right)$$

$$w = w - \eta \cdot \frac{\partial L(w,b)}{\partial w}$$

$$b = b - \eta \cdot \frac{\partial L(w,b)}{\partial b}$$

IF $y_i(w^T x_i - b) \geq 1$:

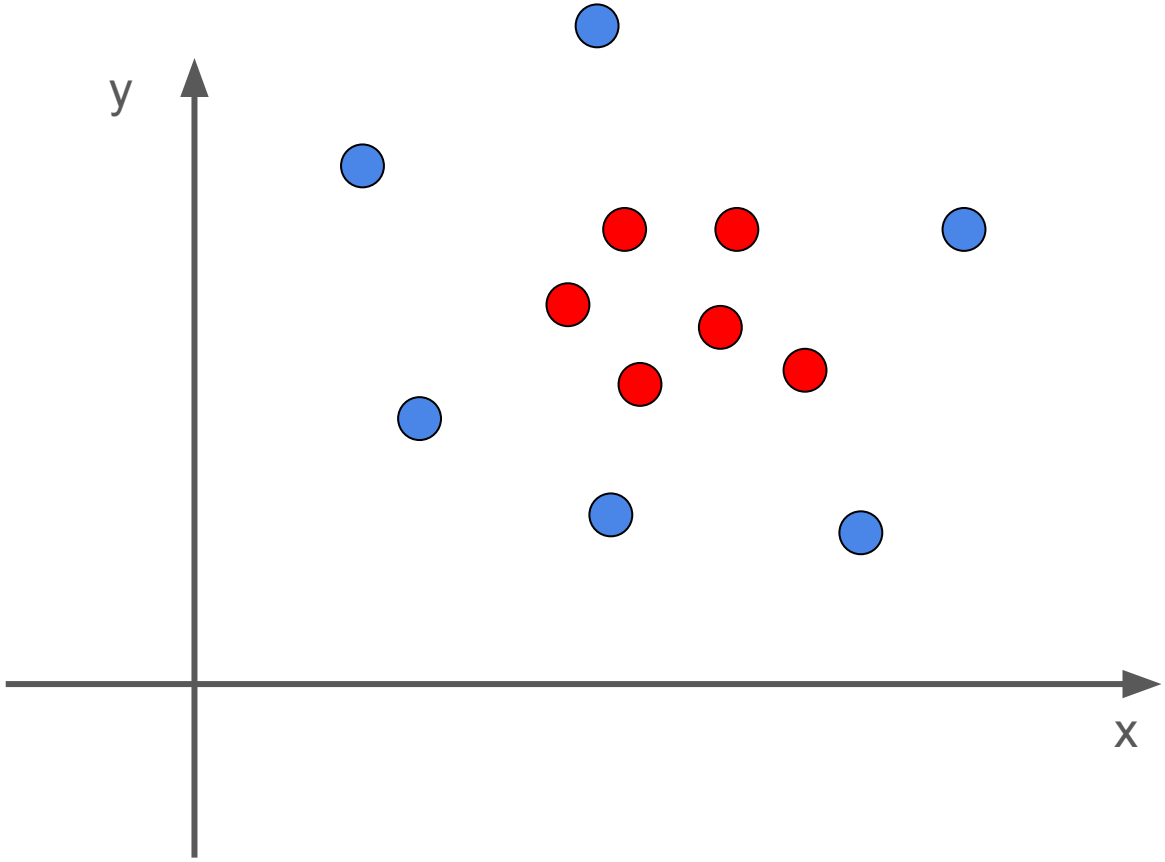
$$\frac{\partial L(w,b)}{\partial w} = 2\lambda w$$

$$\frac{\partial L(w,b)}{\partial b} = 0$$

ELSE $y_i(w^T x_i - b) < 1$:

$$\frac{\partial L(w,b)}{\partial w} = 2\lambda w - y_i x_i$$

$$\frac{\partial L(w,b)}{\partial b} = y_i$$


y
x