Automatic Controls exam aids

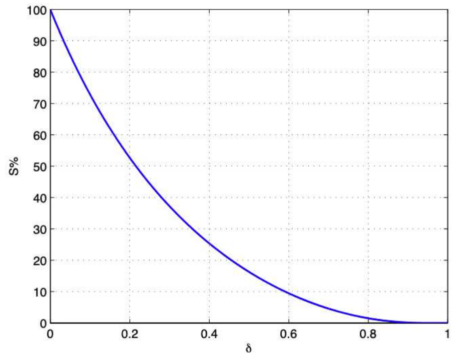
- **Second order systems**
  - Step response

$$y(t) \quad = 1 - \tfrac{e^{-\delta \omega_n t}}{\sqrt{1-\delta^2}} \; \sin(\omega\, t + \varphi)$$

$$\begin{aligned} &\omega := \omega_n \sqrt{1-\delta^2} \\ &\sigma := \delta \omega_n \\ &\varphi := \arccos \delta = \arctan \tfrac{\sqrt{1-\delta^2}}{\delta} \end{aligned}$$

  - Overshoot

$$S\% = 100\,(y_{\max} - 1) = 100\; e^{\frac{-\pi\delta}{\sqrt{1-\delta^2}}}$$



  - 5% settling time

$$T_a = \frac{3}{\delta \omega_n}$$

- **Mason's formula**. Given a block scheme, with input X and output Y, the *transfer function* $G = \dfrac{Y}{X}$ is

$$G = \frac{1}{\varDelta} \sum_{i \in \mathcal{P}} P_i\, \varDelta_i$$

where the determinant is calculated as

$$\varDelta := 1 - \sum_{i \in \mathcal{J}_1} A_i + \sum_{(i,j) \in \mathcal{J}_2} A_i\, A_j - \sum_{(i,j,k) \in \mathcal{J}_3} A_i\, A_j\, A_k + \dots ,$$

- **Root locus.**
  - Asymptotes. Intersection with the real axis at

$$\sigma_a = \frac{1}{n-m} \left(\sum_{i=1}^{m} z_i - \sum_{i=1}^{n} p_i \right)$$

Angles with the real axis:

$$\vartheta_{a,\nu} = \begin{cases} \dfrac{(2\,\nu + 1)\,\pi}{n-m} & K_1 > 0, \\[2ex] \dfrac{2\,\nu\,\pi}{n-m} & K_1 > 0. \end{cases} \qquad \nu = 0,\, 1,\, \dots\, ,\, n{-}m{-}1$$

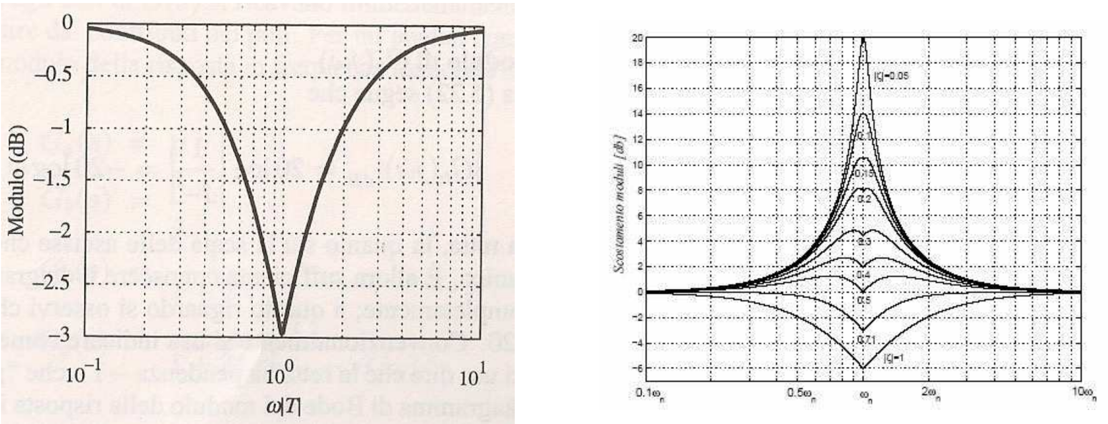
  - Points of the locus. A point s belongs to the locus if

$$(2\,\nu + 1)\,\pi + \sum_{j=1}^{m} \arg(s - z_j) - \sum_{j=i}^{n} \arg(s - p_j) \; ,$$

for some integer ν. This point corresponds to a value of the gain K_1 given by $|K_1| = \dfrac{\prod_{i=1}^{n} \eta_i}{\prod_{i=1}^{m} \lambda_i}$,

Bode diagrams.

- Phases asymptotic Bode diagrams.
  - First order terms: $\pm 90^o$ phase shift. Slope change between $\omega_a = \frac{1}{4.81\tau}$, $\omega_b = \frac{4.81}{\tau}$.
  - Second order terms: $\pm 180^o$ phase shift. Slope change between $\omega_a = \frac{\omega_n}{4.81^\delta}$, $\omega_b = 4.81^\delta \omega_n$.
- Deviations of the exact modulus Bode diagrams from the asymptotic ones.



(a) First order terms.

(b) Second order terms.

Figure 1: Error between exact and asymptotic module Bode diagrams.