Exam of Automatic Controls. July 13$^{\text{th}}$, 2020
Duration: 135 mins

Solve the following problems. Laude is granted if more than 34 points are gained, including the max 3 points from the (optional, upon student's request) oral exam.

1) Provide an asymptotic stability criterion for a discrete time system described by the transfer function
$$G(z) = \frac{b_0\, z^m + b_1\, z^{m-1} + \cdots + b_m}{z^n + a_1\, z^{n-1} + \cdots + a_n}$$
Pts: 2

2) State the Nyquist criterion for OL unstable systems. **Pts: 2**

3) Show that high loop gain decreases the effect of parametric uncertainties on the output response. Show what happens when the parametric uncertainty is in the sensor TF. **Pts: 2**

4) Calculate the Laplace transform of
$$x(t) = t^2 + e^{-2t} \cos 5(t-1)$$
Pts: 2

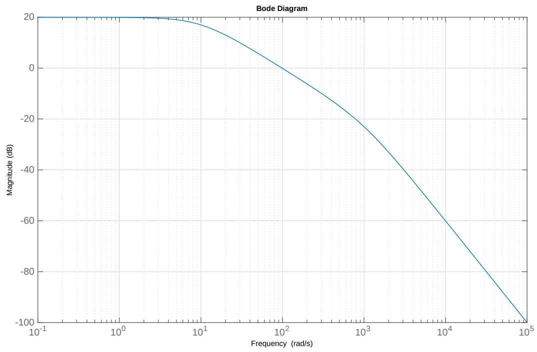
5) Calculate the response to the unitary step of the system
$$G(s) = \frac{s-1}{s+2}.$$
Check the correctness of your result with the initial and final value theorem. **Pts: 2**

6) Write the transfer function of a 2$^{\text{nd}}$ order system whose output response $y(t)$ to a unitary step has
   - $\lim_{t\to\infty} y(t) = 2$,
   - has a max overshoot $S = 30\%$,
   - has pseudo-oscillations with period $T = 0.1s$

Pts: 2

7) Plot the asymptotic Bode diagrams of the transfer function
$$G(s) = \frac{s(s+50)}{(s+1)(s^2+4s+100)}.$$
Pts: 2

8) Consider the following Bode diagrams.



Find a 1^{st} order approximation. **Pts: 2**

9) Draw the Nyquist plot of the system

$$G(s) = \frac{s+1}{(s+10)(s^2+4s+100)},$$

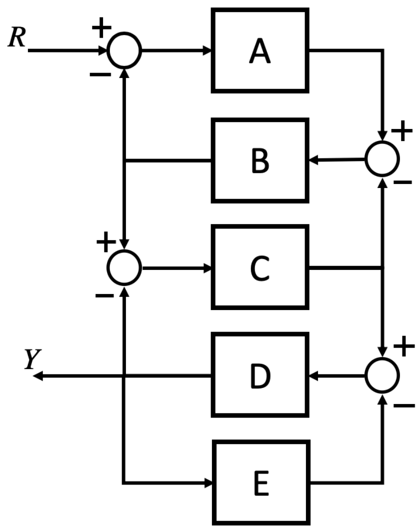
and mention the plotting rules you used. **Pts: 2**

10) Consider the system

$$G(s) = -\frac{s+1}{(s+10)(s^2+4s+100)}.$$

Use the Routh criterion to find the max value of a proportional controller such that the closed-loop system is asymptotically stable. **Pts: 2**

11) Reduce the following block diagram and find the transfer function $G = \frac{Y}{R}$.



Check the correctness of the result with the Mason's formula. **Pts: 3**

12) Consider a mass-spring-damper system. Calculate the speed $v(0^+)$ when the a constant force F is applied and the initial position and speed are $p(0)$ and $v(0)$, respectively. **Pts: 3**

13) Write the differences equation of a 1^{st} order digital filter that attenuates the signal

$$u(t) = A \sin \omega t, \ \ \omega \in [10, 50]\text{rad/s},$$

of at least a factor 10, without introducing a delay larger than $\tau_{\max} = 0.2s$ in $[10, 50]$rad/s. For convenience, discretize the filter with the bilinear discretization method. **Pts: 3**

14) Consider the system

$$G(s) = \frac{10}{s^3 + 3s^2 + 3s + 1}.$$

Design a lag network such that the phase margin is 60°. **Pts: 3**

15) Consider a car with mass $M = 1000$ Kg. Its aerodynamic is such that the drag force can be sufficiently well described by $F_{\text{drag}} = bv$, where v is the vehicle speed in m/s and $b = 200$ Ns/m. Design a speed controller such that

(a) a zero steady-state speed error is achieved in presence of a constant speed reference and of a road grade, while a steady-state speed error smaller than 5 m/s is achieved for acceleration references of max 2 m/s^2.

(b) the closed-loop system exhibits a maximum overshoot $S_{\max} = 10\%$ and a settling time at 5% not larger than 2s.

Pts: 5