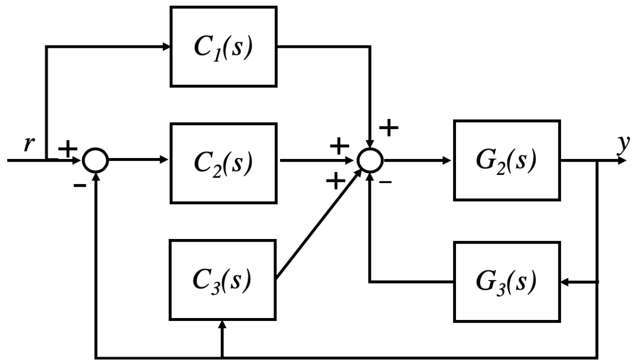
Exam of Automatic Controls. June 8^{th}, 2020
Duration: 1hr

Solve the following problems.

a1) Reduce the following block diagram and find the transfer function $G(s) = \frac{Y(s)}{R(s)}$. Show the intermediate steps. **Pts: 0.1**

$G(s) = \dots$



a2) Verify your result with the Mason formula. **Pts: 0.05**

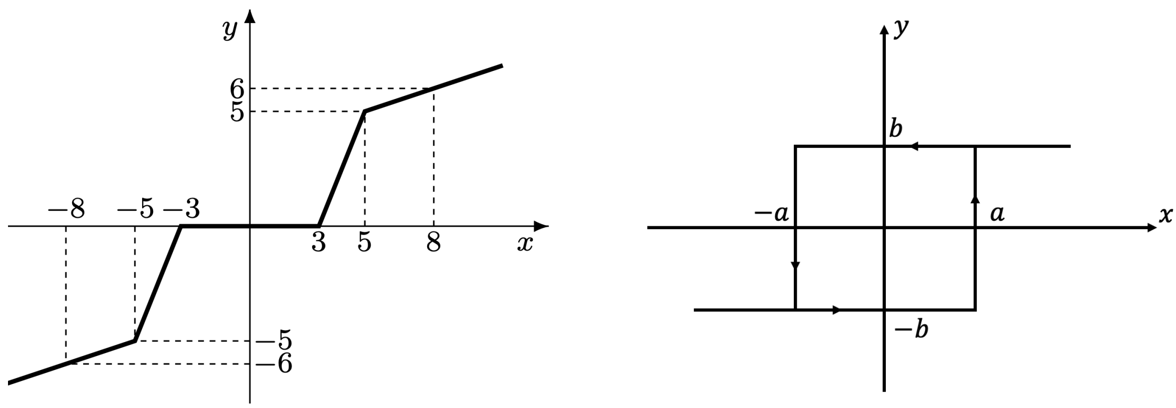
b) Consider the system

$$G(s) = \frac{K}{s(s+3)}, \quad K = 1. \tag{1}$$

b1) By using the root locus, show that the resulting closed-loop system is asymptotically stable for any proportional feedback controller. Find the maximum controller gain such that the CL poles are real. **Pts: 0.1**

b2) Use the root locus to design a controller and choose a constant reference such that permanent oscillation are established, with frequency $\omega = 5.19$ rad/s and unitary amplitude. Explain which drawing rules you have used and how. **Pts: 0.2**

b3) Consider the following two nonlinearities.



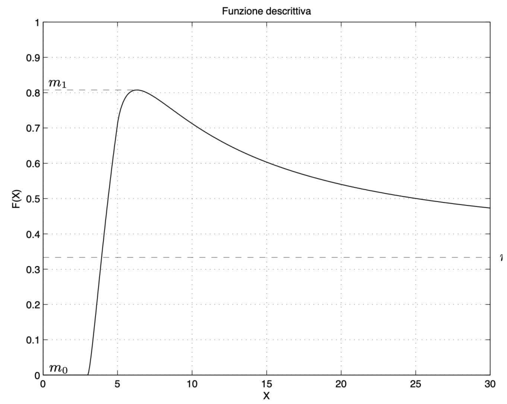
(a) Threshold with partial saturation

(b) Relay with hysteresis

Figura 1: Nonlinearities

Show that the threshold with the partial saturation in closed loop with $G(s)$ cannot enable a limit cycle, while the relay with hysteresis can. Rigorously motivate your answer with plots. **Pts: 0.1**

Hint The qualitative plot of the describing function of the threshold with partial saturation in Figure 1a is



while the relay with hysteresis in Figure 1b has the following describing function

$$F(X) = \frac{4b}{\pi X}\left[\sqrt{1-\left(\frac{a}{X}\right)^2} - j\frac{a}{X}\right], \quad X > a.$$

b4) Design the parameters of the relay with hysteresis (i.e., a and b), in order to obtain a limit cycle with the same amplitude and frequency as in problem b2). **Pts: 0.1**

b5) So far you have shown two ways (i.e., solutions of problems b2) and b4)) to obtain permanent oscillations for the system $G(s)$.
Assume $K \in \left[1, \bar{K}\right]$ in (1). Which of the two methods does still lead to permanent oscillations? Motivate your answer. **Pts: 0.1**

c) Consider the system

$$G(s) = \frac{s(s-5)}{(s+1)(s^2+s+25)}.$$

c1) Plot the asymptotic Bode diagrams of modulus and phase (stepped diagram). **Pts: 0.15**

c2) Plot the linear approximation of the phase diagram. **Pts: 0.05**

c3) Show the deviation of the exact modulus diagram from the asymptotic at the breaking points and sketch the exact modulus diagrams. **Pts: 0.05**