Tech Tip: Springs & Dampers, Part Two
Attack of the Units

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After understanding last month's tech tip, you know how to pick ride frequencies for your racecar, and calculate the spring rate needed for the chose frequency. Now, what do you do about the anti-roll bars.

But first, we have had a couple emails asking about units and frequency calculations, so that will be explained further, and then we will explore how to develop baseline spring rates for anti-roll bars (ARB's).

If you don't watch your units, you can't have any pudding.

When calculating any parameters on the car, you must be sure the units are correct. Just because your physics teacher is not overseeing your calculations does not mean you can slack on the units.

One of the biggest mistakes people make is confusing weight and mass. Weight is a force, mass is an inertia- do not forget that. Once you have decided on a ride frequency and want to figure out what spring rate you need units errors will put you far off target. An example is shown below where weight is used in place of mass.

Equation for calculating spring rate:

$$K_s = 4\pi^2 f_r^2 m_{sm} MR^2$$

K_s = Spring rate (N/m)
m_{sm} = Sprung mass (kg)
f_r = Ride frequency (Hz)
MR = Motion ratio (Wheel/Spring travel)

SI Correct example:
$m_{sm} = 200$ kg $f_r = 2$ Hz $MR = 1.25$

$K_s = 4\pi^2(2 \text{ Hz})^2(200 \text{ kg})(1.25)^2 = 49348$ N/m $= 49.3$ N/mm

SI Incorrect example:
$m_{sm} = 1960$ N $f_r = 2$ Hz $MR = 1.25$

$K_s = 4\pi^2(2 \text{ Hz})^2(1960 \text{ N})(1.25)^2 = 483611$ N/s^2 = Wrong and not useful

English Correct example:
$m_{sm} = 440$ lbm $f_r = 2$ Hz $MR = 1.25$

$K_s = 4\pi^2(2 \text{ Hz})^2/386.4*(440 \text{ lbm})(1.25)^2 = 282$ lbf/in

English Incorrect example:
$m_{sm} = 440$ lbf $f_r = 2$ Hz $MR = 1.25$

$K_s = 4\pi^2(2 \text{ Hz})^2(440 \text{ lbf})(1.25)^2 = 108566$ lbf/s^2 = Wrong and not useful

As you can see, with incorrect units, the results you get are useless. Be careful to use weight/force when needed and use mass when needed- the two are not interchangeable.

Roll

Similar to choosing ride frequencies for bump travel, a roll stiffness must be chosen next. The normalized roll stiffness number is the roll gradient, expressed in degrees of body roll per "g" of lateral acceleration. A lower roll gradient produces less body roll per degree of body roll, resulting in a stiffer vehicle in roll. Typical values are listed below:

- 0.2 – 0.7 deg/g for stiff higher downforce cars
- 1.0 – 1.8 deg/g for low downforce sedans

A stiffer roll gradient will produce a car that is faster responding in transient conditions, but at the expense of mechanical grip over bumps in a corner.

Once a roll gradient has been chosen, the roll gradient of the springs should be calculated, the anti-roll bar stiffness is used to increase the roll gradient to the chosen value. The roll gradient is usually not shared equally by the front and rear. At OptimumG, we call the roll gradient distribution the Magic Number (Milliken calls it Total Lateral Load Transfer Distribution). The Magic Number is expressed as the percentage of the roll gradient taken by the front suspension of the car.

As a baseline, use 5% higher Magic Number than the static front weight distribution. Roll gradients are degrees of body roll per g of lateral acceleration. Roll rates are Newton-meters of torque per degree of body roll or ARB twist. The following equations do no take into account roll due to the tires.

Roll gradient of ride springs:

$$\frac{\phi_r}{A_y} = \frac{-W \times H}{K_{\phi F} + K_{\phi R}}$$

H = Cg to Roll axis dist (m)
W = Vehicle weight (N)
ϕ_r/A_y = Roll gradient from ride springs (deg/g)

$$K_{\phi F} = \frac{\pi (t_f^2) K_{LF} K_{RF}}{180(K_{LF} + K_{RF})}$$

$K_{\phi F}$ = Front roll rate (Nm/deg roll)
t_f = Front track width (m)
K_{LF} = LF Wheel rate (N/m)
K_{RF} = RF Wheel rate (N/m)

Remember that wheel rate is spring rate/ MR^2 ; the effect of the spring at the wheel

$$K_{\phi R} = \frac{\pi (t_r^2) K_{LR} K_{RR}}{180(K_{LR} + K_{RR})}$$

$K_{\phi R}$ = Rear roll rate (Nm/deg roll)
t_r = Rear track width (m)
K_{LR} = LR Wheel rate (N/m)
K_{RR} = RR Wheel rate (N/m)

Total ARB roll rate needed to increase the roll stiffness of the vehicle to the desired roll gradient:

$$K_{\phi A} = \frac{\pi}{180}\left[\frac{K_{\phi DES}K_T(t^2/2)}{[K_T(t^2/2)\pi/180 - K_{\phi DES}]}\right] - \frac{\pi K_W(t^2/2)}{180}$$

$K_{\phi A}$ = Total ARB roll rate needed (Nm/deg roll)
$K_{\phi DES}$ = Desired total roll rate (Nm/deg roll)
K_W = Wheel rate (N/m)
K_T = Tire rate (N/m)
t = Average track width between front and rear (m)

$$K_{\phi DES} = WH/(\phi/A_y)$$

W = Weight of vehicle (N)
H = Vertical distance from roll center axis to Cg (m)
ϕ/A_y = Desired total roll gradient, chosen earlier (deg/g)

Front and Rear Anti-Roll Bar stiffness:

$$K_{\phi FA} = K_{\phi A}N_{mag}MR_{FA}^2/100$$

$K_{\phi FA}$ = FARB roll rate (Nm/deg twist)
$K_{\phi A}$ = Total roll rate (Nm/deg roll)
N_{mag} = Magic Number (%)
MR_{FA} = FARB Motion ratio

$$K_{\phi RA} = K_{\phi A}(100-N_{mag})MR_{RA}^2/100$$

$K_{\phi RA}$ = RARB roll rate (Nm/deg twist)
$K_{\phi A}$ = Total roll rate (Nm/deg roll)
N_{mag} = Magic Number (%)
MR_{RA} = RARB Motion ratio

Remember, the chassis acts as a torsional spring in roll. It is worth comparing the roll rate of your suspension to the roll rate of your chassis- if the chassis twists as much as the suspension, it could be a larger area of concern than the suspension. With steady state roll angles different front to rear, or different roll frequencies front to rear, chassis torsion will be induced- this should be kept in mind.

Try this on an excel spreadsheet and have fun. Email us for comment and questions.

Next time we will speak about the *3rd spring* and introduce damping.