The Rolling Quarter Car Model
a Method to Incorporate Dynamic Tire Response in Grip Optimization

2014-01-0156
Published 04/01/2014

Austin Gurley
Auburn University

CITATION: Gurley, A., "The Rolling Quarter Car Model a Method to Incorporate Dynamic Tire Response in Grip Optimization," SAE Technical Paper 2014-01-0156, 2014, doi:10.4271/2014-01-0156.

Abstract

Selection of springs and dampers is one of the most important considerations when finalizing a race car suspension design. It is also one of most complex due to the dynamic interaction of the vehicle with the ground. Current tuning methods for spring and dampers' effect on vehicle ride can be based on simplified dynamic models of the vehicle, such as the quarter-car model. While efficient computationally, the traditional quarter-car model does not account for the non-linear variation in grip seen by a fluctuating contact-patch. Both amplitude and frequency of suspension oscillation contribute to loss of tire grip. The method can be improved by incorporation of a dynamic tire model, though resulting in non-linear effects. An improved 'rolling quarter-car' model is created, which includes the effect of dynamic tire forces in the analysis of improved grip. Using typical Formula SAE race car, characteristics as a test case, a linearized dynamic model is made. The effect of suspension parameters on the dynamic tire forces produced are surveyed. Improvements made by the new model are presented.

Introduction

Formula SAE is a collegiate design competition in which student teams build open-wheel, formula style racing cars. The races in which the cars compete are autocross format events; tracks with many technical maneuvers are set in (typically) a large flat area such as a parking lot or an airport runway. Formula SAE cars use racing tires and are capable of achieving high lateral and longitudinal acceleration on these courses (e.g. 1.5 g's or more laterally). The tracks are mostly flat and smooth.

A dominant rule of the competition is that cars can only compete for a single year before being retired. As a result, most teams design, build, test, and race each car all in a single year's time. Most teams have a limited budget (compared to professional race teams) and limited access to testing time. With this restraint, modeling and simulation can greatly aid the design process. Fortunately, as academic affiliates, FSAE teams have access to computing resources that are near or may even exceed those of professionals. One particularly invaluable tool, critical to the model developed herein, is access to data for the tires used in the competition. A tire testing consortium makes data available to FSAE team consortium members [1].

Because of competition rules and the typical dimensions of FSAE vehicles, certain parameters are limited in the suspension design. The first consideration is a mandated two inches of wheel travel - a very large range considering the small size of the cars and their typically high spring rates. With the very flat race tracks this puts special constraints on damper selection. The cars also typically have high wheel-set to chassis weight ratios, such that wheel-set masses cannot be neglected as in some models of passenger cars.

Overview of Suspension Design Goals

The dynamic vehicle is a complex system, and while it can be fully modeled - as is done in some commercial simulators - analysis of the vehicle is often performed by simplified or partial models of the system. Generally two types of models are made - one type concerned with motions in the ground plane, and the other with motion perpendicular to it. Motions of consideration in-plane are the longitudinal, lateral, and yawing accelerations, as well as the forces that produce them. Motions of consideration normal to the ground plane include the vertical motion of the chassis (heave), and of the individual wheels (for a typical independent suspension). The pitch and roll of the car are also considered in the 'perpendicular' class, since those motions can be simply defined by the vertical motion of the four corners of the car in their various modes [2]. While the two modes of motion can be studied independently, closer approximations to the real vehicle will couple the modes or combine them into one, albeit more complex, model.

Paper Outline

The model presented in this paper is a ride (out-of-plane) model concerned only with the road profile *motion*. The unique improvement with the new model is that it couples this vertical motion to in-plane tire *force* generation, leading to more useful design results. The presentation of the new model is made as follows: First, a review of the traditional quarter-car model is given. A formulation is made that allows nonlinearities in that model. Existing optimization techniques using that model are then presented, showing the current state of this form of analysis. Second, a fully non-linear and transient tire model is presented, derived from TTC data. Dynamic tire behavior is reviewed, using that model to show the effects of oscillation amplitude and frequency on tire force generation. The model is reduced to a polynomial form for efficient simulation. Third, the rolling quarter car model is presented. A new method for design optimization is given, and the useful results of that method are confirmed using the non-linear tire model derived from data. To make the model useful in initial design, a linear/algebraic form of the model is made. Lastly, the results and ramifications of the method are discussed.

The Quarter Car Ride Model

In the literature, it is very common to begin vertical suspension dynamics design with the consideration of a 'quarter-car' model. This model isolates a corner of the vehicle to observe response to vertical road input. The variables encompassed include a tire-spring rate, the mass of a wheel-set, the installed spring and damper, and one quarter of the vehicle's sprung mass (Figure 1). This model is used to see the response of a wheel to vertical road inputs, isolated from the rest of the vehicle. While this model neglects some parts of the vehicle such as anti-roll bars, it is used because it can be analyzed in both time and frequency to see the effect of springs, dampers, and the masses on the motion response of the wheel-set and chassis to vertical inputs. Though simple, it is a fairly accurate model of ride dynamics [3]. In the present analysis only the dampers are optimized.

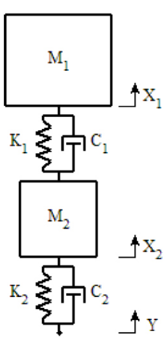

Figure 1. Schematic of a quarter car model.

Linear Model Formulation

The quarter car model can be represented by the state space formulation:

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{c_1}{m_1} & \frac{k_1}{m_1} & \frac{c_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{c_1}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{c_1}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{k_2}{m_2} & \frac{c_2}{m_2} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$$

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} \tag{1}$$

The output z_1 is the chassis excitation amplitude, z_2 is the wheel excitation amplitude, and z_3 is the tire deflection.

One way to utilize this model is to apply a time varying input from the ground, and review the transient response of the vehicle. This input could be either a step input or an actual ground profile of a track if known. However, the inputs in time can be very complex, variant, and difficult to interpret [4]. Therefore, it is useful to analyze the vehicle response by means of a frequency domain approach, giving a picture of the reaction of the vehicle to inputs at every frequency within the range practical motion. For a typical vehicle, the range of meaningful frequencies that could excite the vehicle range from 0.5 to 30 Hz [4].

The road profile used is from the very common model proposed by Wong [5] used as described by Kowalczyk [6]. The model follows the equation

$$\Omega \equiv amplitude(\omega_s) \;=\; surface_{factor} * \left(\frac{\omega_s}{speed}\right)^{-1} \tag{2}$$

This PSD is easily converted to displacement amplitude, and the spatial frequency replaced by an equivalent temporal frequency [5].

$$\omega_t \;=\; v * \omega_s \left\{\frac{\mathrm{m}}{\mathrm{s}} * \frac{\mathrm{cycles}}{\mathrm{s}}\right\} \tag{3}$$

This is used as the input amplitude at each frequency analyzed: correspondingly the higher frequencies have lower amplitude input as is seen in real driving conditions [4,8].

Non-Linear Model Formulation

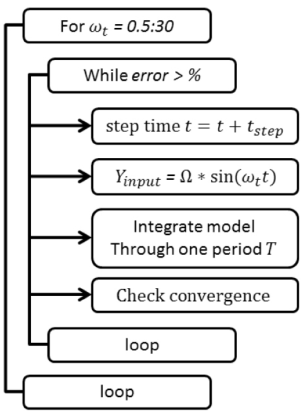

Figure 2. Programming diagram for the nonlinear frequency response technique.

While a linear model has an analytical solution for frequency domain response, the tire model this work will incorporate has nonlinear elements. While the concept of frequency response is still valid, a numerical method must be used. This method has been implemented as shown in Figure 2.

A model is created using the equations of motion for the quarter car and numerically integrated. The input is held as a constant sinusoid at the ground. Nonlinearities such as damping forces are allowed to update at each time step, and the system is allowed to oscillate until the model reaches a steady state response. A phase-plane loop comparison gives a visual indication of the convergence (Figure 3).

A comparison between the numerical and analytical solution of a constant amplitude input are shown in Figure 4 for identical input parameters. Though the solutions are not in perfect agreement, the response method is seen to be a close approximation, and can be used to review the nonlinear response that could not otherwise be modeled.

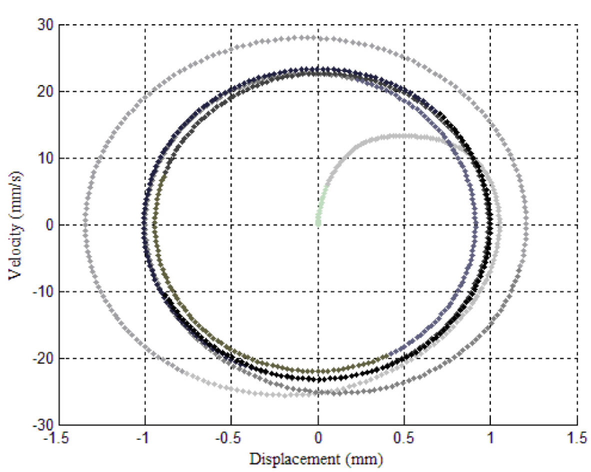

Figure 3. Phase-plane convergence example showing sprung mass(X1) oscillation.

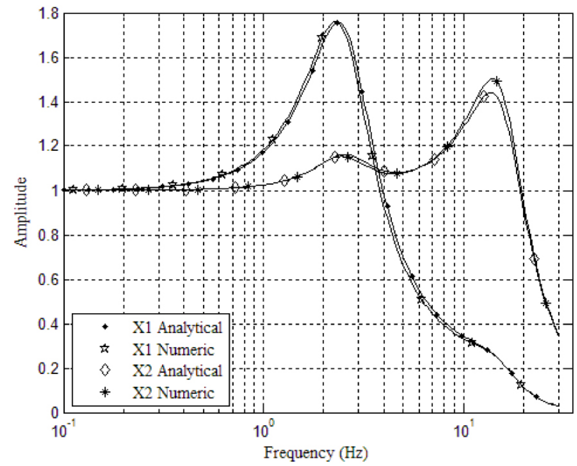

Figure 4. Comparison of a nonlinear and linear frequency response output with identical input parameters. This demonstrates the feasibility of the nonlinear method.

Existing Optimization Methods

As mentioned previously, the optimization cost functions of interest are generally the excitation of the sprung mass:

$$Stability\ Metric\colon \min(\mathrm{mean}\left(\left(\frac{X_1}{Y}\right)\right) \tag{4}$$

and the contact patch deflection:

$$Deflection\ Metric\colon \min\left(\mathrm{mean}\left(\frac{X_2 - Y}{Y}\right)\right) \tag{5}$$

It has been given previously that minimizing the excitation of the two will produce improvements in vehicle stability and grip [7] [8]. In several common and recognized books on vehicle dynamics, this claim has been made with reference to the characteristics of tires, though there has not been shown in those sources a direct correlation between the minimization of tire deflection and grip potential using a tire model or experimental data in the frequency response test [3, 7, 8]. This assumption will be reconsidered later.

A suitable method for optimization using the metrics (4) and (5) is to minimize the RMS amplitude of the response throughout all frequencies. This has been proposed as the best way to maximize the typical 'Grip' of the car [7].

In this manner, the Deflection and Stability criteria are found for the range of spring/damping rates of interest as in Figure 5.

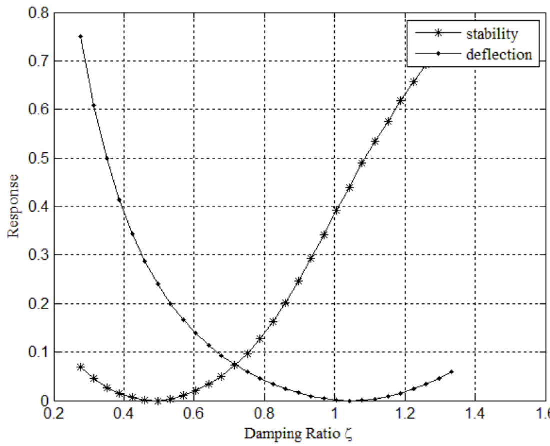

Figure 5. Damping optimization showing stability and deflection metrics as a function of damping ratio.

It is useful to note that some compromise should often be made in the spring/damper selection even if only one criterion is the priority. For example; starting with the damping rate for optimal Grip, the rate can be reduced from the optimal with little loss in Grip and much gain in Stability (Figure 6).

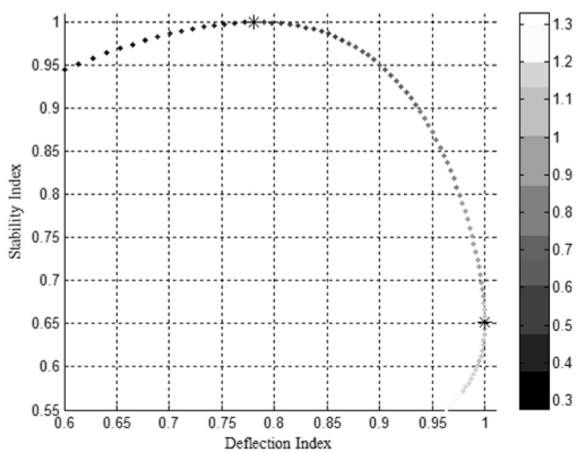

Figure 6. Optimization for a range of damping ratios

Full Nonlinear Tire Model

Steady-State Model

Departing from the quarter car analysis for now, the general dynamic characteristics of a typical FSAE tire are presented. The particular tire used will remain unnamed in compliance with the TTC agreement with Calspan. The model created is a pragmatic model (based on correlations of inputs and outputs) not a physical one (based on geometry and materials simulation such as a Finite Element model) [9]. It is made accurate by confirmation and comparison to real data from the Tire Testing Consortium.

An accurate steady state model of the FSAE tire can be created using a Pacejka magic formula model. During modeling, the author found full multivariable Pacejka models unreliable in predicting any response to load, inclination, or pressure. However, the characteristic magic formula for lateral force as a function of slip (alone) fits well to each sweep of slip angles (Figure 7) [10].

$$F_{y_{ss}}(\alpha)|_{F_z=C_1,\gamma=C_2,P=C_3} = D\sin\big(C\,\mathrm{atan}\big(B\alpha - E(B\alpha - \mathrm{atan}(B\alpha))\big)\big) \tag{6}$$

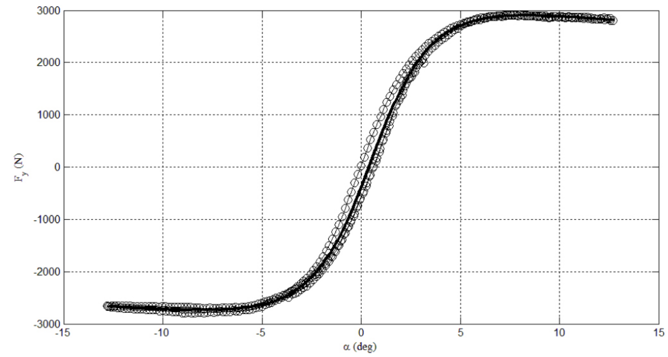

Figure 7. Comparing the steady-state model to raw data.

In creating both the lateral force and self-aligning torque models, a genetic algorithm was developed and the magic formula equation used to fit each sweep of slip angles. A cubic spline interpolation is used between various inclination angles, loads, and internal pressures. The modeled tire is shown below, considering the relationship between lateral force, normal load, and slip angle (Figure 8 and Figure 9). Pressure, temperature, and inclination angle are held constant in these plots.

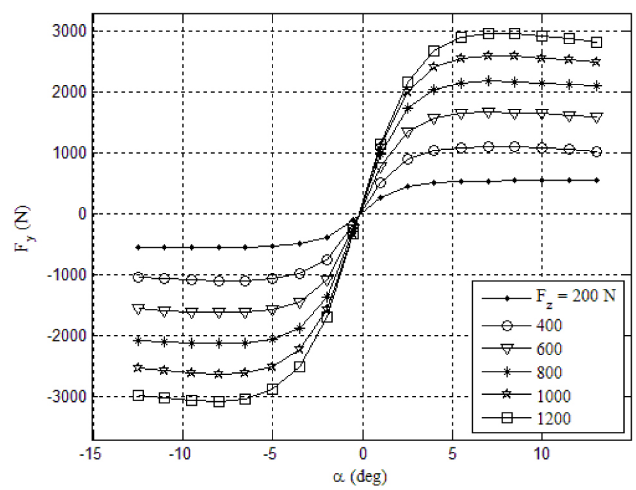

Figure 8. Lateral force curves for different normal loads. (note: tire data plots have been rescaled for propriety)

This steady state lateral force model fits the data on average within 17 lbf[1] throughout the range of loads, inclination angles, and slip angles tested. This error is comparing the model to raw data.

1. Error is given in lbf, not percent, to prevent mistaken values where the lateral force approaches zero.

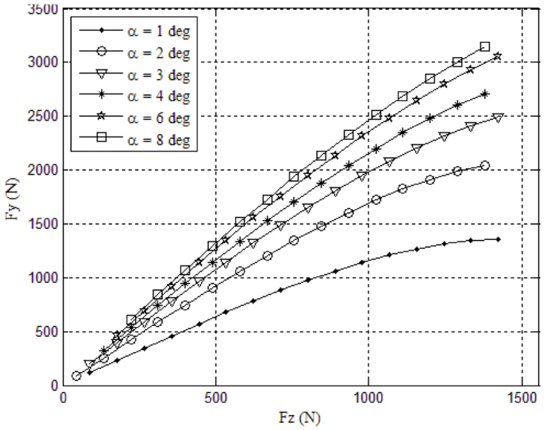

Figure 9. Typical load sensitivity of an FSAE tire.

Dynamic Tire Model (Improvement to the Steady-State Formulation)

A first order dynamic model is created to improve the modeling of transient tire force generation. Tire sidewalls must deflect in order for slip and therefore forces to develop. This has been shown to occur over a distance known as the relaxation length denoted σ [10]. The relationship between the actual force and the steady-state predicted force is given by:

$$F_y + \frac{\sigma^*}{V}\left(\frac{\partial Fy}{\partial t}\right) = F_{y_{ss}} \tag{7}$$

Each sweep of slip angles is reviewed again, this time estimating a constant relaxation length for each sweep, and fitting the offset of the slip curves at 0 degrees slip to find the initial relaxation length σ_0. The relaxation value for each slip sweep is determined by integrating equation (7) through the slip angle sweep with a trial value of σ_0. The value of σ_0 is iterated minimizing the error between model and data. A polynomial fit of the values is made to create σ_0 as a function of F_z.

The actual effective relaxation length considering the nonlinear tire curve, call it σ^*, is a function of both load and slip angle [11]. An attempt was made to model this effect using the equation given by Rill [12].

$$\sigma^* = \frac{\sigma_0}{C_{\alpha_0}}\frac{dF_y}{d\alpha_i} + V\frac{C_y}{K_y} \tag{8}$$

The parameters are easily fit since the instantaneous cornering stiffness (α_i) as well as the initial cornering stiffness ($C_{\alpha 0}$) of the tire are known from the steady-state model. Note that, for this pragmatic model, the actual values of C_y and K_y are not needed: the ratio C_y/K_y is found by again using an iterative method. It was found that the incorporation of Rill's slip-dependent relaxation did not decrease model error. While experiment has shown that relaxation does in fact vary with slip angle [11, 13], the effect will not be utilized here as it could not be reliably modeled.

In validation, three techniques in total were compared to test for improvement over the steady state model. First, a constant relaxation for all loads and slips was used. Second, relaxation length that varies with load only was tested. Third, a function of both slip and load was implemented. The resulting error upon fitting the three models is presented in Table 1 below. The error of each is given, as well as the relative improvement of each model over the steady state form. While all relaxation length models show large improvement over the steady state model, there is still residual error likely due to unaccounted effects of temperature and wear effects. The limits of the Magic Formula curve also could also cause some discrepancy. Relaxation as a function of load was chosen for this work.

Table 1. The reduction of modeling error (compared to a Steady State (SS) model) with relaxation model complexity

Method	Mean Model Error	$\frac{Error}{SS\ Error}$
$\sigma^* = 0$ (SS model)	16.88 lbf	100%
$\sigma^* = Constant$	10.18	60.3%
$\sigma^* = f(Fz)$	9.40	55.7%
$\sigma^* = f(Fz, \alpha)$	9.63	57.1%

It is clear visually that the data fit improves with the addition of the transient effect: the first order lag in force generation more faithfully reproduces the physical data, as can be seen in Figure 10.

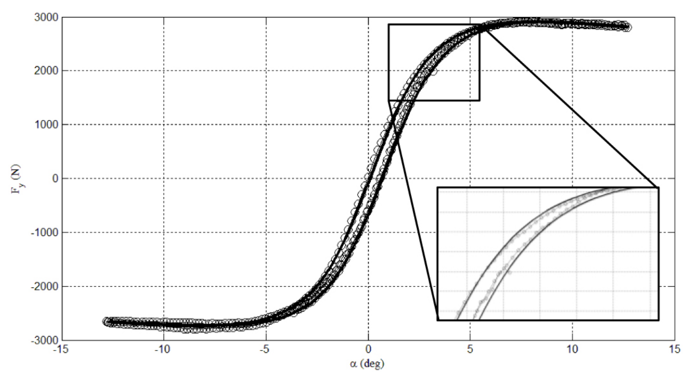

Figure 10. Comparing the dynamic model to raw data. Note the improved capture of the lateral force 'lag'.

Review of Dynamic Tire Effects

Before continuing, the assumptions made in this analysis are listed for review:

1. The Magic Formula fits steady state lateral force vs slip angle adequately
2. Relaxation length varies only with load
3. Relaxation scales by *∂Fy/∂t* (i.e. the effect, due to change in slip or load, is equivalent)
4. Relaxation length is independent of velocity

Assumption 1 is accepted in the literature as a respectable empiric method [10]. The validation of assumption 2 was presented in the previous section. Assumptions 3 and 4 are based on the physical interpretation of relaxation as in the development of the stretched string model [10, 11].

It is known that a sinusoidal fluctuation of load on a tire produces a reduction of lateral force compared to an equivalent constant load, even if the fluctuating load and the constant load have the same average [10, 14]. This is due to the load saturation effect of the tire. A visualization of this effect, holding a constant slip angle, is given below (Figure 11). The data shown is from the tire model presented previously, at a low frequency of load oscillation (<0.5 Hz). The vertical load is fluctuating sinusoidally through time, the (steady state) lateral force at a given load is found, and thus the time-dependent lateral force is known, and is seen to be far from perfectly sinusoidal. Because the lateral force loses more grip with deceased load than it gains with increased load, the average lateral force is reduced. This component of grip loss happens regardless of the rate of oscillation, and is considered the 'static' loss.

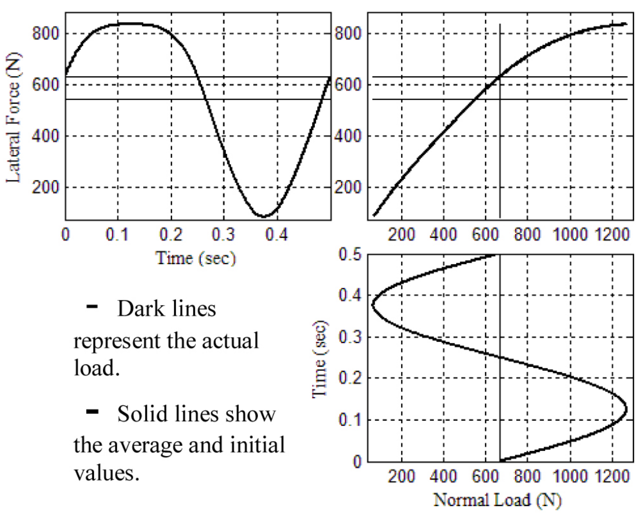

Figure 11. The steady-state lateral response to constant amplitude load fluctuation. Average lateral force during oscillation is less than it would be with a constant normal load - even if the load has the same average in each case.

Because the tire has a dynamic response as well, the frequency of this oscillation will also affect the forces. Again using the dynamic tire model the tire is subjected to oscillating vertical load at various frequencies. Increased frequency can be seen to decrease the average lateral force, as shown in Figure 12.

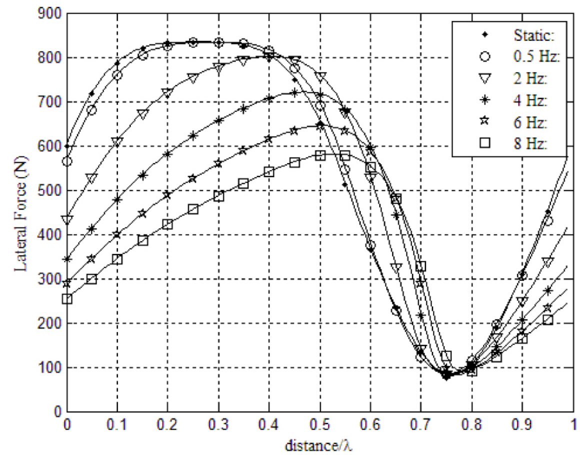

Figure 12. The dynamic lateral response to a range of normal load oscillation frequencies. Higher frequencies reduce average lateral force.

Reduced (Constant Slip) Tire Model

The proposed simulation will require numeric integration. Calling on the full nonlinear tire model is computationally expensive, so it is desirable to reduce the complexity of the tire model if such a reduction can be accomplished without significant loss in resolution. Knowing the analysis will be performed holding constant slip angles, polynomial relationships are preprocessed before the analysis. Of interest are lateral force and relaxation length as functions of load. The relationships between F_y and F_z is seen to fit well to a 3[rd] order polynomial with small error in the range of F_z considered (Figure 13).

$$F_y|_{\alpha=6} = -2.9 \times 10^{-6}\, F_z^3 - 6.1 \times 10^{-4} F_z^2 + 1.26 F_z \tag{9}$$

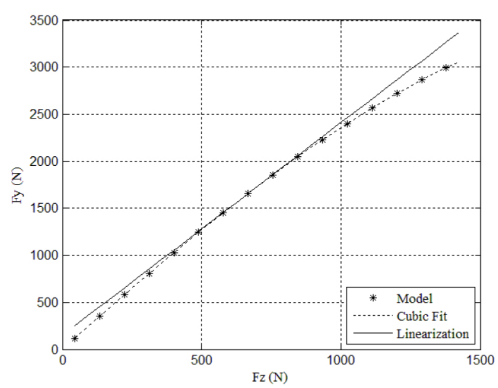

Figure 13. Polynomial fit of Load Sensitivity to the fully nonlinear tire model, holding slip angle at 6 degrees.

The relationship between σ and F_z fits to a second order polynomial with small error (Figure 14).

$$\sigma|_{\alpha=6} = -1.467 \times 10^{-4} F_z^2 + 0.1555 F_z \tag{10}$$

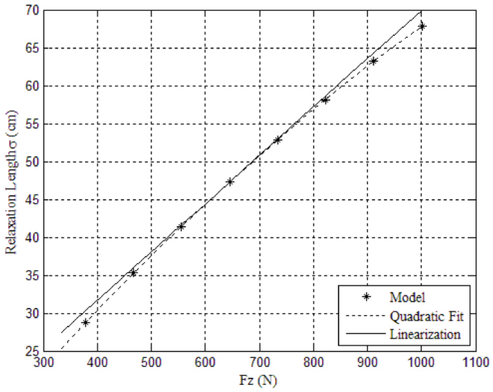

Figure 14. Polynomial fit of Relaxation Length to the fully nonlinear tire model.

The polynomial form as given above was used in the nonlinear frequency response, reducing computation time by a factor of 90.

Rolling Quarter Car Model

Both the static and dynamic losses of the tire a produce a significant lowering of average tire force through a sinusoidal input. These effects will now be presented in light of the quarter car model. Considering only the static losses, it is reasonable to minimize the load fluctuation in order to maximize grip; the frequencies of higher excitation will create greater losses than those regions with less load fluctuation, almost exactly proportional to the difference in fluctuation amplitude [10]. However, the dynamic losses provide a different solution. *Excitation at higher frequencies produces more loss for a given excitation amplitude than those at low frequencies*. To account for this, a new definition of 'Grip' is proposed. The implementation of this new definition is presented here.

Non-Linear Results

Using the non-linear quarter car model, and again sweeping through a range of damping rates, the frequency response is calculated as before. At each frequency, tire load fluctuation is calculated. Transient response is then allowed to stabilize such that a sinusoidal lateral force results. For this stable lateral force fluctuation, the average lateral force through the oscillation is recorded. This is repeated at each frequency of interest (from 0.5 to 30 Hz).

A new criterion is now suggested and compared for a 'Grip' metric. Taking the mean tire force during each oscillation as the index, a new optimization curve is created using the criterion:

$$Grip\ Metric : \max\left(\mathrm{mean}\left(F_y\right)\right) \tag{11}$$

This test, sweeping through the same set of damping ratios and finding the optimized rate, has been repeated at multiple (constant) slip angles, with consistent results up to 6 degrees slip (the typical peak of lateral force, as seen in Figure 8)

Figure 15 shows the results for comparison. The response is unit-less here for visual comparison; though it could be represented in units of force.

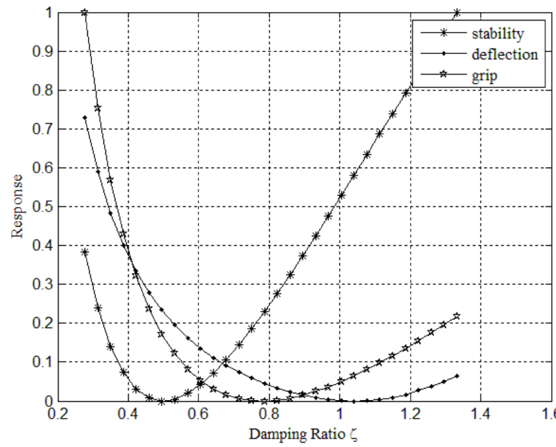

Figure 15. Damping optimization showing excitation as a function of damping ratio, including the new 'Grip' criteria representing average lateral force derived from the full non-linear tire model.

Linear Model

To facilitate a linear model implementation, the equations can be linearized about an operating point $\bar{x}$ using the Taylor series expansion truncated to the second term. In this case the static tire load is the operating point: $\bar{x} = F_{z0} = 660\ \mathrm{N}$. This yields practical results since the tire losses can be given as polynomial functions of those equations [10].

$$f(x) = f(\bar{x}) + \frac{df}{dx}|_{x=\bar{x}}\,(x-\bar{x}) + \frac{d^2f}{dx^2}|_{x=\bar{x}}\,(x-\bar{x})^2 + \cdots \tag{12}$$

$$f(x) \approx f(\bar{x}) + \frac{df}{dx}|_{x=\bar{x}}\,(x-\bar{x}) \tag{13}$$

This produces a linearized lateral force;

$$F_y|_{\alpha=1} \approx 2.259F_z + 33.99 \tag{14}$$

As well as a linearized relaxation length:

$$\sigma|_{\alpha=6} \approx 0.1115F_z + 2.4566 \tag{15}$$

The accuracy can be seen to hold within a fair range of the operating average (Figure 13 and Figure 14). Note that although these can be individually linearized the model is still nonlinear in time as the relaxation length and *∂Fy/∂t* are multiplied in the dynamic lateral force equation (Equation 7). This difficulty has been overcome previously by assuming a sinusoidal load and representing the response as a Fourier series [10]. Considering both static and dynamic loss of the tire due to a sinusoidal input in time, the relationships can be given. The only terms needed are the load saturation curvature (*b*) which is the highest order term of the derivative of (9), and relaxation stiffness (*c*) which is the linear term from (15).

$$F_{y,static.loss} = b * Fy_{ss} * \overline{F_z}^2 \tag{16}$$

$$F_{y,dynamic.loss} = \frac{C_{F\alpha 0}}{2}\left(\frac{\omega_s^2 c^2 \overline{F_z}^2}{1+\omega_s^2 c^2 F_{z0}^2}\right)\alpha$$

$$b = 8.7\times10^{-6} \qquad c = 0.1115 \tag{17}$$

The linear state space model is now used to determine excitation. Because the excitation is linear, all optimization metrics in the analysis can be post-processed (Figure 16).

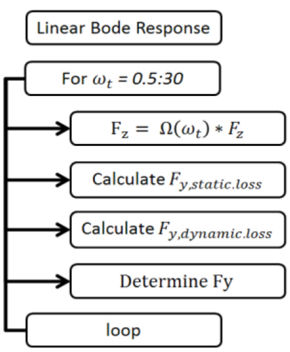

Figure 16. Programming diagram for the post-processed linear 'Grip' optimization cost generation.

The road profile amplitude simply scales the excitation amplitude by multiplication at each frequency. The lateral force losses are calculated using the sinusoidal equations given above, making the Grip optimization criterion:

$$Grip\ Metric:$$

$$\min\left(\operatorname{mean}\left(\frac{1}{4}b\overline{F_z}^2 + \frac{1}{2}\left(\frac{\omega_s^2 c^2 \overline{F_z}^2}{1+\omega_s^2 c^2 F_{z0}^2}\right)\right)\right) \tag{18}$$

As was done with the nonlinear model, the excitation can be examined for a range of damping rates to determine an optimum value. This response agrees well with the results used in the fully non-linear model, as can be seen by comparison of Figure 17 with Figure 15. The final optimal damping rates from the model are given in table 2. The values the model returns are highly dependent on the road profile used; an important consideration when implementing this method.

Table 2. Optimal damping ratio results for each metric and comparison of the linear and nonlinear model results.

Metric	Linear Model	Nonlinear Model	% Difference
Stability	0.4962%	0.4918%	0.14%
Deflection	1.041%	1.028%	0.01%
Grip	0.759%	77.7%	1.8%

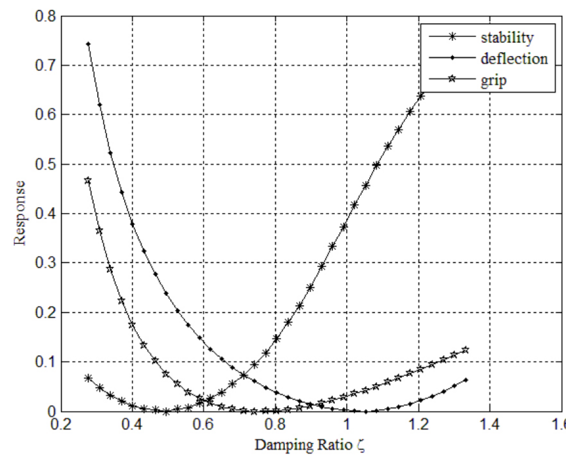

Figure 17. Damping optimization showing normalized excitation as a function of damping ratio, including the new 'Grip' criteria representing average lateral force derived from the reduced post-processed tire model.

Conclusion

The new linear model can now be used to visualize the effect quarter car model parameters have on the tire grip. While the trends of the data are similar, the actual optimized values differ. Damping rates in particular are sensitive to the new analysis. Considering the accessibility of damping adjustment in most racing applications this is an important consideration for design. Other design variables maintain their previously trends but show increased sensitivity to tire vertical stiffness. The effect of the *b* and *c* constants were obviously not in the previous model design statement: The rolling quarter car model offers a means to compare dynamic tire characteristics in suspension design.

A detailed tire model was developed, including dynamic effects, from test data. A method for tuning suspension parameters based on the quarter car model was improved by the consideration of the tire as a part of the system. A simple, linear model was used to show the varying effect of system parameters, such as masses and spring rates, on optimization criteria. Such a model is beneficial as it allows rapid iteration of design parameters and does not require numerical methods (such as numeric integration) for solution. Though computationally expensive, the non-linear model remains useful as it allows non-linear damping and other effects to be included in the analysis. In both models, it was seen that more consideration of the tire effects should be made in the design optimization than has been considered in previous work. The non-linear effects of oscillation amplitude and frequency can affect the optimization results significantly; especially for racing vehicles.

Perhaps the most important design implication from this model is that design damping rates are different than those given by the common deflection criteria. Typically the new value are higher than the original, consistent with studies performed using real road profiles and full vehicle simulation [15].

It is noteworthy that the effects of the rolling quarter car model are most prominent in the linear (low slip) region of the tire operating range. This is due to the reduction of relaxation length with increased slip. As it affects the linear 'response' operation, the model will improve predictions of trim stability and control more than steady-state handling [16].

Future work should explore methods for acquisition of the *b* and *c* parameters which are the minimal required inputs for this model to be useful in the suspension design process. Also, better modeling of the reduction in relaxation length with slip angle should be considered.

Acknowledgments

This analysis, and much success of the Auburn FSAE team, would not be possible without the Tire Testing Consortium and Calspan. Thanks to the organizers for a valuable tool. Thanks also to Matthew Heffernan and Dr. Peter Jones for their discussion and review of this work. Figures were produced in GNU Octave and MATLAB.

References

1. Kasprzak, E. and Gentz, D., "The Formula SAE Tire Test Consortium-Tire Testing and Data Handling," SAE Technical Paper 2006-01-3606, 2006, doi:10.4271/2006-01-3606.
2. Segers, J., "Analysis Techniques for Racecar Data Acquisition," SAE International, Warrendale, PA, ISBN 978-0-7680-1655-0, 2008, doi:10.4271/R-367.
3. Rajamani R., Vehicle Dynamics and Controls, New York: Springer, 2006.
4. Kasprzak, J. and Floyd, R., "Use of Simulation to Tune Race Car Dampers," SAE Technical Paper 942504, 1994, doi:10.4271/942504.
5. Wong J. Y., Theory of Ground Vehicles, New York: Wiley, 1993.
6. Kowalczyk, H., "Damper Tuning with the use of a Seven Post Shaker Rig," SAE Technical Paper 2002-01-0804, 2002, doi:10.4271/2002-01-0804.
7. Dixon J. C., "Ride and Handling," in *The Shock Absorber Handbook*, West Sussex, John Wiley & Sons, Ltd., 2007, pp. 105-134.
8. Jazar R. N., Vehicle Dynamics: Theory and Applications, New York: Springer, 2008.
9. Zegelaar P. W., The Dynamic Response of Tyres to Brake Torque and Road Unevennesses (PhD. Dissertation), Delft: Delft University of Technology, 1998.
10. Pacejka, H., "Tire and Vehicle Dynamics," Society of Automotive Engineers, Inc. and Butterworth Heinemann, Warrendale, PA, ISBN 978-0-7680-1126-5, 2002.
11. Higuchi A., Transient Response of Tyres at Large Wheel Slip and Camber, Delft: Delft University Press, 1997.
12. Rill G., "First Order Tire Dynamics," in *III European Conference on Computational Mechanics*, Lisbon, 2006.
13. van der Jagt P., The Road to Virtual Vehicle Prototyping: new CAE models for accelerated vehicle dynamics development, Eindhoven: T.U. Eindhoven, 2000.
14. U.S. Department of Transportation, Mechanics of Pneumatic Tires, Clark S. K., Ed., Ann Arbor, 1981.
15. Rijk S. v., "Optimization of the Damper Settings of the URE05e," Eindhoven University of Technology Department of Mechanical Engineering, Eindhoven, 2010.
16. Olsson H., Hugon E. and Rouelle C., "Efficient Acquisition of Tire Data and Its Application in Vehicle Handling Simulation," *Tire Science and Technology*, vol. 40, no. 4, pp. 234-245, 2012.

Contact

Austin Gurley is a graduate student at Auburn University. He held the position of suspension designer and vehicle dynamicist for two years on the Auburn FSAE Team, and was the Chief Chassis Engineer for the 2013 Auburn Formula SAE team. His current research interests involve race vehicle dynamics and advanced composites manufacturing. He can be contacted via email at arg0007@auburn.edu

APPENDIX

NOMENCLATURE

$m_1 = Sprung\ Mass\ (kg)$
$m_2 = Unsprung\ Mass\ (kg)$
$k_1 = Installed\ Spring\ Rate\ (N/m)$
$k_2 = Tire\ Spring\ Rate\ (N/m)$
$c_1 = Installed\ Damping\ Rate\ \left(\frac{Ns}{m}\right)$
$c_2 = Tire\ Damping\ Rate\ \left(\frac{Ns}{m}\right)$
$x_1 = Sprung\ Mass\ Displacement\ (m)$
$x_2 = Unsprung\ Mass\ Displacement\ (m)$
$y = Road\ Input\ Displacement\ (m)$
$z_1 = Chassis\ Excitation\ (m)$
$z_2 = Wheel\ Excitation\ (m)$
$z_3 = Tire\ Deflection\ (m)$
$\Omega = Road\ Profile\ Amplitude\ Model$
$\omega_s = Spatial\ Frequency\ \left(\frac{rad}{m}\right)$
$\omega_t = Temporal\ Frequency\ \left(\frac{rad}{s}\right)$
$v = ground\ speed\ \left(\frac{m}{s}\right)$
$F_y = Lateral\ Force\ (N)$
$F_{y_{ss}} = Lateral\ Force, Steady\ State\ (N)$
$\alpha = Slip\ Angle\ (rad\ or\ degrees)$
$F_z = Normal\ Load\ on\ Tire$
$\sigma_0 = Relaxation\ Length\ at\ 0\deg Slip\ \ (m)$
$\sigma^* = Instantaneous\ Relaxation\ Length\ (m)$
$C_y = Tire\ Carcass\ Lateral\ Damping\ Rate\ \left(\frac{Ns}{m}\right)$
$K_y = \ Tire\ Carcass\ Lateral\ Spring\ Rate\ (\frac{N}{m})$
$\sigma = Relaxation\ Length\ (m)$
$\alpha_i = Instant\ Cornering\ Stiffness\ \left(\frac{\Delta N}{\Delta rad}\right)$
$b = Load\ Saturation\ Curvature\ \left(\frac{\Delta N}{\Delta N^2}\right)$
$c = Relaxation\ Stiffness\ \left(\frac{\Delta m}{\Delta N}\right)$
$F_{z0} = Normal\ Load\ Fluctuation\ Mean\ (N)$
$\bar{F_z} = Normal\ Load\ Fluctuation\ Amplitude\ (N)$

The Engineering Meetings Board has approved this paper for publication. It has successfully completed SAE's peer review process under the supervision of the session organizer. The process requires a minimum of three (3) reviews by industry experts.

ISSN 0148-7191

http://papers.sae.org/2014-01-0156