ChassisSim v3 Tyre Model Parameterisation.

You will notice in the tyre tab of ChassisSim that you have a tab that says click here to import the ChassisSim v3 approximation optimisation/ opt results. This is a text file that approximates the load, camber and temperature characterisitics of the tyre model. It is a very powerful tool not just to understand the tyre model but to really refine the results you have.

The purpose of this article is to describe the format of this file and what all the variables mean. Once you have a thorough understanding this will become an invaluable tool to understand your tyre.

ChassisSim v3 tyre approximation model file format

The ChassisSim v3 tyre model approximation is an ascii text file with the following format,

[Tyre Force Characteristics]	
No load points=6	- These are the no of load points
Load 1=1500	- Load at Point 1 in N
Tyre Force 1=3800	- Tyre Force at Point 1 in N
Load 2=3000	- Load at Point 2 in N
Tyre Force 2=6000	- Tyre Force at Point 2 in N
Load 3=4900.039083	- Load at Point 3 in N
Tyre Force 3=9100	- Tyre Force at Point 3 in N
Load 4=6500	- Load at Point 4 in N
Tyre Force 4=11200	- Tyre Force at Point 4 in N
Load 5=8200	- Load at Point 5 in N
Tyre Force 5=12000	- Tyre Force at Point 5 in N
Load 6=9900	- Load at Point 6 in N
Tyre Force 6=12000	- Tyre Force at Point 6 in N
[Tyre Temp Characteristics]	
Opt tyre temp=373.359994	- Tyre temperature that peak tyre force is produced in K
Pre slope constant=0	- How tyre force increases – will be documented later.
Post slope constant=0	- How tyre force decreases – will be documented later.
Temp Mode=0	- Varies between square and exponential function mode
Exp pre temp mult=0	- Pre exponential multiplier – will be documented later
Exp post temp mult=0	- Post exponential multiplier – will be documented later
Exp temp scale=50	- Temp scale – This is in K
[Tyre Slip Characteristics]	
Peak slip angle=0.104	- Peak slip angle in radians
No slip points=10	- No of normalised slip points
Norm slip angle 1=0	- Normalised slip angle at Point 1 (unitless)
Norm slip force 1=0	- Normalised slip force at Point 1 (unitless)
Norm slip angle 2=0.16	- Normalised slip angle at Point 2 (unitless)

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
Danny Nowlan, Director mobile: +61 425 219 375

Norm slip force 2=0.2272	- Normalised slip force at Point 2 (unitless)
Norm slip angle 3=0.32	- Normalised slip angle at Point 3 (unitless)
Norm slip force 3=0.4288	- Normalised slip force at Point 3 (unitless)
Norm slip angle 4=0.48	- Normalised slip angle at Point 4 (unitless)
Norm slip force 4=0.6048	- Normalised slip force at Point 4 (unitless)
Norm slip angle 5=0.64	- Normalised slip angle at Point 5 (unitless)
Norm slip force 5=0.87	- Normalised slip force at Point 5 (unitless)
Norm slip angle 6=0.8	- Normalised slip angle at Point 6 (unitless)
Norm slip force 6=0.88	- Normalised slip force at Point 6 (unitless)
Norm slip angle 7=1	- Normalised slip angle at Point 7 (unitless)
Norm slip force 7=1	- Normalised slip force at Point 7 (unitless)
Norm slip angle 8=1.16	- Normalised slip angle at Point 8 (unitless)
Norm slip force 8=0.959	- Normalised slip force at Point 8 (unitless)
Norm slip angle 9=1.32	- Normalised slip angle at Point 9 (unitless)
Norm slip force 9=0.9	- Normalised slip force at Point 9 (unitless)
Norm slip angle 10=1.32	- Normalised slip angle at Point 10 (unitless)
Norm slip force 10=0.87	- Normalised slip force at Point 10 (unitless)
[Tyre Lat Camb Characteristics]	
Opt tyre camb=3.000000	- Neg camber at which pear lateral grip is generated
camb slope=2.000000	- How grip drops off – will be documented later.
[Tyre Long Camb Characteristics]	
Init mult=1.000000	- Global traction longitudinal circle multiplier
camb slope=2.000000	- How grip drops off - will be documented later
mu slope=0.000000	- How grip frops off with load

It goes without saying the comments aren't included in the text file! Now that we know what the file format let us now discuss what each of these mean.

Description of each element

Before we outline what each of the elements of the ChassisSim tyre v3 approximation mean let us discuss what we are trying to represent. The inputs and outputs are shown in equation (1)

$$\begin{bmatrix} F_z \\ T_t \\ V_{tyre} \\ \delta_{camb} \\ \alpha \\ SR \\ P_t \end{bmatrix} \rightarrow \begin{bmatrix} F_y \\ F_x \\ M_T \end{bmatrix} \qquad (1)$$

The variables are,

F_z = Vertical load on the tyre
T_t = Mean carcass temperature of the tyre
V_{tyre} = Rotational velocity of the tyre
δ_{camb} = Camber angle of the tyre.
α = Slip angle of tyre centreline relative to the ground.
SR = tyre slip ratio
P_t = Tyre pressure
F_y = Lateral force of the tyre
F_x = Longitudinal force of the tyre.
M_T = Self aligning torque of the tyre

The tyre model Mathematically well look like this,

$$
\begin{aligned}
M_{TMax} &= C_{M\_MT}(\delta_{camb}, F_z)\cdot fn(F_Z, T_T) \\
F_{MAX} &= C_{Fy\_MT}(\delta_{camb}, F_z)C_{FP}(P_T)\cdot fn(F_Z, T_T) \\
M_T &= fn(\alpha, F_Z, T_T)\cdot M_{TMax} \\
F_y &= fn(\alpha, F_Z, T_T)\cdot F_{MAX} \\
F_x &= \mu_{TC}(\delta_{camb}, L_T)\cdot fn(SR, F_Z, T_T)\cdot F_{MAX}
\end{aligned}
\tag{2}
$$

To help us in the approximation of this let's do a little bit of re-organisation of equation (2). We'll call this equation (2b),

$$
\begin{aligned}
M_{TMax} &= C_{M\_MT}(\delta_{camb}, F_z)\cdot fn(F_Z, T_T) \\
F'_{MAX} &= fn(F_Z, T_T) \\
M_T &= fn(\alpha, F_Z, T_T)\cdot M_{TMax} \\
F_y &= fn(\alpha, F_Z, T_T)\cdot C_{Fy\_MT}(\delta_{camb}, F_z)\cdot F'_{MAX} \\
F_x &= fn(SR, F_Z, T_T)\cdot \mu'_{TC}(\delta_{camb}, F_z)\cdot F'_{MAX}
\end{aligned}
\tag{2b}
$$

Given that we are interested in characterising tyre performance we'll let self-aligning torque slide for now. However the techniques we are going to discuss can readily be applied to self-aligning torque.

The first thing I want to discuss is the determination of $F'(F_z,T_t)$. This is what we are describing with the Tyre Force Characteristics and Tyre temp characteristics heading. This is effectively the traction circle radius of the tyre. As can be seen from the formula this is a function of two variables, F_z (Tyre Force Characteristics) and mean tyre temperature T_T (Tyre Temp Characteristics). To break this down even further this function can be represented by equation (3)

$$
F'_{MAX} = fn(T_t)fn(F_Z) \tag{3}
$$

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
Danny Nowlan, Director mobile: +61 425 219 375

Effectively the traction circle tyre radius is a function of load multiplied by a temperature multiplier. The load function is a 4th order polynomial fit of 6 points over the load range of the tyre assuming a (0,0) point at the origin. This is illustrated below,

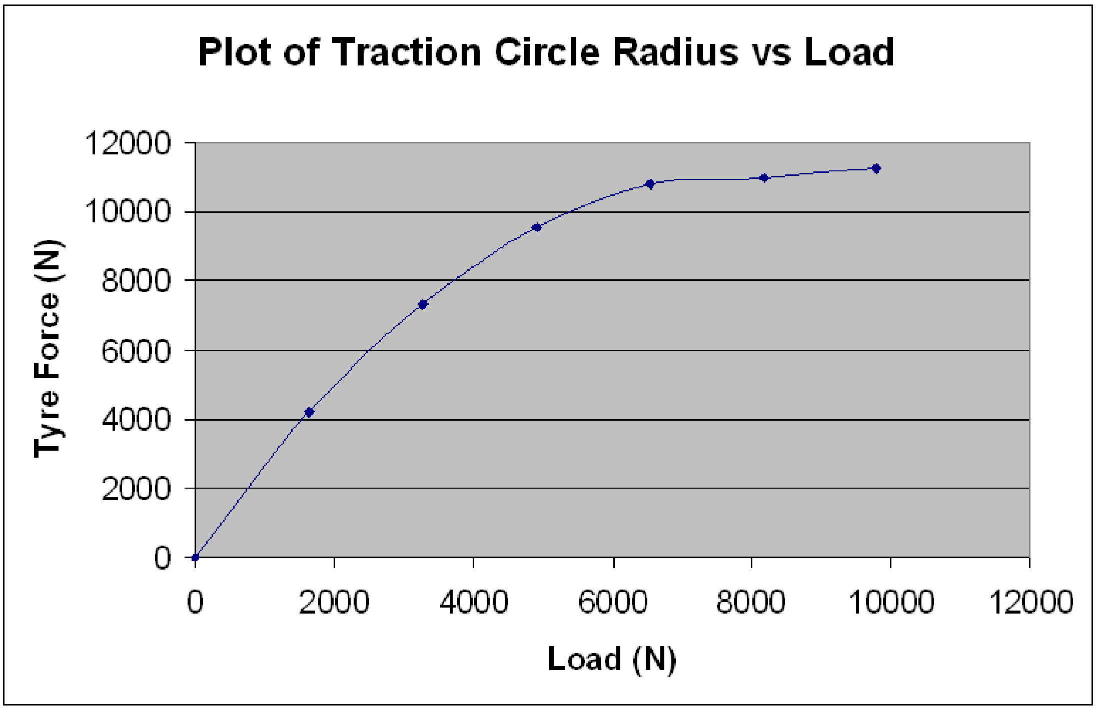

Fig-1 – A six point plot of Tyre force vs load.

This is what is described in the the Tyre Force Characteristics section. All that we are doing is giving it 6 control points and asking it to fit a 4th order curve fit to this. Note an origin of 0 load and 0 force is N. It goes without saying all those forces are in N. The numbers we are feeding in is simply the x-y co-ordinates of Fig-1.

The next point in the approximation of the traction circle is how do we represent the effect of tyre temperature variation. This is what we are describing in the Tyre Temp characteristics. The effect of tyre temperature variation is shown in equation (4),

$$
\begin{aligned}
&fn(T_t) = 1 - k \cdot \left(\frac{T - T_{OPT}}{T_{OPT}} \right)^2 \\
&T \le T_{OPT} \rightarrow k = k_{pre} \\
&T \ge T_{OPT} \rightarrow k = k_{post}
\end{aligned}
\tag{4}
$$

Here the variables are,

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
Danny Nowlan, Director mobile: +61 425 219 375

T_{OPT} = Optimum operating temperature of the tyre in Kelvin. This is the tyre temperature that will generate the most grip. This is the first section of the Tyre Temp Characteristics section.

T = Mean tyre temperature in Kelvin

k_{pre} = Slope of tyre temperature before peak temperature

k_{post} = Slope of tyre temperature after peak temperature.

k_{pre} and k_{post} are the next section described in the Tyre temp Characteristics section. This reflects mathematically what race engineers have known for years. That race tyres are very sensitive to race temperature and if you don't have them operating within a certain temperature bandwidth you might as well pack up and go home. Equation (4) put some substance to this. I should also add that to convert this to a 2D representation (tyre force as a function of load only) k_{pre} and k_{post} is set to 0 as shown in our example.

Table 1 – Typical temperature numbers for the ChassisSim v3 approximation

Parameter	Value
T_{opt}	373.15K
k_{pre}	7
k_{post}	15

So that we are clear these are the numbers you are putting into the v3 approximation file,

[Tyre Temp Characteristics]
Opt tyre temp=373.359994 - T_{opt}
Pre slope constant=0 - k_{pre}.
Post slope constant=0 - k_{post}.

The other option the user has is to represent the temperature is an exponential function. Mathematically this looks like the following,

$$fn(T_t) = k_{MULT} + (1 - k_{MULT})e^{k\cdot\left(\frac{T - T_{OPT}}{T_{SCALE}}\right)^2}$$
$$T \le T_{OPT} \rightarrow k = k_{pre}, k_{MULT} = k_{\exp\_pre}$$
$$T \ge T_{OPT} \rightarrow k = k_{post}, k_{MULT} = k_{\exp\_post}$$

This is used to model extreme tyre temperature sensitivity. Some good start values for this are,

Table 1b – Typical temperature numbers for the ChassisSim v3 exp function

Parameter	Value
T_{opt}	373.15K
k_{pre}	4

k_{post}	4
k_{PRE_MULT}	0.7
k_{POST_MULT}	0.8
T_{scale}	50

In the ini file the values for this would be

Exp pre temp mult=0.7 - Pre exponential multiplier – will be documented later
Exp post temp mult=0.8 - Post exponential multiplier – will be documented later
Exp temp scale=50 - Temp scale – This is in K

When starting out I would suggest to leave the slope numbers as zero then as you get more experience these can be modifed to suit.

The next point in the parameterisation is that know we know what the traction circle radius is doing how does this vary against slip angle and slip ratio. This is something I have put considerable thought to for a long time. I've played with the magic formula representation, and polynomial representations. Ultimately though the best representation I have found is the representation of slip angle and slip ratio is best characterised in equation (5)

$$fn(\alpha, F_Z, T_T) = fn\left(\frac{\alpha}{\alpha_{peak}}\right)$$
$$\alpha_{peak} = \alpha_0 \cdot (1 + k_T T_T) \cdot (1 + k_L \cdot F_Z) \tag{5}$$

Here we have

$fn(\alpha, F_Z, T_T)$ = This is a function that is bounded between –1 and +1

α_{peak} = Peak slip angle.

α_0 = Peak slip angle at zero load and nominal temperature.

k_T = This is how normalised peak slip varies with temperature.

k_L = This how normalise peak slip varies with load.

For slip ratio we simply substitute slip angle for slip ratio. Equation (5) has its origins in the Hugo-Radt model presented in Milliken[1] but with corrections for load and temperature. To represent $fn\left(\frac{\alpha}{\alpha_{peak}}\right)$ I could follow the heard and use a Magic Formula representation. I'm not discounting this but I have discussed in a number of different articles in the past my concern about how this behaves in the post slip region. Consequently this function is much better represented using a look up table. This gives the user considerable flexibility in modelling what is going on. An example of such a function is shown in Fig-2 below,

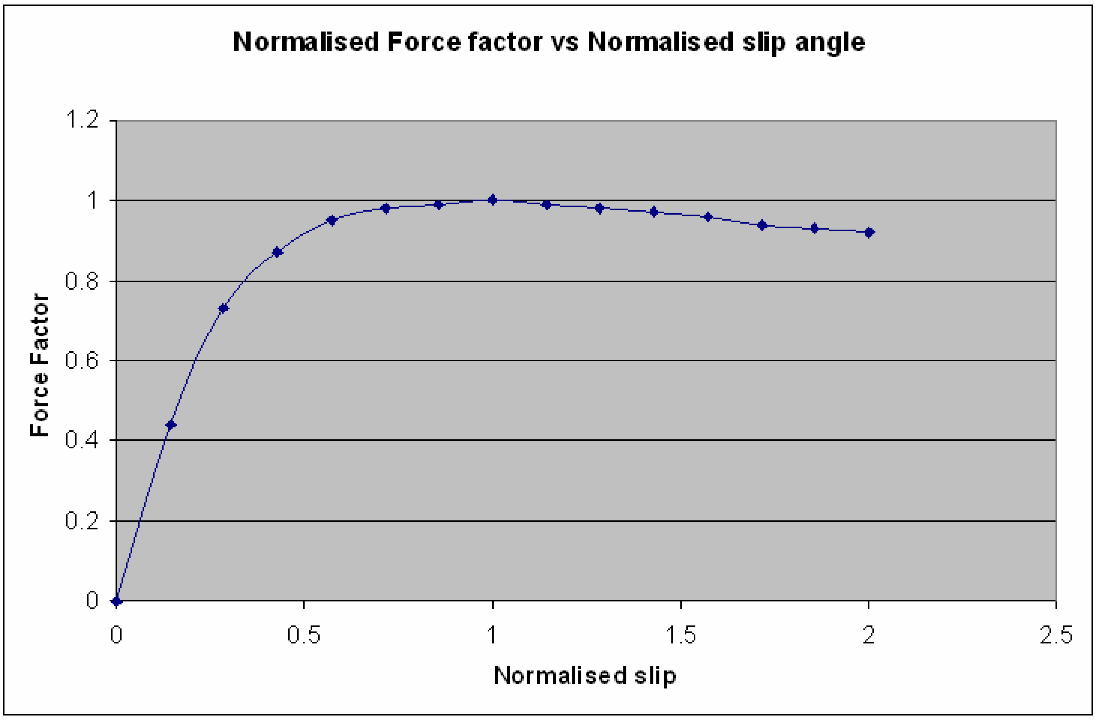

Fig-2 – Plot of fn'(a) vs normalised slip angle

The user enters these settings in the Tyre Slip Characteristics. The peak slip angle is entered as radians and the points of the normalised slip curve shown in Fig – 2 are listed in the sample file illustrated in the file format section. Recalling the file format we have,

[Tyre Slip Characteristics]
Peak slip angle=0.104 - α_{peak} (radians)
No slip points=10 - No of normalised slip points
Norm slip angle 1=0 - These are the values of Fig-2 above
Norm slip force 1=0
Norm slip angle 2=0.16
Norm slip force 2=0.2272
Norm slip angle 3=0.32
Norm slip force 3=0.4288
Norm slip angle 4=0.48
Norm slip force 4=0.6048
Norm slip angle 5=0.64
Norm slip force 5=0.87
Norm slip angle 6=0.8
Norm slip force 6=0.88
Norm slip angle 7=1
Norm slip force 7=1
Norm slip angle 8=1.16

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
Danny Nowlan, Director mobile: +61 425 219 375

Norm slip force 8=0.959
Norm slip angle 9=1.32
Norm slip force 9=0.9
Norm slip angle 10=1.32
Norm slip force 10=0.87

The next step in the parameterisation is to describe the effect of camber and how the traction circle ellipse varies as a function of camber and load. This is what is described in the Tyre Lat Camb Characteristics and Tyre Long Characteristics. This is outlined in equation (6),

$$\begin{aligned} C_{Fy\_MT}(\delta_{camb}, F_z) &= 1 - sf\_c\_y \cdot \frac{(\delta_{camb} - \delta_{OPT})^2}{100} \\ \mu'_{TC}(\delta_{camb}, F_z) &= \mu_{MULT} \cdot \left(1 - sf\_c\_x \cdot \frac{\delta_{camb}{}^2}{100}\right) \\ sf\_c\_y &= sf\_c\_y\_0 + k\_c\_y \cdot F_z \\ sf\_c\_x &= sf\_c\_x\_0 + k\_c\_x \cdot F_z \\ \mu_{MULT} &= \mu_0 + k_u \cdot F_z \\ \delta_{OPT} &= \delta_0 + k_\delta F_z \end{aligned} \qquad (6)$$

Where,

δ_{OPT} = Camber at which most lateral grip is generated.
δ_{camb} = Negative camber of the tyre

The power of equation (6) is that we can effectively roll in both camber sensitivity and the traction ellipse in one fell swoop. Furthermore since were also taking into account Load we can track how the traction ellipse and camber sensitivity. While this is not perfect it certainly is much more appropriate than some other parameterisations that have a linear approximation of camber and tell you to run a modern day racing slick at 20 deg of camber!

To help you get going with this I've presented some numbers that have worked well in some open wheeler modelling I have done. These are shown in table 1.4

Table 2 – Typical temperature numbers for the ChassisSim v3 approximation

Parameter	Value
sf_c_y_0	2
sf_c_x_0	2
k_c_y	0
k_c_x	0
μ_0	1

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
Danny Nowlan, Director mobile: +61 425 219 375

k_u	0
δ_{opt}	3
k_δ	0

The numbers presented in Table 2 apply for GT tyres. So that we are crystal clear then the numbers you are putting into the ChassisSim tyre file are,

[Tyre Lat Camb Characteristics]
Opt tyre camb=3.000000 - δ_{OPT}
camb slope=2.000000 - sf_c_y.
[Tyre Long Camb Characteristics]
Init mult=1.000000 - μ_0
camb slope=2.000000 - sf_c_x
mu slope=0.000000 - k_u

Conclusion

What we have described is an invaluable tool for characterising your racing tyre. While no approximation is perfect this gives you a very powerful tool to describe different racing tyres and refining your model.

ABN 3517 202 936 9
ChassisSim Technologies™
9 Harriet Street Marrickville
NSW 2204 Australia
Danny Nowlan, Director mobile: +61 425 219 375